Vector :-

Vector : magnitude + direction → eg. (40)km/hr in North direction → velocity ✓

Scaler : magnitude → eg. 40km/hr → speed

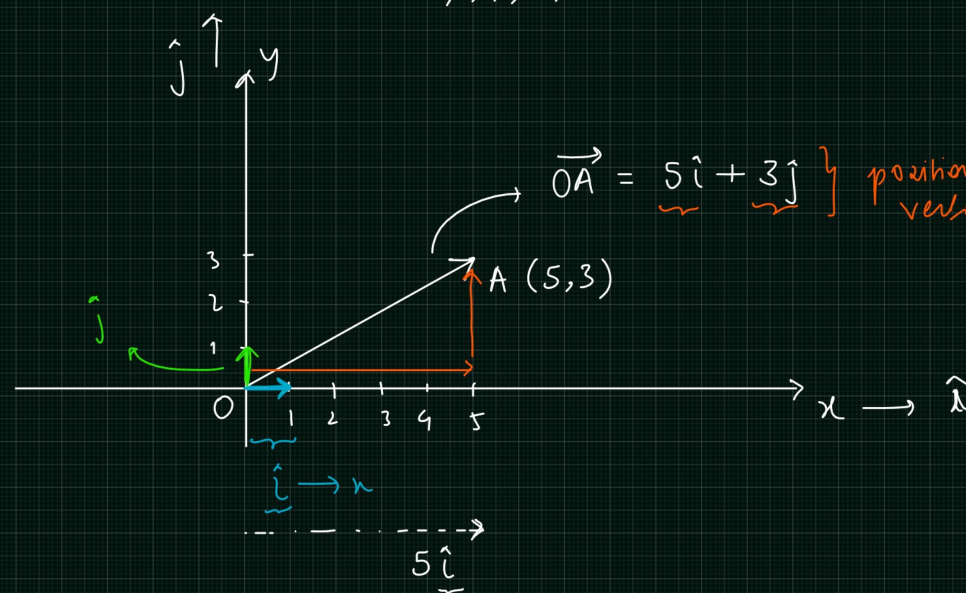
gravity $9.8\ m/s^2$ ↓

directional

Unit vector ⇒ vector with mag = 1

$\hat{i} = 1\hat{i}$

$= 5\hat{i}$



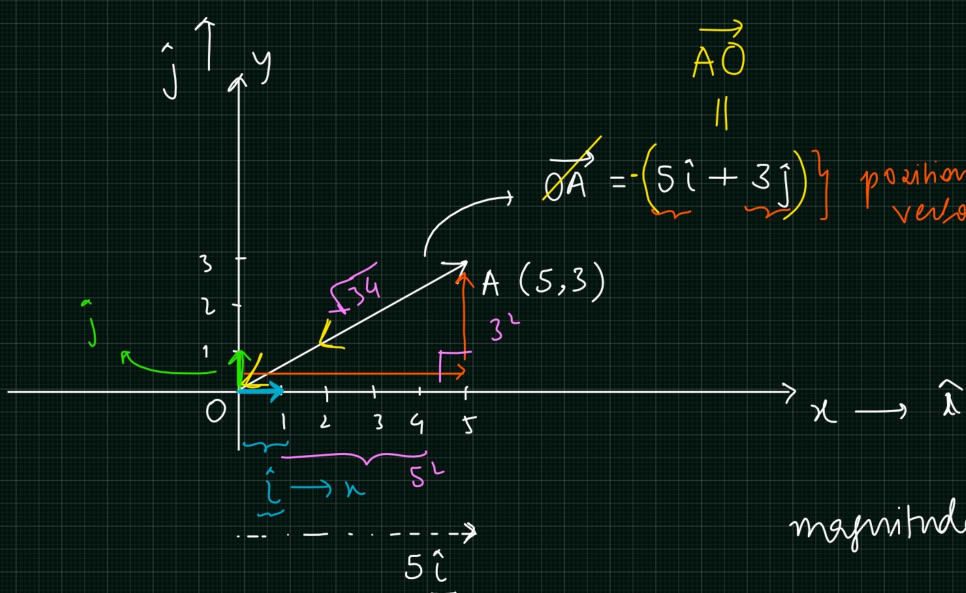
$\vec{OA} = 5\hat{i} + 3\hat{j}$ } positional vector

$\boxed{\vec{OA} = -\vec{AO}}$

$\vec{AO}$

$\|$

$\cancel{\vec{OA}} = -(5\hat{i} + 3\hat{j})$ } positional vector



magnitude $\vec{OA} = |\vec{OA}|$

$= \sqrt{5^2 + 3^2}$

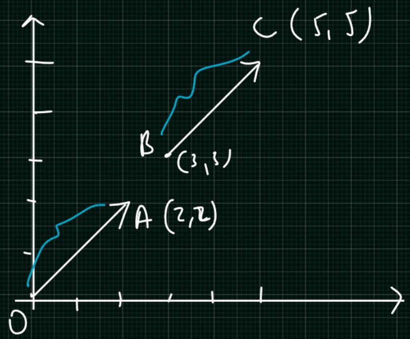
$= \sqrt{25 + 9}$

$= \sqrt{34}$

$\vec{AB} = x\hat{i} + y\hat{j}$ $\qquad$ $|\vec{AB}| = \sqrt{x^2+y^2} \neq 1.$

$$\vec{CD} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}} = \underbrace{\frac{x}{\sqrt{x^2+y^2}}}_{x}\hat{i} + \underbrace{\frac{y}{\sqrt{x^2+y^2}}}_{y}\hat{j}$$

$$|\vec{CD}| = \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}} = \sqrt{\frac{x^2+y^2}{x^2+y^2}} = 1.$$

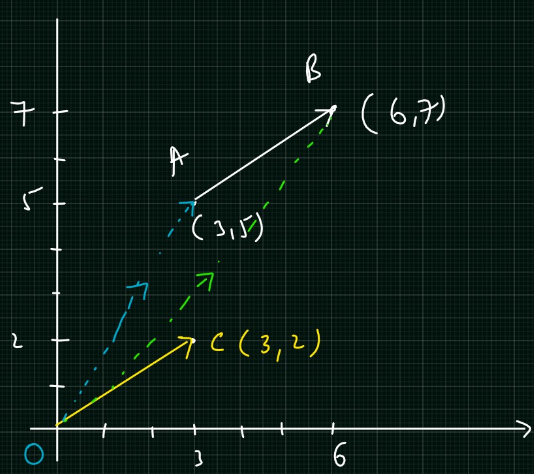


$\vec{OA} = 2\hat{i} + 2\hat{j}$

$\vec{BC} =$

When I can say 2 vectors are same?

↳ when magnitude & direction are same ✓



$\vec{OA} = 3\hat{i} + 5\hat{j}$

$\vec{OB} = 6\hat{i} + 7\hat{j}$

$\vec{AB} =$ destination − source

$= \vec{OB} - \vec{OA}$

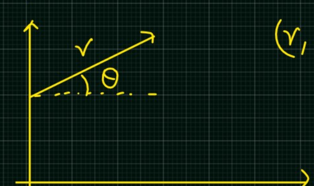
$= 6\hat{i} + 7\hat{j} - (3\hat{i} + 5\hat{j})$

$\vec{AB} = \underbrace{3\hat{i} + 2\hat{j}}$

Cartesian Coordinate

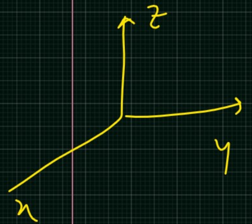
(x, y)

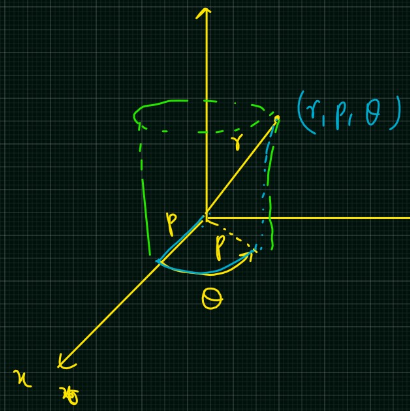
Polar coordinate



(r, θ)

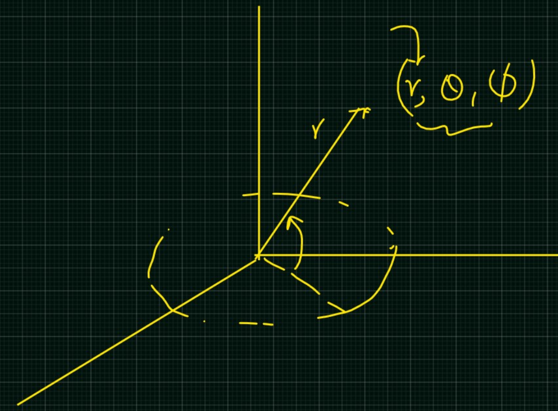
Cartecian cylindrical coordinate system Spherical coordinate





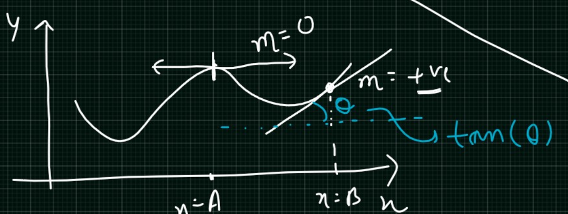
(r, ρ, θ)

$(\hat{r}, \hat{\rho}, \theta)$



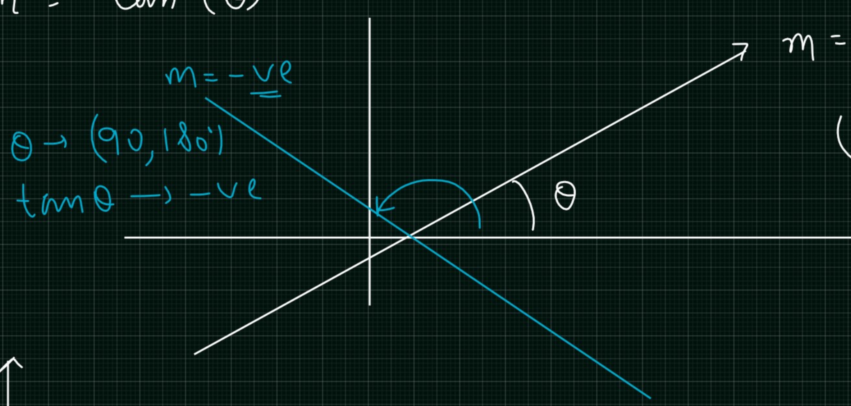
→ Differentiation
→ Concept of maxima & minima

Differentiation → Graphically → slope of the tangent at a point on the graph



$$\frac{dy}{dx} = \frac{\text{change in } y}{\text{small change in } x}$$

slope = m = $\tan(\theta)$

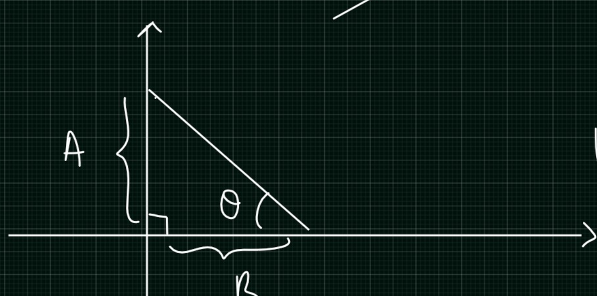


m = +ve

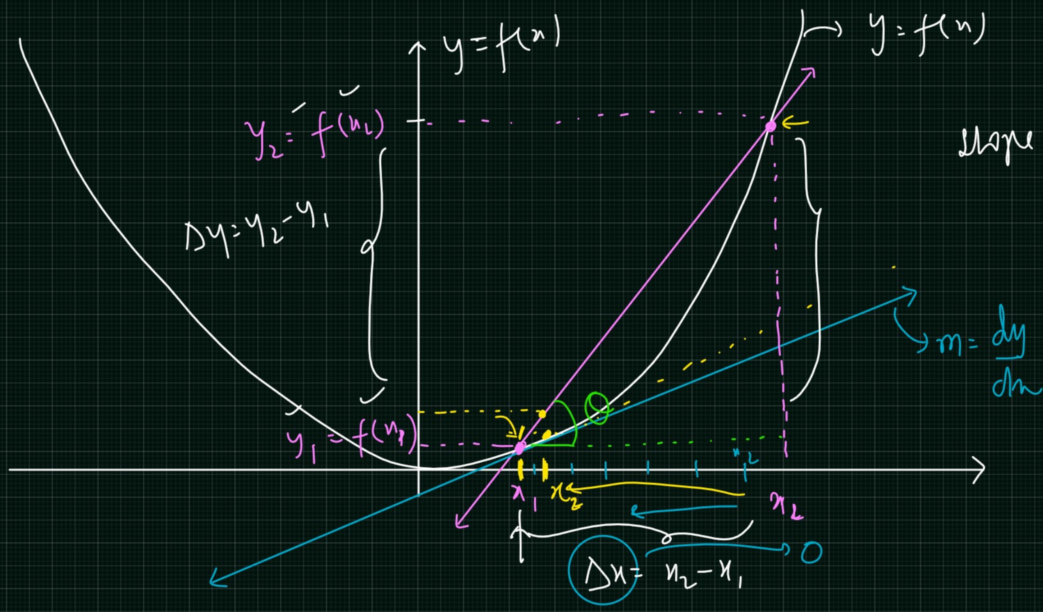
(0, 90)

tan (0, 90)

+ve



$$\tan\theta = \frac{A}{B} = \frac{\text{perpendicular}}{\text{base}}$$



$$\text{slope} = \tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

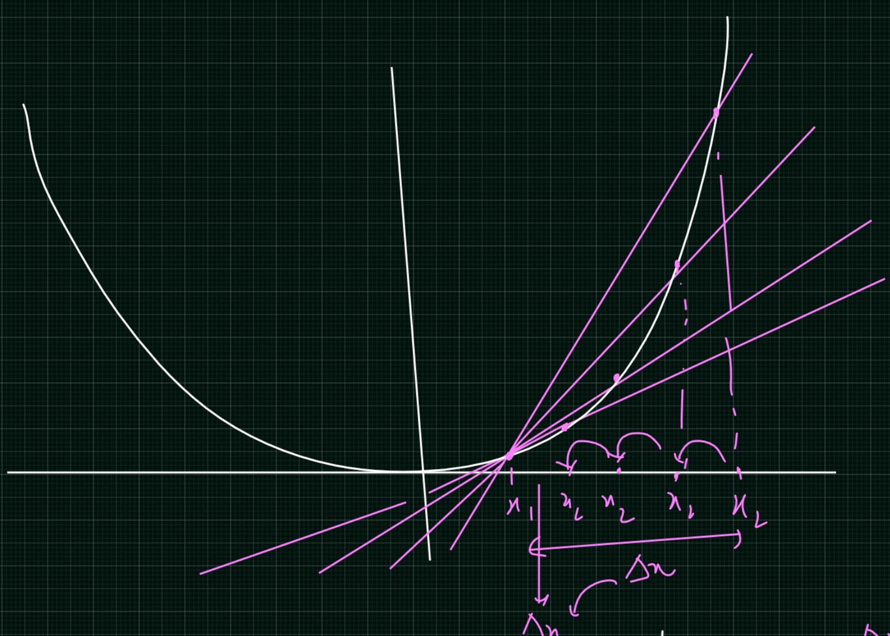
$$= \frac{\text{change in } y}{\text{change in } x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{\text{change in } y}{\text{small change in } x}$$

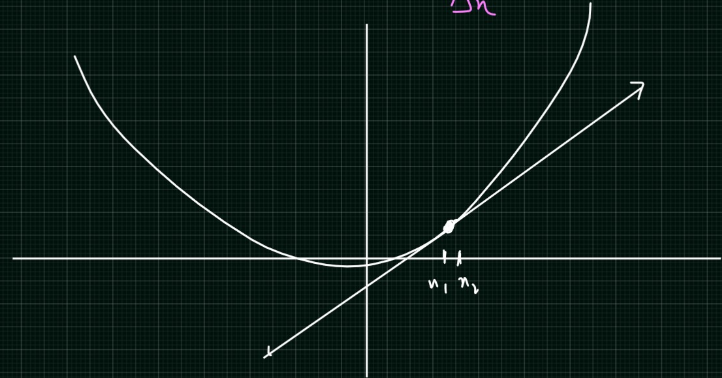
$$w = w - \eta \frac{\partial e}{\partial w}$$

$$e(w, b)$$

$$\frac{\partial e}{\partial w} \qquad \frac{\partial e}{\partial b}$$



$$\lim_{\Delta x \to 0}$$



$\Delta x \to 0 = 0.001$

$$\boxed{\text{slope} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$$

$$\text{slope} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y_2 - y_1}{\Delta x}$$

$$\Rightarrow \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x_2) - f(x_1)}{\Delta x}$$

$x_2 = \underbrace{x_1 + \Delta x}$

first principle of differentiation

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}}$$

```
def diff(x, f):
    delta_x = 0.00001
    return (f(x + delta_x) - f(x)) / delta_x
```

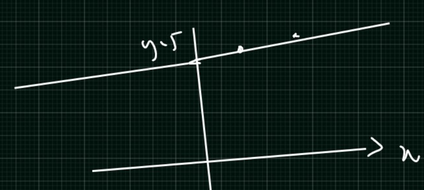
$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x$$

$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$$

$$\boxed{y = x^n \Rightarrow \frac{dy}{dx} = n x^{(n-1)}}$$

$$y = x^{-1} \Rightarrow -1 x^{-1-1} = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

$$\boxed{y = e^x \Rightarrow \frac{dy}{dx} = e^x}$$



$$y = 5 \Rightarrow \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y_2 - y_1}{\Delta x} = \frac{5 - 5}{\Delta x} = \frac{0}{\Delta x}$$

$$\boxed{y = \text{const} \Rightarrow \frac{dy}{dx} = 0}$$

Sum rule

$$y = u(x) \pm v(x)$$

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

Example

$$y = x^2 + x^3$$

$$\frac{dy}{dx} = 2x + 3x^2$$

product rule

$$y = u(x) \cdot v(x)$$

$$\frac{dy}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

$$y' = u' \cdot v + u \cdot v'$$

$$y = (x+1) \cdot (x+2)$$

$$= (1+0) \cdot (x+2) + (x+1) \cdot (1+0)$$

$$= (x+2) + (x+1)$$

$$= 2x+3$$ ✓

division

$$y = \frac{u(x)}{v(x)} \quad \text{when } v(x) \neq 0$$

$$\frac{dy}{dx} = \frac{u' \cdot v - u \cdot v'}{v^2}$$

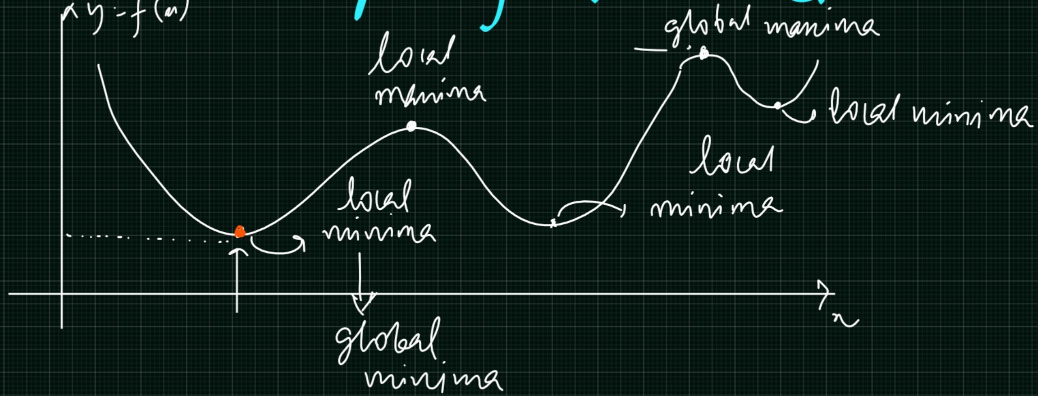
$$y = \frac{(x+1)}{(x+2)}$$

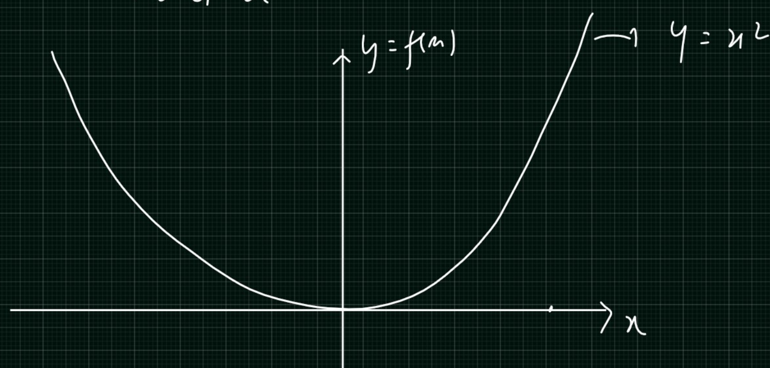
$$u' = 1+0 = 1$$

$$v' = 1+0 = 1$$ ✓

$$\frac{1 \cdot (x+2) - (x+1) \cdot 1}{(x+2)^2} = \frac{(x+2) - (x+1)}{(x+2)^2} =$$

Concept of Maxima & Minima





$y = x^2$

step 1 $\qquad \frac{dy}{dx} = \frac{df(x)}{dx} = \frac{d x^2}{dx} = 2x$

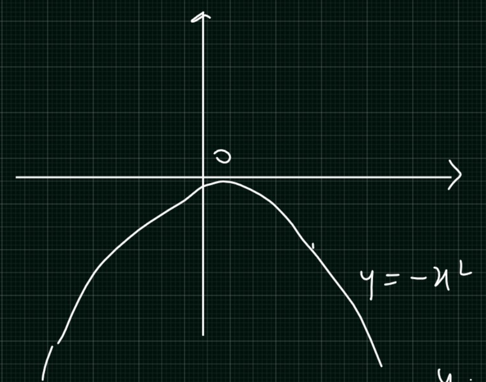
step 2 $\qquad \frac{dy}{dx} = 0 \Rightarrow 2x = 0 \Rightarrow \boxed{x = 0}$ critical point

step 3 $\qquad \frac{d^2y}{dx^2} \Rightarrow \frac{dy}{dx} = 2x \Rightarrow \frac{d^2y}{dx^2} = 2$

— if $\left.\frac{d^2y}{dx^2}\right|_{x=0} > 0$ at $x = 0 \Rightarrow$ point of minima

if $\left.\frac{d^2y}{dx^2}\right|_{x=0} < 0$ at $x = 0 \Rightarrow$ point of maxima

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = 0$ at $x = 0 \Rightarrow$ sadal point or $\underbrace{\text{point of inflexion}}_{\text{neither maxima nor minima}}$



$$\frac{dy}{dx} = -2x$$

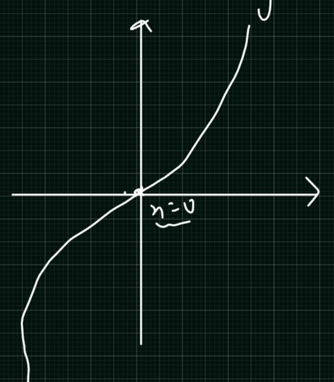
$$\frac{dy}{dx} = 0 \Rightarrow -2x = 0 \Rightarrow x = 0$$

$$y = -2^2 = -4$$

$$\frac{d^2y}{dx^2} = -2$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=0} \Rightarrow -2 < 0 \Rightarrow x = 0 \Rightarrow \text{point of maxima}$$

$y = x^3$ find, maxima or minima -

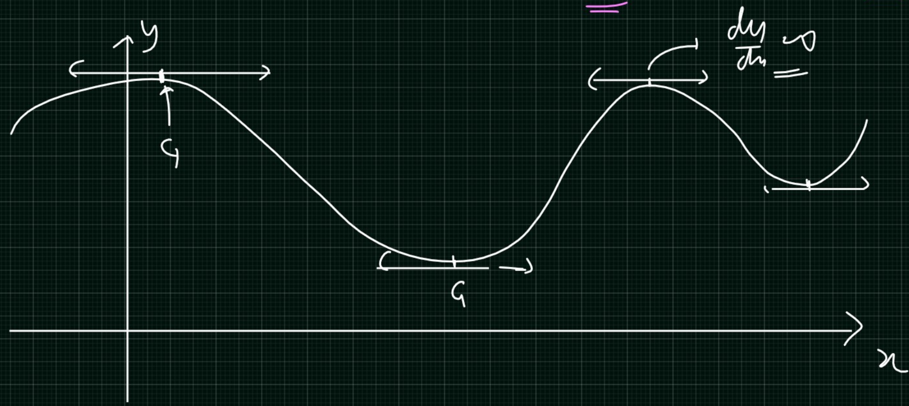


$$\frac{dy}{dx}, \quad \frac{dy}{dx} = 0 \rightarrow c_1, c_2, \quad \left.\frac{d^2y}{dx^2}\right|_{x=c_1} > 0$$

$$3 \cdot 2 \cdot x = 6x$$

$$\frac{dy}{dx} = 3x^2 \qquad \frac{dy}{dx} = 0 \Rightarrow 3x^2 = 0 \Rightarrow \boxed{x = 0} \qquad \frac{d^2y}{dx^2} = 6x$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=0} \Rightarrow \left.6x\right|_{x=0} = 0 \quad \text{saddle}$$



maxima → 2
minima → 2

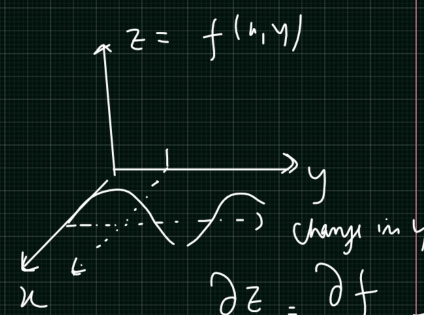
loss f^n $\quad f(w_1, w_2, w_3 \quad b_1, b_2 .. b_3)$

Partial derivatives

$$f(x,y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = 2x + 0 = 2x$$

$$\frac{\partial f}{\partial y} = 0 + 2y = 2y$$



$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} \checkmark$$

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \checkmark$$

$$\boxed{\begin{aligned}\frac{\partial f}{\partial x} &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} \\ \frac{\partial f}{\partial y} &= \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}\end{aligned}}$$