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Office
14: 12-13:00
15: 13:30-14:30
BA 6193

Test: Fri, Sep 30

20160914

Absolute Values

Definition

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Theorem

The Triangle Inequality

$$|a+b| \leq |a| + |b|$$

Proof

Cases:
1. $a \geq 0, b \geq 0$
2. $a < 0, b < 0$
3. $a \geq 0, b < 0$
4. $a < 0, b \geq 0$

1. $|a| = a, |b| = b \wedge |a+b| \geq 0 \Rightarrow |a+b| = a+b = |a|+|b|$
2. $|a| = -a, |b| = -b \wedge a+b < 0 \Rightarrow |a+b| = -(a+b)$.

$\Rightarrow |a|+|b| = -a-b = |a+b|$ $\square$

3. Assume $a \geq -b \Leftrightarrow a+b \geq 0 \Rightarrow |a+b| = a+b$.

$|a| = a \wedge |b| = -b \Rightarrow a+b = |a|-|b| < |a|+|b|$.

$\Rightarrow |a+b| < |a|+|b|$

$\square$

ASSUME $a+b<0 \Rightarrow |a+b| = -a-b = -|a| + |b| < |a|+|b|$

$\Rightarrow \quad |a+b| < |a|+|b| \quad \square$

4. CONSIDER 3 WITH $a := b \wedge b := a$

HENCE, $|a+b| < |a| + |b| \quad \square$

<u>INDUCTION</u>

$$s(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad (S)$$

BASE CASE: $n=1$, $S = 1 = \frac{1 \cdot 2}{2}$.

INDUCTIVE STEP: ASSUME (S) FOR $n=k$.

CONSIDER $s(k+1)$.

$$s(k+1) = \sum_{i=1}^{k} i + (k+1)$$

$$= \frac{k(k+1)}{2} + (k+1) \Big| \quad \text{FROM IND. STEP}$$

$$= \frac{(k+1)(k+2)}{2},$$

which is EXACTLY (S) FOR $k+1$.

SINCE (S) is TRUE FOR $n=1$,

$$\forall k \in \mathbb{N} : s(n) = \frac{n(n+1)}{2}.$$

LET $c(n) = \sum_{i=1}^{n} i^3$.

(S3): $c(n) = \dfrac{n^2(n+1)^2}{4}$

BASE CASE: $n=1 \Rightarrow c(1) = 1 = \dfrac{1 \cdot 2^2}{4}$.

ASSUME (S3) HOLDS FOR $n=k$.

CONSIDER
$$c(k+1) = \sum_{i=1}^{k} i^3 + (k+1)^3$$
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3 \quad \Big| \text{ FROM IND. S.}$$
$$= (k+1)^2\left(\frac{k^2}{4} + k+1\right)$$
$$= \frac{(k+1)^2(k+2)^2}{4},$$

WHICH IS EXACTLY (S3) IF $n=k+1$.

SINCE (S3) HOLDS FOR $n=1$,

$$\forall n \in \mathbb{N}: c(n) = \frac{n^2(n+1)^2}{4}.$$