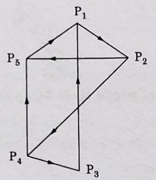
6 Graphs

6.1 Write the adjacency matrix representation of the graph given in figure.



[1998 : 2 M]

6.2 Lt G be an undirected graph. Consider a depth-first traversal of G, and let T be the resulting depth-first search tree. Let u be a vertex in G and let v be the first new (unvisited) vertex visited after visiting u in the traversal. Which of the following statements is always true?

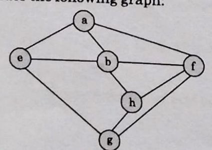
(a) {u,v} must be an edge in G, and u is a descendant of v in T
(b) {u,v} must be an edge in G, and v is a descendant of u in T
(c) If {u,v} is not an edge in G then u is a leaf in T
(d) If {u,v} is not an edge in G then u and v must have the same parent in T [2000 : 2 M]

6.3 Consider an undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let d(r, u) and d(r, v) be the lengths of the shortest paths from r to u and v respectively in G. If u is visited before v during the breadth-first traversal, which of the following statement is correct?

(a) d(r, u) < d(r, v) (b) d(r, u) > d(r, v)
(c) d(r, u) ≤ d(r, v) (d) None of these

[2001 : 2 M]

6.4 Consider the following graph:



Among the following sequences:

1. a b e g h f 2. a b f e h g
3. a b f h g e 4. a f g h b e

Which are depth first traversals of the above graph?

(a) 1, 2 and 4 only (b) 1 and 4 only
(c) 2, 3 and 4 only (d) 1, 3 and 4 only

[2003 : 1 M]

6.5 Let G = (V, E) be an undirected graph with a subgraph $G_1 = (V_1, E_1)$. Weights are assigned to edges of G as follows:

$$w(e) = \begin{cases} 0 \text{ if } e \in E_1 \\ 1 \text{ otherwise} \end{cases}$$

A single-source shortest path algorithm is executed on the weighted graph (V, E, w) with an arbitrary vertex v_1 of V_1 as the source. Which of the following can always be inferred from the path costs computed?

(a) The number of edges in the shortest paths from v_1 to all vertices of G
(b) G_1 is connected (c) V_1 forms a clique in G
(d) G_1 is a tree [2003 : 2 M]

6.6 Let G = (V, E) be a direction graph with n vertices. A path from v_i to v_j in G is sequence of vertices $(v_i, v_{i+1}, \ldots, v_j)$ such that $(v_k, v_{k+1}) \in E$ for all k in i through j – 1. A simple path is a path in which no vertex appears more than once.

Let A be an n × n array initialized as follow:

$$A[j, k] = \begin{cases} 1 \text{ if } (j, k) \in E \\ 0 \text{ otherwise} \end{cases}$$

Consider the following algorithm:

for i = 1 to n
for j = 1 to n
for k = 1 to n
A [j, k] = max (A[j, k], (A[j, i] + A[i, k]));

Which of the following statements is necessarily true for all j and k after terminal of the above algorithm?

(a) A[j, k] ≤ n
(b) If A [j, j] ≥ n – 1, then G has a Hamiltonian cycle
(c) If there exists a path from j to k, A[j, k] contains the longest path lengths from j to k
(d) If there exists a path from j to k, every simple path from j to k contain most A[j, k] edges

[2003 : 2 M]

6.7 In a depth-first traversal of a graph G with n vertices, k edges are marked as tree edges. The number of connected components in G is

(a) k (b) k + 1
(c) n – k – 1 (d) n – k [2005 : 1 M]

6.8 Let G be a directed graph whose vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j iff either j = i + 1 or j = 3i. The minimum number of edges in a path in G from vertex 1 to vertex 100 is

(a) 4 (b) 7
(c) 23 (d) 99 [2005 : 2 M]

6.9 Let T be a depth first search tree in a undirected graph G. Vertices u and v are leaves of this tree T. The degrees of both u and v in G are at least 2. Which one of the following statements is true?

(a) There must exist a vertex w adjacent to both u and v in G
(b) There must exist a vertex w whose removal disconnects u and v in G
(c) There must exist a cycle in G containing u and v
(d) There must exist a cycle in G containing u and all its neighbours in G [2006 : 2 M]

6.10 Consider the depth-first-search of an undirected graph with 3 vertices P, Q, and R. Let discovery time d(u) represent the time instant when the vertex u is first visited, and finish time f(u) represent the time instant when the vertex u is last visited. Given that

d(P) = 5 units, f(P) = 12 units
d(Q) = 6 units, f(Q) = 10 units
d(R) = 14 unit, f(R) = 18 units

Which one of the following statements is TRUE about the graph?

(a) There is only one connected component
(b) There are two connected components, and P and R are connected
(c) There are two connected components, and Q and R are connected
(d) There are two connected components, and P and Q are connected [2006 : 2 M]

6.11 What is the largest integer m such that every simple connected graph with n vertices and n edges contains at least m different spanning trees?

(a) 1 (b) 2
(c) 3 (d) n [2007 : 2 M]

6.12 A depth-first search is performed on a directed acyclic graph. Let d[u] denote the time at which vertex u is visited for the first time and f[u] the time at which the dfs call to the vertex u terminates. Which of the following statements is always true for all edges (u, v) in the graph?

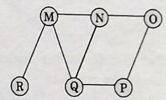
(a) d[u] < d[v] (b) d[u] < f[v]
(c) f[u] < f[v] (d) f[u] > f[v]

[2007 : 2 M]

6.13 The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity.

(a) Θ(n) (b) Θ(m)
(c) Θ(m + n) (d) Θ(mn) [2008 : 1 M]

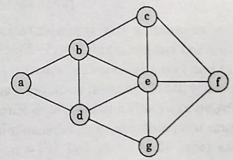
6.14 The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(a) MNOPQR (b) NQMPOR
(c) QMNPRO (d) QMNPOR

[2008 : 1 M]

6.15 Consider the following sequence of nodes for the undirected graph given below:



1. a b e f d g c 2. a b e f c g d
3. a d g e b c f 4. a d b c g e f

A Depth First Search (DFS) is started at node a. The nodes are listed in the order they are first visited. Which all of the above is (are) possible output(s)?

(a) 1 and 3 only (b) 2 and 3 only
(c) 2, 3 and 4 only (d) 1, 2 and 3 only

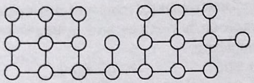
[2008 : 2 M]

6.16 Let G of be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?

(a) $\Theta(n)$ (b) $\Theta(n + m)$
(c) $\Theta(n)^2$ (d) $\Theta(m^2)$

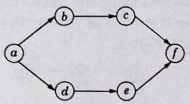
[2014 (Set-1) : 1 M]

6.17 Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is.



[2014 (Set-3) : 1 M]

6.18 Consider the following directed graph:



The number of different topological ordering of the vertices of the graph is______.

[2016 (Set-1) : 1 M]

6.19 Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n-th vertex in this BFS traversal, then the maximum possible value of n is ______.

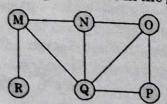
[2016 (Set-2) : 1 M]

6.20 In an adjacency list representation of an undirected simple graph $G = (V, E)$, each edge (u, v) has two adjacency list entries: $[v]$ in the adjacency list of u, and $[u]$ in the adjacency listof v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If $|E| = m$ and $|V| = n$, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?

(a) $\Theta(n^2)$
(b) $\Theta(n + m)$
(c) $\Theta(m^2)$
(d) $\Theta(n^4)$

[2016 (Set-2) : 2 M]

6.21 The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below?



(a) MNOPQR
(b) NQMPOR
(c) QMNROP
(d) POQNMR

[2017 (Set-2) : 1 M]

6.22 Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements:

I. No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D).

II. For every edge (u, v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

[2018 : 2 M]

6.23 The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is ______.

[2018 : 2 M]

6.24 An articulation point in a connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components.

Let T be a DFS tree obtained by doing DFS in a connected undirected graph G.

Which of the following options is/are correct?

(a) Root of T is an articulation point in G if and only if it has 2 or more children.
(b) A leaf of T can be an articulation point in G.
(c) Root of T can never be an articulation point in G.
(d) If u is an articulation point in G such that x is an ancestor of u in T and y is a descendent of u in T, then all paths from x to y in G must pass through u.

[2021 (Set-1) : 2 M]

6.25 Let $U = \{1, 2, 3\}$. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any $A, B \in 2^U$, (A, B) is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subseteq B$ or $B \subseteq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by $B(A)$.

If ϕ denotes the empty set, then the cardinality of $B(\phi)$ is ______.

[2023 : 2 M]

6.26 Let $G(V, E)$ be an undirected and unweighted graph with 100 vertices. Let $d(u, v)$ denote the number of edges in a shortest path between vertices u and v in V. Let the maximum value of $d(u, v)$, $u, v \in V$ such that $u \neq v$, be 30. Let T be any breadth-first-search tree of G. Which ONE of the given options is CORRECT for every such graph G?

(a) The height of T is exactly 15.
(b) The height of T is exactly 30.
(c) The height of T is at least 15.
(d) The height of T is at least 30.

[2025 (Set-1) : 2 M]

6.27 Which of the following statements regarding Breadth First Search (BFS) and Depth First Search (DFS) on an undirected simple graph G is/are TRUE?

(a) A DFS tree of G is a Shortest Path tree of G.
(b) Every non-tree edge of G with respect to a DFS tree is a forward/back edge.
(c) If (u, v) is a non-tree edge of G with respect to a BFS tree, then the distances from the source vertex s to u and v in the BFS tree are within ±1 of each other.
(d) Both BFS and DFS can be used to find the connected components of G.

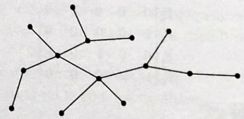
[2025 (Set-2) : 1 M]

6.28 Consider the following algorithm someAlgo that takes an undirected graph G as input.

someAlgo (G)

1. Let v be any vertex in G. Run BFS on G starting at v. Let u be a vertex in G at maximum distance from v as given by the BFS.
2. Run BFS on G again with u as the starting vertex. Let z be the vertex at maximum distance from u as given by the BFS.
3. Output the distance between u and z in G.

The output of someAlgo(T) for the tree shown in the given figure is ______. (Answer in integer)



[2025 (Set-2) : 2 M]

■■■■

Answers Graphs

6.1	Sol.	6.2	(c)	6.3	(c)	6.4	(d)	6.5	(b)	6.6	(d)	6.7	(d)	6.8	(b)	6.9	(d)
6.10	(d)	6.11	(c)	6.12	(d)	6.13	(c)	6.14	(c)	6.15	(b)	6.16	(c)	6.17	(19)	6.18	(6)
6.19	(31)	6.20	(b)	6.21	(d)	6.22	(a)	6.23	(80)	6.24	(a)	6.25	(5040)	6.26	(c)	6.27	(b, c, d)
6.28	(6)																

Explanations Graphs

6.1 Sol.

Adjacency Matrix

$$\begin{array}{c|ccccc} & P_1 & P_2 & P_3 & P_4 & P_5 \\ \hline P_1 & 0 & 1 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 1 & 0 \\ P_3 & 1 & 0 & 0 & 0 & 0 \\ P_4 & 0 & 0 & 1 & 0 & 1 \\ P_5 & 1 & 1 & 0 & 0 & 0 \end{array}$$

6.2 (c)

Draw some random graph and try to verify the options.

6.3 (c)

If u is visited before v, $d(r, u) \le d(r, v)$ i.e., r to u is having shortest or equal length compared from r to v.

6.4 (d)

Consider each search

I. Search (a) = b add (b) a,b
Search (b) = e add (e) a, b, e
Search (e) = g add (g) a, b, e, g
Search (g) = h add (h) a, b, e, h
Search (h) = f add (f) a, b, e, h, f

II. Search (a) = b add (b) a, b
Search (b) = f add (f) a, b, f
Search (f) = failure because there is no edge connected to e.

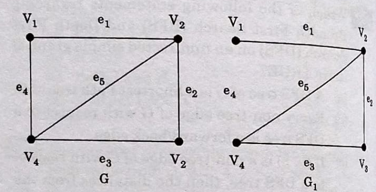
III. Search (a) = b add (b) a, b
Search (b) = f add (f) a, b, f
Search (f) = h add (h) a, b, f, h
Search (h) = g add (g) a, b, f, h, g
Search (g) = e add (e) a, b, f, h, g, e

IV. Search (a) = f add (f) a, f
Search (f) = g add (g) a, f, g
Search (g) = h add (b) a, f, g, h
Search (h) = b add (b) a, f, g, h, b
Search (b) = e add (e) a, f, g, h, b, e

6.5 (b)

Let G = (V, E) be an undirected graph $G_1 = (V, E_1)$ such that $E_1 \subseteq E$ and $V_1 \subseteq V$

Consider an example:



Consider the w (e) =1 if $e \notin E_1$ it means the cost of V_1 to V_4 is only 1 other edges having cost 0. It is notated that G_1 is connected. V_1 does not form of any clique in G a well as G_1 is not a tree. So choice (b) is correct.

6.6 (d)

for i = 1 to n
for j = 1 to n
for k = 1 to n
A[j, k] = max(A[j, k], (A[j, i] + A[i, k]))

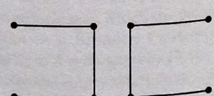
A be $n \times n$ array and A[j, k] = 1 if $(j, k) \in E$.

So there exist a path from j to k and path must contain A[j, k] edges.

6.7 (d)

Number of vertices = n
Number of edges = k
Number of connected components = n – k

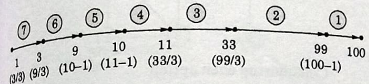
Ex. 8 vertex with 6 edges



So components = 8 – 6 = 2.

6.8 (b)

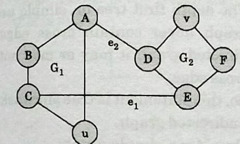
In graph G there is a directed edge between i to j when j is either $i + 1$ or $3i$.



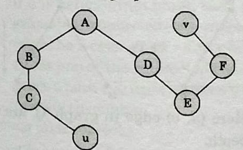
Since minimum value is finding, so we need to make edge which maximum difference in i and j here (99 – 33) = 66 is maximum. So 7 edges are needed.

6.9 (d)

Take two connected graph and connect through two edge:



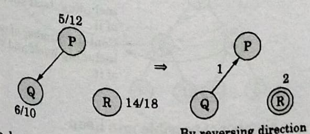
Depth first tree of graph 'G'.



- Option (a) is false, since u and v are not adjacent to single vertex.
- Option (b) is false, since removal of single vertex in G cannot is connect (u) and (v).
- Option (c) is false, since there exist a cycle but not containing both.

6.10 (d)

Consider graph with P, Q and R as vertices and apply dFS algorithm since visiting and leaving time given:



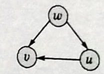
So here 2 strongly connected components are present and P and Q connected.

6.11 (c)

If a graph contain n vertices and n edges and it is simple connected graph then it forms a cycle. Atleast 3 vertices should participate hence the number of spanning trees will be atleast 3.

6.12 (d)

Consider a random graph



Only option (d) is satisfied since finishing time of vertex 'u' is always greater than finishing time of vertex 'v'.

If we visit vertex 'v' from vertex 'w' then vertex 'u' option (a) is give wrong answer.

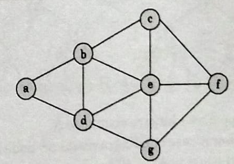
6.13 (c)

The most efficient algorithm for finding the number of connected components (articulation point) in an undirected graph on n vertices and m edges using depth-first search takes O(m + n) time. Assume $n \le m$.

6.14 (c)

The BFS using Queue data structure is QMNPRO

6.15 (b)



DFS orders:

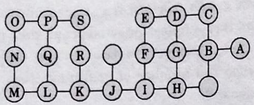
1. a–b–e–f–(d is not depth first order)
2. a–b–e–f–c–g–d is correct DFS order.
3. a–d–g–e–b–c–f is correct DFS order.
4. a–d–b–c– (g is not depth first order).

∴ 2 and 3 are correct DFS orders.

6.16 (c)

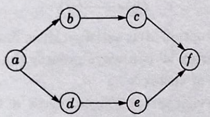
Tightest upper bound on running time of DFS is $\Theta(n)^2$.

6.17 (19)



Maximum DFS recursion depth:
A → B → C → D → E → F → G → H → I → J →
K → L → M → N → O → P → Q → R → S = 19

6.18 (6)

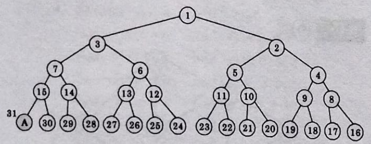


Number of topological orders: 6

a	b	c	d	e	f
a	b	d	e	c	f
a	d	b	c	e	f
a	d	b	c	e	f
a	d	e	b	c	f
a	d	b	e	c	f

6.19 (31)

In worst case binary tree with height 4 (i.e., distance from root node) look like.

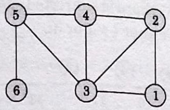


Suppose we have to visit node A.
In worst case of BFS traversal n^{th} node (A) will be the last node at 4^{th} level i.e., we have to visit $2^{4+1} - 1 = 31$ node.
So value of n will be 31.

6.20 (b)

By using BSF (Breadth First Search) traversal we can set the twin pointer in each entry in each adjacency list.
So it will take $\Theta(m + n)$ times (since adjacency list are using).

6.21 (d)

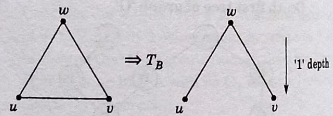


Considering each option:

- After vertices 'M' and 'N', vertex 'O' can't be traversed.
- After visiting vertex 'M', vertex 'O' should be traversed.
- After 'N' either of the vertices 'O' or 'P' should be traversed.

6.22 (a)

- The depth first tree on simple undirected graph never contain cross edge but on directed graph it may or may not contain cross edge.
  So, this statement is true since asked about undirected graph.
- Consider a simple graph:

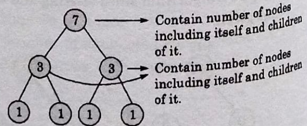


Here (u, v) edge in graph 'G' are on same depth.
Hence it is not always true that $|u_i - u_j| = 1$.
But $|u_i - u_j| \le 1$ i.e. 1 and 0. So, false

Note: To prove statement false one case is sufficient but to prove statement true we need to take care each and every case.

6.23 (80)

Structure of min-heap will be: Since complete Binary Tree

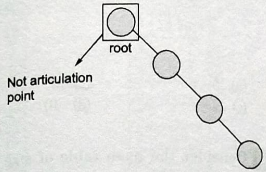


Number of possible min heap

$$= \frac{7!}{7 \times 3 \times 3 \times 1 \times 1 \times 1} = 80$$

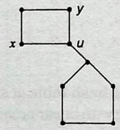
6.24 (a)

(a) True



We need at least 2 children so that root is articulation point.

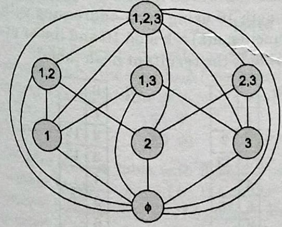
(b) False: This can never happen. Leaf will always have degree = 1.
(c) False: Check option (a) for more information.
(d) False: Below is the reasoning to show how this is false.



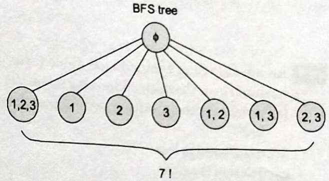
If u is articulation point, then removing u generates 2 connected components, now there might be a case when x and y will belong to either one of the connected component and hence a path will exist between them without passing through u.
Option (a) is correct.

6.25 (5040)

$U = \{1, 2, 3\}$ graph according to description



$B(\phi)$ denotes the number of possible orderings when BFS is started (from ϕ).
Since, ϕ must be connected to all the nodes.



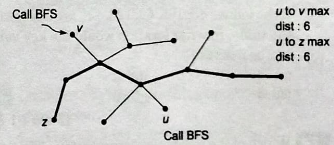
6.26 (c)

BFS traversal spanning tree is the shortest path spanning of undirected unweighted graph.

6.27 (b, c, d)

- Every non tree edge of G is back/forward edge.
- BFS tree is shortest path tree; so that every tree edge (u, v) with respect to BFS tree of graph G : $\text{dist}(u) - \text{dist}(v) = \pm 1$.
- Both BFS and DFS can used for to test connected components of undirected graph.

6.28 (6)



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