MAT 157 120916

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* Marks:

Test	%	
1	25	Sep – Oct
2	30	Nov – Dec
3	30	Spring
Problems	15	

- Foundations of Analysis:

1. Numbers: $\mathbb{N}$, $\mathbb{N}_0$

$\mathbb{Z}$

$\mathbb{Q}$

$\mathbb{R}$

;; not just the distance.
if the universe is finite,
$\mathbb{R}$ is not suitable.
QM is formulated based on $\mathbb{R}$ –
not necessary.

;; closure is implied.

	$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
P1. $(a+b)+c = a+(b+c)$, $\forall a,b,c \in *$	✓	✓	✓	✓	✓
P2. $\exists\, 0$ :: $a+0=a$, $\forall a \in *$	✗	✓	✓	✓	✓
P3. $\forall a, \exists -a$ :: $a+(-a)=0$	✗	✗	✓	✓	✓
P4. $a+b = b+a, \forall a,b$	✓	✓	✓	✓	✓
P5. $(ab)c = a(bc)$ $\forall a,b,c$	✓	✓	✓	✓	✓
P6. $\exists\, 1$ :: $a \cdot 1 = a$, $\forall a$	✓	✓	✓	✓	✓
P7. $\forall a \neq 0$, $\exists a^{-1}$ $a \cdot a^{-1} = 1$	✗	✗	✗	✓	✓
P8 $ab = ba$, $\forall a,b$	✓	✓	✓	✓	✓
P9. $a(b+c) = a\cdot b + a \cdot c$	✓	✓	✓	✓	✓

T. $\mathbb{R} \setminus \mathbb{Q} \notin \varnothing$.

PR. Assume $\sqrt{2} = \frac{a}{b}$ where $\gcd(a,b) = 1$.

$2b^2 = a^2 \Rightarrow a^2$ is even.

$\Rightarrow a$ is even.

$\Rightarrow \exists k, b^2 = 2k^2 \Rightarrow b^2$ is even

$\Rightarrow b$ is even.

$\Rightarrow \gcd(a,b) \neq 1$. #

a:

:: even

$\exists k \in \mathbb{Z},\ a = 2k$

:: odd

$\exists k \in \mathbb{Z},\ a = 2k+1$

1.1. ORDERING

Let P be THE SET OF POSITIVE NUMBERS $(P \subset \mathbb{Q} \vee P \subset \mathbb{R})$

P10. $\forall a,\ a \in P \vee -a \in P \vee a = 0$.

P11. $\forall a, b \in P,\ a+b \in P$

P12. $\forall a, b \in P,\ ab \in P$

:: $a > b$ IFF $a - b \in P$

$a \geq b$ IFF $a - b \in P \vee \{0\}$