HW-10

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Q10.1:-

a) findmin($A_{1\sim n}$) = kthsmall($A_{1\sim n}$, 1) $\longleftrightarrow$ MIN $\leq_p$ KSM

b) findmax($A_{1\sim n}$) = kthsmall($A_{1\sim n}$, n) $\longleftrightarrow$ MAX $\leq_p$ KSM

c) Input :- A seq. $A_{1\sim n}$ of n quantifiable elements
Output :- $\frac{1}{2}(a'_{\lfloor \frac{n}{2} \rfloor} + a'_{\lceil \frac{n}{2} \rceil})$ where $A'_{1\sim n}$ is the sorted list of A in asc order

Algorithm:-

median($A_{1\sim n}$)
  $A'_{1\sim n}$ = sort($A_{1\sim n}$, asc)
  if n is odd, return $a'_{(\frac{n+1}{2})}$
  else, return $(a'_{\frac{n}{2}} + a'_{\frac{n}{2}+1})/2$

d)
$$\text{median}(A_{1\sim n}) = \begin{cases} KSM(A_{1\sim n}, \frac{n+1}{2}) & \text{if } n \text{ is odd} \\ \frac{1}{2}(KSM(A_{1\sim n}, n/2)) + KSM(A_{1\sim n}, \frac{n}{2}+1) & \text{otherwise} \end{cases}$$

e)
$$\text{median}(A_{1\sim n}) = \begin{cases} KLG(A_{1\sim n}, \frac{n+1}{2}) & \text{if } n \text{ is odd} \\ \frac{1}{2}(KLG(A_{1\sim n}, n/2) + KLG(A_{1\sim n}, \frac{n}{2}+1) & \text{otherwise} \end{cases}$$

Q10.2:-

a) $SKSS_{min} \leq_p SKSS \longleftrightarrow SKSS_{min}(A_{1\sim n}, K) = -1 \times SKSS(A'_{1\sim n}, K)$
where $\forall i \in \{1, \ldots, n\}, (a'_i \in A') = -1 \times (a_i \in A)$

$SKSS_{min}$_rd($A_{1\sim n}$, K)
  for i = 1 ~ n
    $a_i = -1 \times a_i$
  ans = SKSS_algo($A'_{1\sim n}$, K)
  return $-1 \times$ ans.

b) First by negation technique

$SKSS \leq_p SKSS_{min} \longleftrightarrow SKSS(A_{1\sim n}) = -1 \times SKSS_{min}(A'_{1\sim n})$
where $\forall i \in \{1,\dots,n\}, (a'_i \in A') = -1 \times (a_i \in A)$

Second by complement technique

$SKSS \leq_p SKSS_{min} \longleftrightarrow SKSS(A_{1\sim n}, K) = \sum_{i=1}^{n} a_i - SKSS_{min}(A_{1\sim n}, n-K)$

c) SKSP.rd$(A_{1\sim n}, K)$
for $i = 1 \sim n$
$a_i = \log a_i$
ans = SKSS.algox$(A_{1\sim n}, K)$
return 2^{ans}

d) $SKSS \equiv_p SKSP$ because $SKSS \leq_p SKSP$ & $SKSP \leq_p SKSS$
$SKSP(A, 3) = 2^4 = 16$
$SKSS(A', 3) = \log 16 = 4$

e) SKSPmin.rd$(A_{1\sim n}, K)$
for $i = 1 \sim n$
$a_i = 1/a_i$
ans = SKSP.algox$(A_{1\sim n}, K)$
return 1/ans

f) $SKSP(A, 3) = 1/0.04 = 25.0$ $SKSP_{min}(A', 3) = 1/25 = 0.04$

g) SKSPmin.rd$(A_{1\sim n}, K)$
for $i = 1 \sim n$
$a_i = \log a_i$
ans = SKSSmin.algox$(A_{1\sim n}, K)$
return 2^{ans}

h) $SKSP_{min}(A,3) = 2^{-2} = 0.25$

$SKSS_{min}(A',3) = \log 0.25 = -2$

Q10.3:-

a) NAS.rd($A_{1\sim n}$)
    for $i = 1 \sim n$
        $a_i = -1 \times a_i$
    return NDS.algox($A_{1\sim n}$)

b) $NDS(A) = NAS(A') = 4$
&
$NDS(A') = NAS(A) = 3$

c) $NAS(A) = \begin{cases} 0 & \text{if } n = 0 \text{ or } 1 \\ n-1-NDS(A) & \text{if } n > 1 \end{cases} \Longleftrightarrow NAS \le_p NDS$

$NAS(A) = \begin{cases} 0 & \text{if } n = 0 \text{ or } 1 \\ n-1-NAS(A) & \text{if } n > 1 \end{cases} \Longleftrightarrow NDS \le_p NAS$

d) $NDS(A_{1\sim n}) = n-1-NAS(A_{1\sim n}) = 8-1-4 = 3$

Q10.4:-

a) downup($A_{1\sim n}$)
    $A' =$ sort($A_{1\sim n}$, asc)
    $p = \lceil n/2 \rceil$
    for $i = 1$ to $\lfloor \frac{n}{2} \rfloor$
        $o_{2i-1} = a'_{p-i+1}$
        $o_{2i} = a'_{p+i}$
    if n is odd, $o_n = a'_1$
    return $O_{1\sim n}$

b) updown($A_{1\sim n}$)

$A'_{1\sim n}$ = sort($A_{1\sim n}$, asc)

for i = 1 to $\lfloor n/2 \rfloor$

$O_{2i-1} = a'_i$

$O_{2i} = a'_{i+\lfloor n/2 \rfloor}$

if n is odd, $O_n = a'_n$

return $O_{1\sim n}$

d) updown($A_{1\sim n}$)

$A'_{1\sim n}$ = sort($A_{1\sim n}$, asc)

$O_1 = a'_1$

for i = 1 to $\lceil n/2 \rceil - 1$

$O_{2i} = a'_{2i+1}$

$O_{2i+1} = a'_{2i}$

if n is even, $O_n = a'_n$

return $O_{1\sim n}$

Q10.5:-

a) downup($A_{1\sim n}$)

$A'_{1\sim n}$ = sort($A_{1\sim n}$, desc)

for i = 1 ~ $\lceil n/2 \rceil - 1$

swap($a'_{2i} = a'_{2i+1}$)

return $A'_{1\sim n}$

b)

10	9	8	7	6	5	4	3	2	1

10	8	9	6	7	4	5	2	3	1

c) O(nlogn)

d) downup($A_{1\sim n}$)

$A_{2\sim n}$ = updown($A_{2\sim n}$)

if $a_1 < a_2$, swap(a_1, a_2)

return $A_{1\sim n}$

f) upupdown($A_{1\sim n}$)

A' = sort($A_{1\sim n}$, asc)

$r = n \% 3$

for $i = 1$ to $\lfloor n/3 \rfloor$

$a_{3i-2} = a'_{r+i}$

$a_{3i-1} = a'_{\lceil n/3 \rceil r + i}$

$a_{3i} = a'_{2\lceil n/3 \rceil r + i}$

if $r = 1$, $a_n = a'_1$

if $r = 2$

$a_{n-1} = a'_1$

$a_n = a'_2$

return $a_{1\sim n}$

g) $n \% 3 = 0$

$n \% 3 = 1$

&

$n \% 3 = 2$

Q10.6:

a)

A	a_7	a_1	a_4	a_2	a_6	a_3	a_5
C	\$189K	\$566K	\$145K	\$300K	\$450K	\$620K	\$800K
Q	7000	20000	5000	10000	15000	20000	25000
U	\$27	\$28	\$29	\$30	\$30	\$31	\$32
X	1	1	1	1	8/15	0	0

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b) FKPmin$(A_{1\sim n}, m)$

$X = \text{FKP.algo}(A_{1\sim n}, \sum_{i=1}^{n} q_i - m)$

return $1 - x_i$ for $i = 1 \sim n$

c) 52000 stocks can be obtained with a max. \$1670K. Now return the total stock price subtracted by \$1670K, which is \$1434K.

e) FKP$(A_{1\sim n}, m)$

$X = \text{FKPmin.algo}(A_{1\sim n}, \sum_{i=1}^{n} q_i - m)$

return $1 - x$, for $i = 1 \sim n$

Q10.21:-

a) Proof:-

$$\binom{n+1}{2} = \frac{(n+1)!}{2(n-1)!} = \frac{(n+1)(n)\cancel{(n-1)!}}{2\cancel{(n-1)!}} = \frac{n(n+1)}{2} = TRN(n)$$

b) Proof:-

$$\binom{n+2}{3} = \frac{(n+2)!}{3!(n-1)!} = \frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{3!\cancel{(n-1)!}} = \frac{n(n+1)(n+2)}{6}$$

$$= THN(n)$$

c) Proof:-

$$\frac{1}{4}\binom{2n+2}{3} = \frac{1}{4}\left(\frac{(2n+2)!}{3!(2n-1)!}\right) = \frac{1}{4}\left(\frac{(2n+2)(2n+1)(2n)\cancel{(2n-1)!}}{6\cancel{(2n-1)!}}\right)$$

$$= \frac{4(n+1)(2n+1)n}{\cancel{(4)}(6)}$$

$$= \frac{n(n+1)(2n+1)}{6} = PRN(n)$$

d) Proof:-

$$\binom{n+2}{3}+\binom{n+1}{3} = \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{6\cancel{(n-1)!}} + \frac{(n+1)(n)(n-1)\cancel{(n-2)!}}{6\cancel{(n-2)!}}$$

$$= \frac{n(n+1)(2n+1)}{6} = PRN(n)$$

e) Proof:-

$$\binom{n+3}{4} = \frac{(n+3)!}{4!(n-1)!} = \frac{(n+3)(n+2)(n+1)(n)\cancel{(n-1)!}}{24\cancel{(n-1)!}}$$

$$= \frac{(n)(n+1)(n+2)(n+3)}{24} = STH(n)$$

f) Proof:-

$$\binom{n+1}{2}^2 = \left(\frac{n(n+1)}{2}\right)^2 = SCB(n)$$

g) $SCB(n) = TRN(n)^2$

$TRN(n) = \sqrt{SCB(n)}$

h) Proof:-

$SQN(n) = 2\,TRN(n) - n$

$$TRN(n) = \frac{SQN(n)+n}{2}$$

i) $SQN(n) = 2\binom{n+1}{2} - n \longleftrightarrow SQN \leq_p BNC$

j) Proof:-

$$SEN(n) = 2 \cdot TRN(n)$$

$$TRN(n) = \frac{SEN(n)}{2}$$

k) Show:-

$$SEN(n) = 2\binom{n+1}{2} \iff SEN \leq_p BNC$$

l) Proof:-

$$SEN(n) = SQN(n) + n$$
$$SQN(n) = SEN(n) - n$$

Since, $SQN \leq_p TRN$ & $TRN \leq_p SQN$, $SQN \equiv_p TRN$