Suppose $f(x)$ is given, and suppose

$$f(x) = f(a) + R.$$

Thus, $R = f(x) - f(a) = \int_a^x f'(t)\,\mathrm{d}t$.
Note that

$$\int_a^x f'(t)\,\mathrm{d}t = [f'(t)(t-x)]_a^x - \int_a^x f''(t)(t-x)\,\mathrm{d}t = 0 - f'(a)(a-x) - \int_a^x f''(t)(t-x)\,\mathrm{d}t.$$

Therefore, for $f(x) = fax) + R_1$ we obtain that

$$f(x) = f(a) + f'(x)(x-a) + \int_a^x f''(t)(x-t)\,\mathrm{d}t.$$

Similarly, for $R_2 = \int_a^x f''(t)(x-t)\,\mathrm{d}t$, we get that

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \int_a^x f'''(t)\frac{(x-t)^2}{2}\,\mathrm{d}t. \tag{1}$$

We therefore can prove by induction that

$$f(x) = f(a) + f'(a)(x-a) + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!} + \int_a^x f^{n+1}(t)\frac{(x-t)^n}{n!}\,\mathrm{d}t, \tag{2}$$

and thus $f(x) = P_n(x) + R_n(x)$ for $R_n(x) = \int_a^x f^{(n+1)}(t)\frac{(x-t)^n}{n!}\,\mathrm{d}t$.
If, for example, $f(x) = \sin x$, then for all $k \in \mathbb{Z}^+$ and for all $x \in D(f)$, then $\left|f^{(k)}(x)\right| \leq 1$, and hence $\left|R_n(x)\right| \leq \int_a^x \frac{(x-t)^n}{n!}\,\mathrm{d}t = \frac{(x-a)^{n+1}}{(n-1)!}$. In this way, if we are to compute the approximation of $\sin(1)$ to within 0.001, then any $n \in \mathbb{Z}^+$ such that $(n+1)! > 1000$ will suffice.

0.1 Lagrange Form

Assume that a bound for $f^{(n+1)}$ is known on the interval from a to x. Therefore, $m \leq f^{(n+1)}(t) \leq W$ for any t between a and x, which means that

$$\begin{aligned} \left|R_n(x)\right| &\leq \int_a^x M\frac{(x-t)^n}{n!}\,\mathrm{d}t \qquad (3)\\ &\leq M\frac{\left|x-a\right|^{n+1}}{(n+1)!}. \qquad (4)\end{aligned}$$

If $f^{(n+1)}$ is continuous, then the IVT shows that there exists $z \in \mathbb{R}$ between $a$and x such that $R_n(x) = f^{(n+1)}(z)\frac{|x-a|^{n+1}}{(n+1)!}$.
Another way to see it is by looking at $P_{n+1}(x)$.
This form of a remainder is called a **Lagrange form**.
Suppose now that that $f(x) = e^x$.
Then $R_n(1) = \frac{e^z}{(n+1)!}$ for some $z \in [0, 1]$.
Since $\log(4) > 1$, we know that $e < 4$, and thus $R_n(1) < \frac{4}{(n+1)!}$.
Therefore, $f(1) = 2^2/3 + R_3(1)$.