Quasidiagonality and the classification of nuclear C^*-algebras

Aaron Tikuisis
a.tikuisis@abdn.ac.uk

University of Aberdeen

Joint work with Stuart White and Wilhelm Winter

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n \in \mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C^*_r(G)$ is quasidiagonal, then G must be amenable (equivalently, $C^*_r(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Theorem (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \; (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \; (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \; \left(= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)\right), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \; (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \ (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Definition (N. Brown)

A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \ (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has *decomposition rank at most d* if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has *decomposition rank at most d* if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd}
A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\
& F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] &
\end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

Quasidiagonal traces and covering dimension

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that all traces on A are quasidiagonal.

Quasidiagonal traces and covering dimension

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that all traces on A are quasidiagonal.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that all traces on A are quasidiagonal.

Quasidiagonal traces and covering dimension

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that all traces on A are quasidiagonal.

Quasidiagonal traces and covering dimension

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that all traces on A are quasidiagonal.

Finite decomposition rank is a powerful property. Characterizing this property is desirable, particularly for Elliott algebras:

Definition

An *Elliott algebra* is a C^*-algebra that is simple, separable, nuclear, unital, and infinite dimensional.

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Finite decomposition rank is a powerful property. Characterizing this property is desirable, particularly for Elliott algebras:

Definition

An *Elliott algebra* is a C^*-algebra that is simple, separable, nuclear, unital, and infinite dimensional.

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Finite decomposition rank is a powerful property. Characterizing this property is desirable, particularly for Elliott algebras:

Definition

An *Elliott algebra* is a C^*-algebra that is simple, separable, nuclear, unital, and infinite dimensional.

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Finite decomposition rank is a powerful property. Characterizing this property is desirable, particularly for Elliott algebras:

Definition

An *Elliott algebra* is a C^*-algebra that is simple, separable, nuclear, unital, and infinite dimensional.

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

$$M_2 = \left(\begin{array}{c|c} \mathbb{C} & \mathbb{C} \\ \hline \mathbb{C} & \mathbb{C} \end{array}\right)$$

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

$$M_4 = \left(\begin{array}{cc|cc} \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \end{array}\right)$$

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

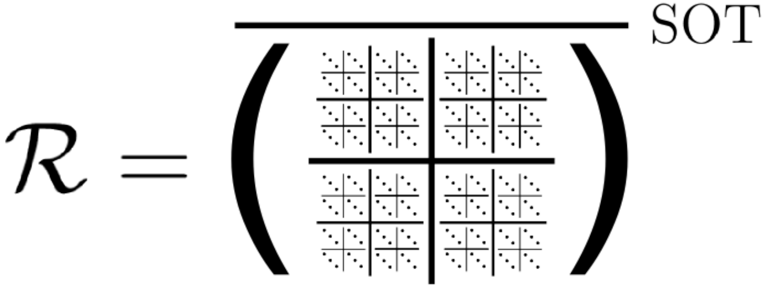
$$M_8 = \left(\begin{array}{cc|cc|cc|cc}
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} \\ \hline
\mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C} & \mathbb{C}
\end{array}\right)$$

The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

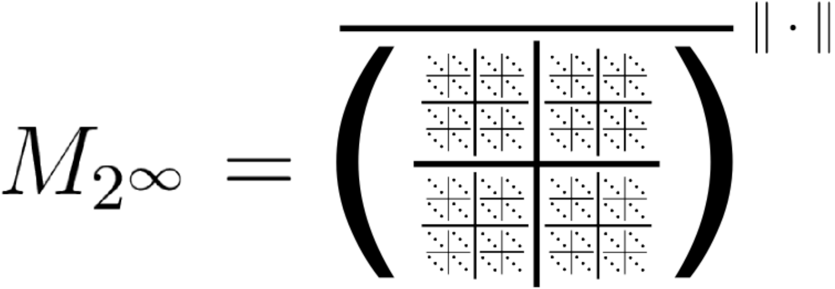


The Jiang–Su algebra $\mathcal{Z}$

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

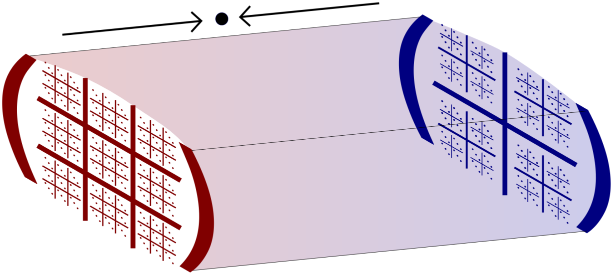
$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

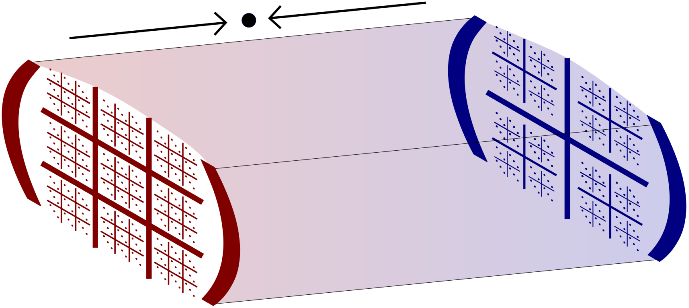


Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.

$\mathcal{Z} =$ 

$\mathcal{Z} =$ 

$\mathcal{Z}$ is an Elliott algebra with unique trace and no nontrivial projections, which satisfies $K_*(\mathcal{Z}) \cong K_*(\mathbb{C})$.

Quasidiagonal traces and covering dimension

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact, it is ≤ 1).

Quasidiagonal traces and covering dimension

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact, it is ≤ 1).

Quasidiagonal traces and covering dimension

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact, it is ≤ 1).

Quasidiagonal traces and covering dimension

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
(ii) A has finite nuclear dimension if and only if A is $\mathcal{Z}$-stable.

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact, it is ≤ 1).

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact it is ≤ 1).

Sketch (of (ii)). Quasidiagonal traces provide maps $A \to \mathcal{Z}_\omega$ realizing any trace, factoring through matrix algebras.

Patch these together along $\partial_e T(A)$ to get a map $\Psi : A \to (A \otimes \mathcal{Z})_\omega$, factoring through finite dimensional C^*-algebras.

Show that Ψ is 2-coloured equivalent to the canonical embedding $A \to (A \otimes \mathcal{Z})_\omega$, yielding $\mathrm{dr}(A) \leq 1$.

Quasidiagonal traces and covering dimension

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact it is ≤ 1).

Sketch (of (ii)). Quasidiagonal traces provide maps $A \to \mathcal{Z}_\omega$ realizing any trace, factoring through matrix algebras.

Patch these together along $\partial_e T(A)$ to get a map $\Psi : A \to (A \otimes \mathcal{Z})_\omega$, factoring through finite dimensional C^*-algebras.

Show that Ψ is 2-coloured equivalent to the canonical embedding $A \to (A \otimes \mathcal{Z})_\omega$, yielding $\mathrm{dr}(A) \leq 1$.

Quasidiagonal traces and covering dimension

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact it is ≤ 1).

Sketch (of (ii)). Quasidiagonal traces provide maps $A \to \mathcal{Z}_\omega$ realizing any trace, factoring through matrix algebras.

Patch these together along $\partial_e T(A)$ to get a map $\Psi : A \to (A \otimes \mathcal{Z})_\omega$, factoring through finite dimensional C^*-algebras.

Show that Ψ is 2-coloured equivalent to the canonical embedding $A \to (A \otimes \mathcal{Z})_\omega$, yielding $\mathrm{dr}(A) \leq 1$.

Theorem (Bosa-Brown-Sato-T-White-Winter)

Let A be a $\mathcal{Z}$-stable Elliott algebra, such that the extreme boundary $\partial_e T(A)$ of the tracial state simplex is compact. Then:
(i) A has finite nuclear dimension (in fact, it is ≤ 1).
(ii) If all traces on A are quasidiagonal then A has finite decomposition rank (in fact it is ≤ 1).

Sketch (of (ii)). Quasidiagonal traces provide maps $A \to \mathcal{Z}_\omega$ realizing any trace, factoring through matrix algebras.

Patch these together along $\partial_e T(A)$ to get a map $\Psi : A \to (A \otimes \mathcal{Z})_\omega$, factoring through finite dimensional C^*-algebras.

Show that Ψ is 2-coloured equivalent to the canonical embedding $A \to (A \otimes \mathcal{Z})_\omega$, yielding $\mathrm{dr}(A) \leq 1$.

Classification of C^*-algebras

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

$$\mathrm{Ell}(A) := (K_0(A), K_0(A)_+, [1_A]_{K_0(A)}, K_1(A), T(A), \rho_A : T(A) \times K_0(A) \to \mathbb{R}).$$

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the Universal Coefficient Theorem (UCT). Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

$$\mathrm{Ell}(A) := \big(K_0(A), K_0(A)_+, [1_A]_{K_0(A)}, K_1(A), T(A), \rho_A : T(A) \times K_0(A) \to \mathbb{R}\big).$$

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the Universal Coefficient Theorem (UCT). Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Classification of C^*-algebras

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

$$\mathrm{Ell}(A) := \big(K_0(A), K_0(A)_+, [1_A]_{K_0(A)}, K_1(A), T(A), \rho_A : T(A) \times K_0(A) \to \mathbb{R}\big).$$

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the Universal Coefficient Theorem (UCT). Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

$$\mathrm{Ell}(A) := \big(K_0(A), K_0(A)_+, [1_A]_{K_0(A)}, K_1(A), T(A), \rho_A : T(A) \times K_0(A) \to \mathbb{R}\big).$$

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the Universal Coefficient Theorem (UCT). Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Classification of C^*-algebras

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

$$\mathrm{Ell}(A) := \big(K_0(A), K_0(A)_+, [1_A]_{K_0(A)}, K_1(A), T(A), \rho_A : T(A) \times K_0(A) \to \mathbb{R}\big).$$

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the Universal Coefficient Theorem (UCT). Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Definition (Rosenberg–Schochet)

A separable C^*-algebra A satisfies the Universal Coefficient Theorem (UCT) if, for every σ-unital C^*-algebra B,

$$0 \to \mathrm{Ext}(K_*(A), K_{*+1}(B)) \to KK(A, B) \to \mathrm{Hom}(K_*(A), K_*(B)) \to 0$$

is an exact sequence.

The Universal Coefficient Theorem

Theorem (Gong–Lin–Niu, Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Suppose that all traces on A and B are quasidiagonal. Then $A \cong B$ if and only if $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$. Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Definition (Rosenberg–Schochet)

A separable C^*-algebra A satisfies the Universal Coefficient Theorem (UCT) if, for every σ-unital C^*-algebra B,

$$0 \to \mathrm{Ext}(K_*(A), K_{*+1}(B)) \to KK(A, B) \to \mathrm{Hom}(K_*(A), K_*(B)) \to 0$$

is an exact sequence.

The Universal Coefficient Theorem

Definition (Rosenberg–Schochet)

A separable C^*-algebra A satisfies the Universal Coefficient Theorem (UCT) if, for every σ-unital C^*-algebra B,

$$0 \to \mathrm{Ext}(K_*(A), K_{*+1}(B)) \to KK(A,B) \to \mathrm{Hom}(K_*(A), K_*(B)) \to 0$$

is an exact sequence.

One use of the UCT is to lift an isomorphism $K_*(A) \to K_*(B)$ to an invertible element of $KK(A,B)$.

Another use arises through Stable Uniqueness Theorems.

The Universal Coefficient Theorem

Definition (Rosenberg–Schochet)

A separable C^*-algebra A satisfies the Universal Coefficient Theorem (UCT) if, for every σ-unital C^*-algebra B,

$$0 \to \mathrm{Ext}(K_*(A), K_{*+1}(B)) \to KK(A,B) \to \mathrm{Hom}(K_*(A), K_*(B)) \to 0$$

is an exact sequence.

One use of the UCT is to lift an isomorphism $K_*(A) \to K_*(B)$ to an invertible element of $KK(A,B)$.

Another use arises through Stable Uniqueness Theorems.

The Universal Coefficient Theorem

Definition (Rosenberg–Schochet)

A separable C^*-algebra A satisfies the Universal Coefficient Theorem (UCT) if, for every σ-unital C^*-algebra B,

$$0 \to \mathrm{Ext}(K_*(A), K_{*+1}(B)) \to KK(A, B) \to \mathrm{Hom}(K_*(A), K_*(B)) \to 0$$

is an exact sequence.

One use of the UCT is to lift an isomorphism $K_*(A) \to K_*(B)$ to an invertible element of $KK(A, B)$.

Another use arises through Stable Uniqueness Theorems.

The Universal Coefficient Theorem

Stable Uniqueness Theorem (Lin, Dadarlat–Eilers)

If A is an Elliott algebra which satisfies the UCT, $\phi, \psi : A \to \mathcal{Q}_\omega$ are homotopic *-homomorphims, and $\iota : A \to \mathcal{Q}$ is a unital *-homomorphism, then

$$\iota^{\oplus n} \oplus \phi \quad \text{and} \quad \iota^{\oplus n} \oplus \psi$$

become approximately unitarily equivalent, as $n \to \infty$.

The Universal Coefficient Theorem

Stable Uniqueness Theorem (Lin, Dadarlat–Eilers)

If A is an Elliott algebra which satisfies the UCT, $\phi, \psi : A \to \mathcal{Q}_\omega$ are homotopic *-homomorphims, and $\iota : A \to \mathcal{Q}$ is a unital *-homomorphism, then

$$\iota^{\oplus n} \oplus \phi \quad \text{and} \quad \iota^{\oplus n} \oplus \psi$$

become approximately unitarily equivalent, as $n \to \infty$.

The Universal Coefficient Theorem

Stable Uniqueness Theorem (Lin, Dadarlat–Eilers)

If A is an Elliott algebra which satisfies the UCT, $\phi, \psi : A \to \mathcal{Q}_\omega$ are homotopic *-homomorphims, and $\iota : A \to \mathcal{Q}$ is a unital *-homomorphism, then

$$\iota^{\oplus n} \oplus \phi \quad \text{and} \quad \iota^{\oplus n} \oplus \psi$$

become approximately unitarily equivalent, as $n \to \infty$.

The Universal Coefficient Theorem

Stable Uniqueness Theorem (Lin, Dadarlat–Eilers)

If A is an Elliott algebra which satisfies the UCT, $\phi, \psi : A \to \mathcal{Q}_\omega$ are homotopic *-homomorphims, and $\iota : A \to \mathcal{Q}$ is a unital *-homomorphism, then

$$\iota^{\oplus n} \oplus \phi \quad \text{and} \quad \iota^{\oplus n} \oplus \psi$$

become approximately unitarily equivalent, as $n \to \infty$.

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C_r^*(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C_r^*(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C_r^*(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C_r^*(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C^*_r(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C^*_r(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C^*_r(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C^*_r(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C_r^*(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C_r^*(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C_r^*(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C_r^*(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C_r^*(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

Tu proved that $C_r^*(G)$ satisfies the UCT whenever G is an amenable group(oid).

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$.

Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$.

Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$.

Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.

Main result

Theorem (T–White–Winter)

Let A be a nuclear C^*-algebra which satisfies the UCT and let $\tau \in T(A)$ be a faithful trace. Then τ is quasidiagonal.

Hence A is quasidiagonal.

Corollary (with Elliott–Gong–Lin–Niu)

Let A and B are Elliott algebras with finite nuclear dimension, which satisfy the UCT. Then $A \cong B$ if and only $\mathrm{Ell}(A) \cong \mathrm{Ell}(B)$.

Moreover, both A and B are inductive limits of subhomogeneous C^*-algebras.