SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note:

1. $\sqrt{xy} = \sqrt{x} * \sqrt{y}$ 2. $\sqrt{(x/y)} = \sqrt{x} / \sqrt{y} = (\sqrt{x} / \sqrt{y}) * (\sqrt{y} / \sqrt{y}) = \sqrt{xy} / y$

SOLVED EXAMPLES

Ex. 1. Evaluate $\sqrt{6084}$ by factorization method .

Sol. **Method:** Express the given number as the product of prime factors.

Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get:

$6084 = 2^2 \times 3^2 \times 13^2$

$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$

$$\begin{array}{r|l} 2 & 6084 \\ \hline 2 & 3042 \\ \hline 3 & 1521 \\ \hline 3 & 507 \\ \hline 13 & 169 \\ \hline & 13 \end{array}$$

Ex. 2. Find the square root of 1471369.

Sol. **Explanation:** In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.

Now, bring down the next period *i.e.,* 47.

Now, trial divisor is 1 x 2 = 2 and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is 12 x 2 = 24 and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period *i.e.,* 69.

Now, the trial divisor is 121 x 2 = 242 and the trial dividend is 7269. So, we take 3as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

$$\begin{array}{r|l} 1 & 1471369\ (1213 \\ & \underline{1} \\ 22 & \ \ 47 \\ & \ \ \underline{44} \\ 241 & \ \ \ 313 \\ & \ \ \ \underline{241} \\ 2423 & \ \ \ \ 7269 \\ & \ \ \ \ \underline{7269} \\ & \ \ \ \ \ \ x \end{array}$$

Ex. 3. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$ **.**

Sol. Given expression = $\sqrt{248 + \sqrt{51 + 13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16.$

Ex. 4. If $a * b * c = \sqrt{(a + 2)(b + 3)} / (c + 1)$***, find the value of*** $6 * 15 * 3$.

Sol. $6 * 15 * 3 = \sqrt{(6 + 2)(15 + 3)} / (3 + 1) = \sqrt{8 * 18} / 4 = \sqrt{144} / 4 = 12 / 4 = 3.$

Ex. 5.*Find the value of* $\sqrt{25/16}$.

Sol. $\sqrt{25 / 16} = \sqrt{25} / \sqrt{16} = 5 / 4$

Ex. 6.*What is the square root of 0.0009?*

Sol. $\sqrt{0.0009} = \sqrt{9 / 1000} = 3 / 100 = 0.03.$

Ex. 7.*Evaluate* $\sqrt{175.2976}$.

Sol. Method: We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \sqrt{175.2976} = 13.24$

1	175.2976 (13.24
	1
23	75
	69
262	629
	524
2644	10576
	10576

x

Ex. 8.*What will come* in *place of question mark* in *each of the following* questions?

(i) $\sqrt{32.4 / ?} = 2$ ***(ii)*** $\sqrt{86.49} + \sqrt{5 + (\,?\,)^2} = 12.3.$

Sol. (i) Let $\sqrt{32.4 / x} = 2$. Then, $32.4/x = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1$.

(ii) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$.

Then, $9.3 + \sqrt{5+x^2} = 12.3 \Leftrightarrow \sqrt{5+x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2.$

Ex.9. Find the value of $\sqrt{\mathbf{0.289 / 0.00121}}$.

Sol. $\sqrt{0.289 / 0.00121} = \sqrt{0.28900/0.00121} = \sqrt{28900/121} = 170 / 11$.

Ex.10. If $\sqrt{1 + (x / 144)} = 13 / 12$***, the find the value of x.***

Sol. $\sqrt{1 + (x / 144)} = 13 / 12 \Rightarrow (1 + (x / 144)) = (13 / 12)^2 = 169 / 144$

$\Rightarrow x / 144 = (169 / 144) - 1$

$\Rightarrow x / 144 = 25/144 \Rightarrow x = 25$.

Ex. 11. Find the value of √3 up to three places of decimal.

Sol.

1	3.000000 (1.732
	1
27	200
	189
343	1100
	1029
3462	7100
	6924

$\therefore \sqrt{3} = 1.732$.

Ex. 12.***If √3 = 1.732, find the value of*** $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ ***correct to 3 places of decimal.*** **(S.S.C. 2004)**

Sol. $\sqrt{192} - (1 / 2)\sqrt{48} - \sqrt{75} = \sqrt{64 * 3} - (1/2) \sqrt{16 * 3} - \sqrt{25 * 3}$

$= 8\sqrt{3} - (1/2) * 4\sqrt{3} - 5\sqrt{3}$

$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732$

Ex. 13. Evaluate: $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Sol. Given exp. $= \sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

$\therefore$ Given exp $= \sqrt{(95 * 85 * 18900) / (17 * 19 * 21)} = \sqrt{5 * 5 * 900} = 5 * 30 = 150$.

Ex. 14.***Simplify*****:** $\sqrt{[(12.1)^2 - (8.1)^2] / [(0.25)^2 + (0.25)(19.95)]}$

Sol. Given exp. $= \sqrt{[(12.1 + 8.1)(12.1 - 8.1)] / [(0.25)(0.25 + 19.95)]}$

$= \sqrt{(20.2 * 4) /(0.25 * 20.2)} = \sqrt{4 / 0.25} = \sqrt{400 / 25} = \sqrt{16} = 4$.

Ex. 15. ***If x = 1 + √2 and y = 1 - √2, find the value of (x^2+ y^2).***

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2 * 3 = 6$.

Ex. 16. ***Evaluate: √0.9 up to 3 places of decimal.***

Sol.

9	0.900000(0.948
	81
184	900
	736
1888	16400
	15104

$\therefore \sqrt{0.9} = 0.948$

Ex.17. If √15 = 3.88, find the value of √ (5/3).

Sol. $\sqrt{(5/3)} = \sqrt{(5 * 3) / (3 * 3)} = \sqrt{15} / 3 = 3.88 / 3 = 1.2933.... = 1.29\overline{3}$.

Ex. 18. Find the least square number which is exactly divisible by 10,12,15 and 18.

Sol. L.C.M. of 10, 12, 15, 18 = 180.Now, 180 = 2 * 2 * 3 * 3 *5 = $2^2 * 3^2 * 5$.

To make it a perfect square, it must be multiplied by 5.

$\therefore$ Required number = $(2^2 * 3^2 * 5^2) = 900$.

Ex. 19. Find the greatest number of five digits which is a perfect square.

(R.R.B. 1998)

Sol. Greatest number of 5 digits is 99999.

3	99999(316
	9
61	99
	61
626	3899
	3756
	143

$\therefore$ Required number == (99999 - 143) = 99856.

Ex. 20. Find the smallest number that must be added to 1780 to make it a perfect square.

Sol.

4	1780 (42
	16
82	180
	164
	16

$\therefore$ Number to be added = $(43)^2 - 1780 = 1849 - 1780 = 69$.

Ex. 21. √2 = 1.4142, find the value of √2 / (2 + √2).

Sol. $\sqrt{2} / (2 + \sqrt{2}) = \sqrt{2} / (2 + \sqrt{2}) * (2 - \sqrt{2}) / (2 - \sqrt{2}) = (2\sqrt{2} - 2) / (4 - 2)$

$= 2(\sqrt{2} - 1) / 2 = \sqrt{2} - 1 = 0.4142.$

22. If x = (√5 + √3) / (√5 - √3) and y = (√5 - √3) / (√5 + √3), find the value of ($x^2 + y^2$).

Sol.

$x = [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})] * [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} + \sqrt{3})] = (\sqrt{5} + \sqrt{3})^2 / (5 - 3)$

$=(5 + 3 + 2\sqrt{15}) / 2 = 4 + \sqrt{15}.$

$y = [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})] * [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} - \sqrt{3})] = (\sqrt{5} - \sqrt{3})^2 / (5 - 3)$

$=(5 + 3 - 2\sqrt{15}) / 2 = 4 - \sqrt{15}.$

$\therefore \quad x^2 + y^2 = (4 + \sqrt{15})^2 + (4 - \sqrt{15})^2 = 2[(4)^2 + (\sqrt{15})^2] = 2 * 31 = 62.$

Ex. 23. Find the cube root of 2744.

Sol. **Method:** Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors. Resolving 2744 as the product of prime factors, we get:

2	2744
2	1372
2	686
7	343
7	49
	7

$2744 = 2^3 \times 7^3.$

$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14.$

Ex. 24. By what least number 4320 be multiplied to obtain a number which is a perfect cube?

Sol. Clearly, $4320 = 2^3 * 3^3 * 2^2 * 5.$

To make it a perfect cube, it must be multiplied by $2 * 5^2$ i.e,50.