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## Project: Part 1

24-677 Special Topics: Linear Control Systems

Prof. D. Zhao

**Due: Nov 3, 2020, 9:50 am. Submit within the deadline.**

- Your online version and its timestamp will be used for assessment.
- We will use [Gradescope](#) to grade. The link is on the panel of CANVAS. If you are confused about the tool, post your questions on Campuswire.
- Submit **your\_controller.py** to Gradescope under **Programming-P1** and your solutions in **.pdf** format to **Project-P1**. Insert the performance plot image in the **.pdf**. We will test **your\_controller.py** and manually check all answers.
- We will make extensive use of Webots, an open-source robotics simulation software, for this project. [Webots is available here for Windows, Mac, and Linux](#).
- For Python usage with Webots, please see [the Webots page on Python](#). Note that you may have to reinstall libraries like `numpy`, `matplotlib`, `scipy`, etc. for the environment you use Webots in.
- Please familiarize yourself with Webots documentation, specifically their [User Guide](#) and their [Webots for Automobiles section](#), if you encounter difficulties in setup or use. It will help to have a good understanding of the underlying tools that will be used in this assignment. To that end, completing at least [Tutorial 1](#) in the user guide is highly recommended.
- If you have issues with Webots that are beyond the scope of the documentation (e.g. the software runs too slow, crashes, or has other odd behavior), please let the TAs know via Campuswire. We will do our best to help.
- We advise you to start with the assignment early. All the submissions are to be done before the respective deadlines of each assignment. For information about the late days and scale of your Final Grade, refer to the Syllabus in Canvas.



$$\ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m} \left( \cos \delta \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right)$$

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}, s_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\ddot{x} = \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \longrightarrow \text{Longitudinal}$$

$$\ddot{\psi} = \frac{2l_f C_\alpha}{I_z} \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{2l_r C_\alpha}{I_z} \left( -\frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right)$$

$$\dot{s}_1 = A_1 s_1 + B_1 u \text{ and } \dot{s}_2 = A_2 s_2 + B_2 u$$

$$\dot{X} = \dot{x} \cos \psi - \dot{y} \sin \psi$$

$$\dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi$$

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} A_1 \end{bmatrix}_{(4 \times 4)} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}_{(4 \times 1)} + \begin{bmatrix} B_1 \end{bmatrix}_{(4 \times 2)} \begin{bmatrix} \delta \\ F \end{bmatrix}_{(2 \times 1)}$$

$$\ddot{x} = \dot{\psi} \dot{y} + \frac{1}{m}(F - fmg)$$

$$u_1 = \delta$$

$$u_2 = F$$

$$x_1 = x \quad \dot{x}_1 = x_2$$

$$x_2 = \dot{x} \quad \dot{x}_2 = \dot{\psi} \dot{y} - fg + \frac{1}{m} u_2$$

$$\dot{X} = \begin{bmatrix} x_2 \\ \dot{\psi} \dot{y} - fg + \frac{1}{m} u_2 \end{bmatrix}$$

$$\overline{x}_2 = 0,$$



$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

$$\dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} d \\ F \end{bmatrix}$$

$$x_1 = y$$

$$x_2 = \dot{y}$$

$$x_3 = \psi$$

$$x_4 = \dot{\psi}$$

$$u = \begin{bmatrix} d \\ F \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_4 \dot{x} + \frac{2C_n}{m} \left( \cos \delta \left( \delta - \frac{x_2 + l_f x_4}{\dot{x}} \right) - \frac{x_2 - l_r x_4}{\dot{x}} \right)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{2l_f C_n}{I_z} \left( \delta - \frac{x_2 + l_f x_4}{\dot{x}} \right) - \frac{2l_r C_n}{I_z} \left( -\frac{x_2 - l_r x_4}{\dot{x}} \right)$$

$$\text{E1: } x_2=0, x_4=0, \delta=0, (|\delta| \leq \pi/6)$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & \frac{-2C_n \left( \frac{1}{\dot{x}} + \frac{\cos \delta}{\dot{x}} \right)}{m} & 0 & \frac{2C_n \left( \frac{l_r}{\dot{x}} - \frac{l_f \cos \delta}{\dot{x}} \right)}{m} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_n(l_r - l_f)}{I_z \dot{x}} & 0 & -\frac{2C_n(l_f^2 + l_r^2)}{I_z \dot{x}} \end{bmatrix}$$



$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ \frac{m \left( x_a \left( \cos \delta - \sin \delta \left( \delta - \frac{x_2 + l_f x_4}{x} \right) \right) \right)}{m} & 0 \\ 0 & 0 \\ \frac{2 C_a l_f}{I_z} & 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-g c_n}{m \dot{x}} & 0 & \frac{2 C_a (l_r - l_f)}{m \dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2 C_a (l_r - l_f)}{I_z \dot{x}} & 0 & -\frac{2 C_a (l_f^2 + l_r^2)}{I_z \dot{x}} \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ \frac{2 C_a}{m} & 0 \\ 0 & 0 \\ \frac{2 C_a l_f}{I_z} & 0 \end{bmatrix}$$

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-g c_n}{m \dot{x}} & 0 & \frac{2 C_a (l_r - l_f)}{m \dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2 C_a (l_r - l_f)}{I_z \dot{x}} & 0 & -\frac{2 C_a (l_f^2 + l_r^2)}{I_z \dot{x}} \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ \frac{2 C_a}{m} & 0 \\ 0 & 0 \\ \frac{2 C_a l_f}{I_z} & 0 \end{bmatrix} \begin{Bmatrix} \delta \\ F \end{Bmatrix}$$

