Exam 3 Notes

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Chapter 15 - Multiple Integration

15.1 - Two-Variable Integration Over Rectangles

$$\left(\int_c^d \left(\int_a^b f(x,y)dx\right)dy\right)$$

$$f_{avg} = \frac{1}{A(R)}\iint_R f(x,y)\ dA$$

$$\iint_R f(x,y)\ dA \pm \iint_R g(x,y)\ dA = \iint_R f(x,y) \pm g(x,y)\ dA$$

$$\iint_R cf(x,y)\ dA = c\iint_R f(x,y)\ dA$$

$$\forall (x,y) \in \mathbb{R}^2,\quad f(x,y) \geq g(x,y) \implies \iint_R f(x,y)\ dA \geq \iint_R g(x,y)\ dA$$

$V = \iint_R f(x,y) - g(x,y)\ dA$ where $f(x,y)$ is top surface, $g(x,y)$ bottom

Fubini's Theorem: If f is continuous on the rectangle $R = \{(x,y) \mid [a,b] \times [c,d]\}$, then:

$$\iint_R f(x,y)\ dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy$$

Partial wrt y: $A(x) = \int_c^d f(x,y)dy \qquad (A(x)$ possibly in terms of $x)$

Partial wrt x: $\int_a^b A(x)dx\ = \int_a^b \int_c^d f(x,y)dydx$

15.2 - Double Integrals Over More General Regions

Vertically Simple (top, bottom terms of x)

$D = \{(x,y) \mid [a,b] \times [g_1(x), g_2(x)]\}$

If f is continuous on D, $\iint_R f(x,y)\ dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx$

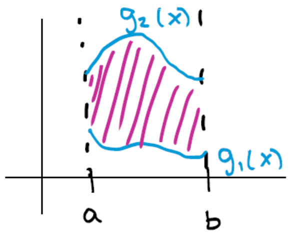
Type I Region: Vertically Simple

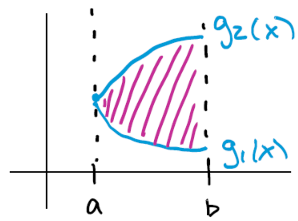
y is bounded by functions of x

Type I: (Top and Bottom in terms of x)

$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

examples





Horizontally Simple (left, right terms of y)

$D = \{(x, y) \mid [h_1(y), h_2(y)] \times [c, d]\}$

If f is continuous D, $\iint_D f(x,y)\ dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy$

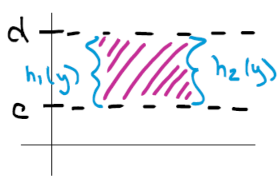
Type II Region: Horizontally Simple

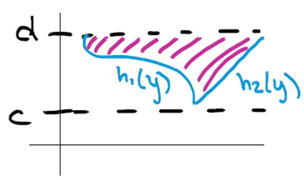
x is bounded by functions of y

Type II: (Left and Right in terms of y)

$$D = \{(x,y) \mid c \le y \le d,\ h_1(y) \le x \le h_2(y)\}$$

examples





With these regions, we are bounded on the left & right by functions of y!