FOURIER-WAVELET REGULARIZED DECONVOLUTION

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ABSTRACT

This paper discuss about image deblurring algorithm "Fourier-Wavelet Regularized Deconvolution Algorithm(ForWaRD)". At first, we explore the existing deblurring techniques and their strengths, limitations. Later, we arrive at the efficient Forward algorithm and its implementation details. The ForWaRD deblurring technique achieves noise regularization from blurred images through Fourier filtering and Wavelet denoising. We briefly discuss about these Fourier and Wavelet domains and see the application of ForWaRD over Embedded platform. Later, the implementation result of forward algorithm in Matlab is compared with traditional approaches. Finally, this paper ends with future prospects of ForWaRD algorithm.

1. INTRODUCTION

Deconvolution is an algorithmic process to reverse the convolution effects. If we take a picture with the lens defocused, the result is a convolution of focused image with the Point Spread Function(PSF) of defocus. To recover the image, the PSF estimation is a crucial step. The image restoration process can be specified as,

$$f(x) * g(x) + n(x) = h(x) \tag{1}$$

Where, * represents the convolution of the original signal f(x) with the psf g(x), n(x) is the Additive white Guassian Noise and h(x) is the blurred image.

To estimate the original signal f(Images are indeed signals with parameter over space, while most other signals are measure of a parameter over time), we can perform the naive deconvolution. As, the convolution in the spatial domain is equal to the multiplication in the frequency domain, we can perform the inverse of g to get the estimate. In this case, the Mean Squared Error(MSE) between the actual image and the estimated result is large. Thus, making the estimate unsatisfactory.

This paper introduces the different estimation techniques based on shrinkage of individual components in Fourier and Wavelet domains. Exploring through the strengths and limitations in both the domains, we arrive at the efficient image deblurring algorithm, 'Forward'[1]. In section 3.1, we go through the Implementation of algorithm on GPU's[2].

2. FOURIER AND WAVELET DOMAINS

2.1. Wiener Estimation in Fourier Domain

Wiener deconvolution, introduced by Norbert Wiener for automatic aiming and firing of anti-aircraft guns during world war 2 is the application of wiener filter to the noise problems inherent in deconvolution. The Wiener filter decreases image noise by performing estimation with desired noiseless image.

If we are familiar with g used in the above convolution equation, we can perform deterministic deconvolution. Otherwise, we need to estimate it.

2.1.1. FoRD:

The Deconvolution in Fourier domain is called FoRD (Fourier Based Regularized Deconvolution). Image in spatial domain is converted to frequency by applying Fourier transform and the result is given to filter. The estimate is obtained after inverse fourier transforming the filtered result to spatial domain.

If we have some knowledge of the type of noise the signal is in convolution with and the power spectral density(psd) of the original signal, we can improve the estimate of f through the Wiener Deconvolution[4]. We can estimate the psd of original image from the observation or we can take it directly from original image. A frequently used technique is to assume the involved noise is of type Gaussian and a large Signal to Noise Ratio(SNR).Then, the wiener filter can perform the optimal calculations and minimizes the

MSE. There are different FoRD techniques like LTI Wiener deconvolution, Tikhonov regularized deconvolution[3]. We concentrate on the LTI wiener estimation.

Shrinkage in the Fourier domain can be achieved by reducing the colored noise i.e, reducing the spectral density, S over the energy signal specified in (2)

$$\frac{G^*(\omega)}{\mid G(\omega) \mid^2 + \frac{S_n(\omega)}{S_f(\omega)}} \tag{2}$$

The Strength of FoRD is it can economically represent the colored noise, but not the special signals(signals with singularities) as the energy of these signals spread over many Fourier coefficients. Thus, FoRD produces sharp images, but with noise.

2.2. Wavelet Domain

A Mother wavelet is a basic function that is isolated over time dimension and in frequency. The difference between Wavelet transformations from Fourier is wavelets are measured in both time and frequency, whereas Fourier transform is measured in either one of them. For example, If a signal is changing with time, Fourier Transform cannot tell at what time change occurred, whereas Wavelet Transform can.[9]

Given the observation, wavelet transform applies subbanding and down-sampling over the image. The wavelet coefficients are the output values from each level after subbanding.

Shrinking the wavelet transform removes noise or undesired signal[5]. We can shrink the wavelet coefficients of the transform either by applying hard/soft thresholding algorithm. Hard thresholding sets any coefficient less than or equal to the threshold to zero. Simple shrinkage in wavelet domain gives excellent results.

To estimate the signal from wavelet Domain, there are various available wavelets filters like Haar, Daubechies etc.,[4] The question to which wavelet algorithm to use depends on the application.

Though wavelet transformations can provide economic representation for signals with singularities it cannot economically represent the colored noise. Eventually, the MSE obtained via wavelet shrinkage is unsatisfactory with large value.

3. FORWARD ESTIMATION

3.1. ForWaRD Algorithm

As seen before, shrinkage in any single domain cannot yield good results as no single transform domain can represent both colored noise and smoothness. The Fourier wavelet Regularized Deconvolution Algorithm relies on both fourier and wavelet domains to achieves this for estimating the signal.

ForWaRD algorithm estimation is based on employing Fourier transformations after operator inversion to economically represent the colored noise and then decreasing the leaked noise with Wavelet Transformations which uses wide range of smoothness classes.

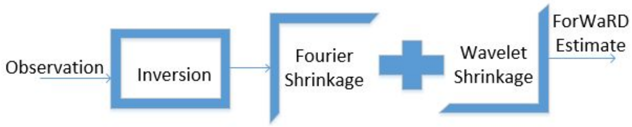

Fig. 1. Fourier-Wavelet Regularized Deconvolution

Working:

The Forward algorithm can be explained in 3 steps.

Step1: Operator Inversion
Finding the inverse transform of g from the naive deconvolution technique specified in (1). For the psf creation in Matlab, we can use Various types of available filters like Box car Blur of 9x9,3x3,6x6 or Adjustable Box car Blur, Dilation-homogeneous operator, Low pass filter used by Kalifa et al. in [3]. etc., and the noise is considered as the additive white Gaussian noise(AWGN).

Step2: Fourier Shrinkage
During operator inversion, Some of the Fourier noise coefficients are amplified. Small amount of Wiener like fourier shrinkage is sufficient to attenuate these amplified signals with minimal loss of signal components.

Step3: Wavelet Shrinkage
The leaked noise after step2 that Fourier shrinkage failed to attenuate, is sent through the oracle wavelet shrinkage to extract the original signal. This process can be further simplified into 4 steps.

i. Choosing the wavelet function
Various wavelet filters like Haar or Daubechies as specified in sec. 3.2 can be used. But, The author[1] used both the wavelet tranforms(WT1,WT2) of Daubechies type, one is

of 6 decomposition levels and other is of 2 levels. The algorithms used for WT1,WT2 can be found in [8]
ii. Perform Thresholding[6] algorithm over the wavelet coefficients. Ex: Threshold factor of 3σ
iii. Pass the signal to Wavelet Domain Wiener Filter(WDWF). Wiener filter is applied over wavelet coefficients after step ii. WDWF helps in decreasing the MSE.
iv. The inverse wavelet transform[7] retrieves the desired signal with little loss of details.
(An interesting observation of the algorithm is signals with more economical wavelet representations requires less Fourier shrinkage).

3.2. FoRWaRD on GPUs

3.2.1. Real-time HD video Deblurring

The goal[2] is to implement the forward algorithm in embedded system that deblurs HD video at 30 frames per second. The current implementation of forward involves wavelet denoising that utilizes stationary wavelet transform which takes huge memory bandwidth.

The basic forward algorithm[1] is concentrated on deblurring gray scale images. In this application, the major challenge apart from minimizing memory bandwidth requirements, parallelizing huge computations, is to deblur 1080x1920 color frames to meet the goal of 30frames/sec. To reach the desired goal, a hardware accelerator is needed.

In [2], the implementation is concentrated over a GPU platform to restructure the video to users. GPU shows 10 times more peak memory bandwidth compared to its counterpart FPGU which is a major factor to consider while moving 30 frames/sec between memory and device.

The real time Hd video deblurring primarily uses 2 main algorithms. one is FoRWaRD and another is demosaicking algorithm. The fig. 2 shows the detailed process of deblurring using ForWaRD algorithm. Demosiaciking is the image processing technique which takes color input frame in bayer format and estimates missing pixels, so that each pixel has a combination of RGB to form the full image. These algorithms make use of surrounding pixel information to derive the particular pixel details.

The DDGPU library based on CUDA programming model is used to implement the above 2 specified algorithms. During the development process, the authors [2] modified these algorithms to reach the desired speed. DDGPU library preallocates the device memory space of 1GB to prevent time to create and free memory spaces while the frame is being processed. This library is tested on various graphics cards like GTX 580, GTX 670, GTX 780. Out of all, GTX 780 showed good performance rate of 23 frames/second.

The following notations are used in the figure 2.

PSF	Point Spread Function
FFT	Forward Fourier Transform
IFFT	Inverse FFT
WT	Wavelet Transform
IWT	Inverse WT
MAD	Median Absolute Deviation

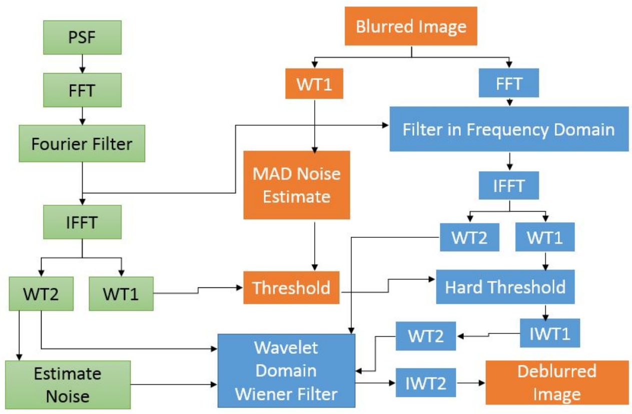

Fig. 2. ForWaRD algorithm Implementation.

The ForWaRD implementation has majorly 3 sections of which crucial is deblurring section. For numerical robustness, the Pre-compute section and Deblurring section are computed in parallel.

1. Pre-compute (Green Boxes)
Knowing the amount of camera lens defocus, This section computes the psf noise estimations and initial threshold. Once these values are generated, they do not change while system is operating.From fig. 2, The estimated noise from Haar wavelet transform(WT2) and the computed threshold from the Daubechies 4 wavelet transform(WT1) are given as inputs to the WDWF.[2]

2. Threshold updating (Orange Boxes)
In practice, the noise variance values are unknown and have to be reliably estimated using the median estimator on the wavelet coefficients of the blurred image after applying demosaicking algorithm. The MAD Noise Estimate sort the pixel values by finding the median to update the threshold in deblurring section.

3. Deblurring (Blue Boxes)
This is the main section of algorithm, that contains the Fourier filtering followed by Wavelet denoising. One wavelet transform goes directly to WDWF while other wavelet transform

is thresholded and after an inverse, forward transforms goes finally to WDWF. The outputs from the precompute and threshold sections are also given as inputs to WDWF. After acquiring all the necessary inputs, inverse wavelet transform from WDWF leads to the FoRWaRD estimate.

3.3. Other Applications

Forward algorithm can be applied in various space variant systems where image resolution varies as the space/fixation point varies. For example, Satellite imagery, Aiming of guns, Seismic deconvolution, Image database retrieval, Image Compression.

4. RESULTS OF FORWARD ALGORITHM

Forward performance is tested in Matlab over a 2d image signal by taking a Boat image as the original signal, 9x9 Box car blur and BSNR(Blurred Signal to Noise Ratio) as 40db. Fig. 3 shows the original signal, convoluted signal, Wiener estimate and ForWaRD estimate.

The ForWaRD estimate with a SNR of 26.09dB is a good improvement over Wiener estimate of 23.32dB. Wiener estimate has ripples whereas ForWaRD estimate smooothens the regions and most of the edges also well preserved.

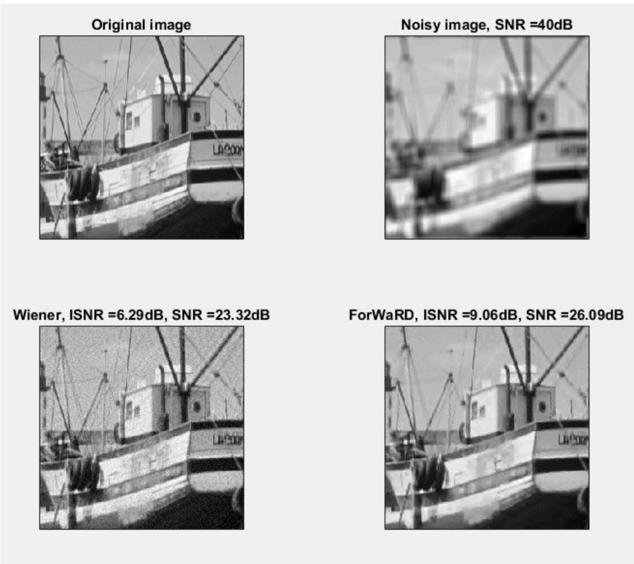

Fig. 3. Boat Image estimations

5. CONCLUSION

We have seen the estimation of traditional techniques and the ForWaRD improvements over them. Implementation of a Forward algorithm in Matlab over various blurred images resulted in better visual quality with less MSE as compared with traditional Wiener filter estimates. As observed from real time HD video deblurring, The forward algorithm with few modifications can run at greater speeds on desktop graphics cards. For image restoration, Forward deblurring is better than many traditional restoring techniques and code is also easily available for desired modifications to implement on FPGA in embedded scale graphics hardware.

Yet, One interesting research possibility for the current FoRWaRD algorithm is to first work with wavelet domain to estimate f(x,y)$\times$g(x,y) from the noisy observation h(x,y) and then invert the convolution operator. This technique is called VWD (Vaguelette-Wavelet Decomposition). However VWD like WVD[10] (Wavelet Vaguelette Decomposition) technique is not suitable for Box car blur convolution operator.

6. REFERENCES

[1] R. Neelamani, H. Choi, and R. Baraniuk, ‘Forward: Fourier-wavelet regularized deconvolution for ill-conditioned systems,” *IEEE Signal Processing, IEEE Transactions*, vol. 52, no. 2, pp. 418–433, Feb. 2004.

[2] J. Dysart, B. Brockman, S. Johnes, F. Bacon “Embedded Real-time HD Video Deblurring,” *IEE High Performance Extreme Computing Conference (HPEC)*, pp. 1–6, Sept. 2014.

[3] C. Gonzalez, E. Woods, “Digital Signal Processing,” ,Pearson, 2008.

[4] T. Young,J. Gerbrands,J. van Vliet, “Fundamentals of Image Processing,” Delft University of Technology.

[5] Y. Nievergelt, “Wavelets Made Easy,” Birkhaeuser, 1999.

[6] J. Kalifa and S. Mallat, “Thresholding estimators for linear inverse problems,’Ann. Statist.,’, vol. 31,no. 1, Feb 2003.

[7] S. Mallat, A Wavelet Tour of Signal Processing. New York: Academic, 1998.

[8] http://dsp.rice.edu/software/forward

[9] http://www.tutorialspoint.com/dip

[10] D. L. Donoho, Nonlinear solution of linear inverse problems by wavelet-vaguelette decomposition, Appl. Comput. Harmon. Anal., vol. 2, pp. 101126, 1995.