In [ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import Latex
```

In [ ]:
```python
T, Y = np.meshgrid(np.arange(0, 5, 0.25), np.arange(-3, 3, 0.25))
dYdT = (np.sin(T)) * (np.cos(Y))  # put f(t, y) here to find slope
U = 1 / (1 + dYdT**2)**0.5 * np.ones(T.shape)  # Normalizes the arrows to se
V = 1 / (1 + dYdT**2)**0.5 * dYdT

plt.figure()
plt.title('Direction Field for ' + r'$\frac{dy}{dt} = (\sin{t})(\cos{y})$')
Q = plt.quiver(T, Y, U, V)  # draws the arrows at (X, Y) with slope dYdX
```

In [ ]:
```python
T, Y = np.meshgrid(np.arange(0, 5, 0.25), np.arange(-3, 3, 0.25))
dYdT = (Y ** 2) - 1  # put f(t, y) here to find slope
U = 1 / (1 + dYdT**2)**0.5 * np.ones(T.shape)  # Normalizes the arrows to se
V = 1 / (1 + dYdT**2)**0.5 * dYdT

plt.figure()
plt.title('Direction Field for ' + r'$\frac{dy}{dt} = Y^2 - 1$')
Q = plt.quiver(T, Y, U, V)  # draws the arrows at (X, Y) with slope dYdX
```

Direction Field Analysis

The direction field corresponds to the differential equation $\frac{dy}{dt} = y^2 - 1$

Observations:

1. Equilibrium Points: The equation has equilibrium points at $y = -1$ and $y = 1$, where $\frac{dy}{dt} = 0$. These are horizontal lines on the graph.
2. Behavior Based on Initial Value:
   - If $y(0) > 1$, then $\frac{dy}{dt} > 0$, and y increases without bound as $t \to \infty$. Thus, $y \to \infty$.
   - If $-1 < y(0) < 1$, then $\frac{dy}{dt} < 0$, and y decreases toward $y = -1$ as $t \to \infty$.
   - If $y(0) < -1$, then $\frac{dy}{dt} > 0$, and y increases toward $y = -1$ as $t \to \infty$.
3. Stability:
   - The equilibrium point at $y = -1$ is **(asymptotically) stable**, meaning solutions starting near this value tend to approach it.
   - The equilibrium point at $y = 1$ is **unstable**, meaning solutions starting near this value tend to move away from it.

Summary of Dependency on Initial Value:

The long-term behavior of $y(t)$ depends on the initial value $y(0)$:

- For $y(0) > 1: \; y(t) \to \infty$.
- For $-1 < y(0) < 1: \; y(t) \to -1$.
- For $y(0) < -1: \; y(t) \to -1$.