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MAT 157 Problem Set I 02016/09/19

Let $a, b, x, y \in \mathbb{R}$.
Let $\mathbb{F}$ be a field.

Lemma 0.1. *Cancellation Property*

$$\forall a, b, c \in \mathbb{F} : a + c = b + c \Leftrightarrow a = b \tag{1}$$
$$\forall a, b, c \in \mathbb{F}, c \neq 0 : ac = bc \Leftrightarrow a = b \tag{2}$$

Proof. Suppose $a + c = b + c$.

$$\begin{aligned}
\exists\ (-c) : c + (-c) = 0 &\quad \text{Existence of an Additive Inverse} && (3)\\
\Rightarrow\ (a + c) + (-c) = (b + c) + (-c) &\quad \text{Definition of } = && (4)\\
\Rightarrow\ a + (c + (-c)) = b + (c + (-c)) &\quad \text{Associative Law} && (5)\\
\Rightarrow\ a + 0 = b + 0 &\quad \text{Existence of an Additive Inverse} && (6)\\
\Rightarrow\ a = b &\quad \text{Existence of an Additive Identity} && (7)\\
& && (8)
\end{aligned}$$

Suppose now $ac = bc$.

$$\begin{aligned}
\exists\ c^{-1} : cc^{-1} = 1 &\quad \text{Existence of an Additive Inverse} && (9)\\
\Rightarrow\ (ac)c^{-1} = (bc)c^{-1} &\quad \text{Definition of } = && (10)\\
\Rightarrow\ a(cc^{-1}) = b(cc^{-1}) &\quad \text{Associative Law} && (11)\\
\Rightarrow\ a \times 1 = b \times 1 &\quad \text{Existence of a Multiplicative Inverse} && (12)\\
\Rightarrow\ a = b &\quad \text{Existence of an Additive Identity} && (13)
\end{aligned}$$

□

Lemma 0.2. $\forall a \in \mathbb{F} : a \times 0 = 0$

Proof.

$$\begin{aligned}
0 + 0 &= 0 &\quad \text{Existence of an Additive Identity} && (1)\\
\Rightarrow a \times (0 + 0) &= a \times 0 + a \times 0 &\quad \text{Distributive Law} && (2)\\
&= a \times 0 &\quad \text{Definition of } = && (3)\\
(a \times 0 + a \times 0) - (a \times 0) &= a \times 0 - (a \times 0) &\quad \text{Definition of } = && (4)\\
\Rightarrow a \times 0 + (a \times 0 - a \times 0) &= 0 &\quad \text{Associative Law} && (5)\\
& &\quad \text{and Existence of an Additive Inverse} &&\\
\Rightarrow a \times 0 + 0 &= 0 &\quad \text{Existence of an Additive Inverse} && (6)\\
\Rightarrow a \times 0 &= 0 &\quad \text{Existence of an Additive Identity} &&
\end{aligned}$$

□

Lemma 0.3. $\forall a, b \in \mathbb{R} : (-a)b = -ab$

Proof.

$$
\begin{aligned}
a + (-a) &= 0 && \text{Existence of an Additive Inverse} && (1)\\
\Rightarrow ab + (-a)b &= ba + b(-a) && \text{Commutative Law} && (2)\\
\Rightarrow b(a + (-a)) &= b \times 0 && \text{Distributive Law} && (3)\\
& && \text{and Existence of an Additive Inverse} && (4)\\
&= 0 && \text{Lemma 0.2} && (5)\\
\Rightarrow ab + (-a)b &= 0 && \text{Definition of } = && (6)\\
\Rightarrow (-a)b + ab &= 0 && \text{Commutative Law} && (7)\\
(-a)b + ab - ab &= 0 - ab && \text{Definition of } = && (8)\\
\Rightarrow (-a)b + 0 &= -ab && \text{Existence of an Additive Inverse} && (9)\\
& && \text{and Existence of an Additive Identity} && \\
&= (-a)b && \text{Existence of an Additive Identity} && (10)
\end{aligned}
$$

□

Corollary 0.3.1. $\forall a \in \mathbb{R} : -b = (-1)b$

Proof. From Lemma 0.3, if $a = 1$, then $(-1)b = -1 \times b$

$$
\begin{aligned}
-1 \times b &= -b \times 1 && \text{Commutative Law} && (1)\\
\Rightarrow (-1)b &= -b && \text{Definition of } = && (2)\\
& && \text{and Existence of a Multiplicative Identity} &&
\end{aligned}
$$

□

Lemma 0.4. $-(-a) = a$

Proof.

$$
\begin{aligned}
a + (-a) &= 0 && \text{Existence of an Additive Inverse} && (1)\\
(-1)(a + (-a)) &= (-1)0 && \text{Definition of } = && (2)\\
(-1)a + (-1)(-a) &= 0 && \text{Distributive Law} && (3)\\
& && \text{and Lemma 0.2} && \\
\Leftrightarrow -a - (-a) &= 0 && \text{Corollary 0.3.1} && (4)\\
a + (-a - (-a)) &= a + 0 && \text{Definition of } = && (5)\\
(a - a) - (-a)) &= a && \text{Associative Law} && (6)\\
& && \text{and Existence of an Additive Identity} && (7)\\
0 - (-a) &= a && \text{Existence of an Additive Inverse} && (8)\\
-(-a) &= a && \text{Existence of an Additive Identity} && (9)
\end{aligned}
$$

□

1

Theorem 1.1. $a > 0 \land x > y \Rightarrow ax > ay$

Proof.

$$
\begin{aligned}
x > y &\Leftrightarrow x - y \in \mathbb{P} && \text{Definition of } \mathbb{P} \text{ and } > && (1)\\
a \in \mathbb{P} &\Rightarrow a(x-y) \in \mathbb{P} && \text{Multiplicative Closure of } \mathbb{P} && (2)\\
a(x-y) = ax - ay \in \mathbb{P} &\Rightarrow ax - ay > 0 && \text{Distributive Law} && (3)\\
&\Leftrightarrow ax > ay && \text{Definition of } > && (4)
\end{aligned}
$$

□

Theorem 1.2. $a < 0 \wedge x > y \Rightarrow ax < ay$

Proof.

$$
\begin{aligned}
a < 0 &\Rightarrow -a \in \mathbb{P} && \text{Trichotomy Law} && (1)\\
x > y &\Leftrightarrow x - y \in \mathbb{P} && \text{Definition of } \mathbb{P} \text{ and } > && (2)\\
\Rightarrow \ (-a)(x-y) &= (-a)x + (-a)(-y) \in \mathbb{P} && \text{Distributive Law, Definition of } - && (3)\\
& && \text{and Multiplicative Closure of } \mathbb{P} &&\\
&= -ax - a(-y) && \text{Lemma 0.3} && (4)\\
-ax - a(-y) &= -ax - (-y)a && \text{Commutative Law} && (5)\\
-ax - (-y)a &= -ax + ya && \text{Lemma 0.4} && (6)\\
\Rightarrow -ax + ay &= ay - ax && \text{Commutative Law} && (7)\\
&\Rightarrow ay - ax \in \mathbb{P} && \text{Multiplicative Closure of } \mathbb{P} && (8)\\
&\Rightarrow ay > ax && \text{Definition of } > && (9)\\
&\Leftrightarrow ax < ay && \text{Definition of } < && (10)
\end{aligned}
$$

□

Theorem 1.3. $-0 = (-1)0$

Proof. From Corollary 0.3.1, if $a = 0$, by Existence of an Additive Inverse $\exists -0 : -0 = (-1)0$. □

Theorem 1.4. $\forall x, y \in \mathbb{R} : x, y > 0 \wedge \frac{1}{x} < \frac{1}{y} \Rightarrow x > y$

Proof.

$$
\begin{aligned}
x, y > 0 &\Rightarrow x, y \in \mathbb{P} && \text{Definition of } > && (1)\\
&\Rightarrow xy \in \mathbb{P} && \text{Multiplicative Closure of } \mathbb{P} && (2)\\
\tfrac{1}{x} < \tfrac{1}{y} &\Rightarrow \tfrac{1}{y} - \tfrac{1}{x} \in \mathbb{P} && \text{Definition of } < && (3)\\
&\Rightarrow xy(\tfrac{1}{y} - \tfrac{1}{x}) \in \mathbb{P} && \text{Multiplicative Closure of } \mathbb{P} && (4)\\
&\Rightarrow (xy)\tfrac{1}{y} - (xy)\tfrac{1}{x} \in \mathbb{P} && \text{Distributive Law} && (5)\\
&\Rightarrow (xy)\tfrac{1}{y} - (yx)\tfrac{1}{x} \in \mathbb{P} && \text{Commutative Law} && (6)\\
&\Rightarrow x(y \times \tfrac{1}{y}) - y(x \times \tfrac{1}{x}) \in \mathbb{P} && \text{Associative Law} && (7)\\
&\Rightarrow x \times 1 - y \times 1 \in \mathbb{P} && \text{Existence of an Additive Inverse} && (8)\\
&\Rightarrow x - y \in \mathbb{P} && \text{Existence of a Multiplicative Identity} && (9)\\
&\Rightarrow x > y && \text{Definition of } > && (10)
\end{aligned}
$$

□

Theorem 1.5. $x^2 = y^2 \Leftrightarrow x = y \vee x = -y$

Lemma 1.5.1. $x = y \vee x = -y \Rightarrow x^2 = y^2$

Proof. Suppose $x = y$.

$$x \times x = x \times y \quad \text{Definition of } = \tag{1}$$
$$y \times y = x \times y \quad \text{Definition of } = \tag{2}$$
$$\Rightarrow x^2 = y^2 \quad \text{Transitive Law} \tag{3}$$

Suppose $x = -y$.

$$x \times x = x \times (-y) \quad \text{Definition of } = \tag{4}$$
$$(-y)(-y) = x \times (-y) \quad \text{Definition of } = \tag{5}$$
$$(-y)(-1)y = (-1)(-y)y \quad \text{Corollary 0.3.1 and Commutative Law} \tag{6}$$
$$= -(-y)y \quad \text{Corollary 0.3.1} \tag{7}$$
$$= y^2 \quad \text{Lemma 0.4} \tag{8}$$
$$\Rightarrow x^2 = y^2 \quad \text{Transitive Law} \tag{9}$$

Lemma 1.5.2. $\forall a, b \in \mathbb{R} : ab = 0 \Leftrightarrow a = 0 \vee b = 0$

Proof. By Commutative Law and Lemma 0.2, $a = 0 \Rightarrow ab = ba = b \times 0 = 0$.

Similarly, $b = 0 \Rightarrow ab = a \times 0 = 0$. If $ab = 0$ and $b \neq 0$, $\exists\ b^{-1} : abb^{-1} = 0 \times b^{-1}$, hence by Commutative Law and Existence of a Multiplicative Inverse $a \times 1 = b^{-1} \times 0$, then by Existence of a Multiplicative Identity and Lemma 0.2 $a = 0$.

If $ab = 0$ and $a \neq 0$, $\exists\ a^{-1} : a^{-1}ab = a^{-1} \times 0$, hence by Commutative Law and Lemma 0.2 $aa^{-1}b = 0$, then by Existence of a Multiplicative Inverse $1 \times b = 0$, and by Commutative Law and Existence of a Multiplicative Identity $b \times 1 = b = 0$.

If $a = 0 \wedge b = 0$, then by Lemma 0.2 $ab = 0 \times 0 = 0$ □

Lemma 1.5.3. $\forall a, b \in \mathbb{F} : (a + b)(a - b) = a^2 - b^2$

Proof.

$$(a + b)(a - b) = (a + b)a + (a + b)(-b) \quad \text{Distributive Law} \tag{1}$$
$$= (a(a + b)) - (b(a + b)) \quad \text{Commutative Law} \tag{2}$$
$$= (a^2 + ab) - (ba - b^2) \quad \text{Distributive Law} \tag{3}$$
$$= a^2 + (ab - (ba - b^2)) \quad \text{Associative Law} \tag{4}$$
$$= a^2 + ((ab - ba) - b^2) \quad \text{Associative Law} \tag{5}$$
$$= a^2 + ((ab - ab) - b^2) \quad \text{Commutative Law} \tag{6}$$
$$= a^2 + (0 - b^2) \quad \text{Existence of an Additive Inverse} \tag{7}$$
$$= a^2 - b^2 \quad \text{Existence of an Additive Identity} \tag{8}$$

□

If $x^2 = y^2$, by Cancellation Property and Lemma 1.5.3 $(x-y)(x+y) = 0$.

Therefore, by Lemma 1.5.2 and Cancellation Property $x = y \vee x = -y$ □

Theorem 1.6. $x^3 = y^3 \Leftrightarrow x = y$

Proof. By 1.5, if $x = y$, then $x^2 = y^2$.

$$
\begin{aligned}
x^2 = y^2 &\Rightarrow x^3 = xy^2 && \text{Definition of} = && (1)\\
y^2 = x^2 &\Rightarrow y^3 = xy^2 && \text{Definition of} = && (2)\\
&\Rightarrow x^3 = y^3 && \text{Transitive Law} && (3)\\
& && && (4)
\end{aligned}
$$

Lemma 1.6.1. $x^3 - y^3 = (x-y)(x^2 + xy + y^3)$

Proof.

$$
\begin{aligned}
(x-y)(x^2+xy+y^2) &= x^3 + x^2y + && \text{Distributive Law} && (1)\\
&+xy^2 - yx^2 - yxy - y^3 && && \\
&= x^3 + x^2y + xy^2 - && \text{Commutative Law} && (2)\\
&-x^2y - xy^2 - y^3 && && \\
&= x^3 + (x^2y - x^2y) + && \text{Commutative Law} && (3)\\
&+(xy^2 - xy^2) - y^3 && && \\
&= x^3 + 0 + 0 - y^3 && \text{Existence of an Additive Inverse} && (4)\\
&= x^3 - y^3 && \text{Existence of an Additive Identity} && (5)
\end{aligned}
$$

□

Definition 1.1. A number $x \in \mathbb{R}$ is called a *postive square root* of a number $a \in \mathbb{R}$ if $x \in \mathbb{P}$ and $x^2 = a$. x is denoted as $\sqrt{a}$.

Suppose now that there is an equation $ax^2 + bx + c$ with $a, b, c, x \in \mathbb{R} \wedge a \neq 0$.

Definition 1.2. A *discriminant* Δ *of a quadratic* is defined as $\Delta = \sqrt{b^2 - 4ac}$.

Definition 1.3. A number $x \in \mathbb{R}$ such that $ax^2 + bx + c = 0$ is called the real root of the equation.

Remark. For a real root of the equation to exist, Δ must be an element in $\mathbb{P}$.

If $x^3 = y^3$, by Lemma 1.6.1, Cancellation Property and Lemma 1.5.2 $x = y \vee x^2 + xy + y = 0$. Consider the case when $x^2 + xy + y = 0$. Note that it is an equation in the form $ax^2 + bx + c$, hence, by the remark above, $\Delta = \sqrt{-3} \notin \mathbb{R}$. Therefore, the real root of this equation does not exist. Hence, $x = y$ is the only case satisfying the conditions.

□

Lemma 1.7. $\forall a \in \mathbb{R} : a^2 \geq 0$

Proof. Suppose that $a > 0$. Therefore, $a \in \mathbb{P}$ by Definition of $\mathbb{P}$. Hence, $a^2 \in \mathbb{P}$ by Multiplicative Closure of $\mathbb{P}$.
Suppose now that $a < 0$. Therefore, $-a \in \mathbb{P}$ by Trichotomy Law . Hence, $(-a)(-a) \in \mathbb{P}$ by Multiplicative Closure of $\mathbb{P}$. But by Commutative Law , Lemma 0.3 and Lemma 0.4, $(-a)(-a) = a^2$. Thus,
$\forall a \in \mathbb{R}, a < 0 : a^2 > 0$.
Suppose now that $a = 0$. Thus, by Lemma 0.2 $aa = 0 \times 0 = 0$.

□

Lemma 1.8. $\forall a \in \mathbb{P} : a^{-1} \in \mathbb{P}$

Proof. Suppose it is not the case, i.e. $\exists\ a^{-1} < 0$. Therefore, $-a^{-1} \in \mathbb{P}$ by Trichotomy Law . But by Lemma 0.3, Commutative Law and Existence of an Additive Inverse $a(-a^{-1}) = -1 \notin \mathbb{P}$, which contradicts Multiplicative Closure of $\mathbb{P}$. Thus, $a^{-1} \in \mathbb{P}$ □

Theorem 1.9. *AM-GM Inequality*

$$\forall a, b \in R, a \geq 0, b \geq 0 : \sqrt{ab} \leq \frac{a+b}{2}$$

Proof.

Lemma 1.9.1. $(a-b)^2 = a^2 - 2ab + b^2$

Proof.

$$\begin{aligned}
&= (a-b)a - (a-b)b && \text{Distributive Law} && (1)\\
& && \text{Commutative Law , Lemma 0.3.1} && (2)\\
&= a(a-b) - b(a-b) && \text{Commutative Law} && (3)\\
&= a^2 - ab - ba + b^2 && \text{Distributive Law} && (4)\\
& && \text{Lemma 0.3, Lemma 0.4} && (5)\\
&= a^2 - ab - ab + b^2 && \text{Commutative Law} && (6)\\
&= a^2 - 2ab + b^2 && \text{Definition of} = && (7)
\end{aligned}$$

□

Lemma 1.9.2. $4ab \leq (a+b)^2$

Proof.

$$
\begin{aligned}
(a+b)^2 \in \mathbb{P} & & \text{Lemma 1.7} \quad (1)\\
&= (a+b)a + (a+b)b & \text{Distributive Law} \quad (2)\\
&= a(a+b) + b(a+b) & \text{Commutative Law} \quad (3)\\
&= a^2 + ab + ba + b^2 & \text{Distributive Law} \quad (4)\\
&= a^2 + ab + ab + b^2 & \text{Commutative Law} \quad (5)\\
&= a^2 + 2ab + b^2 & \text{Definition of} = \quad (6)\\
\Rightarrow (a+b)^2 - 4ab &= a^2 - 2ab + b^2 & \text{Commutative Law} \quad (7)\\
&= (a-b)^2 \geq 0 & \text{Lemma 1.9.1, Lemma 1.7} \quad (8)\\
&\Leftrightarrow (a+b)^2 - 4ab \geq 0 & \text{Definition of} = \quad (9)\\
&\Leftrightarrow (a+b)^2 \geq 4ab & \text{Existence of an Additive Inverse} \quad (10)\\
& & \text{Existence of an Additive Identity}\\
& & \text{Additive Closure of } \mathbb{P}
\end{aligned}
$$

□

Lemma 1.9.3. $\forall a, b \in R, a \geq 0, b \geq 0 : 4ab \leq (a+b)^2 \Rightarrow \sqrt{ab} \leq \frac{a+b}{2}$

Proof. If $a = 0 \vee b = 0$, LHS $\Leftrightarrow 0 \leq b^2 \vee 0 \leq a^2$, RHS $\Leftrightarrow 0 \leq \frac{b}{2} \vee 0 \leq \frac{a}{2}$, which are both trivially true from the condition that $a, b \geq 0$.

Suppose now that $a > 0 \wedge b > 0$.

$(a+b)^2 - 4ab = (a+b-2\sqrt{ab})(a+b+2\sqrt{ab}) \in P$. Since $a, b \in \mathbb{P}$, by Lemma 1.8 $(a+b+2\sqrt{ab})^{-1} \in \mathbb{P}$ and hence by Multiplicative Closure of $\mathbb{P}$ $a+b-2\sqrt{ab} \in \mathbb{P}$. Thus, $\frac{a+b}{2} \geq \sqrt{ab}$ is implied by $4ab \leq (a+b)^2$ □

Therefore, from Lemma 1.9.2 and Lemma 1.9.3,

$$\forall a, b \in R, a \geq 0, b \geq 0 : \sqrt{ab} \leq \frac{a+b}{2}$$

□

2

2.1

If $x = -2 \wedge y = 1$, $x^2 > y^2$ but $x < y$.

2.2

$\sqrt{12} = 2\sqrt{3}$.
Suppose that $\sqrt{3}$ is rational, i.e. $\exists\ a, b \in \mathbb{Z}, (a, b) = 1 : \sqrt{3} = \frac{a}{b}$.
Therefore, $a^2 = 3b^2$. Hence, $3|a^2$. But then $3|a$, and thus $9|a^2$. Assume $\exists k \in \mathbb{Z} : a^2 = 9k$. So $b^2 = 3k$, and, similarly, $3|b$. Therefore (a, b) is at least 3, which is a contradiction. Since there are no a, b satisfying the conditions, there are no $2a, b$ such that $2\sqrt{3} \in \mathbb{Q}$. Ergo, $\sqrt{12}$ is irrational.

2.3

If $a = 0 \wedge b = \sqrt{3}$, $a + b = \sqrt{3}$, which is irrational, but $0 \in \mathbb{Q}$.

2.4

If $a = 1 \wedge b = -1$, $\min(|a|, |b|) = 1$, while $|1 - 1| = 0$, which is a contradiction.

3

Suppose $\exists\ a, b \in \mathbb{Z}, (a, b) = 1 : \sqrt{3} + \sqrt{5} = \frac{a}{b}$.
Therefore, $5 = \frac{a^2}{b^2} - 2 \times \sqrt{3} \times \frac{a}{b} + 3$, or $2 = \frac{a^2}{b^2} - 2 \times \sqrt{3} \times \frac{a}{b}$. Hence, $\frac{1}{2 \times \frac{a}{b}}(\frac{a^2}{b^2} - 2) = \sqrt{3}$. LHS implies that $\sqrt{3}$ is rational, which is a contradiction. Thus, $\sqrt{3} + \sqrt{5}$ is irrational.

4

4.1

$= 2 - \sqrt{3}$, since $4 > 3$

4.2

$(\sqrt{5} + \sqrt{3})^2 - 25 = 8 + 2\sqrt{15} - 25 = 2\sqrt{15} - 7 = \sqrt{60} - \sqrt{49} > 0$
Hence, $= \sqrt{5} + \sqrt{3} - 5$

4.3

Since $17 > 12$, $\sqrt{17} - \sqrt{12} > 0$, and the dominator becomes $\sqrt{12} + 1 - \sqrt{17}$.
$1 + 12 + 2\sqrt{12} - 17 = 4\sqrt{3} - 4 = 4(\sqrt{3} - \sqrt{1}) > 0$.
$10 - (\sqrt{5} + 1)^2 = 10 - (6 + 2\sqrt{5}) = 4 - 2\sqrt{5} = 2(\sqrt{4} - \sqrt{5}) < 0$.
Therefore, $= \frac{\sqrt{5}+1-\sqrt{10}}{1+\sqrt{12}-\sqrt{17}}$.

4.4

$3 - \frac{6^5}{5^5} = 3 - \frac{36 \times 36 \times 6}{625 \times 5} = 3(1 - \frac{12 \times 36 \times 6}{625 \times 5}) = 3(\frac{3125 - 432 \times 6}{3125}) = 3(\frac{3125 - 2592}{3125}) > 0$
Therefore, $= 3^{\frac{1}{5}} - \frac{6}{5}$

5

Theorem 5.1. $\forall n \in \mathbb{N} : \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Proof.

$$1^3 = \tfrac{1^2(1+1)^2}{4} \qquad \text{Base Case} \tag{1}$$

Assume that:

$$\exists k \in \mathbb{N} : \textstyle\sum_{i=1}^{k} i^3 = \tfrac{k^2(k+1)^2}{4} \qquad \text{Inductive Step} \tag{2}$$

Consider the case n=k+1:

$$\textstyle\sum_{i=1}^{k} i^3 + (k+1)^3 = \tfrac{k^2(k+1)^2}{4} + (k+1)^3 \qquad \text{Inductive Hypothesis} \tag{3}$$

$$\Leftrightarrow (k+1)^2 \tfrac{k^2+4k+4}{4} = \tfrac{(k+1)^2(k+2)^2}{4} \qquad \text{Rearrangement} \tag{4}$$

which is exactly the hypothesis in case $n = k + 1$.

Therefore, if the hypothesis is true in case $n = k$, it is true for $n = k + 1$.
But the hypothesis holds in case $n = 1$, hence

$$\forall n \in \mathbb{N} : \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

. $\square$

6

6.1

Consider the trivial case of two professors, Prof Y and Prof Z. After the statement of Professor X, Prof Y thinks that Prof Z must resign, while at the same time Prof Z knows that Prof Y should leave his place. Since both of them are not aware of their own mistakes, they would not resign in the first meeting. In the second meeting, however, they can infer the existence of their own mistakes, and hence both must resign.

In general case, the reasoning is similar. We are given that any Professor knows about 16 mistakes which are not their own. Hence, during the first 16 meetings no one would resign, but when the meeting finishes, everyone must go.

6.2

The statement of Professor X has given more information to each individual about how much the others know about his own possible mistakes, of which he was not aware before the statement.