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OFFICE HOURS:
1.30 – 2.30 TUE
OR BY APPOINTMENT

TUTORIALS START AFTER THE FIRST WEEK

MAT 240 ALGEBRA I

GRADING:
H/W 20% | 9 ASSIGNMENTS
TERM TEST 30% | MID-OCTOBER
FINAL EXAM 50%

FRIEDBERG – INSEL – SPENCE 4TH EDITION

PATH:
MAT 240
247
347

MAT 240:

- ALGEBRA
- ANALYSIS
- GEOMETRY
- LOGIC
- SET THEORY
- TOPOLOGY

COURSE STRUCTURE:

0. FIELDS, COMPLEX NUMBERS
1. VECTOR SPACES
2. LINEAR MAPS
3. LINEAR SYSTEMS OF EQUATIONS
4. DETERMINANTS
5. EIGENVALUES AND EIGENVECTORS

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1. SET NOTATION

NOTION A SET IS A COLLECTION OF ELEMENTS.

e.g. $\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

$\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$

$\mathbb{R}$; a SET OF DECIMAL EXPANSIONS
; BUT $0.9999\ldots = 1$

$\mathbb{C}$

NOTATION $x \in A$ ~ "x is an element of A"

$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$

$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$

$A \times B = \{(x, y) \mid x \in A \text{ AND } y \in B\}$

$\bigcap_{\alpha \in \Lambda} A_\alpha = \{x \mid x \in A_\alpha \ \forall \alpha \in \Lambda\}$

$\bigcup_{\alpha \in \Lambda} A_\alpha = \{x \mid \exists A_\alpha \ x \in A_\alpha\}$

$\bigtimes_{\alpha \in \Lambda} A_\alpha = \{(\overline{x_\alpha}) \mid x_\alpha \in A_\alpha\}$

$A \setminus B = \{x \in A \mid x \notin B\}$

$A \subset B$ $A \subseteq B$

A PROPER SUBSET AN IMPROPER SUBSETS

$\emptyset$ EMPTY SET

NOTE

WHEN LISTING ELEMENTS OF SETS, ORDERING OR REPETITIONS DON'T MATTER.

? WHAT ARE OTHER MATHEMATICAL DATA STRUCTURES, e.g. a SEQUENCE?

2. FIELDS

NOTION A FIELD IS A SET WITH ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION DEFINED ON ITS ELEMENTS.

e.g. NOT A FIELD: $\mathbb{N}_0$, $\mathbb{Z}$ (NO MULTIPLICATIVE INVERSE)

A FIELD: $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

FIELDS

DEFINITION A FIELD IS A SET F WITH TWO BINARY OPERATIONS:

$$+ : F \times F \to F \mid (a,b) \mapsto a+b$$
$$\cdot : F \times F \to F \mid (a,b) \mapsto a \cdot b$$

AND AT LEAST TWO DISTINCT ELEMENTS: $0, 1 \in F \ (0 \neq 1)$

SUCH THAT THE FOLLOWING PROPERTIES ARE SATISFIED FOR ALL $a, b, c \in F$

	$+$	$\cdot$
COMMUTATIVITY	$a+b=b+a$	$a \cdot b = b \cdot a$
ASSOCIATIVITY	$(a+b)+c = a+(b+c)$	$(a \cdot b)c = a \cdot (b \cdot c)$
EXISTENCE OF THE NEUTRAL ELEMENT	$a+0=a$	$a \cdot 1 = a$
EXISTENCE OF THE INVERSE ELEMENT	$\forall a \exists b : a+b=0$	$\forall a : a \neq 0 \ \exists b : a \cdot b = 1$
DISTRIBUTIVITY	$\forall a,b,c : \ a \cdot (b+c) = a \cdot b + a \cdot c$	

EXAMPLE $\mathbb{Z}_2 = \{0, 1\}$ WITH ADDITION AND MULTIPLICATION

$0+0=0$, $0+1=1+0=1$, $1+1=0$

$0 \cdot 0 = 0$, $0 \cdot 1 = 1 \cdot 0 = 0$, $1 \cdot 1 = 1$

$\mathbb{Z}_3 = \{0, 1, 2\}$ w/ $\oplus$ AND $\odot$

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

•	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ w/ $\oplus$ and $\odot$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	1	2	3	4
2	2	2	4	1	3
3	3	3	1	4	2
4	4	4	3	2	1

? PROVE THAT $\mathbb{Z}_p$ IS A FIELD IFF $p \in \mathbb{P}$