1 Review

We have show that a Jordan Canonical Form is unique using dot diagrams for T_{K_λ} and any eigenvalue λ.

For this purpose we have found a cycle basis for K_λ, and, ordering subbases by their sizes with the subbasis of greatest size being the first, we can construct a dot diagram, which has several nice properties.

For example, the number of dots in the diagram is equal to the dimension of K_λ, and the number of dots in the first s rows is equal to the nullity of $(T - \lambda I)^s$.

We now can finde a Jordan Canonical basis for the sequences such that, for example, $\ker(A - 2i) = \operatorname{span}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$

Taking $(A - 3I)v = \begin{pmatrix}1\\-1\\-1\end{pmatrix}$, we can solve for v, and thus obtain a cycle basis. Having done that, the JCF is easily found.

Suppose now a dot diagram is given. Starting in the top left, we obtain thatfor vectors v_1 and v_2 such that $(A - I)v_1$ and $(A - I)v_2$ are in $\ker(A - I) \cap \operatorname{im}(A - I)$. We already saw that $\operatorname{rank}(A - I) = 2$, and hence these form a matrix.

We now solve for v_1 and v_2 by noting that they are eigenvectors to obtain $\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ and $v_2 = \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$.

Extending $(A - I)v_1$, $(A - I)v_2$ to a basis of $\ker(A - I)$, eg by taking $v_3 = \begin{pmatrix}0\\0\\0\\1\\-1\end{pmatrix}$, we can see that $[L_A]_\beta$ is in JCF.

We can therefore formulate a general strategy for finding Jordan Canonical basis:

- Figure out dot diagrams
- For a fixed eigenvalue λ, working from left to right in the 1st row of the dot diagram (among initial eigenvectorrs.
- Solve for the end vectors by considering the system of the form $(T - \lambda I)^{l-1}v =$ initial vectors.

Example 1.1

For the first two initial vectors we have that $(T - \lambda I)^2 v_1$ and $(T - \lambda)^2 v_2 \in \ker(T - \lambda) \cap (\operatorname{im}(T - \lambda I)^2)$.