Supermarket Sweep

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a.

We start first with a list of the nodes given in the Supermarket Sweep.csv file, each wrapped in a custom Item object that holds its x, y, and price attributes.

If two items are on the same x-aisle, the shortest distance from one to the other is the absolute value of the difference between their y-coordinates.

If two items are on different x-aisle, the shortest distance from one to the other is either by moving up to the end of the aisle, across to the other item's aisle, and then down to the item, or by moving down to the start of the aisle, across to the other item's aisle, and then up to the item. An algorithm should choose the shortest of these two options, which could be done by computing both and using their minimum.

We then divide the distance by the contestant's speed, 10 feet per second, to convert this distance to time.

Then two seconds is added to each time between nodes to account for the time it takes for a contestant to pick up the j^{th} node.

We implemented this in a short nested loop in Python below:

```
# initialize an empty two-dimensional list
d = [[0 for i in range(len(item_list))] for i in range(len(item_list))]
# iterate through all item objects twice for all pairings
for i in range(len(item_list)):
    for j in range(i,len(item_list)):
        item_i = item_list[i]
        item_j = item_list[j]

        # if item i and j share an aisle:
        if item_i.x == item_j.x:
            d[i][j] = (abs(item_i.y - item_j.y) / 10)
            d[j][i] = (abs(item_i.y - item_j.y) / 10)
        # if they don't share an aisle:
        else:
            dist_x = abs(item_i.x - item_j.x)
            dist_y = min(((110 - item_i.y) + (110 - item_j.y) ), item_i.y + item_j.y)
            d[i][j] = ((dist_x + dist_y) / 10)
            d[j][i] = ((dist_x + dist_y) / 10)
```

b.

For the sake of our formulation, we altered the d_{ij} matrix conceptually to include pickup time when necessary. The below addition to our previous loop accounts for this:

```python
        if i!=j:
            d[j][i]+=2
            d[i][j]+=2
```

Then, since the end node we use in our formulation doesn't have an entry in the list of items, we duplicate the first row and column of the matrix to the last row and column since the end node is the same as the start node, which exists as the first item in the item list shown. If we previously added 2 to one of these entries, we want to remove it since the last node does not represent an item that needs time for collecting.

```python
# copy the first column to the end
d.append(d[0])
# for each column, copy the first row to the end
for i in range(len(d[0])):
    # check if we didn't already add the 2 seconds
    if i in [0, len(d[0])-1]:
        d[i].append(d[i][0])
    # if we did, remove it
    else:
        d[i].append(d[i][0] - 2)
```

Data Placeholders:

n represents the number of nodes, with node 1 being the start node, nodes 2,3,. . . ,n being item nodes, and n+1 being the end node which shares the attributes of the start node.

T represents the maximum time the contestant is given to shop.

C represents the maximum amount of items the contestant can put in their cart.

v_i represents the value of node i. $\forall i \in [\![1, n]\!]$

d_{ij} represents the minimum time it takes to move from node i to node j. If node j represents an item and not a start/end point, it will include the 2 seconds to add that item to the cart. $\forall i = 1, 2, ..., n$ $\forall j \in [\![2, n+1]\!]$

Decision Variables:

$x_{ij} = 1$ if node j follows node i in the chosen path, 0 otherwise. $\forall i \in [\![1, n]\!]$ $\forall j \in [\![2, n+1]\!]$

y_j represents the running time to follow the path from the start until node j. $\forall j \in [\![1, n+1]\!]$

$t_{ij} = y_j$ if node j follows node i in the chosen path, 0 otherwise. $\forall i \in [\![1, n]\!]$ $\forall j \in [\![2, n+1]\!]$

$$\max_{x,y,t} \quad \text{score} = \sum_{i=1}^{n} v_i \sum_{j=2}^{n+1} x_{ij}$$

$$\text{s.t.} \quad (1) \quad y_1 = 0$$

$$(2) \quad \sum_{j=2}^{n+1} x_{1,j} = 1$$

$$(3) \quad \sum_{j=2}^{n+1} x_{ij} \leq 1 \qquad \forall i \in [\![2, n]\!]$$

$$(4) \quad \sum_{i=1}^{n} x_{ij} \leq 1 \qquad \forall j \in [\![2, n]\!]$$

$$(5) \quad \sum_{i=1}^{n} x_{i,n+1} = 1$$

$$(6) \quad t_{ij} \leq T x_{ij} \qquad \forall i \in [\![1, n]\!] \qquad \forall j \in [\![2, n+1]\!]$$

$$(7) \quad y_j = \sum_{i=1}^{n} t_{ij} \qquad \forall j \in [\![2, n+1]\!]$$

$$(8) \quad \sum_{k=2}^{n+1} t_{jk} = y_j + \sum_{k=2}^{n+1} d_{jk} x_{jk} \qquad \forall j \in [\![1, n]\!]$$

$$(9) \quad x_{ii} = 0 \qquad \forall i \in [\![1, n]\!]$$

$$(10) \quad \sum_{i=1}^{n} x_{ij} = \sum_{k=2}^{n+1} x_{jk} \qquad \forall j \in [\![2, n]\!]$$

$$(11) \quad \sum_{i=1}^{n} \sum_{j=2}^{n} x_{ij} \leq C$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in [\![1, n]\!] \qquad \forall j \in [\![2, n+1]\!]$$

$$t_{ij} \geq 0 \qquad \forall i \in [\![1, n]\!] \qquad \forall j \in [\![2, n+1]\!]$$

Constraint Explanations:

(1) The path begins at node 1 with a time of 0.

(2) Node 1 has exactly one destination node directly after it in the path.

(3) Nodes 2 through n may have at most one destination node directly after it in the path.

(4) Nodes 2 through n may have at most one origin node directly before it in the path.

(5) Node n+1 has exactly one origin node directly before it in the path.

(6) If the direct path from node i to node j exists, than $t_i j$ is upper bounded at T. Otherwise, it is constrained to equal 0.

(7) Each running time y_j is set as the sum over all i of $t_i j$ for all destination nodes j, of which at most one is nonzero.

(8) We define the sum over destination nodes k of $t_j k$ as the sum of the running total at node j plus the additional time added by the chosen destination.

(9) No node may follow itself in the path.

(10) Nodes that are not entered may not be exited, and nodes that are entered must be exited.

(11) The amount of item nodes in the path must be less than C.

c.

Path Taken:

Node 1: Start Node (0,0)

Node 2: Coffee Beans: $6.99 at (0,15)

Node 3: K-Cups: $10.99 at (0,35)

Node 6: Granola: $5.49 at (0,100)

Node 26: Shampoo: $8.99 at (40,100)

Node 31: Trash Bags: $8.99 at (50,95)

Node 36: Air Freshner: $6.99 at (60,75)

Node 35: Dog Treats: $3.99 at (60,65)

Node 34: Broom: $13.99 at (60,35)

Node 33: Detergent: $12.99 at (60,20)

Node 40: Redbull (4): $7.99 at (70,30)

Node 39: Gatorade (12): $6.99 at (70,35)

Node 28: Paper Towels: $9.99 at (50,25)

Node 27: Toilet Paper: $7.99 at (50,15)

Node 22: Ibuprofen: $5.49 at (40,20)

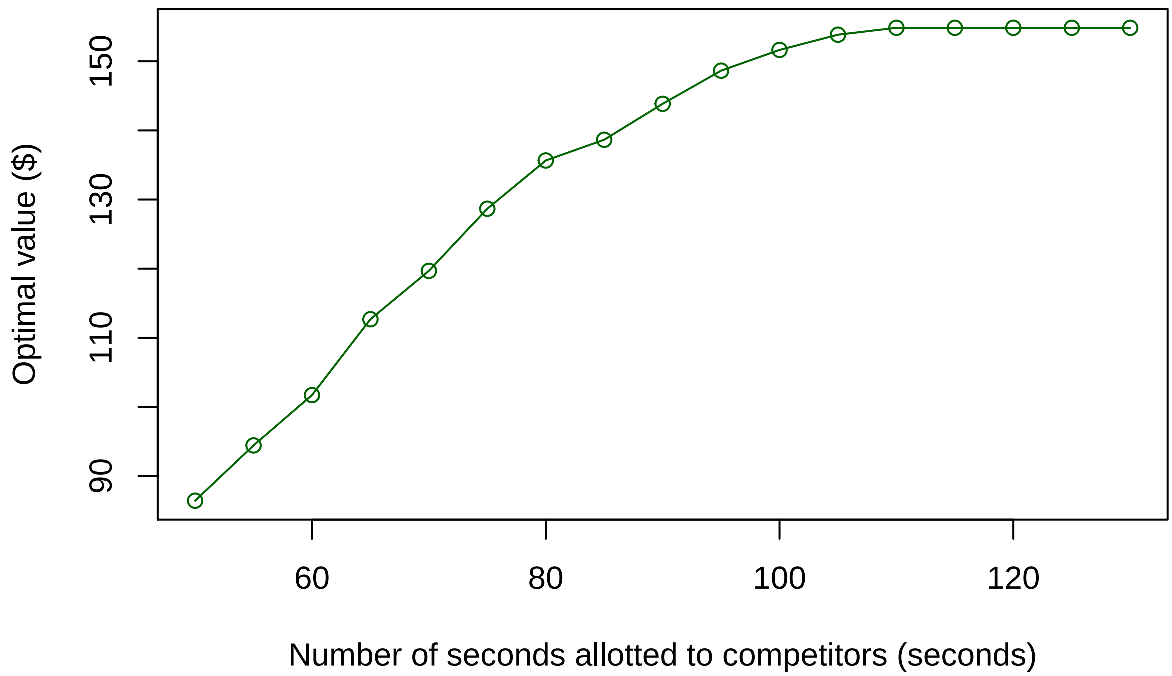
Node 23: Diapers: $25.99 at (40,35)

Back to Start (0,0)

The total value of the optimized cart is $143.85

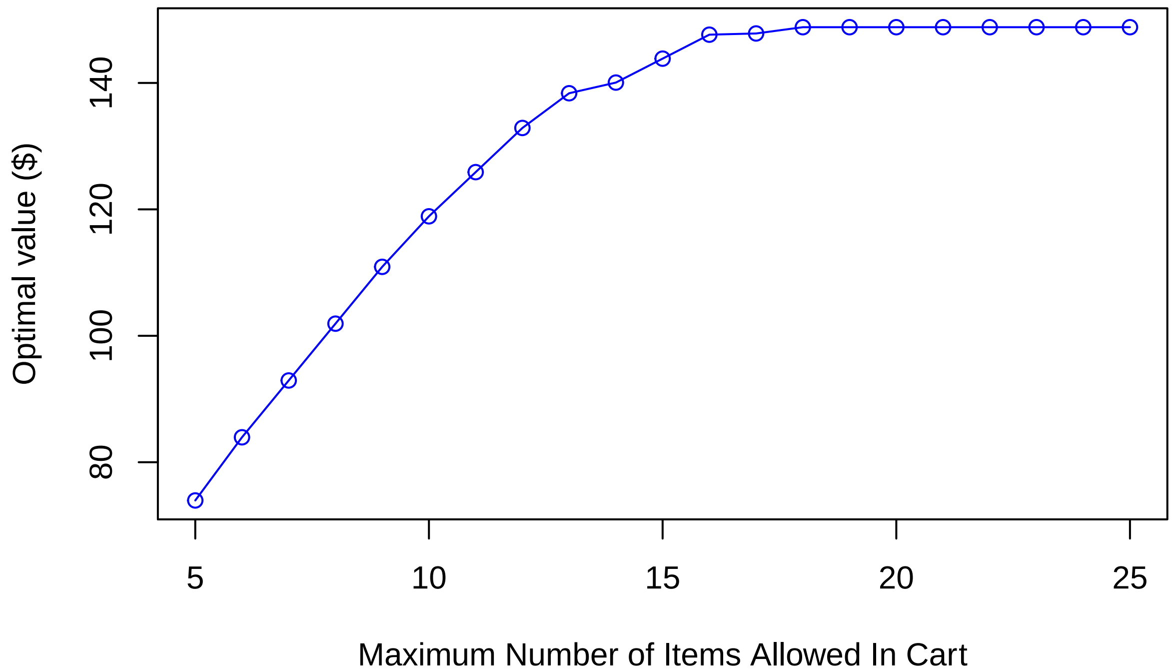
d.

We see from the graph that the optimal value increases as we increase the allotted time that contestant can shop for. This makes sense because the contestant will have more time to pick up the fifteen most expensive items instead of being limited by choosing nearby items. The fifteen most expensive items will give the optimal value of $154.85 which we can see starting at 110 seconds, which shows where additional time is no longer helpful to the contestant. As we increase the allotted time by 5 second increments, we see that the function increases slower and slower until it stops, signifying a concave increasing relationship.



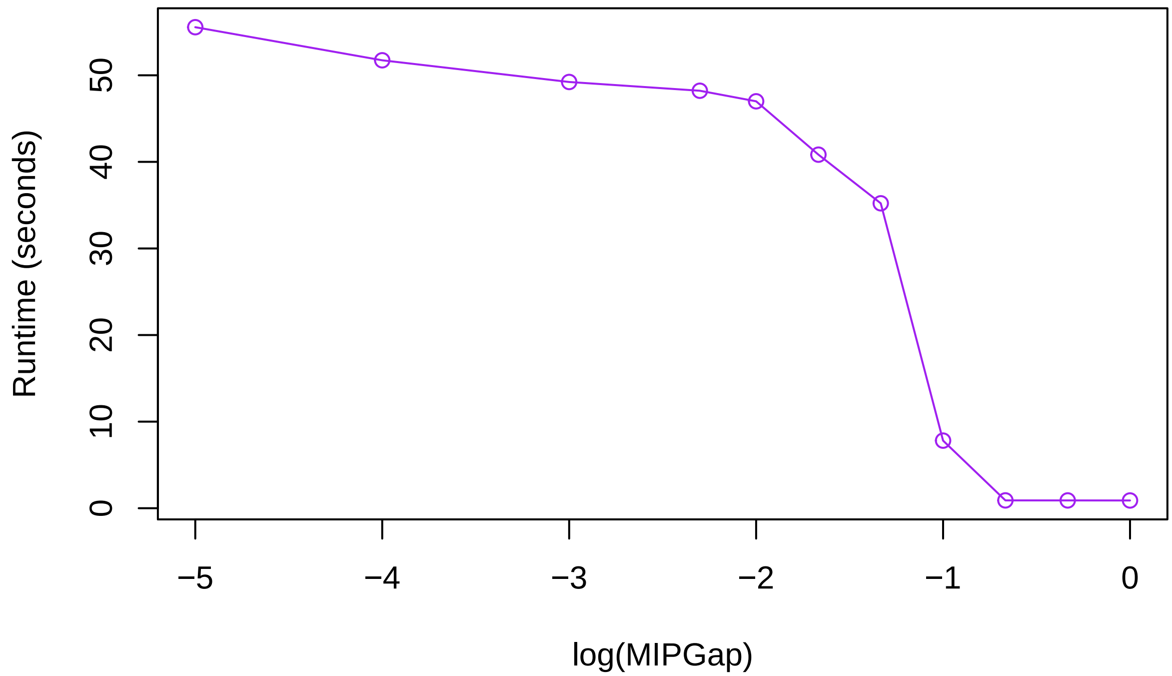
e.

As shown in the graph, the optimal price of the items collected increases as the number of items allowed increases until the capacity of the cart is 18. At this point, the optimal value of the items in the cart plateaus because the contestant does not have the time to go around the store and pick up any more items without sacrificing more expensive ones. It appears that the most money they can make during a 90 second interval is $148.82, reached with a cart capacity of 18 items. Similarly to the last graph, we see a concave increasing relationship, with more significant benefits to the earlier increases in cart capacity.



f.

The MIPGap parameter helps determine when the optimizing function can terminate by giving a minimum relative percent difference from the true optimal value. As we changed this parameter- keeping the cart capacity and allotted time constant- we observe a decrease in the runtime, measured here in seconds. We decided to alter our MIPGap inputs using a log transformation of base 10 to better highlight these multiplicative increments which showed the best results. The graph shows that as we increase the MIPGap parameter, the time to solve the problem drastically decreases from a value consistently within the 45 to 50 second range all the way down to 1 second. On a log scale, we see first a concave and then a convex decreasing function, which highlights an important area of a MIPGap around $10^{-1.7}$ where there is a significant dropoff in runtime before finding a suitable optimal solution.



Runtime (seconds)
0
10
20
30
40
50
−5
−4
−3
−2
−1
0
log(MIPGap)