

Q12.1:-

a)  $x=8, a_8=15$  Input: A, x

swap( $a_x, a_n$ ) &

initialize  $i=1$  &  $j=n-1$

increment  $i$  while  $a_i < a_n$  &

decrement  $j$  while  $a_j > a_n$

swap( $a_i, a_j$ )

increment  $i$  while  $a_i < a_n$  &

decrement  $j$  while  $a_j \geq a_n$

swap( $a_i, a_j$ )

increment  $i$  while  $a_i < a_n$  &

Stop since  $i=j$

swap( $a_i, a_n$ ) since  $a_i \geq a_n$

Output A' &  $x'=5$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

17	12	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

17	12	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

17	12	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

10	12	2	80	20	35	1	17	30	15
----	----	---	----	----	----	---	----	----	----

10	12	2	80	20	35	1	17	30	15
----	----	---	----	----	----	---	----	----	----

10	12	2	80	20	35	1	17	30	15
----	----	---	----	----	----	---	----	----	----

10	12	2	80	20	35	1	17	30	15
----	----	---	----	----	----	---	----	----	----

10	12	2	1	20	35	80	17	30	15
----	----	---	---	----	----	----	----	----	----

10	12	2	1	20	35	80	17	30	15
----	----	---	---	----	----	----	----	----	----

10	12	2	1	20	35	80	17	30	15
----	----	---	---	----	----	----	----	----	----

10	12	2	1	20	35	80	17	30	15
----	----	---	---	----	----	----	----	----	----

10	12	2	1	15	35	80	17	30	20
----	----	---	---	----	----	----	----	----	----

10	12	2	1	15	35	80	17	30	20
----	----	---	---	----	----	----	----	----	----

10	12	2	1	15	35	80	17	30	20
----	----	---	---	----	----	----	----	----	----

b)  $x=3, a_3=2$  Input: A, x

swap( $a_x, a_n$ ) &

initialize  $i=1$  &  $j=n-1$

increment  $i$  while  $a_i < a_n$  &

decrement  $j$  while  $a_j \geq a_n$

swap( $a_i, a_j$ )

increment  $i$  while  $a_i < a_n$  &

Stop since  $i=j$

swap( $a_i, a_n$ ) since  $a_i \geq a_n$

Output A' &  $x'=2$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

17	12	10	80	20	35	1	15	30	2
----	----	----	----	----	----	---	----	----	---

17	12	10	80	20	35	1	15	30	2
----	----	----	----	----	----	---	----	----	---

17	12	10	80	20	35	1	15	30	2
----	----	----	----	----	----	---	----	----	---

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----



c) Input:  $A, x$  [ $x=4, a_4=80$ ]

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

swap( $a_x, a_n$ ) &

17	12	2	10	20	35	1	15	30	80
----	----	---	----	----	----	---	----	----	----

i j ↑

initialize  $i=1$  &  $j=n-1$

increment  $i$  while  $a_i < a_n$  &

17	12	2	10	20	35	1	15	30	80
----	----	---	----	----	----	---	----	----	----

→  $i=j$  ↑

Stop since  $i=j$

swap( $a_{i+1}, a_n$ ) since  $a_i < a_n$

17	12	2	10	20	35	1	15	20	80
----	----	---	----	----	----	---	----	----	----

↑

Output  $A'$  &  $x' = 10$

d)  $x=7$  &  $a_7=1$  Input:  $A, x$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

swap( $a_x, a_n$ ) &

17	12	2	80	20	35	10	15	30	1
----	----	---	----	----	----	----	----	----	---

i j ↑

initialize  $i=1$  &  $j=n-1$

increment  $i$  while  $a_i < a_n$  &

17	12	2	80	20	35	10	15	30	1
----	----	---	----	----	----	----	----	----	---

↑

Stop since  $i=j$

swap( $a_i, a_n$ ) since  $a_i > a_n$

1	12	2	80	20	35	10	15	30	17
---	----	---	----	----	----	----	----	----	----

↑

Output  $A'$  &  $x' = 1$

Q12.3:-

a)  $x=8, a_8=15$ . Input:  $A, x$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

swap( $a_x, a_n$ ) &  $i = \text{the first } a_i$   
from left such that  $a_i \geq a_n$  &

17	12	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

i j ↑

increment  $j$  while  $a_j > a_n$

17	12	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

l → j ↑

swap( $a_i, a_j$ ) &

12	17	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

→ i j ↑

Increment  $i$  by 1

increment  $j$  while  $a_j \geq a_n$

12	17	2	80	20	35	1	10	30	15
----	----	---	----	----	----	---	----	----	----

i → j ↑

swap( $a_i, a_j$ ) &

12	2	17	80	20	35	1	10	30	15
----	---	----	----	----	----	---	----	----	----

→ i j ↑

Increment  $i$  by 1

increment  $j$  while  $a_j \geq a_n$

12	2	17	80	20	35	1	10	30	15
----	---	----	----	----	----	---	----	----	----

i → j ↑

swap( $a_i, a_j$ ) &

increment  $i$  by 1

increment  $j$  while  $a_j \geq a_n$

12	2	1	80	20	35	17	10	30	15
----	---	---	----	----	----	----	----	----	----

→  $i$

$j$

↑

12	2	1	80	20	35	17	10	30	15
----	---	---	----	----	----	----	----	----	----

$i$

→  $j$

↑

swap( $a_i, a_j$ ) &

increment  $i$  by 1

increment  $j$  while  $a_j \geq a_n$  &

12	2	1	10	20	35	17	80	30	15
----	---	---	----	----	----	----	----	----	----

→  $i$

$j$

↑

12	2	1	10	20	35	17	80	30	15
----	---	---	----	----	----	----	----	----	----

$i$

→  $j =$

↑

Stop since  $j = n$

swap( $a_i, a_n$ ) &

Output  $A'$  &  $x = 5$

12	2	1	10	15	35	17	80	30	20
----	---	---	----	----	----	----	----	----	----

↑

b)  $x=3$  &  $a_3=2$  Input:  $A, x$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

swap( $a_x, a_n$ ) &  $i$  = the first  $a_n$

from left such that  $a_n \geq a_n$  &

increment  $j$  while  $a_j \geq a_n$  &

17	12	10	80	20	35	1	15	30	2
----	----	----	----	----	----	---	----	----	---

$i$

$j$

↑

17	12	10	80	20	35	1	15	30	2
----	----	----	----	----	----	---	----	----	---

$i$

→  $j$

↑

swap( $a_i, a_j$ ) &

increment ( $i$ ) by 1

increment  $j$  while  $a_j \geq a_n$  &

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

→  $i$

$j$

↑

1	12	10	80	20	35	17	15	30	2
---	----	----	----	----	----	----	----	----	---

$i$

→  $j =$

↑

Stop since  $j = n$

swap( $a_i, a_n$ ) &

Output  $A'$  &  $x = 2$

1	2	10	80	20	35	17	15	30	12
---	---	----	----	----	----	----	----	----	----

↑

c)  $x=4$ ,  $a_4=80$  Input:  $A, x$

swap( $a_x, a_n$ ) &  $i$  = the first  $a_n$  from

left s.t.  $a_n \geq a_n$  & stop since  $i = n$

Output  $A'$  &  $x = 10$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

17	12	2	10	20	35	1	15	30	80
----	----	---	----	----	----	---	----	----	----

→  $j =$

↑

d)  $x=7$ ,  $a_7=1$  Input:  $A, x$

swap( $a_x, a_n$ ) &  $i$  = the first  $a_n$

from left such that  $a_n > a_n$  &  $j = i+1$

17	12	2	80	20	35	1	15	30	10
----	----	---	----	----	----	---	----	----	----

↑

⊕

17	12	2	80	20	35	10	15	30	1
----	----	---	----	----	----	----	----	----	---

$i$

$j$

↑

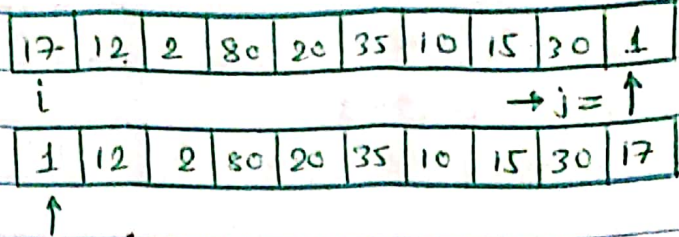


increment  $j$  while  $a_j \geq a_n$  &

Stop since  $j = n$

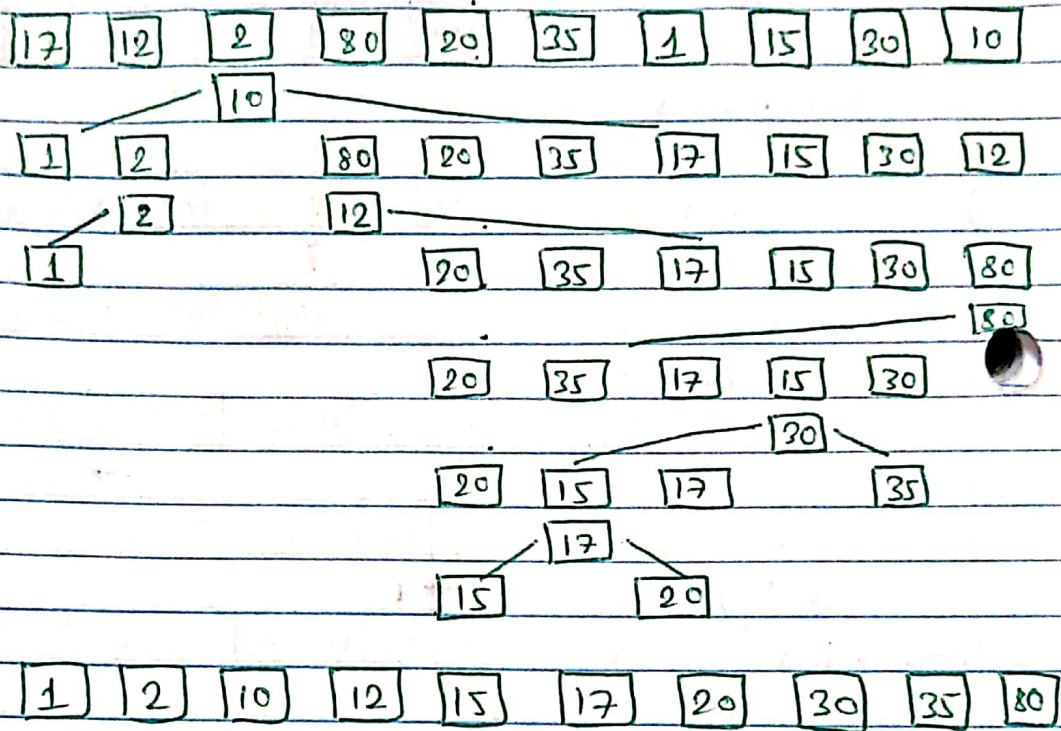
swap  $(a_i, a_n)$  &

Output  $A'$  &  $x = 1$

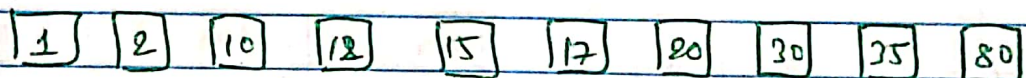


Q12.5:-

a) Last Pivot:-



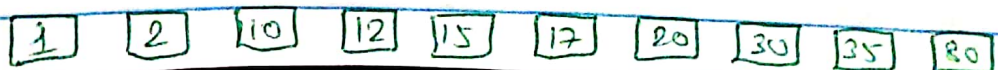
b) First Pivot:-



c) Last 3 Pivots:-



d) First 3 Pivots:-



Q12.6:-

a) Last Pivot :-

1 2 10 80 20 25 17 15 30 12

b) First Pivot :-

1 2 10 15 12 17 20 80 30 25

c) Last 3 Pivot :-

1 2 10 12 15 35 80 17 30 20

d) First 3 Pivot :-

1 2 10 12 20 35 17 15 30 80

Q12.10:-

a) quick.SKSM(s, e, k)

x = partition(A, s, e)

if  $x = n - k + 1$ , return sum(A[n-k+1:n])

if  $x < n - k + 1$ , quick.SKSM(x, e, k)

if  $x > n - k + 1$ , quick.SKSM(s, x-1, k)

b) Worst case =  $\Theta(n^2)$

Best case =  $\Theta(n)$

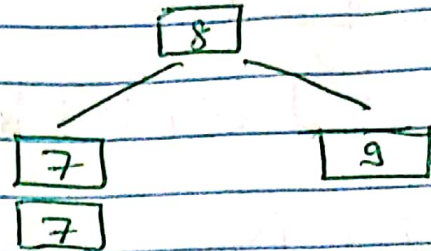
Average case =  $\Theta(n)$

c)  $7 + 8 + 9 = 24$

5 2 9 4 0 8 7 1  
5



1 2 0 4 8 7 9



1 2 0 4 5 7 8 9