1 Mathematics of Flat Linkage Mechanisms

1.1 Introduction

Suppose that a linkage mechanism $ABCDEF$ is given, with each linkage having a known length. Some hinges are fixed and some are allowed to move. Suppose that A and E are fixed, and have coordinates $(0, 0)$ and $(1, 0)$ respectively, while moving points B, C, D and F have coordinates (x_1, y_1),(x_2, y_2), (x_3, y_3) and (x_4, y_4).

The configuration space of a linkage mechanism can be defined as a topological space, with a standard euclidean metric. Since links between each pair of hinges are straight lines, we can easily describe the relations between $\{x_i\}$ and $\{y_i\}$ with equations.

Problem (Direct Problem)**.**

What can we say about classes of configuration spaces from classes of linkage mechanisms up to homeomorphisms?

Problem (Inverse Problem)**.**

Given a class of configuration spaces, what can we say about the corresponding class of linkage mechanisms?

Theorem 1.1 (Thurston)

There exists a flat linkage mechanism such that its configuration space is your signature.

Definition 1.2. L is a linkage mechanism in a general position if $\epsilon_1 l_1 + \epsilon_2 l_2 + \cdots + \epsilon_n l_n \neq 0$ for all $\epsilon_i \in \{-1, 1\}$.

Exercise 1.3. The configuration space Conf L is unique up to the order of linkages.

Theorem 1.4

For all g there exists L_g such that Conf $L = M_g^2$, where M_g is a manifold of genus g.

Theorem 1.5

For all n-dimensional smooth oriented manifolds M^n there exists L_M such that Conf $L_M = \sqcup^k M^n$, where k is some finite positive integer.

Theorem 1.6

For all algebraic varieties X in $\mathbb{R}^n$ there exists L_X such that Conf $L_X = \sqcup^k X$, where k is some finite positive integer.