1 Brain Maths

Modern neuroscience focuses on neurons. Neurons generate electricity for communication.

There are about 10^{11} neurons on average in a human brain. Each neuron consists of two main parts – a body and an axon. The body also have synapses attached to its surface which serve asa communication device for other neurons. Synapses, in turn, connect to dendrites of other neurons.

The body of a neuron has a membrane acting like a capacitor storing electricity for communication. We will denote the potential difference that the body stores as ΔV.

Neurons communicate by utilising spikes in the potential difference. An average neuron has a period of relaxation after each spike of 100 mV over 5 ms.

The usual method of measuring the output of a neuron is as follows: electrodes are inserted inside the body which then measures the potential difference.

How can a neuron control the potential difference?

Membrane contains proteins which act as channels migrating electrically charged ions like K^+, Na^+, Ca^{2+} and Cl^-. There are two aspects affecting the flow of ions – electric gradient and osmosis. The flow would be unrestricted but for pumps also present in the membrane. These pumps utilise internal energy to manage the change in potential difference.

Each pump or channel is affected by the potential difference between the inside and the outside of the cell, concentration of charged ions and neuromediators present in the surroundings.

The spikes result from the sudden opening of channels and the action of pumps. It is worthwhile to note that the action of pumps is much slower than that of channels.

There are several types of spikes. Some neurons give a one-off spike, another rattle a sequence and then sleep, while the other neurons give off spikes on the periodical basis.

How can such a complicated machinery work with such efficiency? How can we predict what effect each spike would bring upon the other neurons?

Each neuron has around 10^4 synapses, which receive the signal along the axons from the body and then transfer the charge to dendrites communicating with other neurons.

One of the first models assumed that each neuron processes all the received signals I_i from the dendrites with some weights w_i assigned, which then bring about post-synaptic effects. If, however, the signal was greater than some well-defined barrier $\widehat{I}$, it was thought to give off a spike itself to communicate the message to other neurons:

$$\sum w_i I_i > \widehat{I} \tag{1}$$

This, however, turned out to be far from what really happens in a brain.

First of all, the barrier $\widehat{I}$ is fuzzy – there is a middle range at which a neuron operates optimally.

Secondly, there are latencies in spikes.

Thirdly, spikes are all different, and dependent on the input signal.

Finally, some neurons give off semi-spikes, while others do not.

Experiments have shown that if we transfer a current meant to decrease the potential difference, neurons give off rebound spikes.

If we give a fast periodic impulse, some neurons give off a spike which is a sum of the outputs that would occur for each individual pulse. Some neurons were responsive to the frequency of the received impulse.

To explain this largely unpredictable behaviour, several tentative fixes were introduced. One example is classification of neurons into resonators and integrators.

To build a model of a neuron, we need to identify main parameters.

First, denote the potential difference as V, and the fraction of channels and pumps of particular type i open as $x_1, \ldots, x_M$, where $\dot{x_i} = g_i(V, x_1, \ldots, x_M)$. The current through the membrane would be $C\dot{V} = I + f(V, x_1, \ldots, x_M)$, where C is the capacitance of the membrane, I is the total magnitude of external currents, and f represents the total magnitude of currents coming through the internal means.

The main purpose of the model is to simulate the actions of a real neuron. However, if parameters are shuffled in without any consideration, obtained models are most often unrepresentative of any physical neuron. To account for this problem, we can study a toy model, which includes only one equation:

$$C\dot{V} = I + f(V)$$