TUTORIAL
1 GB 221

MAT157:

CONSTRUCTION OF REAL NUMBERS II

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$\mathbb{R}$ IS THE SET OF ALL SETS $\alpha \subset \mathbb{Q}$:

(i) IF $x \in \alpha$, $y < x$, THEN $y \in \alpha$

(ii) $\alpha \neq \emptyset$

(iii) $\alpha \neq \mathbb{Q}$

(iv) IF $x \in \alpha$, $\exists y \in \mathbb{Q}.\ y > x;\ y \in \alpha$

A RATIONAL NUMBER r CORRESPONDS TO THE SET:

$$\alpha = \{ x \in \mathbb{Q} \mid x < r \}$$

$$0 = \{ x \in \mathbb{Q} \mid x < 0 \}$$

$a < b$ :: $a \subset b$

LUB(A) :: $\bigcup_{\alpha \in A} \alpha$;; A IS BOUNDED

DEFINE :: $\alpha + \beta = \{ x + y \mid x \in \alpha, y \in \beta \}$

SUPPOSE $z = x + y \in \alpha + \beta$

$w < z$

$w - x < z - x = y \in \beta$

$\Rightarrow w - x \in \beta$, $w = \underbrace{w-x}_{\beta} + \underbrace{x}_{\alpha}$

(ii) EXERCISE

(iii) $\exists$ $a > x$ $\forall x \in \alpha$

$b > y$, $\forall y \in \beta$

$a + b \notin \alpha + \beta$

(iv) SUPPOSE $x + y$ IS THE LARGEST ELEMENT.

$\exists x' > x, x' \in \alpha$

$x' + y \in \alpha + \beta$

$x' + y > x + y$ #

ASSOCIATIVE LAW

$(\alpha+\beta)+\gamma \subseteq \alpha+(\beta+\gamma) \wedge \alpha+(\beta+\gamma) \subseteq (\alpha+\beta)+\gamma$

COMMUTATIVITY

$\alpha+\beta = \beta+\gamma$

ADDITIVE IDENTITY

$\alpha + 0 \subseteq \alpha \quad \forall \alpha \in \mathbb{R} \qquad (x+y<x)$

$\alpha \subseteq \alpha + 0$

$x' > x, \quad x' \in \alpha, \quad x = x' + (x-x')$

$\therefore$ IF $\alpha \in \mathbb{R}$,

$\because -\alpha = \{x \in \mathbb{Q} \mid -x \notin \alpha, \ x \neq LUB(\alpha)\}$

$-\alpha = \{x \in \mathbb{Q} \mid -x > \alpha\}$

THEOREM $-\alpha \in \mathbb{R}$

(i) EXERCISE

(ii) EXERCISE

(iii) EXERCISE

(iv) SUPPOSE $x \in -\alpha$, SO $-x \notin \alpha$.

THEN $-x$, AS A REAL NUMBER, IS $\{r \in \mathbb{Q} \mid r < -x\} \not\supseteq \alpha$.

$\exists \ r \in \mathbb{Q} \ \because \begin{cases} r < -x \\ r \geq \alpha \end{cases}$

IF $r = \alpha$, THEN $\dfrac{r+(-x)}{2} \in \mathbb{Q}$ is $> \alpha$.