

ESAM495 Discussion Summary

Convergent Cross Mapping

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Question 1: Why do we expect Granger Causality to fail in these types of systems? Explain with as much as mathematical precision (or for a small toy example) as possible.

The assumption of separability is violated. In stochastic systems, new information is injected by the noise in the cause variable X , so the cause X and effect Y are separable. However, in deterministic systems, information about the cause X will be redundantly present in the history of the effect Y itself and cannot formally be removed from U (the universe).

Question 2: What does Sugihara say defines causality in deterministic systems?

Time series variables are causally related if they are from the same dynamic system. That is, they share a common attractor manifold (each variable can identify the state of the other).

Question 3: What is required for this definition of causality to ‘work’? Will this work for all deterministic dynamical systems?

The shadow manifolds of the variables have to have the same underlying structure or topology. For example, in Lorenz system, the method will fail to predict Z because its shadow manifold has a completely different structure compared to X ’s and Y ’s.

Question 4: What is the basic idea of the CCM test? What is the basic procedure for this test?

The idea is to see whether the time indices of near-by points on the X manifold can be used to identify near by points on the Y manifold. Given two time series X and Y , the idea is to construct a shadow manifold from X and generate a cross-mapped estimate of Y from the nearest neighbors in the X manifold.

Question 5: What does convergence mean in the context of the CCM test? What is the procedure for testing causality?

Convergence is used to distinguish causation from simple correlation. As the length of time series increases, the attractor is more filled in, resulting in closer nearest neighbors and smaller estimation error. Thus, CCM converging is a necessary condition for causation.

Question 6: Explain what Sugihara and co-authors see for a well-behaved system with unidirectional causality. How can they tell when a direction is casual and when it is not.

They implement the CMM algorithm using X to predict Y and using Y to predict X . If the cross mapping of Y using M_X fails and the cross mapping of X using M_Y succeeds, we say there is a unidirectional causality.

Question 7: Explain what they see when they have a system with an external variable

that forces the system.

Suppose species X and Y do not interact with each other but are both driven by a common environmental variable Z. The cross mapping between X and Y will not be detected because there is no information flow between variables. However, information of the external variable Z can still be recovered from X and Y.

Question 8: What choices does one need to implement CCM and what other factors may cause failure or bad results?

One needs to decide the time lag τ and the number of dimensions for the shadow manifolds (it should be at least big enough to capture the dimensionality of the original system).