

# 1 Sequences and Series

**Definition 1.1.** A sequence is a list of numbers  $a_1, a_2, \dots, a_n, \dots$

Sequences can be thought as a function from  $\mathbb{N} \rightarrow \mathbb{R}$ , with  $na_n \in \mathbb{R}$ .

**Definition 1.2.** Let  $L$  be a number such that for any  $\epsilon_n > 0$  there exists  $N$  such that  $|a_n - L| < \epsilon$  for all values of  $n > N$ .

If  $\{a_n\}$  has a limit  $L$ , we say that it **converges** to  $L$ . If there is no limit, then a sequence is said to diverge.

Take  $c_n$  = For any  $M > 0$ , there exists  $N$  so that  $c_n > M$  and for all  $n > N$ . In this case we say that  $f(n)$  diverges.

Consider now the sequence such that  $h_n = 1 + \frac{1}{n}$  if and only if  $n$  is odd and  $h_n = 0$  if  $n$  is even.

We define  $\lim_{n \rightarrow \infty} h_n = L$ , which means that for a given  $\epsilon > 0$  there exists  $N > 0$  such that  $h_n < L + \epsilon$  for  $n > N$  and for any  $\epsilon > 0$  and  $M$  there exists  $n > M$  such that  $|h_n - L| < \epsilon$ .