SUPPOSE THERE ARE TWO CONSTRUCTIONS OF REAL NUMBERS,

THUS, THERE EXISTS AT LEAST 0 AND 1 IN BOTH
CONSTRUCTIONS.

THEN THERE MUST BE A MAP FROM 1 TO 1 IN
THE OTHER CONSTRUCTION $\Rightarrow$ $\mathbb{Z}$ CORRESPOND
$\Rightarrow$ $\mathbb{Q}$ CORRESPOND
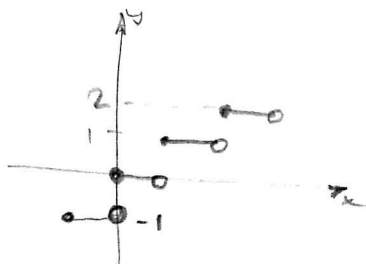$\Rightarrow$ $\mathbb{R}$ CORRESPOND

$\longrightarrow$ READ CH 29 SPIVAK 3E

FUNCTIONS

NOTION

A FUNCTION IS A "RULE"
WHICH SAYS HOW TO TAKE
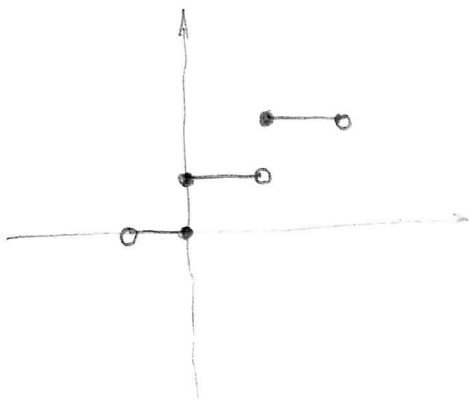CERTAIN NUMBERS AND
FIND THE CORRESPONDING
VALUES OF THE FUNCTION.

① TAKE ANY NO. x AND SQUARE IT. x^2

② $f(x) = \begin{cases} 0, & \text{IF } x \in \mathbb{R} - \mathbb{Q} \\ 1, & \text{IF } x \in \mathbb{Q} \end{cases}$
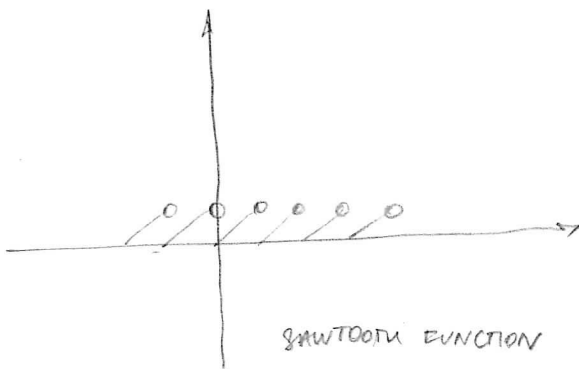
③ FLOOR FUNCTION

EXAMPLE

$$\lfloor x \rfloor = \max \{ m \in \mathbb{Z} \mid m \leq x \}$$

EXAMPLE

$$\lceil x \rceil = \min \{ m \in \mathbb{Z} \mid m \geq x \}$$

$$\lceil x \rceil = \begin{cases} \lfloor x \rfloor + 1, \text{ UNLESS } x \in \mathbb{Z} \\ x, \text{ IF } x \in \mathbb{Z} \end{cases}$$

SAWTOOTH FUNCTION

FRACTIONAL PART

$$\{ x \} = x - \lfloor x \rfloor$$

POLYNOMIALS

RATIONAL FUNCTIONS =
QUOTIENTS OF POLYNOMIALS