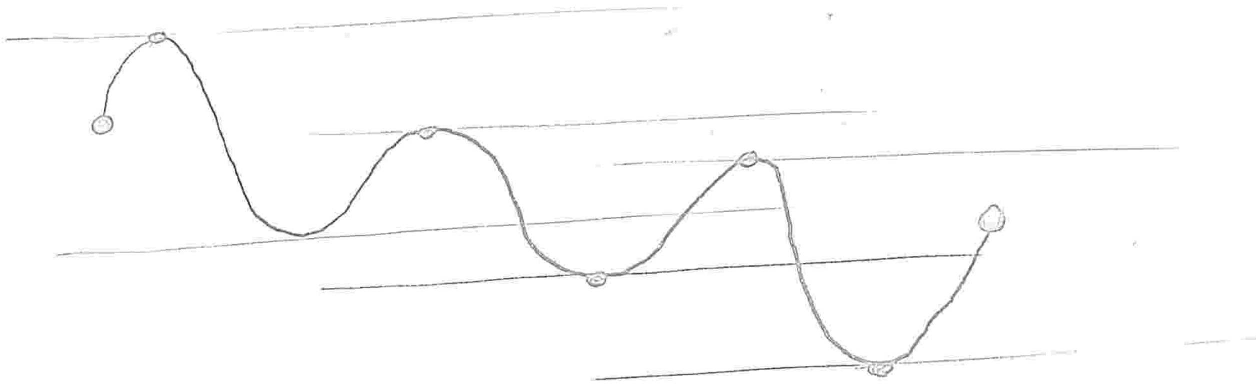
MAT157: 20161121

LOCAL MAXIMA | MINIMA

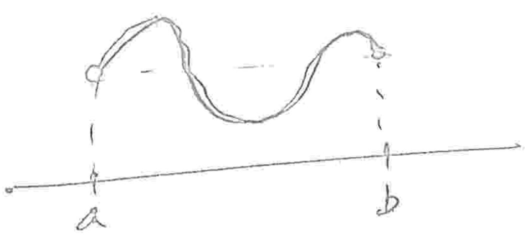


DEFINITION

$f(x)$ HAS A LOCAL MAXIMUM AT $x = a$ IF $\exists\ \delta > 0$ SUCH THAT $\forall\ x \in (a-\delta, a+\delta) \wedge \text{domain}(f)$ $f(x) \leq f(a)$.

$f(x)$ HAS A LOCAL MINIMUM AT $x = a$ IF $\exists\ \delta > 0$ SUCH THAT $\forall\ x \in (a-\delta, a+\delta) \wedge \text{domain}(f)$ $f(x) \geq f(a)$.

ROLLE'S THEOREM



SUPPOSE $f(x)$ IS CONTINUOUS ON $[a, b]$, DIFFERENTIABLE ON (a, b).

SUPPOSE $f(a) = f(b)$

THEN THERE IS $c \in (a, b)$ SO THAT $f'(c) = 0$.

PROOF.

f IS CONTINUOUS ON $[a, b]$

$\Rightarrow$ f has A MAXIMUM AND A MINIMUM.

IF f HAS A MAXIMUM OR A MINIMUM at $c \in (a, b)$, then $f'(c) = 0$, FROM THE PREVIOUSLY PROVEN RESULT.

OTHERWISE, max & min ARE AT ENDPOINTS. BUT $f(a) = f(b)$

$\Rightarrow$ max = min.

$\Rightarrow$ $f(x)$ IS CONSTANT,

$\Rightarrow$ $f'(c) = 0$, $\forall c \in (a, b)$.

MEAN VALUE THEOREM

IF $f(x)$ IS CONTINUOUS ON $[a,b]$, DIFFERENTIABLE ON (a,b), THEN $\exists\, c \in (a,b)$ SUCH THAT

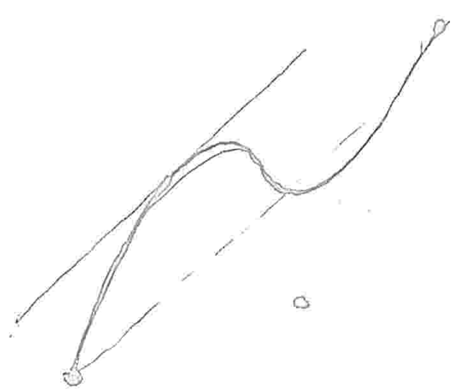
$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

PROOF

DEFINE

$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a)$$

FOR $x \in [a,b]$.



$$h(a) = f(a)$$
$$h(b) = f(a)$$

FROM ROLLE'S THEOREM,

$\exists\, c \in (a,b)$ SUCH THAT $h'(c)=0$,

$$h'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} \Rightarrow$$

$$h'(c)=0 \Leftrightarrow f'(c) = \frac{f(b)-f(a)}{b-a},$$

COROLLARY

IF $f'(x)=0$ AT SOME INTERVAL, THEN $f(x)$ IS CONSTANT ON THE INTERVAL.

PROOF.

SUPPOSE NOT.

CHOOSE a, b IN THE INTERVAL WITH

$$f(a) \neq f(b).$$

MVT $\Rightarrow \exists\, c \in (a,b)$ SUCH THAT

$$f'(c)=0, \quad \frac{f(b)-f(a)}{b-a} \neq 0.$$

COROLLARY

IF $f'(x) = g'(x)$ ON AN INTERVAL, THEN $\exists\, k \in \mathbb{R}$ SUCH THAT $g(x) = f(x)+k$ $\forall x$ IN THE INTERVAL.

PROOF

LET $h(x) = f(x) - g(x)$.

$$h'(x) = f'(x) - g'(x) = 0.$$

$\Rightarrow h(x) = -k$ IS CONSTANT.

$\Rightarrow g(x) = f(x) + k$.

DEFINITION

$f(x)$ IS INCREASING (STRICTLY INCREASING) IF $f(x) > f(y)$ WHENEVER $x > y$, AND $x, y \in \mathrm{DOM}(f)$

$f(x)$ IS DECREASING (STRICTLY DECREASING) IF $f(x) < f(y)$ WHENEVER $x > y$ AND $x, y \in \mathrm{DOM}(f)$.

THEOREM

IF $f(x)$ IS DEFINED ON (a, b)
AND DIFFERENTIABLE THERE, AND $f'(x) > 0$,
$\forall x \in (a, b)$ THEN $f(x)$ IS
INCREASING ON $(a, b$