Introduction to Credit Risk and Credit Risk Modelling:

Extracting Market Implied Probabilities of Default using Option Pricing Theory

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Introduction:

One of the most prominent procedures in order to successfully navigate the intricate landscape of financial markets, is the proper managing of Risk, abstractly defined as the inherent uncertainty and potential for different than expected outcomes associated with any decision or action. As with every aspect in life, financial decisions are based on our expectations about a distribution of future outcomes, and expectation, by default, is far from certainty. Risk, in the financial world, expresses the above dynamic. Given that it is an inherent and indispensable element within the fabric of modern finance, playing a pivotal role in shaping everything; from investment decisions and asset allocation, to trading strategies and market dynamics, it is crucial for the parties involved to correctly manage it.

Managing of Risk includes identification, measurement, monitoring and ultimately controlling of Risk and its different kinds. Risk can be found in a number of different functions and characteristics of the Financial System, with the basic categories being: Market Risk (potential losses in financial instruments or portfolios due to fluctuations in market factors such as Interest Rates, Currencies, Commodities, etc), Liquidity Risk (potential losses by not being able to quickly and cost-effectively convert an asset into cash without significantly affecting its price) and Credit Risk (potential losses due to the counterparty failing to meet their financial obligations). As financial systems become increasingly interconnected and complex, the recognition and proactive management of the above flash points, has become imperative for fostering stability and resilience, and history has shown that when Risk assessment and its practices are not in proper application, utterly devastating consequences might arise, not only for the immediate participants, but also for the economy as a whole (domino effect).

The greatest and most complex -according to many- of all is Credit Risk, which refers to the potential for financial loss arising from the failure of a counterparty to fulfil their contractual obligations, and, in general, mirrors the creditworthiness characteristics of the counterparty. As the concept of Credit is one of the key functional elements of the modern capitalist economies, and given the tremendous gigantism in complexity and size of the modern Credit Markets, being able to manage Credit Risk is more crucial than ever, for any corporation and market participant.

This report offers an introduction to a particular concept of Credit Risk management, the estimation of the Probability of Default of a corporation, through the presentation and practical implementation of the EDF KMV model, which is based on Robert Merton's famous approach on structural Credit Risk and further developed by Moody's Analytics. The above, estimates the Equity implied Probability of Default metric, which quantifies the likelihood that an entity defaults on its Debt obligations. It will be shown that the Probability of Default calculation is of utter importance, and one of the three basic components -alongside Exposure at Default (EAD) and Loss Given Default (LGD)- constituting the Credit Risk of an entity, instrument, or whole portfolio.

What is Credit and Credit Risk?

Credit (IOU) can be simply seen as a promise that a certain sum of money will be paid in the future. One of the most common forms of Credit is created when someone borrows cash from someone else, promising to pay it back in the future, plus some compensation for using it. That is called Debt. Debt is an Asset for the lender, and a Liability to the borrower, and its value is derived from the mutual faith in that promise. However, by definition, a promise doesn't conclude certainty about the obligation taking place.

***Note:** Keep in mind that although lending is the most common source of Credit Risk, it can also be found in several different cases of common as well as more complex transactions: Whenever a financial commitment is being made for the future, there is Credit Risk. For example, when you lose at a bet with your friend and promise to pay him out tomorrow, your friend is exposed to Credit Risk. When an investment bank enters into an Interest Rate Swap agreement with a counterparty, it is also exposed to Credit Risk.

There is the possibility, that the debtor fails to make their contractual obligations, thus not fulfilling the promise. That event is generally called Default. So, an obvious risk is created when an IOU transaction is being made. Credit Risk, namely the potential the debt issuer fails to meet their obligation and the loss associated with that event. Main sources of this kind of risk can be found in a variety of instruments incorporating Credit (liquid or not), such as bank loans, bonds, credit derivatives, structured products, etc. As a result, every major group of market participants is directly exposed to Credit Risk either willingly or unwillingly. However, not all exposure to credit risk is inherently detrimental: Types of Market Participants like banks exist for originating and managing Credit Risk. Others, like Hedge Funds (outright risk takers seeking only commensurate or alpha rates of return for the exposures, taking speculative and leveraged positions in complex Credit Derivatives, Hybrid products, distressed Debt, etc), might actually seek exposure to Credit Risk and consider the possibility of an entity defaulting as an opportunity to deploy, not loose, capital. The latter might sound controversial, but it is not. Modern Credit Markets function, among other purposes, as a mechanism of transferring Credit Risk between counterparties, depending on each participant's risk profile.* Credit Risk as is, does not constitute a problem as long as it is correctly assessed and measured, and thus incorporated into the instruments' valuation.

Credit Markets:

Over the last 30 years, Credit Markets witnessed a breakthrough of changes: more liquid, higher volumes, extreme complexity, globalized and highly interconnected with the world economy. For a long time, they had been an illiquid, private market consisting mostly of Banks, Savings Institutions and Credit Unions, issuing loans, and funding their Loan Book through customer deposits. All the above changed, with the gradual commercialization of the 1) concept of Securitization and the 2) introduction of Credit Derivatives. Securitization is an innovative process that converts a pool of Assets (claims) into a tradable interest-bearing security which is sold to investors. For example, consider a lending institution that wants to remove loans from its portfolio. Here is, briefly, the securitization process: 1) It pools them together and repackages into a separate portfolio, 2) the bank transfers that portfolio to an SPV**, 3) the SPV issues Bonds backed by the portfolio and sells them to investors, while the bank receives the proceedings of the Bonds' issuance, 4) the Cash Flow generated by the underlying portfolio is used to pay interest and principal to the security holders. The above process transforms illiquid assets with long-term cash flows into liquid and tradable securities that are being sold in the financial markets.

That way, Credit Risk is now transferable and institutional investors (Pension, Mutual Funds, Hedge Funds, Insurance companies) had now the ability to add that kind of Risk in their portfolio, through purchasing these Bonds. There are many types of such securities, some of which are complex, namely Asset Backed Securities (ABS), Collateralized Loan Obligations (CLOs) and Collateralized Bond Obligations (CBOs), as well as ABS CDOs (Securities backed by other Securities which are backed by Assets portfolios). The impressive future of these instruments is that they are divided into tranches of different seniorities (Senior, Mezzanine, Equity tranches), each of which representing a different Credit Risk level (thus offering different yield as a compensation). That way, there are examples of Bond Senior Tranches rated with a higher Credit Score than the Assets backing those Senior Tranches.

***Comment:** The complexity of modern Credit Markets can lead to painful accidents, as it was the case in the '08 GFC. However, according to the author's opinion, that's not due to their existence as is, but mostly due to periods of consistent ignorance in Risk Management proper practices by market participants, as a result of incentives distortion and extrapolation of favourable conditions in the future. E.g.: Securitization is a brilliant idea as it is. Rating the mezzanine trench of a mezz ABS CDO created from subprime mortgages as AAA, isn't.

****SPV (Special Purpose Vehicle)**: is a legal entity created solely for a specific and often temporary purpose. It is structured to isolate risk and is distinct from the originator of the assets.

That might seem counterintuitive, but it isn't, if we consider the concept of diversification: The (Credit) Risk of a portfolio of (Credit) Assets, is different than the sum of (Credit) Risk of each Asset comprising the portfolio, as it depends on the correlations of these assets' performances. The above concept, as empirical data from GFC has shown, has its limitations though*, especially in cases of re-securitization. Further analysis would be out of context for this Report.
As Credit took the form of tradable securities via the above instruments, Credit Derivatives came into play, as tools of managing (mitigating, transferring, or isolating) Credit Risk exposure. These instruments are mainly OTC and are usually paired with positions in Credit Instruments (Sovereign Bonds, Corporate Bonds, ABS, etc). For example, CDS (Credit Default Swap) is a derivative contract where the buyer makes periodic payments (premium) to the seller in exchange for protection against a credit event on the underlying Asset. That way someone that has a long position on the underlying asset, can offset Credit Risk (and isolate his exposure to other Risks, like the IR one). The premium paid (as a %), mirrors (implies) the Default Probability of the underlying asset (given that the CDO is liquid enough to be at fair value). Note that Credit Derivatives can be used for speculation too.* Remember, as with every Risk, the participant can choose the amount and direction of his exposure. The complexity involved on the above instruments as well as the use of extensive leverage (and mismatching maturity Balance Sheet), by some Market Participants to fund such positions, can create situations of incentives distortion which can lead to unstabilizing bubbles.

Credit Risk Management

Given the above, the need for proper management of the Credit Risk, as the process of assessing, mitigating, and monitoring the potential risks associated with Credit Transactions, is imminent. First and foremost, Credit Risk management, generally, can be seen as a two-scope procedure: From a bottom-up perspective, there is: 1. Credit Risk management of individual instruments or entities, and 2. there is the Portfolio Credit Risk management, where a portfolio of Credit Assets is constructed using optimization methods for targeted Expected Returns and Credit Risk attached. There is also another one important categorization, as per the Credit instruments applied. There is: 1. Counterparty Credit Risk: which mainly refers to the risk a counterparty in a derivative transaction may not fulfil its obligations (applied in OTC Derivatives where the is no central Clearing Mechanism) and 2. Credit (Default) Risk: focuses on the risk a borrower may fail to fulfil its financial obligations by defaulting on its debt (focused on Loans, Bonds). We will be focusing on the Credit Risk management on entities.

Moving on, it should be mentioned that Credit Risk management has a qualitative and a quantitative approach, regarding the Credit Assessment of an entity. The qualitative can be objective, non-systemized and decisions are the reflection of personal judgment. Classic credit analysis is seen as an expert system that relies above all, on the subjective judgment of trained professionals. Individuals are turned into experts over the course of their careers, gaining additional authority as they acquire experience and demonstrate skill. The Credit assessment analysis of the above approach incudes:

***Speculation example:** You are witnessing the early signs of a Black Swan event about to happen (like a Pandemic). You subsequently believe that Markets will enter a "Flight to Safety" mode, due to perceived corporate Credit Risk rising (=Credit Spreads rising). An idea would be to buy CDS contracts on High Yield Corporate Bonds. (remember: You do not possess the HY CBs, only the CDS). Supposing you are right, the market gets into "Flight to Safety" mode, and Corporate Bonds experience a sell-out, as Sovereign Bonds' yields free fall. The result is a spike at the Credit Spreads, which is mirrored in a substantially higher CDS Premium, which makes your CDS long position far in-the-money. Your PnL skyrockets. However, till that event happens, you keep paying CDS Premiums to the seller. That could wipe out your position even if you were right about the event (but misjudged the timing). (Note that the above is not the author's idea; it's a real trade played out in the outbreak of COVID19 by a well-known Hedge Fund).

****Incentives Distortion example:** in an environment where banks constantly securitize their loan portfolios, transferring Credit Risk away, why should they give attention to proper Credit Risk assessment when originating the loan? It is suggested for the reader to search for the term: NINJA loans.

1. Detailed examination of the financial statements (both historical and projections) of the borrower/ debt issuer. Analysis is being done regarding the financial health of the corporation, as well as their ability to be Profitable, produce Cash Flow, and keep financial leverage in reasonable levels.
2. Strategic Analysis of the company regarding its unique characteristics, competitive advantage, and general strategy (SWOT Analysis, etc.).
3. Industry & Macroeconomic Environment Analysis. Analysis of the direct and indirect environment the company operates. What are the industry's characteristics? In what cycle stage is the industry? (Mature, developing, declining?) What is the level of competition and are there any entry barriers? How is the entity positioned in the industry? Moving on, what is the current and expected Macroeconomic environment situation? In what stage of the economic cycle are we? What are the inflation and growth prospects?
4. Character Assessment of the borrower: Discussion with entity's management. How is the company being run? Through what principles and corporate governance practices? How confident is the management about future progress? What's their reputation "in the street"?

The above process is being applied to all stages of Credit Risk management (loan origination, underwriting monitoring), with a focus on the loan underwriting. Although subjective, it can capture relevant company idiosyncratic characteristics that a quantitative model could miss. The combination of both qualitative and quantitative analysis provides a more comprehensive understanding of a borrower's creditworthiness.
Quantitative Credit Risk Modelling is based in quantitative methods to model Default patterns and overall assess the Credit Risk associated with lending. At its core, it is comprised of 3 important metrics:

1. Exposure at Default: the Gross Exposure of the Creditor in case default happens. It is usually equal to the Face Value of the Debt.
2. 1 Year Probability of Default: The statistical likelihood that default happens in 1 years' time.
3. Loss Given Default: The expected losses in case default happens. Usually, when default happens, Creditors can recover a percentage of the amount (via the collateral, restructuring agreements, court orders, etc). So, there is an expected Recovery Rate (RR) as a percentage of the Exposure. Subsequently, LGD = 1 – RR.

If we multiple the three metrics EAD*PD*LGD, the result is the Expected Losses (EL) metric, which showcases the potential (expected) losses a Creditor might face as a consequence of entering a Credit transaction. Out of the three, the Default Probability is the most difficult -and most important- to assess, and there are many quantitative techniques which try to estimate it, including econometric ones (regression, logit/probit models), simulation models, optimization techniques, as well as hybrid systems. All the above, generally, try to explain historical PD patterns (dependent variable), using several independent variables. *The independent variables could be anything, from company financial statements data (Profitability, Liquidity, Capital Efficiency, CF generation), to hard or soft macroeconomic variables (like GDP Growth, Inflation, Yields, PMI. Economic Sentiment, etc). For example, Altman's Z-score is a widely used one, derived from discriminant analysis.**

***Note:** All these statistical analysis models are based on the intuition that the future will exhibit patterns and behaviours (cause-effect relationships) similar to those observed in the past. In other words, as Mark Twain once said, History never repeats itself, but it does often rhyme.
****Altman's Z-Score**: $Z_i = 1.2A + 1.4B + 3.3C + 0.6D + 1.0E$
A = Working Capital / Total Assets
B = Retained Earnings / Total Assets
C= EBIT / Total Assets
D = Market Value of Equity / Book Value of Total Liabilities
E = Sales/Total Assets
The weights and components of the Z-Score formula were derived through a statistical analysis of a sample of companies that went bankrupt and those that did not. If Z score is above 3, there is no substantial likelihood of default, if Z score is below 1.81, the company is in distress and default is more likely, and if Z score is in between the two values, the company is "in grey zone" and caution is advised.

Other models, include: Springate Model, Ohlson O-Score and Cambridge Model. The first two models use statistical techniques (discriminant analysis, ANOVA, etc), utilizing mostly financial ratios and accounting data to predict the likelihood of severe economic destress (PD variations). The last one (Cambridge Mode), utilizes a neural network, and has a "black box" approach. Last but not least, Banks, especially large ones, have built their own proprietary models (using relevant statistical methods) of estimating PD.

Structural Credit Risk Models

Structural Credit Risk Models constitute a sub-category of the quantitative approach and are based on the idea that the financial structure of a firm determines its likelihood of default. These models typically dynamically incorporate the firm's assets, liabilities, and the value of its equity, trying to model the Default Risk of an entity as a function of changes in the above. More specifically, they define a default event as the point at which the entity's Assets fall below a specified threshold relative to its liabilities, indicating potential financial distress. Subsequently, they provide with a framework which is based on core Financial Theory and considers the incentives dynamics among Equity Holders and Creditors. Structural Credit Risk models, view default not as an exogenous event but as an endogenous outcome driven by the interplay of Asset Value, Equity interests, and Debt. They are dynamic -as values change over time-, and they incorporate stochastic processes to model the evolution of the mentioned variables into the future. They are being used to assess Default Risk, and subsequently calculate Credit Spreads and Premia. The main difficulty can be found in assigning dynamics to unobserved values, like the firm's Asset Value.

Structural Credit Risk modelling first appeared conceptually when Robert C. Merton (on his famous 1974 paper "On the pricing of Corporate Debt: the Risk Structure of Interest Rates"), introduced option pricing theory as a framework to model the Credit Risk of a corporation, presenting the Equity Value, as a European Call Option over the Assets of the entity with one year Maturity, and subsequently utilizing Black-Scholes formula to model the above relationship. The above innovative approach introduces the use of both market data (which by nature are forward looking) and company fundamentals to extract the equity market implied Probability of Default for that specific corporation (by reverse engineering the BLS formula), mirroring the likelihood that it defaults on its Debt, over a time period of one year.

First and foremost, the Equity Holders/Creditors relationship, as well as the concept of Agency Cost should be briefly described, before diving into the model's intuition. As we know, a corporation can fundamentally be described as a mechanism that: 1. Raises Capital (funding) and invests it in Assets, and 2. Through the proper utilization of its Assets, the company tries to produce income from its operations, that is greater than the cost of using these Assets (Cost of Capital). In Corporate Finance, that is: ROIC>WACC*. That way the company produces Economic Value.

The company's funding base is comprised of two main sources: Equity, which represents ownership over the company (residual ownership in Assets, after Liabilities are fully covered) and Debt (Liabilities in general), which represent an obligation to repay a specific amount of money over time, and no ownership is implied. The proportion of Equity and Liabilities funding the Asset Side of the Balance Sheet, designates the Capital Structure of the corporation, as the fundamental accounting equation implies.

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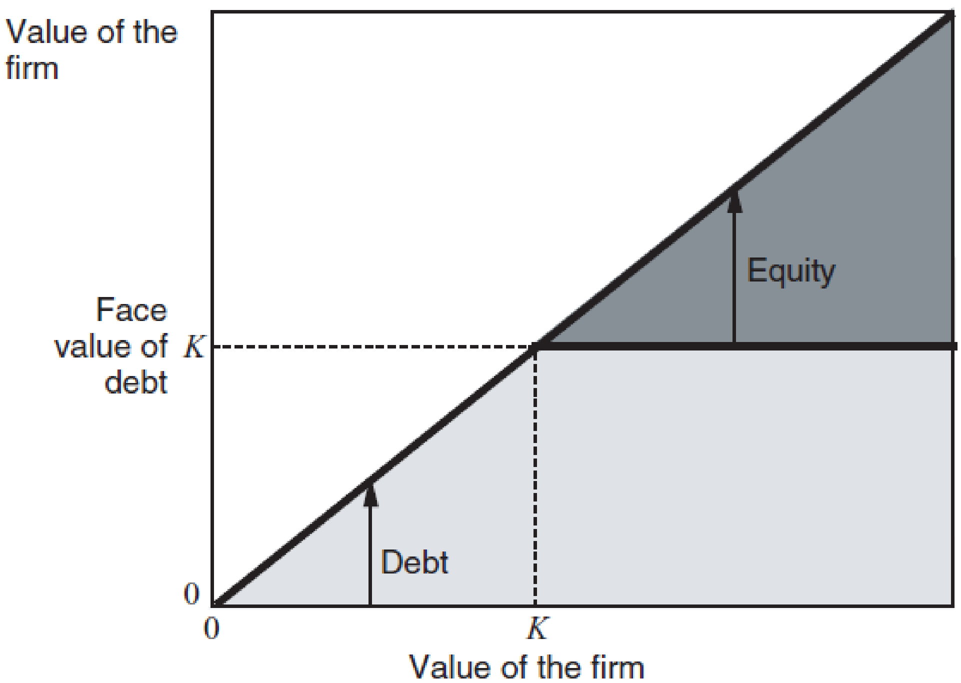
*ROIC (Return on Invested Capital) = NOPLAT/Invested Capital.
NOPLAT is a measure of operating profit (profit attributed to entity's capital providers), less adjusted taxes. (variation of EBIT)
Invested Capital: Assets amount that is being utilized in the company's operations. (=Net working capital+ Non-current Assets)

Equity holders, expect to be compensated via the Net Earnings of the company (either as dividend, or as capital appreciation); that is the income generated from asset utilization, net of all expenses witnessed through the above process. So, they have a residual claim at the company's Earnings and there is no contractual guarantee for any compensation amount, thus being exposed to Earnings variability, both downside and upside. However, their risk/reward potential is not symmetric, as Equity is characterized by the concept of limited liability, which limits their financial loss to the amount they have invested in the company (Equity Value can't turn negative). So, in the worst-case scenario Equity Holders can lose no more than their initial investment.

On the other hand, Debt holders (Creditors) are compensated through interest payments, which are fixed amounts representing the compensation paid by the corporation for the privilege of using their funds. From a PnL perspective, Interest payments are classified as an (financial) expense, just like any other operating expense the company witnessed through the asset utilization process. As a result, it is paid before any compensation gets attributed to Equity holders. Note that due to the fixed nature of the relevant compensation, Debt holders are exposed to downside earnings volatility (operational earnings might not be enough to cover interest payments), but not to upside earnings volatility, as they receive only the predetermined interest payments as compensation, and any excess earnings do not flow to them.

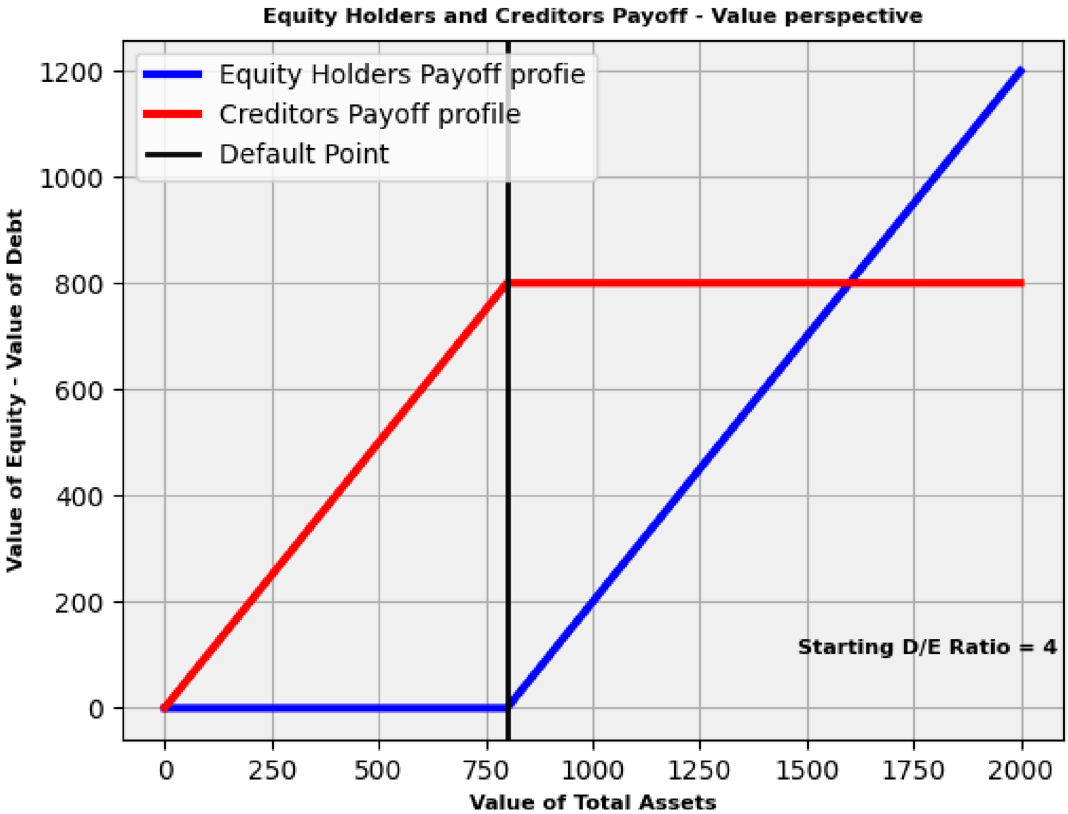


Equity vs Debt Holders: Heads I win, Tails we loose.

From the above, we can clearly witness a disparate risk/return profile amongst Creditors and Equity Holders. In the event of loses occurring, Equity holders "pay first", with the losses eating into retained earnings or shareholder capital. On the contrary, creditors face losses only when the equity of shareholders is depleted. However, in scenarios of profitability, shareholders enjoy the entire upside, while creditors are limited to a prearranged capped sum (the par value of the debt along with interest). Consequently, the payoff structure for shareholders is notably asymmetric, as they risk losing their entire investment but have unlimited potential gains. Creditors also encounter an asymmetric payoff structure, albeit less pronounced and more focused, offering virtually no upside, but also a reduced likelihood of losing their entire investment.

Therefore, we can see that Creditors' and Equity holders' incentives are not always aligned, and situations of conflicting interests might arise. Also, keep in mind that the corporation's management and executives are appointed by the Equity Holders, so the first represent the interests and work on behalf of the latter. Creditors have almost no governance rights over the corporation that they lend to. The above situation is called Agency Cost, and problems might arise due to that.

The most basic implication of the above is in regard to the risk appetite of the two groups. Given that Equity Holders are exposed to the upside potential, but Creditors are not, the corporation's management might go after risky investments with the potential of high reward. That way, Creditors are faced with extra risk, but not extra reward. Creditors are risk-averse, and they do not seek volatility. Equity holders are risk-tolerant, they seek upside volatility.



Another implication of Agency Cost is that, given that Shareholders are in a first-loss position, any additional capital they invest in the corporation is at risk of being lost. So, in financial stress situations, where the corporation is in need of new capital injection, equity holders are less incentivized to inject cash, if there is no potential of recovery. A classic illustration of the above, is when a corporation has negative BV Equity; that is when its Assets' Book Value is lower than its Liabilities (the company owns less than what it owes). If the company lucks of recovery prospects, Equity holders (from a utility maximization perspective), have absolutely no reason to inject new capital (just to pay off the Creditors), resulting in letting the company Default and Creditors face losses.

Equity as a Call Option on the corporation's Assets

Trying to model the above relationship, Robert Merton first showed that Equity Holders can be seen as possessing a European Call Option on the Company's Assets, with a Strike price equal to the level of Debt, a maturity of one year, and a Call premium equal to the Equity Value.

Here is the intuition: At Call Option maturity, the value of the company's Assets will be either greater than, less than, or equal to the debt. Since the call option's strike price is the value of the debt, if Total Assets Value (option underlying asset) is greater than the debt (meaning Equity Value>0), Equity Holders (Call Option buyers) are incentivized to exercise their option, (basically paying out the Debt, thus claiming the Assets' ownership). On the other hand, if the value of the company's Assets is below the value of debt at maturity (meaning Equity<0), Equity Holders have zero incentive to pay-off the debt. That means the call option is out-of-the-money and expires without being exercised. (Equity Holders simply walk away, leaving Creditors with debt that cannot be repaid through the liquidation of the company's assets as TA<L). The call premium is the company's equity, as this is what the Call Option buyer (Equity Holder) pays so as to have the right to buy the Total Assets at the Strike Price (basically pay off the debt and be the sole beneficiary of claims over Asset Value).

In other words, if the buyer (Equity Holder) of the contract does not exercise the Call Option, the corporation defaults. Subsequently, the probability of the company defaulting, is equal to the probability that the above-described Call Option is not exercised at Maturity. As we know, a Call option will not be exercised at Maturity if it's "out of the money"; that is if the value of the underlying asset (corporation's Total Assets) is lower than the strike price (Debt). Another way to say the above is that the one-year Probability of Default is equal to the probability that the corporation's Value of Total Assets will be lower than its Debt, when the option expires. From the above, we can characterize the level of Debt as the "Default Point", in which if Total Assets' Value is lower than that, the company defaults, whereas if Total Asset's Value is higher than that, the Company doesn't default. Subsequently, if we were able to know the future value of the company's Assets in one year, we would be able to know (by comparing it with the Default Point) if it would default or not.

***Note**: It should be reminded that a European Call Option is a derivative instrument between 2 parties, where the contract buyer (long position) is given the option, when the contract expires (matures), to buy an amount of the underlying asset at a predetermined price (Strike Price); and the contract seller (Short position), when the contract matures, is obligated to sell the underlying asset at that predetermined price (Strike), if the buyer exercises his right. Lastly, for that optionality, the contract buyer pays a fee to the seller, which is called the Call Premium. We can observe that there is an asymmetric risk/reward profile between the 2 parties. That's because the Call Option buyer is exposed to unlimited upside potential (spot price has no upside limit), but limited downside potential (max loss is the premium paid), where the Call Option seller is exposed to unlimited downside potential (spot price has no upside limit), and limited upside potential (max profit is the premium received). Another way of saying the above, is that the buyer is exposed to favorable for him underlying asset volatility and protected against unfavorable for him volatility. On the other hand, a Call option seller is exposed to unfavorable volatility, and blocked of favorable volatility.

****Note**: An easy-to-understand example of the above relationship could be explained in the context of borrowing to buy a house. You believe the Athenian housing market has further upside potential, and you want to (long) position yourself accordingly. You find a nice apartment in downtown Athens valued at 500,000 and you want to purchase it. You agree with a Bank that you will put down EUR 200,000 as Equity and the rest of the amount (EUR 300,000) will be borrowed from the bank. Caution: We make the hypothesis that your investment is characterized by limited liability (the bank can claim ownership only of the house, in case of possible inability of yours to meet with your contractual obligations). One year later: Your analysis was wrong, and a housing market decline occurs. The value of the house (underlying Asset) now stands at EUR 250,000. Why would you continue to pay interest and repay the 300,000 loan? Your equity has been wiped out. Your current PnL is EUR -200,000. If you pay back the debt, you will exchange EUR 300,000 of cash to claim ownership of something that is worth 250,000. So, you witness another -50,000 loss. Your total loses amount to EUR -250,000 vs EUR -200,000 if you never paid back the debt and quitclaimed ownership of the house to the bank.

We can observe the Default Point (the company's Book Value of Liabilities is reported in the Balance Sheet), but we can't observe the Value of Total Assets at expiration. In fact, we can't even observe the value of Assets in the present, only the Equity Value (approximated by Market Capitalization). Good or bad, there is no way of predicting the future. However, statistics provide us with tools of estimating the future value of Assets, (more specifically, estimating the distribution of probabilities of its future value, as a function of its value in the present, its drift rate and its volatility). Finaly, from the above we can conclude that the Probability of Default of the corporation in one year is a function of: 1) The Value of Assets in the Present, 2) the Asset Volatility, and 3) the Default Point (we assume that the Drift Rate is equal to the Risk Free Rate). That is the basis of the famous structural approach to Credit Risk, pioneered by Robert Merton.

In this report, we are presenting a commercialized version of Merton's approach, called KMV EDF, which has been developed by Moody's Analytics based on the work of three researchers: Stephen Kealhofer, John McQuown, and Oldrich Vasicek. The above, incorporates a more realistic capital structure of the company (taking into account the existence of different maturities and levels of Debt Seniority), and uses relatively more complex techniques of parameters' estimation. Among others, it assumes that the Default Point is equal to the level of Current Liabilities plus half of the Non-Current Liabilities (DP = CL + 0.5*NCL), instead of Total Debt that the Merton Model assumes. Let's dive into the model:

The fundamental question to be answered is, how we can estimate the above-mentioned Probability of Default. Fortunately, as we have modelled the Equity Value as a Call Option, we can use the Black-Scholes model formula, which calculates the theoretical price of a Call Option as a function of its key components: spot price of the underlying asset, volatility of the underlying asset, Risk Free Rate, time to Maturity and Strike Price. In the renowned Call option valuation formula*, $N(d2)$ represents the probability the European Call Option is in-the-money at expiration; In other words, the probability the underlying asset value (in our case company's Asset Value), is greater than the Strike Price (in our case the Default Point). Consequently, $1 - N(d2)$ is the Probability the Call Option does not get exercised, (in our case, that means the company Defaults). That's the Probability we are looking for (PD). Let's analyze the above step-by-step:

Let us start with the assumption that the fluctuation of Total Assets' value over time takes the form of a random walk. We can model the above, as a Geometric Brownian motion, using the Wiener process. So, we have:

$$dAt = rAtdt + \sigma_A AtdW_t$$

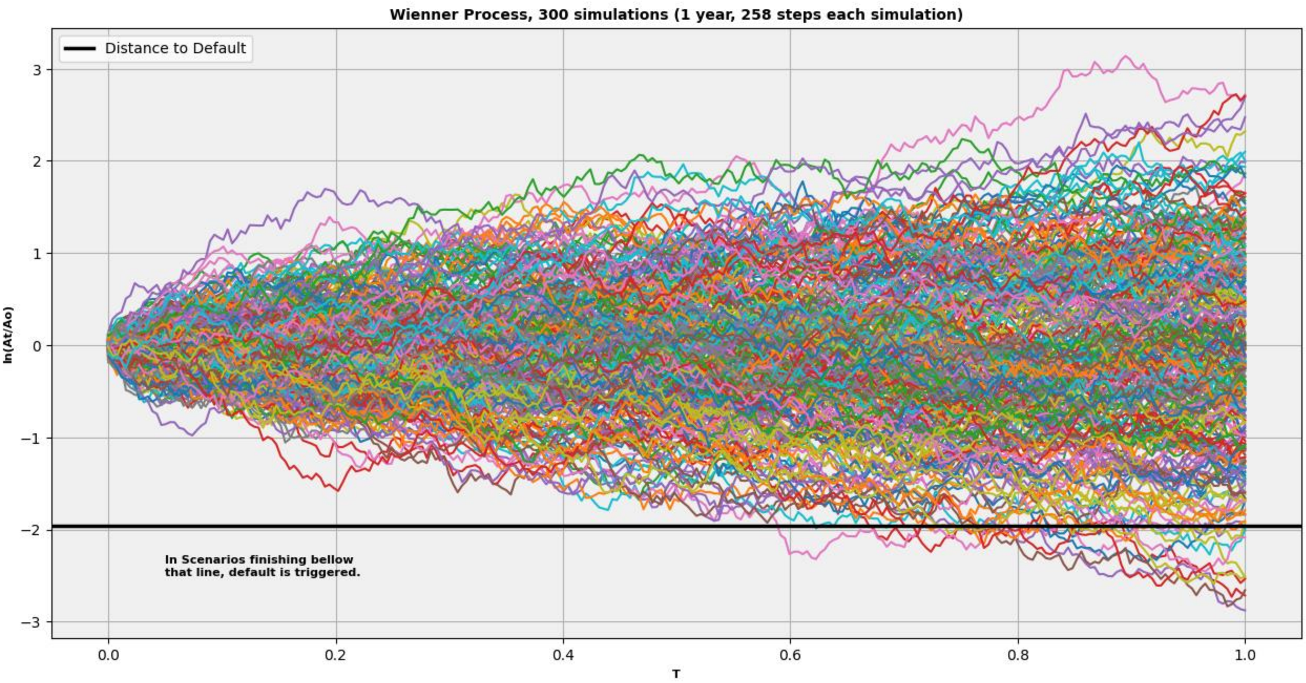
Also, by Ito's Lemma, we can prove that:

$$\ln\frac{A_t}{A_0} = \left(R_f - \frac{\sigma_A^2}{2}\right) * T + \sigma_A * \sqrt{T} * W \Rightarrow \ln At = \ln Ao + \left(R_f - \frac{\sigma_A^2}{2}\right) * T + \sigma_A * \sqrt{T} * W$$

where W represents a stochastic process following the standard normal distribution. Moving on, we are in search of the probability that the Asset Value at T (expiration), is less than the Default Point. So, we have:

***Note:** Bls Formula, $C_0 = f(S_0, \sigma_S, K, R_f, T) = S_0 * N(d1) - K * e^{-R_f * T} * N(d2) \Rightarrow \quad N(d2) = \frac{S_0 * N(d1) - C_0}{K * e^{-R_f * T}}$

$d1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(R_f + \frac{\sigma_S{}^2}{2}\right) * T}{\sigma_S * \sqrt{T}}$, $d2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(R_f - \frac{\sigma_S{}^2}{2}\right) * T}{\sigma_S * \sqrt{T}}$



$$P(A_T \leq DP) = P(\ln A_T \leq \ln DP) = P\left[\ln Ao + \left(R_f - \frac{\sigma_A^2}{2}\right) * T + \sigma_A * \sqrt{T} * W \leq \ln DP\right] =$$

$$P\left[\ln Ao - \ln DP + \left(R_f - \frac{\sigma_A^2}{2}\right) * T \leq -\sigma_A * \sqrt{T} * W\right] = P\left[\frac{\ln\frac{Ao}{DP} + \left(R_f - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}} \leq -W\right] =*$$

$$P\left[\frac{\ln\frac{Ao}{DP} + \left(R_f - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}} \leq -se\right] = P\left[-\frac{\ln\frac{Ao}{DP} + \left(R_f - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}} \geq * \; se\right] = N\left[-\frac{\ln\frac{Ao}{DP} + \left(R_f - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}}\right] =$$

$$\mathbf{PD} = \boldsymbol{N}\left[\frac{\ln \frac{\boldsymbol{DP}}{\boldsymbol{A_0}} - \left(\boldsymbol{R_f} - \frac{\boldsymbol{\sigma_A^2}}{\mathbf{2}}\right) * \boldsymbol{T}}{\boldsymbol{\sigma_A} * \sqrt{\boldsymbol{T}}}\right]$$

***Note:** From Brownian Motion properties, we know that $se_{t1+t2} = \frac{W(t1+t2)-W(t1)}{\sqrt{t2}}$, $(se \sim SND)$

****Note:** Our objective is to estimate the Risk Neutral Probability of Default. The above, is based on the concept of Risk-Neutral valuation. More specifically, a Call Option (which is a contract whose value is a partial function of the underlying asset's price) can be valued in a context where Risk is neutralized. The above can be achieved with a hypothetical portfolio being long positioned in the underlying asset, and short positioned on a Call Option over that underlying asset. By properly adjusting weights (Delta) in the Spot position, the portfolio can yield the same return for every possible scenario of returns in the underlying asset price. So, the investment is risk-free, thus the Risk-Free Rate should be used to price it. If someone wants to estimate the Real Probability of Default, they should use the underlying asset's drift rate.

We have found a formula for the Probability we are looking for, as a function of the Asset Value in the present, the Asset volatility, the Default Point and the Risk Free Rate. However, there is a problem: A_0 *and* σ_A values are not observable. It is true that $A_0 = E_0 + L_0$, where L_0 is the Market Value of Debt. That is unobservable for most corporations with a wide variety of illiquid debt types and maturities. What we can observe though, is the Market Capitalization of Equity (Shares Outstanding*Stock Price), as an approximation of E_0. Is there an equation connecting E_0, A_0 *and* σ_A? Yes: Let's use the BLS formula to model the Call Option of Equity Holders over the Company's Asset Value, with a Strike Price = Default Point, and one year Maturity:

$$Equity\ V_0 = Asset\ V_0 * N(d1) - Default\ Point * e^{-R_f*T} * N(d2) \Rightarrow$$

$$E_0 = A_0 * N(d1) - DP * e^{-R_f*T} * N(d2)$$

$$d1 = \frac{\ln\frac{A_0}{DP} + \left(Rf + \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}}$$

$$d2 = \frac{\ln\frac{A_0}{DP} + \left(Rf - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}}$$

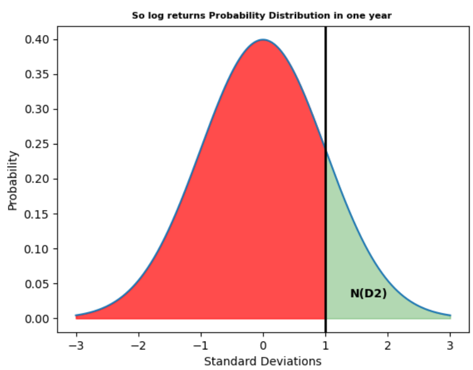
Notice that:

$$\Rightarrow N(-d2) = N\left[-\frac{\ln\frac{A_0}{DP} + \left(Rf - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}}\right] = N\left[\frac{ln\frac{DP}{A_0} - \left(R_f - \frac{\sigma_A^2}{2}\right) * T}{\sigma_A * \sqrt{T}}\right] = PD$$

So, Given than $N(d2)$ is a component of Black-Scholes formula, showing the probability of a Call Option getting exercised*, we can witness that by applying the BLS in Equity over Assets Call Option, the $Probability\ of\ Default$ (=Call Option does not get exercised) is equal to $N(-d2)$. Let's not forget that:

$$P(Call\ Option\ Gets\ Exersised) \equiv P(Call\ Option\ is\ in - the - money) \equiv P(S_T \geq K) = N(d2)$$

***Note:** $N(d2) \rightarrow P(S_T \geq K)$ explanation example:
Consider a Stock Call option with the bellow characteristics:
$S_0 = 28$
$K = 33$
$T = 1Y$
$\ln\frac{S_T}{S_0} \sim N(\mu = 0.04, \sigma = 0.12)$
$\ln\frac{K}{S_0} \approx 16\,\% \rightarrow \log\ return\ needed\ for\ So\ to\ reach\ K\ (and\ thus\ Call\ Option\ to\ get\ exersised).$



Question: Given that $\ln\frac{S_T}{S_0}$ is a random variable, where does the specific $\ln\frac{K}{S_0}$ observation "fall" into our future normal distribution curve?
Answer: We standardize the distance between the expected value (μ) and the specific observation, so as to calculate the probability of it happening. (actually to calculate the probability a random observation from our distribution is of equal or greater distance from the mean). So:

Distance from Expected Value $= \frac{\ln\frac{K}{S_0} - E\left(\ln\frac{S_T}{S_0}\right)}{\sigma} = \frac{0.16 - 0.04}{0.12} = 1$ std away from the mean
So, what is the probability the actual value of $\ln\frac{S_T}{S_0}$ in one year, falls 1 std apart or greater from the expected value? We can answer, through the Cumulative Distribution function: $P(Zo \geq 1) = 1 - P(Zo \leq 1) = N(-1) = 15.87\%$

And the above, applied to our specific Call Option:

$P(Call\ Option\ Gets\ Exersised\ by\ Equity\ Holders) \equiv P(No\ Default) \equiv P(A_T \geq DP) = N(d2)$

$P(Call\ Option\ does\ not\ get\ Exersised\ by\ Equity\ Holders) \equiv P(\ Default) \equiv P(A_T \leq DP) = 1 - \ N(d2) = N(-d2)$

Moving one, since we have two unknowns, $A_0\ and\ \sigma_A$, we need to find a second equation connecting $E_0, A_0\ and\ \sigma_A$, so as to have a 2x2 systems of equations, and thus solve for the unknowns. If we look at our resources carefully, there is another observable metric which is a function of E_0, and haven't been utilized yet: σ_E (Equity Value volatility).

Let us assume that the corporation's Equity Value is also a random variable and follows a Geometric Brownian Motion:

$$dE = \mu_E E dt + \sigma_E E dW \quad (3)$$

Additionally, given than Equity Value is a function of Asset Value, by Ito's Lemma, we can break down the equity differential, as:

$$dE = \left[\frac{\theta E}{\theta t} + \frac{\theta E}{\theta A}\mu_A A + \frac{\Theta^2 * E}{2*\theta*A^2}\sigma_A^2 A^2\right] dt + \frac{\theta E}{\theta A}\sigma_A A dW \quad (4)$$

If we match the second part of each equation and solve for σ_E, we have:

$$\xRightarrow{(3),(4)} \mu_E E dt + \sigma_E E dW = \left[\frac{\theta E}{\theta t} + \frac{\theta E}{\theta A}\mu_A A + \frac{\Theta^2 E}{2\theta A^2}\sigma_A^2 A^2\right] dt + \frac{\theta E}{\theta A}\sigma_A A dW \Rightarrow \cdots$$

$$\Rightarrow \sigma_E = \frac{\theta E}{\theta A} * \frac{A}{E} * \sigma_A$$

Furthermore, if we partially differentiate the Black-Scholes formula for A_0, we have:

$$\frac{\theta E}{\theta A_0} = \frac{\theta\left[A_0 * N(d1) - DP * e^{-R_f * T} * N(d2)\right]}{\theta A_0} = \ N(d1) \qquad \text{(Equity Delta)} \ \ (5)$$

Subsequently, we have:

$$\overset{(5)}{\Rightarrow} \sigma_E = \frac{\theta E}{\theta A} * \frac{A}{E} * \sigma_A \Rightarrow \sigma_E = N(d1) * \frac{A}{E} * \ \sigma_A$$

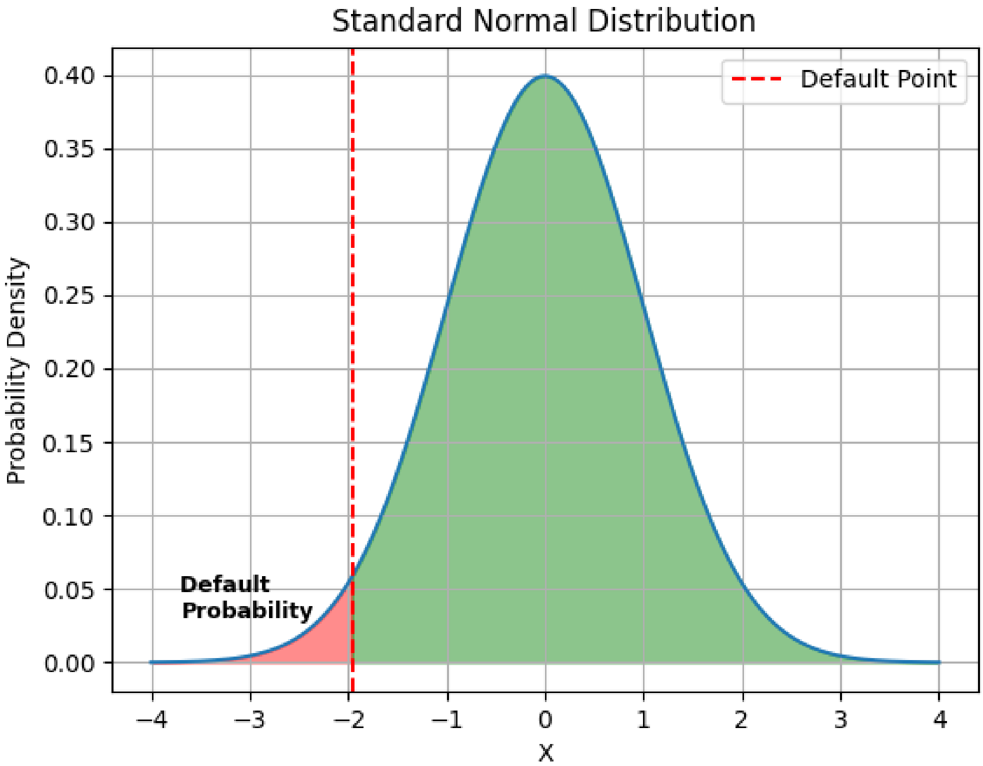
Finally, we have a non-linear $2x2$ system of equations:

$$E_0 = A_0 * N(d1) - DP * e^{-R_f * T} * N(d2)$$

$$\sigma_E = N(d1) * \frac{A}{E} * \ \sigma_A$$

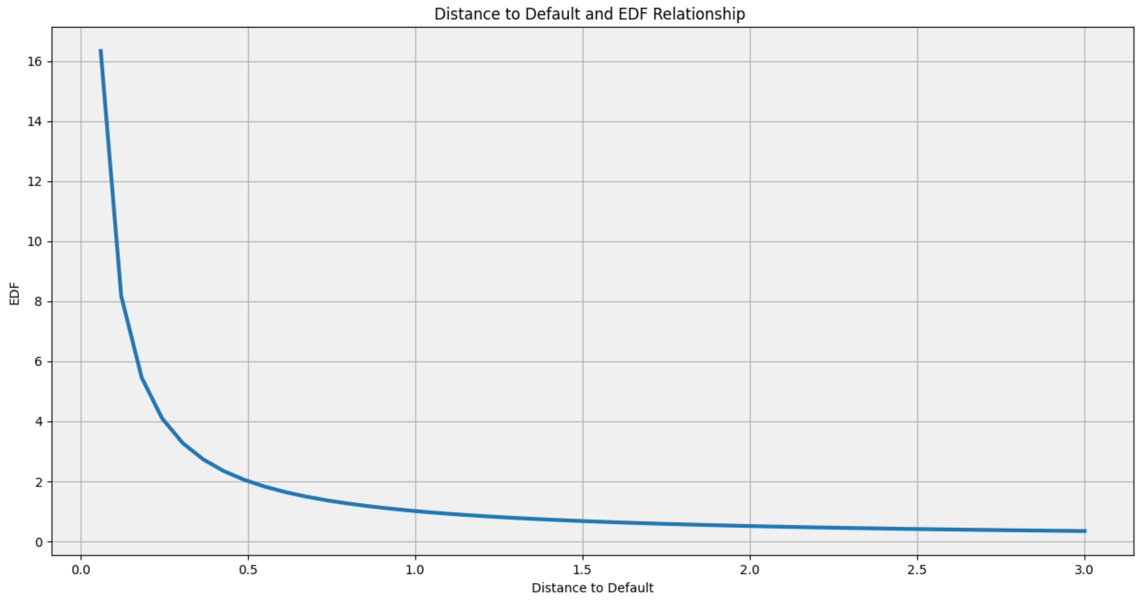
There is a number of ways to find the above A_0 , σ_A unknowns, the most obvious one being, solving the 2x2 system of equations through an optimization process. After solving the equations system and estimating A_0 , σ_A*, we now have at our disposal all three parameters that we need to calculate Probability of Default: Initial Asset Value, Asset volatility, and Default Point. We put the above to the PD formula:

$$PD = N\left[\frac{\ln\frac{DP}{A_0} - \left(Rf - \frac{\sigma_A^2}{2}\right)}{\sigma_A}\right]$$



The value inside the Cumulative Probability Distribution formula is called the Distance-to-Default, and it intuitively answers the following question: How "far away", in standard deviation terms, is the log return needed for A_0 to reach DP, from the expected log return of Assets? As we can intuitively understand, the greater this distance is, the lower is the Probability of Default. Also, we can witness that the higher the volatility of Assets is, the higher the Probability of Default over a period of one year.

***Note:** Please note that the latest version of Moddy's KMV EDF model implements a fairly more complex method of estimating initial Asset Value and Asset Volatility. Especially for the asset volatility, the method uses as an input not only historical equity volatility, but also modelled and forward-looking equity volatility. Also, their model instead of using the Standard Normal distribution, uses an empirical distribution based on their own data (yet quite similar with the snd). Most of the methodology for the above is not publicly available.



Last but not least, it should be mentioned that the KMV approach -and its many variations- to Credit Risk assessment has a number of limitations and weaknesses, and its prediction accuracy, as well as its credit spread differentiations explanatory capability, is being challenged by a number of scholas and market participants. First and foremost, the use of stochastic processes implies a degree of arbitrariness and subjectivity in the estimation of the model's parameters. (for example, there is the implicit assumption of normality of Asset returns). Also, the fact that a universal Default Point (which also implies unchanged capital structure), is defined for all corporations could be seen as problematic, given the operational differences in sectors and geographies (different industries or individual companies may suit for different Default Point parameter).

However, having said the above, its many advantages make this approach a basic one in the field, as it is being widely used by the industry for the estimation of the Probability of Default of a corporation and subsequent Credit Spread extraction. Accurate and timely information from the Equity Markets (which can be observed instantly) provides a continuous credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis. As a result of its dynamic nature, (empirical data have shown that) changes in PD tend to anticipate at least one year earlier than the downgrading of the issuer by rating agencies. The key is in how the PD metric is interpreted by the user. The model actually provides a cardinal rather than an ordinal deterministic ranking of credit quality, so it should be interpreted accordingly. Last but not last, for anyone interested for the pure technical aspect, a python code practically implementing the KMV model has been created, estimating the Probability of Default for PPC SA (Public Power Corporation – DEI), the largest electrical energy producer and supplier in the Balkans (and aggressively expanding). You can check the project here (GitHub Repository)

***Note:** The numerical values of x and y axis of the depicted PD (EDF) and Distance to Default relationship is indicative, not absolute.

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