

Merge sort:

$S = [7, 5, 2, 3, 0, 4, 1, 6]$  input in array

$S_1 = [7, 5, 2, 3]$

$S_{11} = [7, 5]$

7

5

$[5, 7]$

$S_{12} = [2, 3]$

2

3

$[2, 3]$

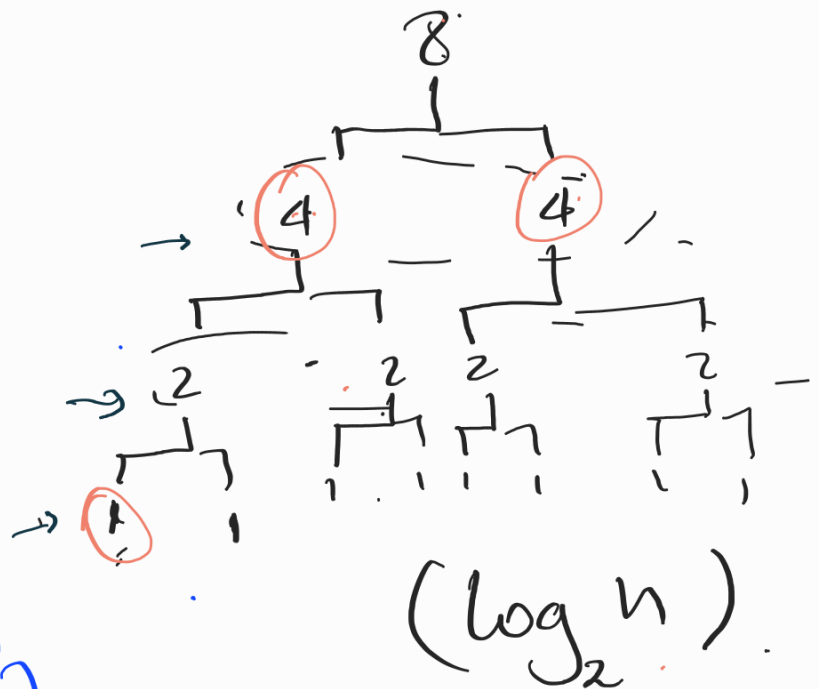
$S_2$

$[0, 1, 4, 6]$

$[0, 1, 2, 3, 4, 5, 6, 7]$

Combine

Recursion: 2k



$\lambda_1$  &  $\lambda_2$

$O(K)$

$$l_1 = [a_1, a_2, \dots, a_k]$$

$$l_2 = [b_1, b_2, \dots, b_k]$$

$$\rightarrow [a_1, a_2, b_1, \dots]$$

Complexity

Combine:  $O(n)$

$$R \times O(s)$$

$$\hookrightarrow O(s) = O(\log n)$$

$$R = \log(n)$$

complexity merge-sort:  $O(n \log(n))$

$n$	$2^n$	$\log_2(2^n)$
5	32	5
4	16	4
3	8	3
2	4	2
1	2	1
0	1	0

$$2^n = \underbrace{2 \times \dots \times 2}_n$$

$\log_2(K) \pm$  # times ...  
we divide  
 $n$  by 2 until  
we get 1  
as result.

$$K/2 / 2 / 2 \dots = 1$$

n times

Memory complexity :- extra memory.

w/o deep copy (pointers):  $O(1) + n$

w/ 1. deep copy of original array  $O(n) + n$

w/ 1. deep copy per recursive call:  $O(n \log(n))$   
 $+ n$   
 $=$

Quick Sort:

5, 9, 7, 2, 3, 10, 0, 4, 1, 8, 6, 11.

pivot = 5

rand(0, n)

$S_1 = [2, 3, 0, 4, 1]$

$S_n = [0, 1]$

$[1]$

$> [0, 1]$

$[0, 1, 2, 3, 4]$

$S_{12} = [3, 4]$

$> [3, 4]$

[4]

$$S_2 = [9, 7, 10, 8, 6, 11] \quad [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$$

$$S_{21} = [7, 8, 6]$$

$$\begin{matrix} [6] \\ [2] \end{matrix} \rightarrow [6, 7, 8]$$

$$S_{22} = [10, 11]$$

$$\begin{matrix} [11] \end{matrix} \rightarrow [10, 11]$$

$$[6, 7, 8, 9, 10, 11]$$

Combine:

$$\underline{S_{left}} + \underline{\text{pivot}} + \underline{S_{right}}$$

"+" = concat.

Complexity

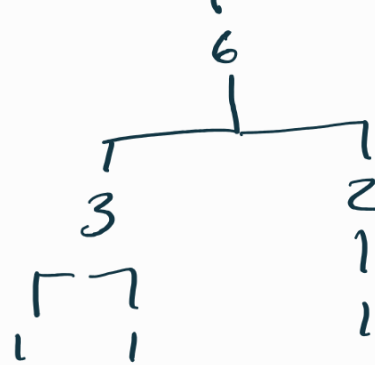
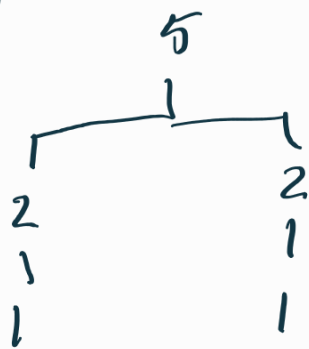
$$R: \log(n)$$

$$O(S): n$$

$$\rightarrow O(n \log n)$$



$3 \times \log 6$



$[11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0]$ .

$[10, \dots, 0]$ .

$[9, \dots, 0]$

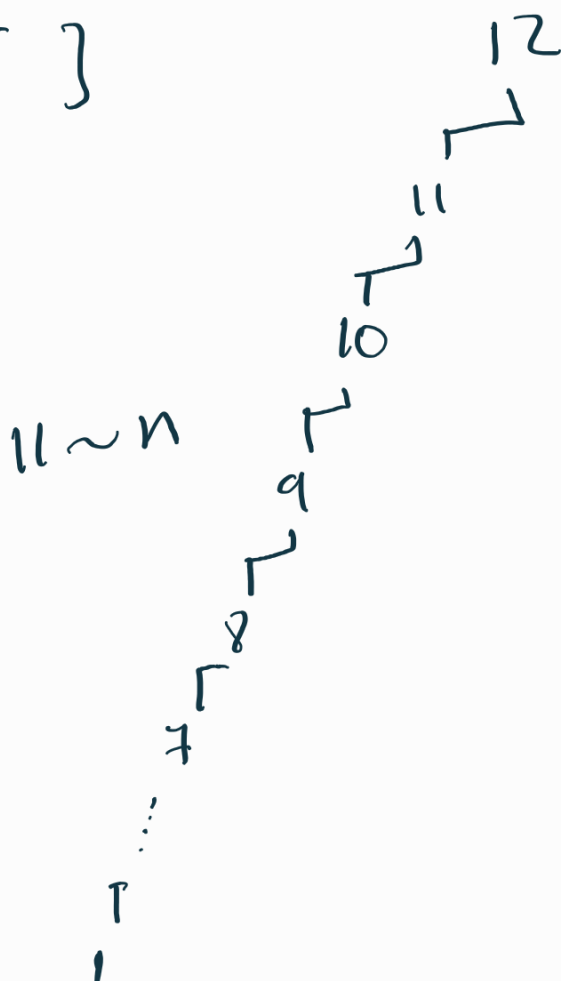
$[8, \dots, 0]$

$\vdots$

$[7]$

$[6]$

$[5]$



$O(n^2)$

<del>1</del>	4	7	<del>11</del>	<del>15</del>
2	5	8	<del>12</del>	<del>19</del>
3	6	9	<del>16</del>	<del>22</del>
<del>10</del>	<del>13</del>	<del>14</del>	<del>17</del>	<del>24</del>
<del>18</del>	21	<del>23</del>	<del>26</del>	<del>30</del>

target: 13

+ 1 python:

