# 1 Mathematics of Flat Linkage Mechanisms

## 1.1 Introduction

Suppose that a linkage mechanism $ABCDEF$ is given, with each linkage having a known length. Some hinges are fixed and some are allowed to move. Suppose that $A$ and $E$ are fixed, and have coordinates $(0,0)$ and $(1,0)$ respectively, while moving points $B, C, D$ and $F$ have coordinates $(x_1, y_1)$,$(x_2, y_2)$, $(x_3, y_3)$ and $(x_4, y_4)$.

The configuration space of a linkage mechanism can be defined as a topological space, with a standard euclidean metric. Since the links between each hinge are straight lines, we can easily write the equations describing the relations between $\{x_i\}$ and $\{y_i\}$.

**Problem** (Direct Problem)**.**

What can we say about classes of configuration spaces from classes of linkage mechanisms up to homeomorphisms?

**Problem** (Inverse Problem)**.**

Given a class of configuration spaces, what can we say about the corresponding class of linkage mechanisms?

**Theorem 1.1** (Thurston)

There exists a flat linkage mechanism such that its configuration space is your signature.

**Definition 1.2.** $L$ is a linkage mechanism in a general position if $\epsilon_1 l_1 + \epsilon_2 l_2 + \cdots + \epsilon_n l_n \neq 0$ for all $\epsilon_i \in \{-1, 1\}$.

**Definition 1.3.** For polyhedral linkage mechanisms in a general position, defined by a set $\{l_1, l_2, \ldots, l_n\}$ of lengths

**Exercise 1.4.** The configuration space $ConfL$ is unique up to the order of linkages.

**Theorem 1.5**

For all $g$ there exists $L_g$ such that $ConfL = M_g^2$, where $M_g$ is a manifold of genus $g$.

**Theorem 1.6**

For all $n$-dimensional smooth oriented manifolds $M^n$ there exists $L_M$ such that $ConfL_M = \sqcup^k M^n$, where $k$ is some finite positive integer.

**Theorem 1.7**

For all algebraic varieties $X$ in $\mathbb{R}^n$ there exists $L_X$ there exists $ConfL_X = \sqcup^k X$, where $k$ is some finite positive integer.