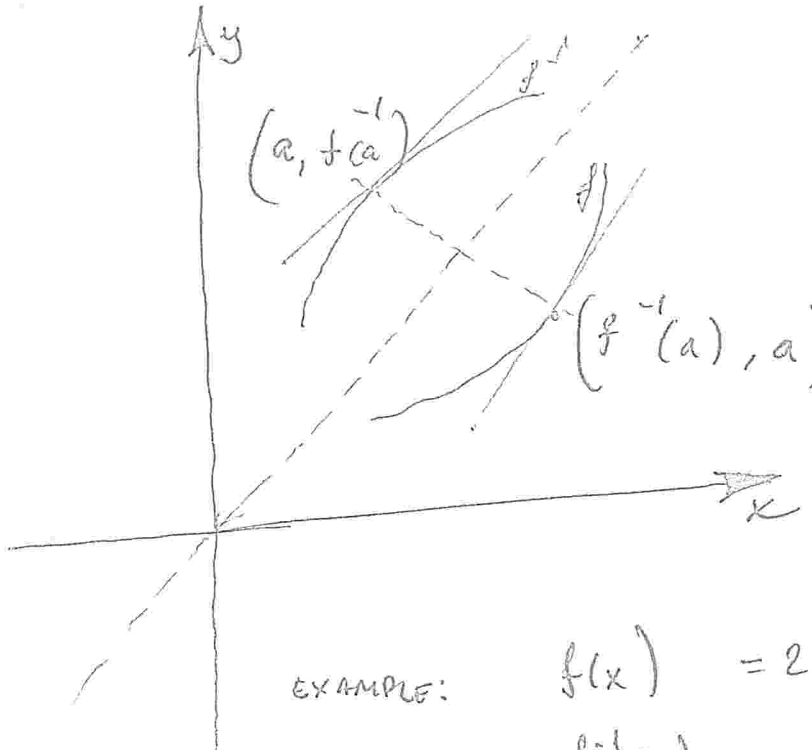
MAT157:

20161205

DERIVATIVE OF f^{-1}



EXAMPLE: $f(x) = 2x+5$

$f^{-1}(x) = \dfrac{x-5}{2}$.

$f(x) = x^2$

$f^{-1} = \sqrt{x}$, $x \geq 0$. $\Rightarrow (f^{-1})' = \dfrac{1}{2\sqrt{x}}$

THEOREM SUPPOSE $f(x)$ IS INJECTIVE AND DIFFERENTIABLE.

IF $f'(f^{-1}(a)) \neq 0$,

THEN $f^{-1}(x)$ IS DIFFERENTIABLE AT $x=a$

AND $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$

PROOF: $\left(f^{-1}\right)'(a) = \lim\limits_{h\to 0} \dfrac{f^{-1}(a+h) - f^{-1}(a)}{h}$.

LET'S WRITE $b = f^{-1}(a)$.

Denote $b + k = f^{-1}(a+h)$.

So

$$\lim_{h\to 0} \frac{f^{-1}(a+h) - f^{-1}(a)}{h} =$$

$$= \lim_{h\to 0} \frac{b+k-b}{h} = \lim_{h\to} \frac{k}{h}.$$

SINCE $b + k = f^{-1}(a+h)$

$f(b+k) = a+h$.

$h = f(b+k) - a = f(b+k) - f(b)$

So $\lim\limits_{h\to 0} \dfrac{k}{h} = \lim\limits_{h\to 0} \dfrac{k}{f(b+k) - f(b)}$.

f^{-1} continuous (thm from last time) $\Rightarrow$

$k \longrightarrow 0$ as $h \longrightarrow 0$.

So

$$\lim_{h\to 0} \frac{k}{f(b+h) - f(b)} =$$

$$= \lim_{k\to 0} \frac{k}{f(b+h) - f(b)} = f'(b)$$

EXAMPLE [illegible]

$f(\theta) = \tan\theta$ ON $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

SINCE 1–1, $f'(\theta) = \sec^2\theta$.

IF $t = \tan\theta$, $\arctan t = \theta$.

We call f^{-1} (arctan)

$$\frac{d}{dt}(\arctan t) = \frac{1}{f'(f^{-1}(t))} =$$

$$= \frac{1}{\sec^2(f^{-1}(t))} = \frac{1}{\sec^2(\arctan t)}$$

$$= \frac{1}{1+t^2}$$