Paper / Subject Code: 49371 / Engineering Mathematics-III

S.E. Computer Science & Engineering (Artificial Intelligence & Machine Learning) / IT (R-2019)
SEMESTER - III - Engineering Mathematics-III QP CODE: 10027291 DATE: 24/05/2023

(Time: 3 hours)   Max. Marks: 80

N.B.   (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $t\sqrt{1+\sin t}$ [5]

(b) Find the constants a, b ,c, d, e if [5]

$f(z) = (ax^3 + b\,x\,y^2 + 3\,x^2 + c\,y^2 + x) + i(d\,x^2\,y - 2\,y^3 + e\,x\,y + y)$ is analytic.

(c) Calculate the Spearman's rank correlation coefficient R [5]

X   :   85 ,   74 ,   85 ,   50,   65,   78,   74,   60,   74,   90

Y   :   78   91,   78,   58,   60,   72,   80,   55,   68,   70

(d)   Find inverse Laplace transform of $tan^{-1}\left(\frac{s+a}{b}\right)$. [5]

Q2.

(a) Find the Laplace transform of $e^{-4t}\int_0^t u\sin 3u\,du$ [6]

(b) find the value of k if the function $f(x) = k\,x^2\,(1-x^3)$, $0 \le x \le 1$.

$F(x) = 0$ otherwise

Is a probability density function. find mean and variance. [6]

(c) Obtain the Fourier series to represent f (x) = x^2 in (0, 2π)

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ ……. [8]

Q3.

(a)   Find the analytic function f (z) = u + iv such that [6]

u +v = $\frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$.

27291   **Page 1 of 2**

(b) Using convolution theorem Find inverse Laplace transform of $\dfrac{s^2}{(s^2+9)(s^2+4)}$. [6]

(c) Fit a second-degree parabolic curve to the following data

Year (x)   : 1974   1975   1976   1977   1978   1979   1980   1981

Production (y) : 12   14   26   42   40   50   52   53.   [8]

Q4.

 (a)   Obtain the   Fourier series to represent f (x) = 9 - x^2 in (-3, 3).   [6]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X :   78 ,   36 , 98 , 25 , 75, 82, 90, 62, 65, 39 .

Y :   84 , 51,   91, 60,   68, 62 , 86, 58, 53, 47.   [6]

(c) Prove that $\int_0^\infty e^{-t} \dfrac{\sin 2t + \sin 3t}{t}$ dt = $\dfrac{3\pi}{4}$.   [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves   3 x^2y + 2x^2 - y^3- 2 y^2 = c. [6]

(b)   If X denotes the outcome when a fair die is tossed, find Moment generating function

   Of X and hence find the mean and variance of X.   [6]

(c)   Obtain the half range cosine series of   f (x) = x ($\pi - x$) in (0, π)

 Hence show that $\dfrac{\pi^4}{96}$ $= \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4}$ ……..   [8]

Q6.(a) Find inverse Laplace transform of $\dfrac{s+29}{(s + 4)(s^2+9)}$.   [6]

(b)   The probability density function of a random variable X is

   X   : 0   1   2   3   4   5   6

  P (X =x) : k   3k   5k   7k   9k   11k   13k

  Find k , p (X < 4) , P (3 < X ≤ 6) .   [6]

(c)   Verify Laplace equation for u = $\left(r + \dfrac{a^2}{r} \right)$ cos θ .also find v and f (z) .   [8]

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