Theorem 0.1

If a map between CW complexes is a *weak homotopy equivalence*, then it is a homotopy equivalence.

Remark 0.2. This does not hold for more general spaces. For example, Cantor's set is not a homotopy equivalence to discrete uncountable space. Similarly, the Warsaw circle, *connected topologist sine curve* is not homotopy equivalent to a point, because the length of the topologist sine curve has an infinitely large length.

Let G be a finite group. What can we say about spaces with the action of G and G-equivalent maps? What can we say about G-isovariant maps, maps which are equivariant and such that for any x, the stabiliser of $f(x)$ is the same as the stabiliser of x?

What if, provided a map between G-CW complexes induces equivariant isomorphisms on homotopy groups, then it's an equivariant homotopy equivalence? Unfortunately, this does not hold. However, we still want to something about equivariant homotopy equivalences.

Definition 0.3. A *model structure* on a category $\mathfrak{C}$, which has all limits and colimits, consists of 3 distinguished classes of morphims: fibrations (F), cofibrations (C), and weak equivalences (W).

We provide the following axioms, along with the property of *lifting*:

- if $g$is a fibration, cofibration or a weak equivalence, and f is a retract of g, then f is also.
- If $g \circ f$ can be composed and 2 out $f, g, g \circ f$ are one of F, C or W, then so is the third.
- every morphism f can be written as $p \circ i$, where p is a fibration, while i is a cofibration, and either one can also be a weak fibration.

Remark 0.4. What is a cofibration? A cofibration can be thought as an inclusion of a subcomplex of a CW complex.

We can now define a standard model structure on Top:

- W are weak homotopy equivalences.
- F are Serre fibrations
- C are whatever that has the lifing property with respect to acyclic fibration

We need some more definitions.

Definition 0.5. An object X is fibrant if $X \to *$ is a fibration.

Definition 0.6. An object X is cofibrant if $\emptyset \to X$is a cofibration.

Now, we have enough to prove the following lemma.

Lemma 0.7

Every object is weak equivalent to one that is both fibrant and cofibrant.

Having defined a cylinder object, we can define left and right homotopies, so that the following can be proven.

Theorem 0.8

If X, Y are both fibrant and cofibrant, then $f, g : X \to Y$ are left homotopy equivalent if and only if they are right homotopy equivalent.

Theorem 0.9

Fibrant-cofibrant objects are weak equivalent if and only if they are homotopy equivalent.

Do equivariant spaces and maps model structure for Top^G?

We can make the following choices:

- F are still equivariant Serre fibrations.
- A choice of W determines C

It turns out that cofibrant spaces are free G-CW complexes.