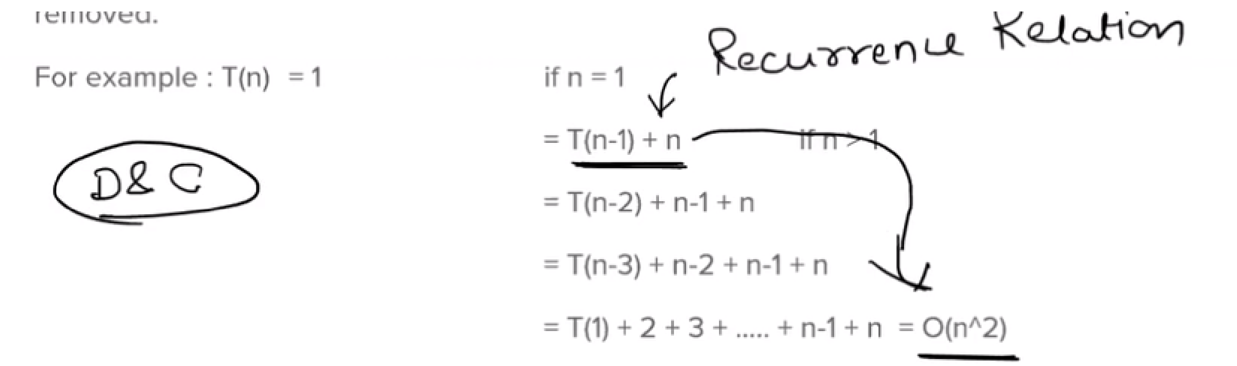
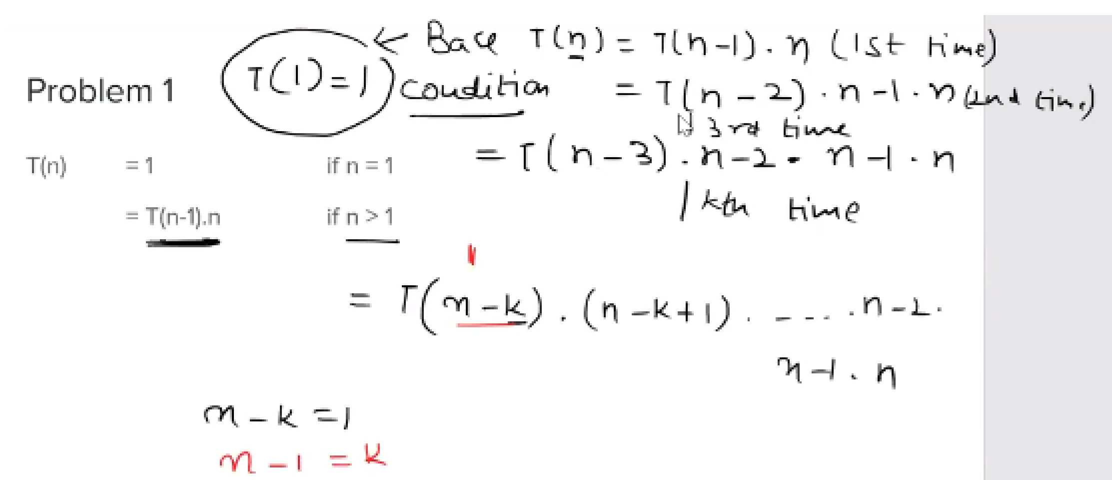
Recurrence relation solving

Substitution Method:

This method simply substitute the given function repeatedly until the given function is removed.



$$n + (n-1) + (n-2) + .. + 1 = n(n+1)/2 = n^2$$



$$T(n-(n-1)) * (n-(n-1)+1) * (n-(n-1)+2)$$

$\cancel{n}-\cancel{n}+1$ $\quad$ $\cancel{n}-\cancel{n}+1+1$ $\quad$ .... $n-1*n$

Base condition $-\underline{T(1)} * 2 * 3 * --- * n-1 * n$

$$1 * 2 * 3 * \ldots * n-1 * n = n!$$

$$O(n!) = O(n^n)$$

Problem 2

T(n)	=	1	if n = 1
	=	T(n/2) + n	if n>1

$T(n) = T(n/2) + n$ (1st time)

↓ 2nd time

$= T\left(\frac{n}{2^2}\right) + \frac{n}{2} + n$

↓ 3rd time

$= T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + \frac{n}{2^1} + \frac{n}{2^0}$

↓ K times

$= T\left(\frac{n}{2^K}\right) + \frac{n}{2^{K-1}} + \frac{n}{2^{K-2}} + \ldots + \frac{n}{2^1} + \frac{n}{2^0}$

$$\frac{n}{2^K} = 1 \Rightarrow n = 2^K$$

$$(\log_2 n = K)$$

$K = \log_2 n$

$$T\left(\frac{n}{2^{\log_2 n}}\right) + \frac{n}{2^{\log_2 n - 1}} + \frac{n}{2^{\log_2 n - 2}} + \ldots + \frac{n}{2^1} + \frac{n}{2^0}$$

$\cancel{n}$ $n^{\log_2 1}$

$$T(1) + n\left(\underbrace{\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2}_{\text{GP series}} + \ldots + \left(\frac{1}{2}\right)^{\log_2 n - 1}\right)$$

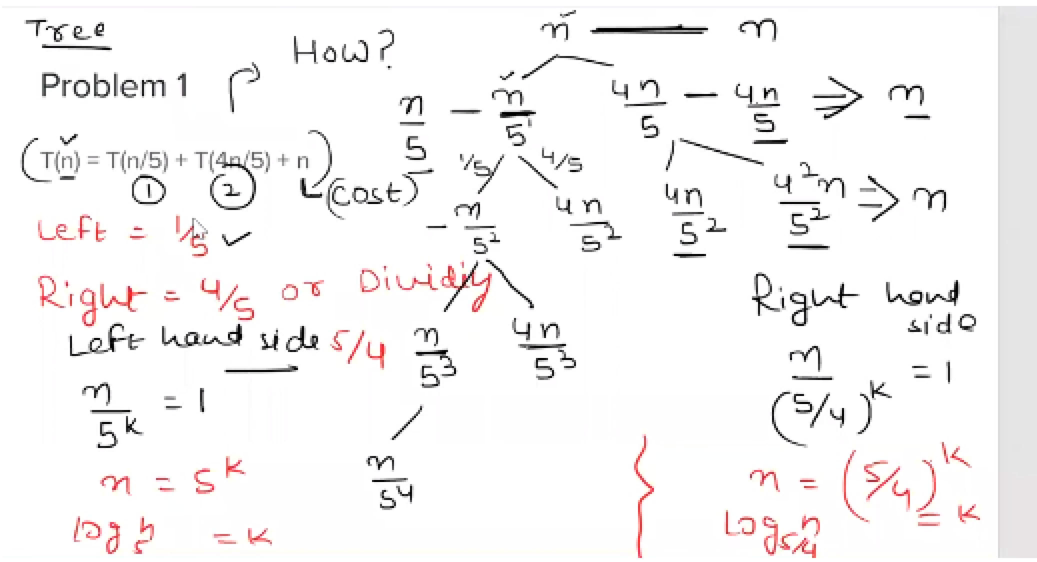
$r = 1/2$

$$1 + n\left(\frac{1\left(1-\frac{1}{2}\right)^{\log_2 n - 1}}{1 - 1/2}\right) \quad \frac{a(1-r)^n}{1-r} \quad r<1$$

$$1 + n\,[1] = O(n)$$

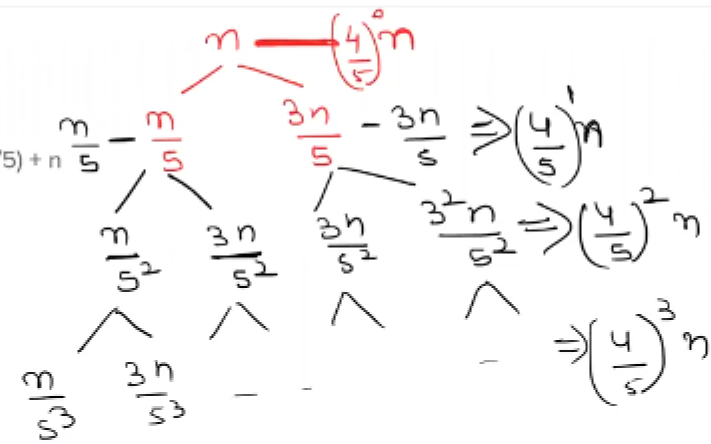
Recursive Tree Method:

- If two or more recursive terms occur, then we go for recursive tree method
- In this particular method we draw recurrence tree and calculate the time taken by every level of the tree. And at the end we add the overall work done at all the levels.
- Keep drawing the tree until we find particular pattern among all the levels.



Problem 2

T(n) = T(n/5) + T(3n/5) + n



Left = 1/5

Right = 3/5

Left $\Rightarrow \frac{1}{5}$

Right $\Rightarrow \frac{3}{5}$ ✓

$$\frac{n}{5^k} = 1 \qquad\qquad \frac{n}{(5/3)^k} = 1$$

$$n = 5^k \qquad\qquad n = (5/3)^k \quad \text{Greater}$$

$$\underline{\underline{(\log_5 n = k)}} \qquad\qquad \rightarrow \underline{\underline{(k = \log_{5/3} n)}}$$

$$\left(\frac{4}{5}\right)^0 n + \left(\frac{4}{5}\right)^1 n + \left(\frac{4}{5}\right)^2 n + \cdots + \left(\frac{4}{5}\right)^k n$$

$$n\left(\left(\frac{4}{5}\right)^0 + \left(\frac{4}{5}\right)^1 + \left(\frac{4}{5}\right)^2 + \cdots + \left(\frac{4}{5}\right)^k\right) =$$

$$n\left(\left(\frac{4}{5}\right)^0 + \left(\frac{4}{5}\right)^1 + \left(\frac{4}{5}\right)^2 + \cdots + \left(\frac{4}{5}\right)^{\log_{5/3} n}\right)$$

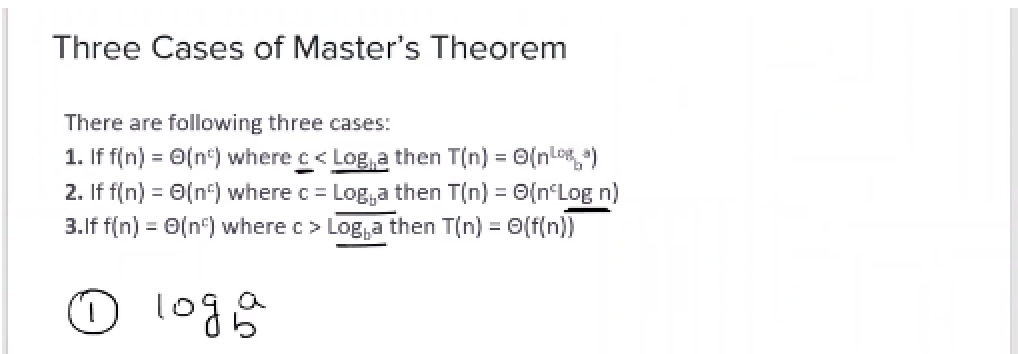
Master’s Theorem

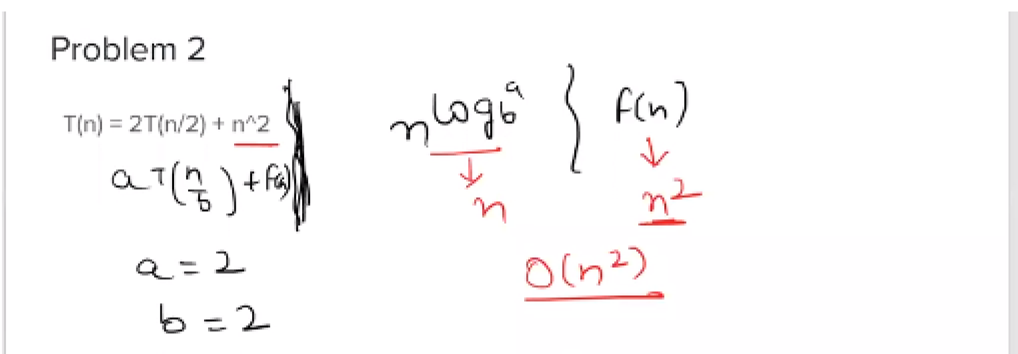
Easiest Method or Direct Method to find out the recurrence relation but can be worked only for the recurrences that can be transformed like:

$$T(n) = aT(n/b) + f(n)$$

where a ≥1 and b>1

f(n) : Positive Function





Problem 3

T(n) = 2T(n/2) + n

$f(n)$

$n^{\log_b a} = n^{\log_2 2} = \underline{n} \mid \underline{n}$

$O(f(n) \cdot \log n)$

$\underline{O(n \log n)}$

1) Master's Theorem applicable or not

2) LHS RHS

$n^{\log_b a}$ $\{$ $f(n)$

LHS $\neq$ RHS

O (Greater value)

LHS = RHS

$O(f(n) \log n)$