

$$W_0 = \{([0], [0])\} \quad (1)$$

$$W_1 = \{([0], [0]), ([0], [1]), ([0], [2])\} \quad (2)$$

$$W_2 = \{([0], [0]), ([1], [0]), ([2], [0])\} \quad (3)$$

$$W_3 = \{([0], [0]), ([1], [1]), ([2], [2])\} \quad (4)$$

$$W_4 = \{([0], [0]), ([1], [2]), ([2], [1])\} \quad (5)$$

- 1  $([0], [0])$  is always an element of the subspace by the subspace test. If it is the only element, the subspace  
2 is the zero subspace.
- 3 If  $[1]$  is included as an element of a tuple inside the subspace, then by properties of the subspace,  
4  $[1] + [1] = [2]$  is an element of some tuple in the subspace as well. Hence,  $[2]$  must be included as an  
5 element of a tuple at the same position as  $[1]$  is included.
- 6 Since  $[0, 0] \in V$  and the position of  $[1]$  in one tuple determines the position of  $[2]$  in the other, while the  
7 positions of  $[1]$  and  $[2]$  determine the position of  $[0]$ , there are at most 3 tuples in  $W$ .
- 8 If there are more than three tuples, by Dirichlet's principle there would be duplicates of elements in some  
9 tuples, which would make it necessary to introduce other duplicates, until all 9 elements are included.