Image-to-Image Regression with Distribution-Free Uncertainty Quantification and Applications in Imaging

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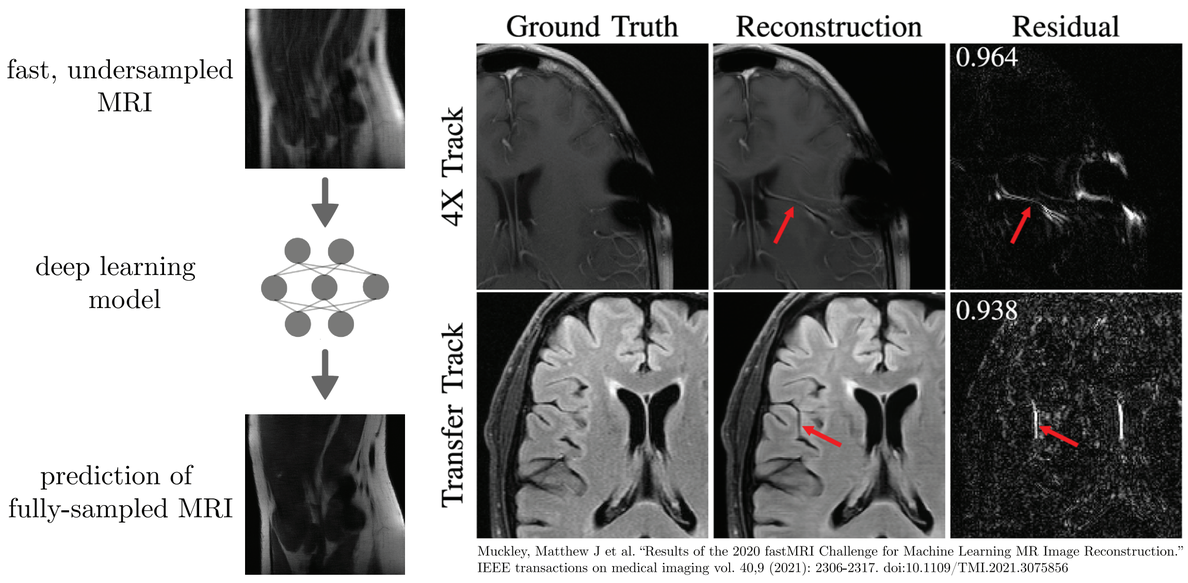
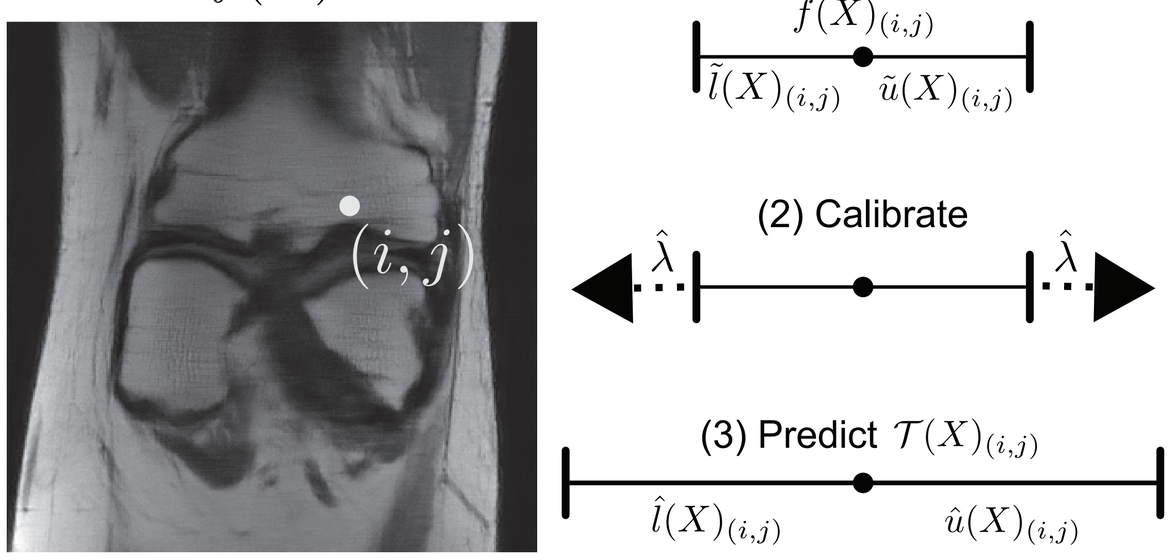
Deep learning advances imaging capabilities, but only if we can trust it!



Muckley, Matthew J et al. "Results of the 2020 fastMRI Challenge for Machine Learning MR Image Reconstruction." IEEE transactions on medical imaging vol. 40,9 (2021): 2306-2317. doi:10.1109/TMI.2021.3075856

This is where conformal helps: we can construct pixel-wise uncertainty intervals



Heuristics notions of uncertainty include:
- softmax scores
- per-pixel Gaussian variance
- magnitude of residual
- quantile estimates (via quantile regression)

Here are the details

Uncertainty Interval

$$\mathcal{T}_\lambda(X)_{(m,n)} = \left[\hat{f}(X)_{(m,n)} - \lambda\tilde{l}(X)_{(m,n)},\ \hat{f}(X)_{(m,n)} + \lambda\tilde{u}(X)_{(m,n)}\right]$$

Risk-control guarantee

$$\mathbb{P}\left(\mathbb{E}\left[L(\mathcal{T}(X), Y)\right] > \alpha\right) \leq \delta$$

$$L(\mathcal{T}(X), Y) = 1 - \frac{\left|\left\{(m,n) : Y_{(m,n)} \in \mathcal{T}(X)_{(m,n)}\right\}\right|}{MN}$$

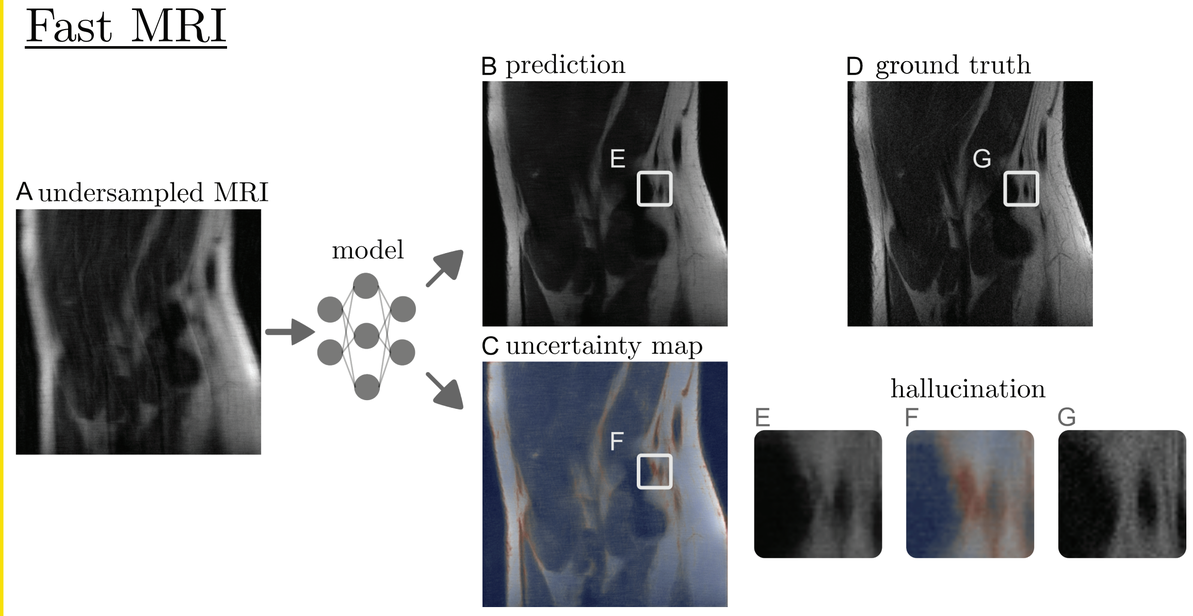
Algorithm 2 Pseudocode for computing $\hat{\lambda}$

Input: Calibration data, (X_i, Y_i), $i = 1, \ldots, n$; risk level α; error rate δ; underlying predictor $\hat{f}$; heuristic lower and upper interval lengths $\tilde{l}$ and $\tilde{u}$; maximum value $\lambda_{\max}$; step size $d\lambda > 0$.
Output: Parameter $\hat{\lambda}$ for computing RCPS.

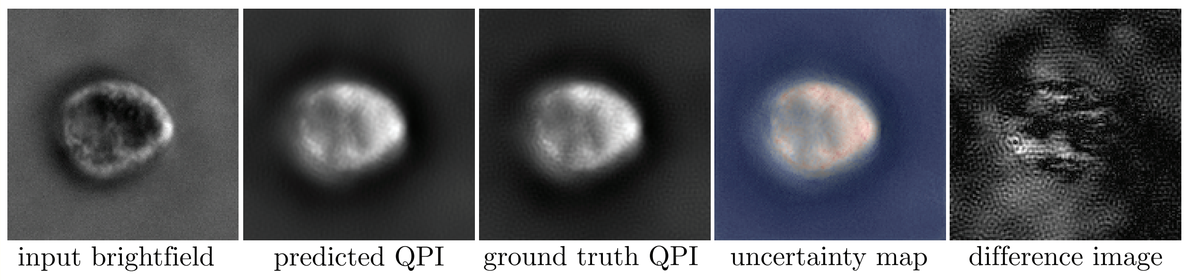
$\lambda \leftarrow \lambda_{\max}$
$r \leftarrow 1$
while $r \leq \alpha$ **do**
  $\lambda \leftarrow \lambda - d\lambda$
  for $i = 1, \ldots, n$ **do**
    $L_i \leftarrow L\left(\mathcal{T}_\lambda(X_i), Y_i\right)$
  $r \leftarrow \frac{1}{n}\sum_{i=1}^{n} L_i + \sqrt{\frac{1}{2n}\log\frac{1}{\delta}}$
$\hat{\lambda} \leftarrow \lambda + d\lambda$

Imaging applications can benefit from conformal

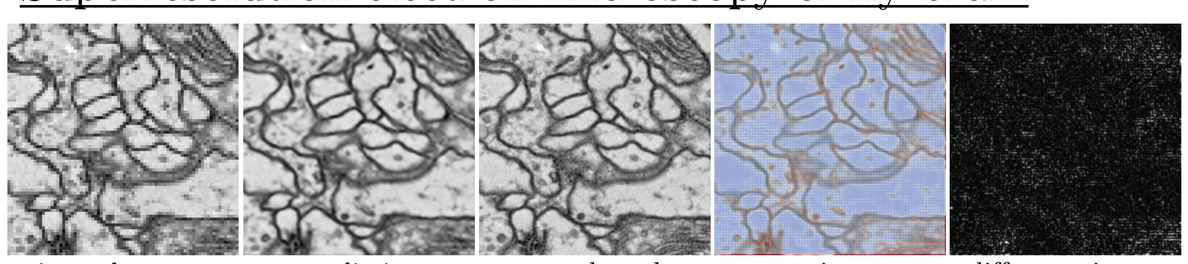
Fast MRI



Phase imaging of leukocytes

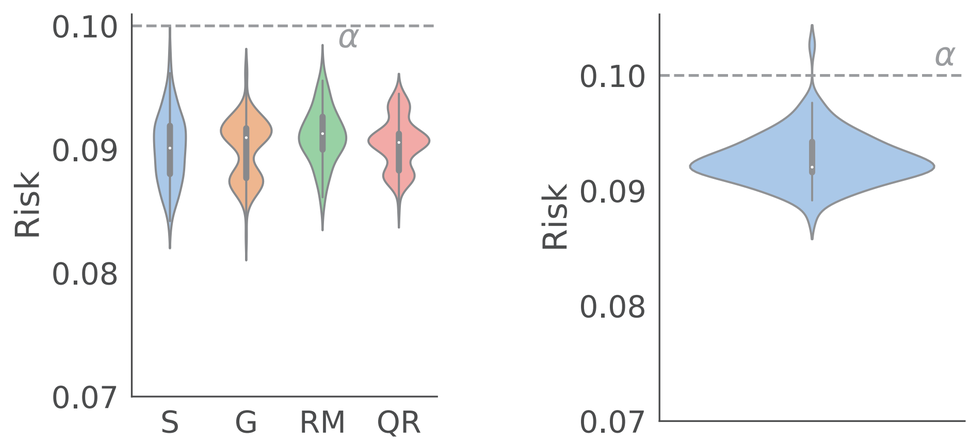


Superresolution electron microscopy of fly brain

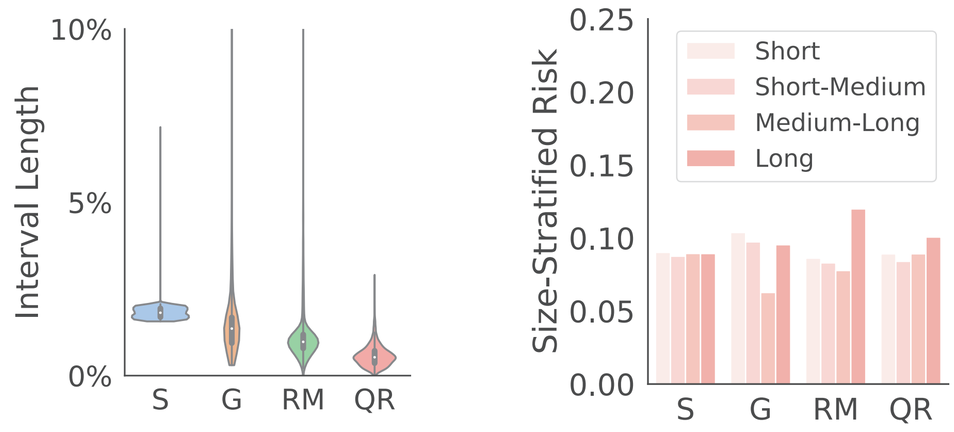


Quantitative evaluation

Risk control: make sure we got what we promised



Set sizes: are our intervals actually informative?



Prediction accuracy: we still want an accurate model!