HW-11

ATTENDER PAL SINGH
U01698955

Q11.1:-

a)

$$verify(DUP.algor, A) = \begin{cases} T & \text{if } isdownup(O) \wedge (sort(A) = sort(O)) \\ F & \text{otherwise} \end{cases}$$

$DUP \in NP$

b)

$$verify(UUD.algor, A) = \begin{cases} T & \text{if } isupdown(O) \wedge (sort(A) = sort(O)) \\ F & \text{otherwise} \end{cases}$$

$UUP \in NP$

c) $verify(CEU.algor, A) = CEU.algor(A) \oplus (NAS(sort(A)) = n-1)$

where $O = CEU.algor(A)$ and $n = |A|$

$CEU \in NP$

d)

$$verify(AVLconst.algor, A) = \begin{cases} T & isAVL(T) \wedge (DFTinorder(T) = sort(A)) \\ F & \text{otherwise} \end{cases}$$

where $T = AVLconst.algor(A)$

$AVLc \in NP$

e) $CVH \in NP$

Q11.3:-

a) Yes, $(x \vee \neg y \vee z) \wedge (x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee \neg z)$ is satisfiable when $(x = T), (y = T)$ and $(z = F)$

$$(T \vee F \vee F) \wedge (T \vee T \vee T) \wedge (F \vee F \vee T) \equiv T$$

Q11.4:- a) No.

Q11.5:-

a) Yes, $S_r = p \vee \neg(q \vee r)$ is satisfiable when $p = T$, S_r is T

Q11.10 :-

a) A purported answer for the SPE_p can be verified in polynomial time $O(|S'|)$.
Hence, $SPE_p \in NP$.

b) $SSE \leq_p SPE_p \longleftrightarrow SSE(A,m) = \log(SPE_p(A', 2^m))$
where, $\forall i \in \{1, \ldots, n\}, a'_i = 2^{a_i}$

$SPE_p \leq SSE \longleftrightarrow SPE_p(A,m) = 2^{SSE(A', \log m)}$
where $\forall i \in \{1, \ldots, n\}, a'_i = \log a_i$

Hence, $SSE \equiv_p SPE_p$.

c) Since $SPE_p \in NP$ and $SPE_p \in NP$-hard by $SSE \leq_p SPE_p$, $SPE_p \in NP$-complete.

d) $SPE_p \leq_p SPM_p \longleftrightarrow SPE_p(A,m) = \begin{cases} S = SPM_p(A,m) & \text{if } \prod_{x \in S} x = m \\ \text{no} & \text{otherwise} \end{cases}$

e) Input :- A set S of n positive numbers, a target m and a lower bound l

Output :- $\begin{cases} \text{True and/or } S' & \text{if } \exists S' \subseteq S \text{ such that } l \leq \prod_{i=1}^{|S'|} s'_i \leq m \\ \text{False} & \text{otherwise} \end{cases}$

f) $SPM_p^{dv} \in NP$

$SPE_p \leq_p SPM_p^{dv} \longleftrightarrow SPE_p(A,m) = SPM_p^{dv}(A,m,m)$

Since, $SPM_p^{dv} \in NP$ and $SPM_p^{dv} \in NP$-hard, $SPM_p^{dv} \in NP$-complete

g) $SPE_p \leq_p SPMin_p \longleftrightarrow SPE_p(A,m) = \begin{cases} S = SP_{min_p}(A,m) & \text{if } \prod_{x \in S} x = m \\ \text{no} & \text{otherwise.} \end{cases}$

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h) Input :- A set S of n positive nos, a target m, and an upper bound u

Output :- $\begin{cases} S' & \text{if } \exists S' \subseteq S \text{ such that } m \le \prod_{i=1}^{|S'|} s_i \le u \\ \text{False} & \text{otherwise} \end{cases}$

Hence, $SP_{minp} \in NP$

i) $SP_{minp}^{dv} \in NP$ & $SPE_p \le_p SP_{minp}$. Thus, $SP_{minp}^{dv} \in$ NP-hard
$\therefore$ $SP_{minp}^{dv} \in$ NP-complete.

j) $SPM_p(A,m) = SP_{minp}(A', 1/m) \longleftrightarrow SPM_p \le_p SP_{minp}$
$SP_{minp}(A,m) = SPM_p(A', 1/m) \longleftrightarrow SP_{minp} \le_p SPM_p$
where, $\forall i = 1 \to n$, $a_i' = 1/a_i$

k) $SPM_p^{dv}(A,m,l) = SP_{minp}^{dv}(A,l,m) \longleftrightarrow SPM_p^{dv} \le_p SP_{minp}^{dv}$
$SP_{minp}^{dv}(A,m,u) = SPM_p^{dv}(A,u,m) \longleftrightarrow SP_{minp}^{dv} \le_p SPM_p^{dv}$

l) $SSM \le_p SPM_p \longleftrightarrow SSM(A,m) = \log(SPM_p(A', 2^m))$
where, $\forall i \in \{1, \dots, n\}$, $a_i' = 2^{a_i}$
$SPM_p \le_p SSM \longleftrightarrow SPM_p(A,m) = 2^{SSM(A', \log m)}$
where, $\forall i \in \{1, \dots, n\}$, $a_i' = \log a_i$
Hence, $SSM \equiv_p SPM_p$.

m) $SS_{min} \le_p SP_{minp} \longleftrightarrow SS_{min}(A,m) = \log(SP_{minp}(A', 2^m))$
where $\forall i \in \{1, \dots, n\}$, $a_i' = 2^{a_i}$
$SP_{minp} \le_p SS_{min} \longleftrightarrow SP_{minp}(A,m) = 2^{SS_{min}(A', \log m)}$
where $\forall i \in \{1, \dots, n\}$, $a_i' = \log a_i$
Hence, $SS_{min} \equiv_p SP_{minp}$.

Q11.11:-

a) $SS_{min} \le_p ZOK_{min}$, $SS_{min}^{dv} \le_p SS_{min}$ where $SSM^{dv} \in$ NP-complete
Hence,
$ZOK_{min} \in$ NP-hard.

b) Input :- A set $A=(C,W)$ of n pair numbers (c_i, w_i)

Output :- $\begin{cases} A' & \text{if } \exists A' \subseteq A \text{ such that } \sum_{i=1}^{|A'|} c_i \leq u \;\&\; \sum_{i=1}^{|A'|} w_i \geq m \\ \text{False} & \text{otherwise} \end{cases}$

c) $ZOK_{min}^{dv} \in NP$

SSE $\in$ NP-complete & $SSE \leq_p ZOK_{min}^{dv}$

Since, $ZOK_{min}^{dv} \in NP$ & $ZOK_{min}^{dv} \in$ NP-hard,

$ZOK_{min}^{dv} \in$ NP-complete.

d) $ZOK \leq_p ZOK_{min}$ & $ZOK_{min} \leq_p ZOK$.

e) $ZOK^{dv} \leq_p ZOK_{min}^{dv}$ & $ZOK_{min}^{dv} \leq_p ZOK^{dv}$.

Q11.12:-

a) $SSE \leq_p ZOKE$. Hence $ZOKE \in$ NP-hard

b) Input :- A set $A=(P,W)$ of n pair numbers (p_i, w_i)

Output :- $\begin{cases} \text{True} & \text{if } \exists A' \subseteq A \text{ such that } l \leq \sum_{i=1}^{|A'|} p_i \;\&\; \sum_{i=1}^{|A'|} w_i = m \\ \text{False} & \text{otherwise} \end{cases}$

c) $ZOKE^{dv} \in NP$

SSE $\in$ NP-complete & $SSE \leq_p ZOKE^{dv}$

$\therefore ZOKE^{dv} \in$ NP-complete.

d) $SSE \leq_p ZOKE_{min}$. Hence, $ZOKE_{min} \in$ NP-hard

e) Input :- A set $A=(C,W)$ of n pair numbers (c_i, w_i)

Output :- $\begin{cases} A' & \text{if } \exists A' \subseteq A \text{ such that } \sum_{i=1}^{|A'|} c_i \leq u \;\&\; \sum_{i=1}^{|A'|} w_i = m \\ \text{False} & \text{otherwise} \end{cases}$

f) $ZOKE_{min}^{dv} \in NP$.
$SSE \in$ NP-complete & $SSE \leq_p ZOKE_{min}^{dv}$, $ZOKE_{min}^{dv} \in$ NP-hard
$\therefore ZOKE_{min}^{dv} \in$ NP-complete.

g) $ZOKE \leq_p ZOKE_{min}$ & $ZOKE_{min} \leq_p ZOKE$

h) $ZOKE^{dv} \leq_p ZOKE_{min}^{dv}$ & $ZOKE_{min}^{dv} \leq_p ZOKE^{dv}$.

Q11.14:-

a) $USSM \in$ NP-hard $\because USSE \leq_p USSM$

b) Input :- a set A of n numbers, a target number m & lower bound l

Output :- $\begin{cases} T & \text{if } \exists X \text{ such that } l \leq \sum_{i=1}^{n} x_i a_i \leq m \text{ where } 0 \leq x_i \text{ integers} \\ F & \text{otherwise} \end{cases}$

c) $USSM^{dv} \in NP$.
$USSE \in$ NP-complete & $USSE \leq_p USSM^{dv}$. $USSM^{dv} \in$ NP-hard
$\therefore USSM^{dv} \in$ NP-complete.

d) $USS_{min} \in$ NP-hard $\because USSE \leq_p USS_{min}$.

e) Input :- a set A of n nos., a target number m & upper bound u.

Output :- $\begin{cases} X & \text{if } \exists X \text{ such that } m \leq \sum_{i=1}^{n} x_i a_i \leq u \text{ where } 0 \leq x_i \text{ integers.} \\ \text{False} & \text{otherwise} \end{cases}$

f) $USS_{min}^{dv} \in NP$
$USSE \in$ NP-complete & $USSE \leq_p USS_{min}^{dv}$, $USS_{min}^{dv} \in$ NP-hard.

$\therefore USS_{min}^{dv} \in$ NP-complete.

g) $USSM \le_p USS_{min}$ & $USS_{min} \le_p USSM$

h) $USSM^{dv} \le_p USS_{min}^{dv}$ & $USS_{min}^{dv} \le_p USSM^{dv}$

Q 11.15:-

a) $USSE \le_p USPE$ & $USPE \le_p USSE$

b) $USSE \equiv_p USPE$

c) Input :- A set P of K positive numbers & $n \in R^+$
Output :- $\prod_{i=1}^{n} p_i x_i$ such that

$$\text{maximize } \prod_{i=1}^{n} p_i x_i$$

$$\text{subject to } \prod_{i=1}^{n} p_i x_i \le m$$

where $0 \le x_i$ integer.

d) $USPM \in$ NP-hard $\because USPE \le_p USPM$.

e) $USSM \le_p USPM$ & $USPM \le_p USSM$

f) Input :- a set P of n positive numbers, a target no. m & a lower bound l.

Output :- $\begin{cases} \text{True} & \text{if } \exists X \text{ such that } l \le \prod_{i=1}^{n} p_i x_i \le m \text{ where } 0 \le x_i \text{ integer} \\ \text{False} & \text{otherwise} \end{cases}$

g) $USPM^{dv} \in NP$ & NP-hard. $\therefore USPM^{dv} \in$ NP-complete.

h) $USSM^{dv}(A, m) = USPM^{dv}(A', 2^m)$
$USPM^{dv}(A, m) = USSM^{dv}(A', \log m)$

$\therefore USSM^{dv} \equiv_p USPM^{dv}$

Q11.16:-

a) Input :- A set P of K positive numbers & $n \in R^+$

Output :- $\prod_{i=1}^{n} p_i x_i$ such that

$$\text{minimize } \prod_{i=1}^{n} p_i x_i$$

$$\text{subject to } \prod_{i=1}^{n} p_i x_i \geq m$$

$$\text{where } 0 \leq x_i \text{ integer.}$$

b) $USP_{min} \in NP\text{-hard} \because USPE \leq_p USP_{min}$

c) $USS_{min}(A, m) = \log USP_{min}(A', 2^m)$

$USP_{min}(A, m) = 2^{USS_{min}(A', \log m)}$

d) Input :- a set P of n +ve nos., a target number m & an upper bound l

Output :- $\begin{cases} \prod_{i=1}^{n} p_i x_i & \text{if } \exists X \text{ such that } m \leq \prod_{i=1}^{n} p_i x_i \leq u \text{ where } 0 \leq x_i \text{ integer} \\ \text{False} & \text{otherwise} \end{cases}$

e) $USP_{min}^{dv} \in NP$ & NP-hard. $\therefore USP_{min}^{dv} \in$ NP-complete

f) $USS_{min}^{dv}(A, m) = USP_{min}^{dv}(A', 2^m)$

$USS_{min}^{dv}(A, m) = USS_{min}^{dv}(A', \log m)$

g) $USPM \leq_p USP_{min}$ & $USP_{min} \leq USPM$

h) $USPM^{dv} \leq_p USP_{min}^{dv}$ & $USP_{min}^{dv} \leq USPM^{dv}$.

Q11.17:-

a) $UKP \in NP\text{-hard} \because USSM \leq_p UKP$ & $USSM \in NP\text{-hard}$

$$USSM \leq_p UKP.$$

b) Input :- A list A of n different items.

Output :- $\begin{cases} \sum_{i=1}^{n} x_i p_i & \text{if } \exists X \text{ such that } \sum_{i=1}^{n} x_i w_i \leq m \land \sum_{i=1}^{n} x_i p_i \geq l \text{ where} \\ \text{False} & \text{otherwise} \qquad 0 \leq x_i \text{ integer} \end{cases}$

c) $UKP^{dv} \in NP$ & NP-hard $\therefore UKP^{dv} \in$ NP-complete.

d) $UKP_{min} \in$ NP-hard

$USS_{min} \in$ NP-hard & $USS_{min} \leq_p UKP_{min}$.

$USS_{min} \leq_p UKP_{min}$.

e) Input :- a list A of n different items.

Output :- $\begin{cases} \sum_{i=1}^{n} x_i c_i & \text{if } \exists X \text{ such that } \sum_{i=1}^{n} x_i w_i \geq m \land \sum_{i=1}^{n} x_i c_i \leq u \\ \text{False} & \text{otherwise} \end{cases}$

f) $UKP_{min}^{dv} \in NP$ & NP-hard $\therefore UKP_{min}^{dv} \in$ NP-complete.

g) $UKP \leq_p UKP_{min}$ & $UKP_{min} \leq_p UKP$

h) $UKP^{dv} \leq_p UKP_{min}^{dv}$ & $UKP_{min}^{dv} \leq_p UKP^{dv}$.

Q11.18 :-

a) $USSE \in$ NP-complete. $USSE \leq_p UKE$

b) Input :- a list A of n different items.

Output :- $\begin{cases} \text{True} & \text{if } \exists X \text{ such that } \sum_{i=1}^{n} x_i w_i = m \land \sum_{i=1}^{n} x_i p_i \geq l \text{ where} \\ \text{False} & \text{otherwise} \qquad 0 \leq x_i \text{ integer} \end{cases}$

c) $UKE^{dv} \in NP$ & NP-hard $\therefore UKE^{dv} \in$ NP-complete

d) $USSE(A, m) = \begin{cases} T & \text{if } UKE_{min}((A, A), m, m) = m \\ F & \text{otherwise} \end{cases} \longleftrightarrow USSE \leq_p UKE_{min}$.

e) Input:- a list A of n items.

Output:- $\begin{cases} \text{True} & \text{if } \exists X \text{ such that } \sum_{i=1}^{n} x_i w_i = m \land \sum_{i=1}^{n} x_i c_i \leq u \text{ where } 0 \leq x_i \text{ integer} \\ \text{False} & \text{otherwise} \end{cases}$

f) $UKE_{min}^{dv} \in NP$ & NP-hard. $\therefore UKE_{min}^{dv} \in$ NP-complete

g) $UKE \leq_p UKE_{min}$ & $UKE_{min} \leq UKE$

h) $UKE^{dv} \leq_p UKE_{min}^{dv}$ & $UKE_{min}^{dv} \leq UKE^{dv}$