

From the EPSRC website: this should address all of the following:

- The area of research
- The need for a Workshop in this area, and the relationship to other events in the UK
- The aims of the Workshop, and its likely level of demand
- The programme content and likely speakers
- Plans for Workshop support
- Plans for publicity
- Plans for Workshop evaluation and follow-on activities

Track record: 2 pages.

The rest of the Case for Support: 8 pages.

Other documents to prepare (no page limitations):

- Justification for resources - mention other planned funding sources
- Pathways to impact
- Workshop timetable

Case for support

1 Research track record

This section is nearly at the 2-page limit -AT

Operator algebras—chiefly C^* -algebras and von Neumann algebras—arose in the 1930's and 1940's during the mathematical rigourisation of quantum mechanics, and their study quickly grew in light of applications to other areas of mathematics, especially the representation theory of groups. Among areas of modern analysis, the subject of operator algebras is one of the most active areas. Deep connections have by now been established between operator algebras and such areas as quantum information, dynamical systems and ergodic theory, algebraic topology, differential geometry, knot theory, number theory, symbolic dynamics, operator theory, and harmonic analysis. In recent decades, two Fields medals have been awarded to mathematicians for work in operator algebras.

Within C^* -algebra theory, the primary drivers of research have been (i) the classification programme for amenable C^* -algebras, and (ii) the varied constructions of C^* -algebras motivated by (iii) applications to wider maths?-JS

The classification of amenable C^* -algebras is a programme with origins in the Connes-Haagerup classification of injective von Neumann factors (Connes was awarded a Fields medal for this work), and in the classification of UHF and AF C^* -algebras, to which O. Bratteli, J. Dixmier, G.A. Elliott, and J. Glimm contributed REF. It spurred the advancement of C^* -algebra K-theory, including the bivariant extensions, KK- and E-theory. Significant advances in the classification programme include the Kirchberg-Phillips classification of purely infinite C^* -algebras REF and a succession of increasingly general classification results for stably finite C^* -algebras REF.

Recently, examples came to light showing that some simple separable amenable C^* -algebras are too topologically high-dimensional to be classified by K-theory and traces. In reaction to these, an outshoot of the classification programme has been the study of regularity for C^* -algebras, captured in the Toms–Winter conjecture REF.

C^* -algebra constructions form a vast research subject, studied both to provide interesting examples of C^* -algebras, and to use these C^* -algebras as tools to shed light on other mathematical areas. Some of the most prominent of these constructions are:

- group C^* -algebras, relating to unitary representation theory and approximation properties of the group;
- crossed products, which encode a dynamical system and its orbit equivalence relation; mention Giordano–(Matui–)Putnam–Skau? -AT
- Roe C^* -algebras associated to a discrete metric space;
- Cuntz–Krieger algebras, graph algebras, and higher-rank versions; these encode symbolic-dynamical properties of combinatorial objects such as graphs.

Coarse geometry originated with John Roe in the late 1980's as a tool in index theory. Is it fair to say that anyone else (eg. Gromov? Margulis? Milnor? Mostow? Švarc? Wolf?) played a role in the beginnings of coarse geometry? -AT Yes, at least - geometric group theory / Gromov / studying discrete groups as metric spaces-JS Since then, coarse approaches have been very successful for rigidity problems in topology (the Novikov conjecture, stable Borel conjecture-JS), geometry (metrics with positive scalar curvature), higher index theory and geometric group theory (work of Higson, Roe, Yu, ... can elaborate on the actual results if helpful).

The central concepts in coarse geometry are the Roe C^* -algebras, associated to a discrete metric space with bounded geometry (in particular, discrete groups)-JS. This C^* -algebra is generated by operators of finite propagation: these operators appear in quantum physics, harmonic analysis, and also recently in quantum information and quantum computing. Operator algebraic and K-theoretic methods are used to study Roe C^* -algebras.

These avenues of research have recently intersected with the classification programme, particularly via the notion of nuclear dimension by Winter and Zacharias. Their work suggests connections between the topological and K-theoretic methods used to study the group and Roe C^* -algebras relevant to higher index theory, and the techniques important in the classification programme. A promising foray recently ... by Erik Guentner, Rufus Willett and Guoliang Yu about dynamic asymptotic dimension (mention here?)-JS. Yes, I think it would be good to mention GWY here. -AT

There are many meetings on C^* -algebras and on coarse geometry, at centres such as MFO (Oberwolfach), BIRS (Banff), and the Fields Institute (Toronto), although few events have specifically focussed on the interaction between these two subjects. C^* -algebra theory has been the topic of two Abel Symposia (2004 and 2015). Recently, the Mittag-Leffler Institute held a programme “Classification of operator algebras: complexity, rigidity, and dynamics” (1-4/16), the Fields Institute hosted a thematic programme “Abstract harmonic analysis, Banach and operator algebras” (1-6/14); the University of Münster held a focus semester on C^* -algebras (4-7/15); there has been a workshop on C^* -algebras and dynamical systems at BIRS (10/14) and a workshop on C^* -algebras at MFO (8/16). Workshop on coarse geometry have been held in Lille (6/12) and Kyoto (12/13). A conference “Non-commutative dimension theories” was held in Hawai’i bringing together researchers in C^* -algebras, K -theory, and coarse geometry. In the near future, there will be a research programme on C^* -algebras at CRM (Barcelona, 2017), and a workshop on C^* -algebras at BIRS (9/2017). **Mention upcoming ICMS workshop? -AT**

I’ll update the list - any preference for EU?-JS

Both C^* -algebras and coarse geometry are increasingly well-represented in the United Kingdom. The interface between the two subjects represents a major point of connection between operator algebras and index theory.

1.1 Investigators

All three investigators work in C^* -algebras. A. Tikuisis has a strong research track record in the classification of C^* -algebras, particularly contributing to the problem of settling the Toms–Winter conjecture [1, 7, 15, 16, 17, 7]. He is the lead applicant and organiser of the upcoming BIRS workshop “Future targets in the classification of C^* -algebras” (9/17). He holds an EPSRC first grant on “Regularity and dimension for C^* -algebras” (EP/N00874X/1).

J. Śpakula is an expert in Roe C^* -algebras with a strong track record REF. **Mention MC-CIG. -AT [No experience in organising conferences.] [We had a 1 month Research-in-Teams at the Erwin Schrödinger Institute in Vienna with Erik Guentner and Rufus Willett in June 2013, officially about nuclear dim and Roe algebras – but it’s probably not very good in mentioning that because we didn’t really progress on this problem and ended up doing something else.]JS**

S. White ... **Point out experience organising more significant conferences. -AT**

A. Tikuisis and S. White are the directors of the Scottish Operator Algebras Research (SOAR) cluster, which hosts twice-annual meetings, attracting important external speakers. Organisation of these regular meetings has given these two investigators experience in coordination and local organisation in Aberdeen.

1.2 Scientific advisory panel

George Elliott, FRSC, has been a professor and Canada Research Chair at the University of Toronto since 2001. He has continually been a leader over the last 40 years in C^* -algebra theory [3, 4, 6, 9, 5, 8], especially in classification, which is regularly called the Elliott programme. He has organised or been on the scientific committee of numerous mathematical events, recently including an Abel Symposium (Hurtigruten, 2015), workshops in BIRS (Banff, 2014 and 2016), a 6-month thematic program at the Fields Institute (Toronto, 2014), an workshop at ICMS (Edinburgh, 2013), and a workshop at MFO (Oberwolfach, 2012). He has held a Tier 1 Canada Research Chair in Mathematics since 2001 (CAD \$200k per annum).

Wilhelm Winter has been a professor at the University of Münster since 2011 (W3 since 2013). His groundbreaking work on covering dimension has brought new life into the structure and classification of C^* -algebras [11, 14, 16, 19, 24, 22, 23, 25] and exposed a new interface for interaction with coarse geometry [25]. He has organised many mathematical events, including a 4-month focus semester on C^* -algebras (Münster, 2015), an EPSRC-funded masterclass and workshop (Glasgow, 2014), a WIMCS-LMS conference (Aberystwyth, 2013), a workshop at ICMS (Edinburgh, 2013), and a workshop at MFO (Oberwolfach, 2012). During previous employment in the UK, he was awarded EPSRC grants EP/G014019/1 (c. £250k) and EP/I019227/1 (c. £350k). He has three projects in the DFG-funded Collaborative Research Centre (SFB) 878 (amount?). **Might need updating. Mention chair? -AT**

Guoliang Yu ... [12, 26, 27]...

2 Description of proposed research and its content

2.1 Background

A C^* -algebra is a norm-closed self-adjoint subalgebra of the set of bounded operators on a complex Hilbert space. Their study lies within the domain of functional analysis, yet has deep connections to other areas of mathematics, especially algebraic topology, group theory, differential geometry, and dynamical systems. C^* -algebras are highly structured objects, yet which can be constructed in a tremendous variety of interesting ways. A classical construction is a *group C^* -algebra*, arising from the unitary representation theory of groups. An extremely versatile C^* -algebra construction is the crossed product, which encode not only a topological space, but also a set of symmetries and/or a time-evolution on the space.

C^* -algebras are often thought of as noncommutative topological spaces, a point of view that begins with the observation, due to Gelfand, that commutative C^* -algebras correspond (contravariantly) to locally compact Hausdorff spaces. In many settings, mathematical objects arise which are degenerate or intractable from a classical topological approach, but become manageable when viewed as noncommutative geometric objects; a prime example is the set of unitary representations of a discrete group, which is poorly-behaved and intractable as a topological space, but which can instead be analysed using a group C^* -algebra. It has been extremely fruitful to generalise topological concepts to the noncommutative setting, and this point of view is central to Connes' programme of noncommutative geometry [2].

K-theory is perhaps the most celebrated import from topology to C^* -algebras theory. This algebraic invariant for topological spaces was developed in the 1960's by M. Atiyah and F. Hirzebruch. In the 1970's, the generalisation of *K-theory* to C^* -algebras quickly proved itself to be an informative tool for distinguishing C^* -algebras, and crucial to developing index theory for noncompact manifolds.

An important problem in C^* -algebraic *K-theory* is to settle the *Baum–Connes conjecture*, which proposes a geometric description of the *K-theory* of reduced group C^* -algebras; this conjecture and its partial confirmations have numerous applications in geometry and topology.

Realising that some core ingredients of the Baum–Connes theory are coarse-geometric in nature lead J. Roe, N. Higson, G. Yu and others to further developments. The heart of the Baum–Connes theory is the existence of an analytic assembly/index map, taking a generalised elliptic operator to its index in the *K-theory* of a “noncommutative space”, an appropriate operator algebra. From the coarse perspective, the C^* -algebras to use are Roe C^* -algebras, associated to discrete metric spaces of bounded geometry (e.g. Cayley graphs of finitely generated groups). The uniform Roe C^* -algebra $C_u^*(X)$ of a discrete metric space X of bounded geometry is the norm closure of the set of operators on $l^2(X)$ with finite propagation (other Roe C^* -algebras arise by related constructions). The coarse Baum–Connes conjecture predicts a geometric description for the *K-theory* of Roe C^* -algebras. Results about the coarse Baum–Connes conjecture have seen powerful consequences in geometry and topology, such as Yu's confirmation of the Novikov conjecture for groups admitting a coarse embedding into a Hilbert space REF, and the nonexistence of metrics with positive scalar curvature on complete Riemannian manifolds with finite asymptotic dimension REF.

In another direction, topological *K-theory* is a core component of the Elliott classification programme for C^* -algebras. This programme focuses on the problem of proving that Elliott algebras (simple, separable, amenable C^* -algebras) are isomorphic, provided that they have (compatibly) isomorphic ordered *K-theory* and trace spaces. In fact, it was famously conjectured by G.A. Elliott that ordered *K-theory* paired with traces is a complete invariant for the class of Elliott algebras, until counterexamples emerged in the work of M. Rørdam [13] and A. Toms [18], building on examples due to J. Villadsen [20, 21]. These examples forced a recognition that classification requires some sort of “low topological dimension” hypotheses; a robust property of this nature is conjectured by A. Toms and W. Winter to exist with diverse characterisations.

The study of “low topological dimension” type properties for C^* -algebras is the subject of C^* -algebra regularity. Featuring prominently in this recent topic is nuclear dimension, a noncommutative generalisation of topological dimension, defined by W. Winter and J. Zacharias [25] by combining H. Lebesgue's covering dimension with E.C. Lance's completely positive approximation property. In light of the major impact made by nuclear dimension in C^* -algebra theory, analogous numerical invariants, called Rokhlin dimension and dynamic asymptotic dimension, have been imported into dynamical systems REF.

The *asymptotic dimension* of a discrete metric space is the coarse analogue of H. Lebesgue's covering dimension, defined by M. Gromov REF. W. Winter and J. Zacharias found an interesting link between nuclear dimension and asymptotic dimension: they showed that the nuclear dimension of the uniform Roe C^* -algebra $C_u^*(X)$ is bounded above by the asymptotic dimension of the underlying space X .

More background, depending on choice of topics in 2.4. -AT

2.2 National importance

As mentioned in the track record, operator algebras is one of the most active areas of modern analysis. Developments in the classification of C^* -algebras have accelerated in the last decade. Major recent events in C^* -algebras have been held at the MFO (8/16), Mittag-Leffler Institute (1-4/16), Abel Symposium (8/15), BIRS (10/14), and the Fields Institute (1-6/14). Two Fields medals (A. Connes and V. Jones) have been awarded for work in operator algebras.

A number of current and recent EPSRC-funded projects concern research under the topics of the proposed workshop; the workshop will give these investigators an opportunity to disseminate their findings. The related EPSRC-funded projects are “ C^* -algebras of semigroups and dynamical systems,” EP/M009718/1 (PI: X. Li), “Regularity and dimension for C^* -algebras” EP/M009718/1 (PI: A. Tikuisis), “Coarse geometry and cohomology of large data sets” EP/I016945/1 (PI: J. Brodzki), “Quantum groups and noncommutative geometry,” EP/L013916/1 (PI: C. Voigt), “The Haagerup subfactor, K-theory and conformal field theory,” EP/J003352/1 (PI: D.E. Evans), and “The Baum-Connes conjecture for translation algebras” EP/J015806/1 (PI: N. Wright). “Geometric and analytic aspects of infinite groups” EP/H027998/1 (PI: C. Drutu)-JS

Mention relation to recent workshops and conferences in UK: CStAR, ICMS, Nottingham. -AT

The workshop will lead to increased interaction between the British research communities in operator algebras and coarse geometry. These are both very active subjects; operator algebras are well-represented in Aberdeen, Belfast, Glasgow, and Newcastle, while Sheffield and Southampton contains researchers in coarse geometry. Through strengthening the connections between UK researchers in these two fields and building dialogue with leading international figures, the United Kingdom will be placed at the forefront of activity concerning the interface of these subjects.

The field of operator algebras has long been a strength within the United Kingdom. Recent appointments in operator algebras in the United Kingdom include X. Li (QMUL), E. Kakariadis (Newcastle), A. Popov (Newcastle), C. Voigt (Glasgow), and M. Whittaker (Glasgow), as well as two of the organisers (J.S., and A.T.). Do Jan and I still count as recent appointments? -AT Gabor Elek, Lukasz Grabowski, Yemon Choi ? all Lancaster, more of a “L2-invariants, measured group theory”-JS This workshop will contribute to the United Kingdom taking a revived leadership position in operator algebras, providing international visibility for U.K.-based researchers.

2.3 Academic impact

A primary objective of the proposed workshop is to foster interaction between C^* -algebra theory and coarse geometry. Exchanges of ideas will have an impact in each of these areas. Researchers with expertise in one of these subjects will be exposed to the major developments in the other. Attention will be paid to open problems in the intersection of the two subjects, leading to collaborations in solving them.

- Say more things here, with minimal repetition from the National Importance section. -AT

2.4 Research objectives

The theme of the workshop will be to advance research in operator algebras and their connections to coarse geometry. Say more general stuff. -AT

The objectives fall under two main headings:

Dimension theories in C^* -algebras, coarse geometry, and dynamics. Inspired by the Lebesgue's classical notion of covering dimension for topological spaces, various notions of dimension have been defined for C^* -algebras, for (coarse) metric spaces, and for group actions on topological spaces:

- Asymptotic dimension for metric spaces is the earliest of these, introduced by Gromov in the '90s in the context of large-scale invariants for groups.
- Nuclear dimension (Winter–Zacharias) is a property of C^* -algebras which is prominent both in the Toms–Winter conjecture and as a key hypothesis in recent far-reaching classification theorems [10].

- Rokhlin dimension for group actions on topological spaces arose in efforts to compute nuclear dimension for crossed product C^* -algebras, and have since found applications to purely dynamical problems [Gabor mentioned some joint work with Gutman and someone – it isn't arXived yet but if we can say this it looks good I think](#) -AT.
- Dynamic asymptotic dimension is complementary form of dimension for group actions on topological spaces, and more generally for topological groupoids, introduced by Guentner, Willett, and Yu, with a view to K-theory computation via approximate Mayer–Vietoris sequences.

Certain connections are already known between these concepts:

- the asymptotic dimension of a discrete group coincides with the dynamic asymptotic dimension of the canonical action of the group on its Stone–Čech compactification;
- the nuclear dimension of a uniform Roe algebra is bounded above by the asymptotic dimension of the underlying metric space;
- the nuclear dimension of a crossed product can be bounded above in terms of either the dynamic asymptotic dimension or the Rokhlin dimension, as well as the topological dimension of the space and in the case of Rokhlin dimension, dimension-related properties of the group.

These relations seem the tip of the iceberg, and the full extent of the connections between these notions of dimension are yet unclear. Some specific problems that beg to be addressed are the following: [Add/delete/change questions on this list?](#) -AT

- what C^* -algebraic property for a uniform Roe algebra encodes the asymptotic dimension of the underlying metric space?
- what is the precise relationship between Rokhlin dimension and dynamic asymptotic dimension?
- is there an analogue of the Toms–Winter conjecture, for group actions on topological spaces?
- what does the nuclear dimension of a reduced group C^* -algebra tell us about the underlying group?

Groups and C^* -algebras arising from them. Group theoretic considerations have shaped the study of operator algebras throughout its history. Many exciting recent developments at this interface consider the study of the following objects:

- (1) Boundary actions of groups, in the sense of Furstenberg, which play a prominent role in recent work by Kalantar, Kennedy, Breuillard, and Ozawa on C^* -simplicity; they also provide interesting new examples of purely infinite classifiable C^* -algebras coming from non-amenable groups.
- (2) Exotic group C^* -algebras, that is, C^* -completions of group algebras which lie between the full and reduced group C^* -algebras. While it has long been known that these can exist, categorical approaches and connections to the Baum–Connes conjecture have spurred new interest in these.
- (3) The action of a group on its Stone–Čech completion, and minimal subsystems thereof. The crossed product by this action yields the uniform Roe algebra, a C^* -algebra which encodes the group up to quasi-isometry; connections have recently been found between pure infiniteness of this crossed product and non-amenability of the group.

TODO: more here. Problems or questions?

2.5 Programme

A number of talks will take place during the workshop; these will be of reasonably limited length to allow time for discussion and collaboration between participants. The talks will be delivered by about 18 invited keynote speakers (including 5? invited ICM speakers) and 8 contributed talks.

The workshop will provide an overview of recent developments in the topics of C^* -algebras and its connections to coarse geometry, and present the important open problems these areas. Contributed talks will give opportunities for postgraduate students and postdoctoral researchers to present their work.

The speakers we plan to invite are [list the speakers].

[Here's some initial ideas. Shall we "overbook" with a list of 20? Maybe canvas our scientific advisory panel for suggestions? At any rate, there's already too many names suggested!](#) -AT

ICM (5 names):

Cuntz (Münster), Izumi (Kyoto), Ozawa (Kyoto), Rørdam (Copenhagen), Schick (Göttingen)

UK (7 names):

Grabowski (Lancaster), Hunton (Durham), Li (Queen Mary University of London), Ying-Fen Lin (Belfast), Whittaker (Glasgow), Zacharias (Glasgow), Wright (Southampton)

Others (12 names):

Eckhardt (Miami), Hirshberg (Ben Gurion), Kennedy (Waterloo), Kerr (Texas A&M), Guentner (Hawaii at Manoa), Matui (Chiba), Niu (Wyoming), Putnam (Victoria), Roe (Penn State), Sierakowski (Wollongong), Strung (IMPAN), Willett (Hawaii at Manoa)

In addition to funding from EPSRC, we are applying for funding from the London Mathematical Society, the Edinburgh Mathematical Society, and the Glasgow Mathematical Journal Trust.

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