1 Curry-Howard Correspondence IV

Curry-Howard has a much deeper meaning than just the correspondence between two logical systems.

Brouwer was mistaken in his belief that the model of classical logic cannot be embedded into the constructive setting.

1.1 λ_μ-Calculus

Let V be a set of variables, let A denote a set of all addresses, and suppose tat Φ is a set of types (formulae). We also introduce two operations: $\to$, $\perp$.

Let $x \in V$ and $a \in A$, and suppose a new context is defined as $M \stackrel{\text{def}}{=} x|(MM)|(\lambda x : \sigma.M)|(\mu a : \not\sigma.M)|([a]M)$.

We can understand an address as a channel device through which information can be transmitted. This operation is akin to exceptional procedures in programming languages.

Now, we introduce two rules:

- $\Gamma, a : \not\sigma \vdash M : \perp$
- $\Gamma \vdash (\mu a : \not\sigma.M) : \sigma$

and

- $\Gamma, a : \not\sigma \vdash M : \sigma$
- $\Gamma, a : \not\sigma \vdash [a]M : \not\sigma$

For example, the Pierce tautology (?) can be written as a λ-term as follows:

$$\lambda x : (p \to q) \to p.\mu a : \not p.[a](x(\lambda z : p.\mu b : \not q.[a]z))$$

This is one of the examples how classical and intuitionistic logic can be put into correspondence.

1.2 λ-Cube

We can draw a range of related logical systems, with the logical model and model of computation represented as a pair, in a figure to obtain a cube.