AMS 209: Homework 6 Report

Due: *November 29, 2017*

Instructor: Dongwook Lee

Panos Karagiannis

Contents

Question 1: Initial guess close to the actual solution

Answer:
We use Newton's method to find a solution to the equation

$$f(x) = e^{2x+3} - e^x$$

,where clearly the root occurs when $x = -3$. We first start by guessing a root that is **close** to the actual solution and we it be equal to `x_0 = -2.7`. As we vary the threshold to be $10^{-4}, 10^{-6}, 10^{-8}$ we respectively get the following results:

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Solution Convergence Summary
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Your converged solution `x` $= -2.9999999999999942$
Solution converged in `Nstep`$= 5$
Threshold value $= 1.0000000000000000E - 004$
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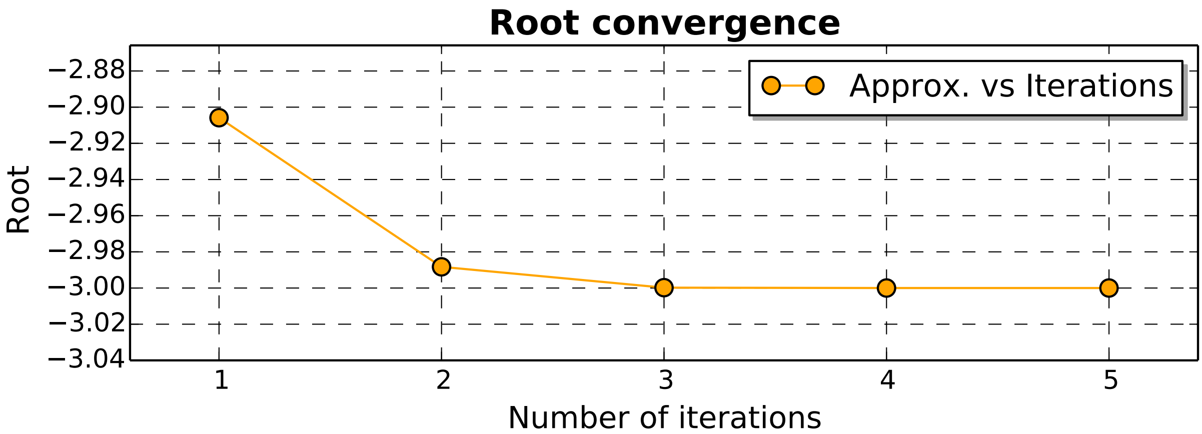

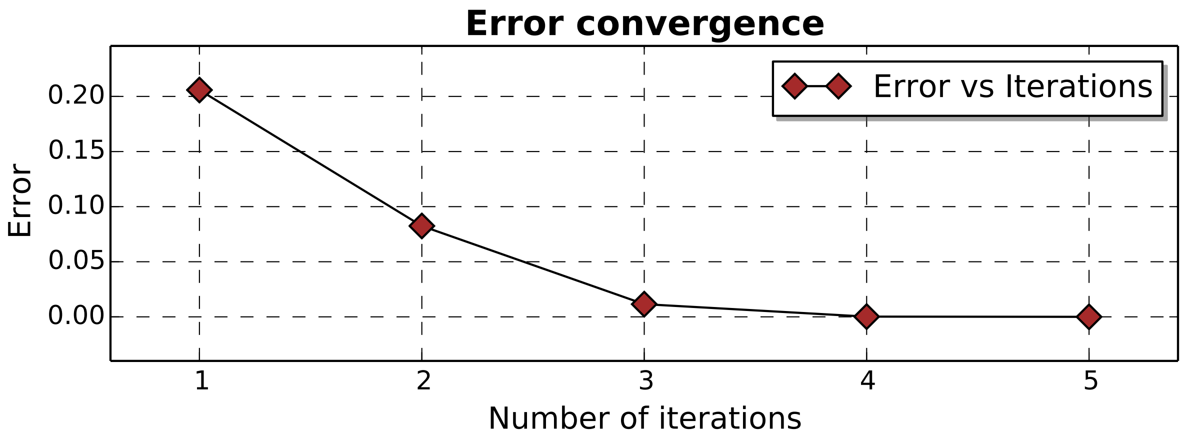

Question 1: Initial guess close to the actual solution continued on next page. . .

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Solution Convergence Summary

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Your converged solution `x` = -2.9999999999999942

Solution converged in `Nstep`= 5

Threshold value = $9.9999999999999995E - 007$

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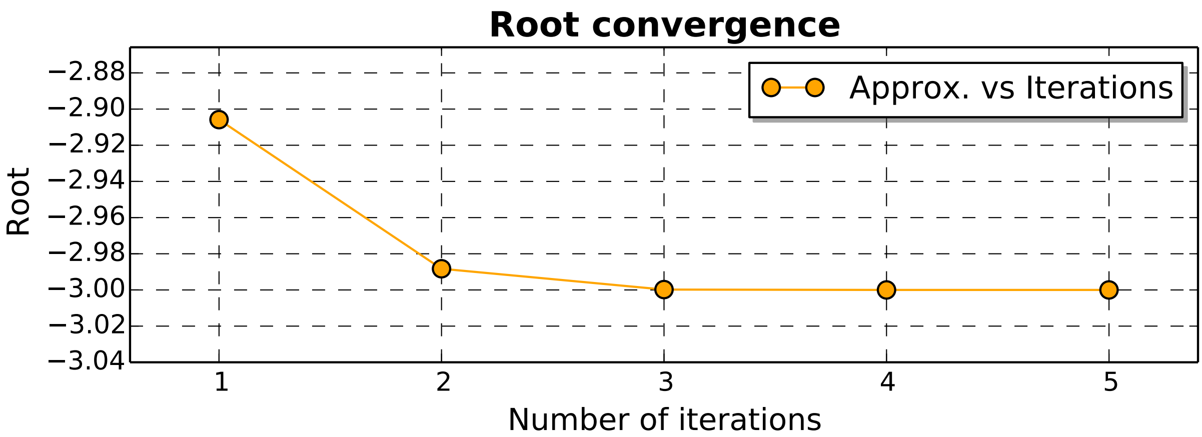

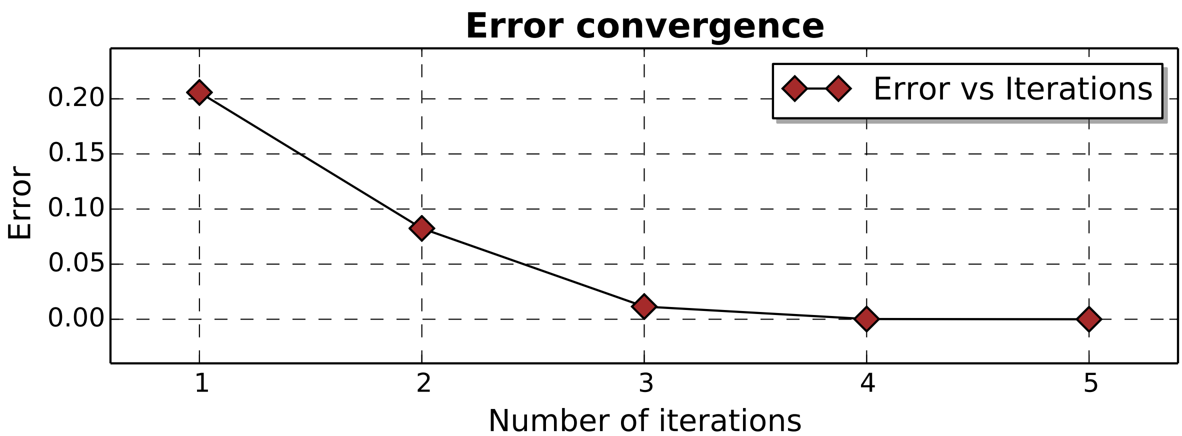

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Solution Convergence Summary

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Your converged solution `x` $= -3.0000000000000000$

Solution converged in `Nstep`$= 6$

Threshold value $= 1.0000000000000000E - 008$

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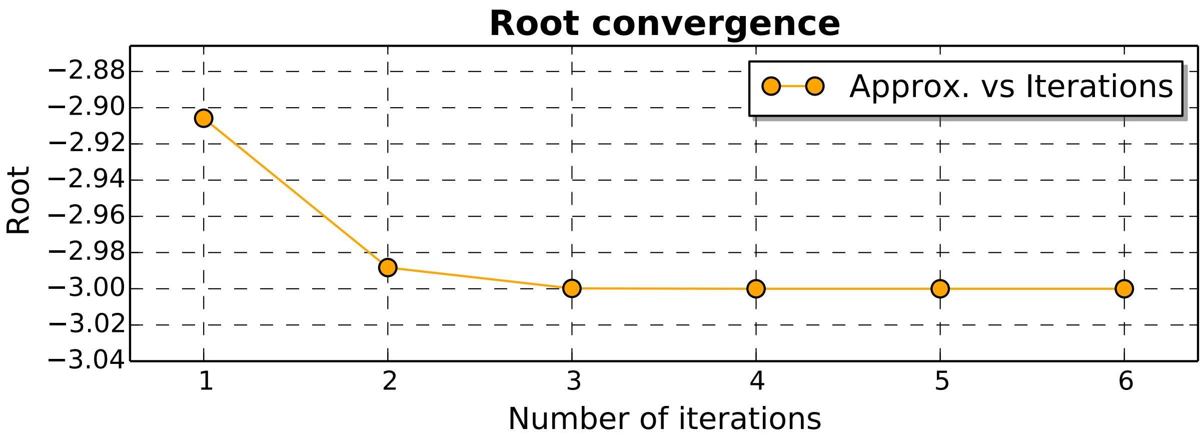

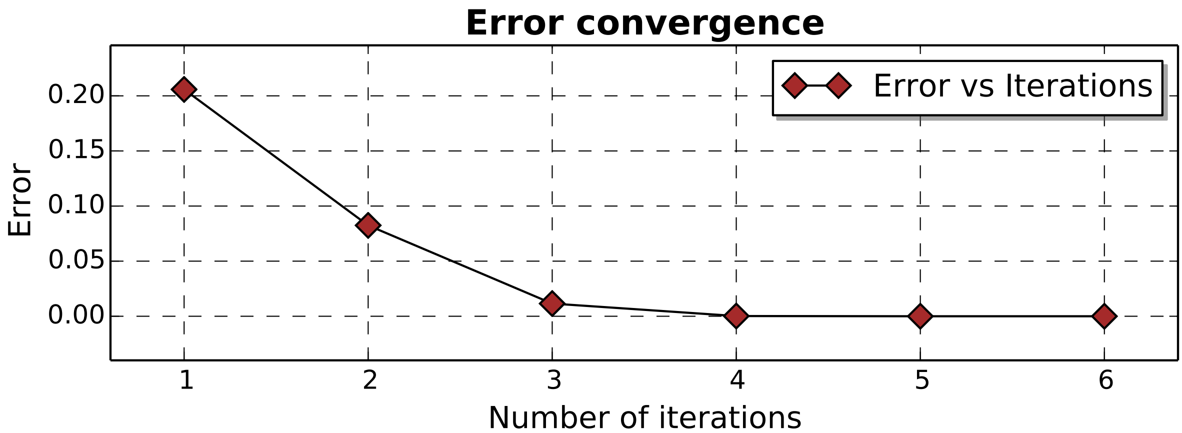

Question 2: Initial guess far from the actual solution

Answer:
We now start at an initial guess that is **further** apart, namely, we set `x_0 = 1` . Then, the results we get for the same thresholds $10^{-4}, 10^{-6}, 10^{-8}$ are, respectively:

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Solution Convergence Summary
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Your converged solution `x` = 2.9999999999963225
Solution converged in `Nstep`= 13
Threshold value = $1.0000000000000000E - 004$
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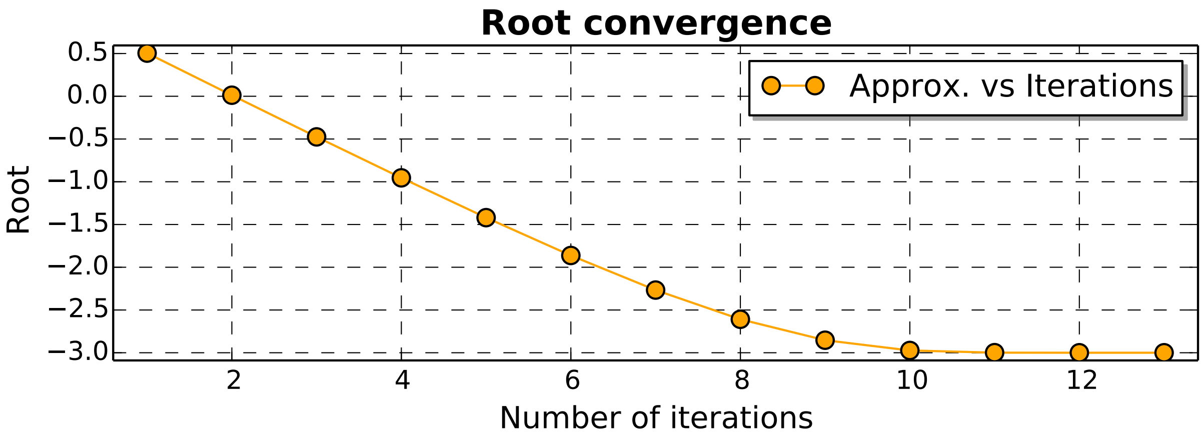

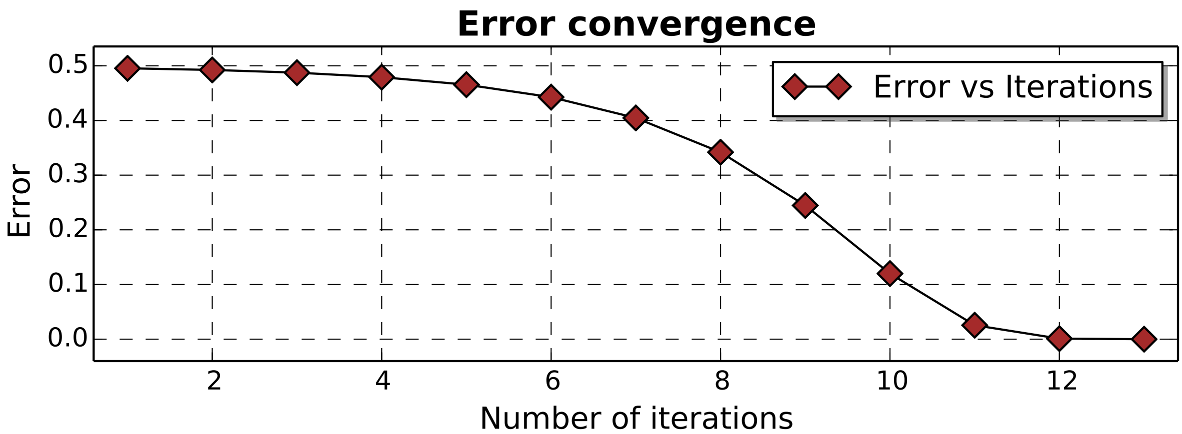

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Solution Convergence Summary

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Your converged solution `x` $= -3.0000000000000000$

Solution converged in `Nstep`$= 14$

Threshold value $= 9.9999999999999995E - 007$

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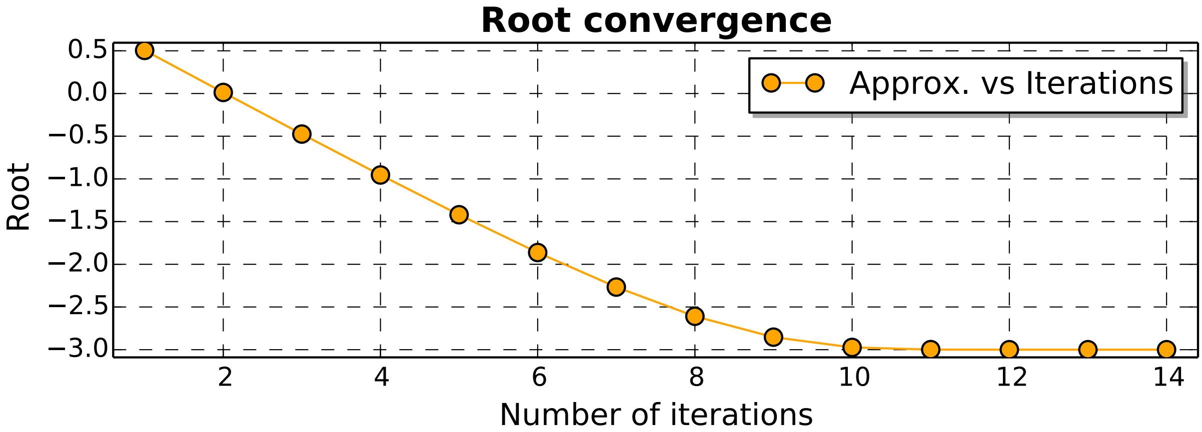

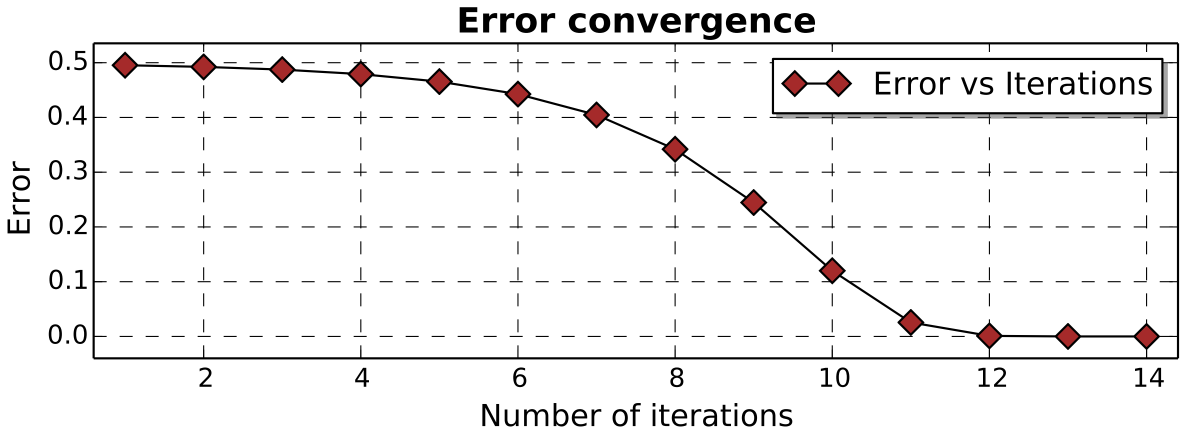

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Solution Convergence Summary
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Your converged solution `x` $= -3.0000000000000000$
Solution converged in `Nstep`$=14$
Threshold value $= 1.0000000000000000E-008$
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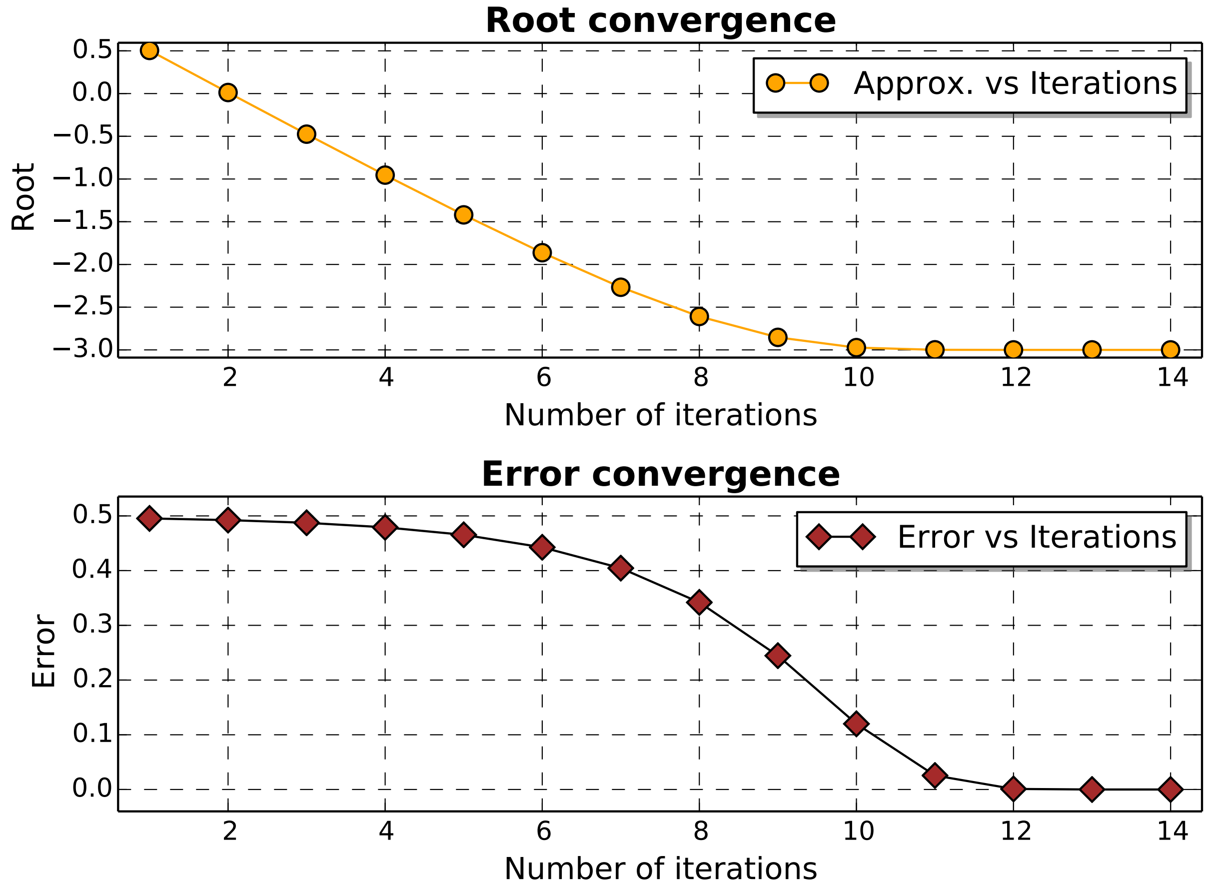

Question 3: Observations and conclusion

Answer:
The most important *observation* between the two initial guesses is that in the first case, where the initial guess is close to the actual root, we approach the solution much faster by using at most 6 iterations. This is not the case when the initial guess is farther from the actual solution, where we see that, in the worst case, 14 steps are required to approximate the solution within a threshold of 10^{-8}.

Question 3: Observations and conclusion continued on next page...

Newton's Method is a relatively good approximation method for finding the solution to a non-linear equation. Generally, it is expected to give **quadratic convergence** provided that the initial guess is sufficiently accurate and that the objective function is **differentiable** at each time step x_t. A weakness of newton's method is that we need to compute the derivative at each step. While this is trivial in the *2-D* case, it becomes much more expensive in the case where we are solving a system of non-linear equations and we need to invert the Jacobian matrix at each time step. To avoid this, in practice we use **quasi-Newton** methods such as Broyden's method.