1 Maps from the Generic Riemann Surface

Key terms: moduli spaces, smooth projective curves, least ramification, primitive faithful group, simple vs finite groups

Consider maps from the generic Riemann surface.

Denote a Riemann surface of genus g as χ_g.

Suppose that χ_g is *generic*. For example, for $g = 0$, anything is generic. If $g = 1$, $y^2 = x(x-1)(x-t)$ is generic.

Let $\mathfrak{m}_g$ be a moduli space of genus g of smooth projective curves over T_k.

Then $\dim \mathfrak{m}_g = \begin{cases} 3g-3, g \geq 2 \\ 4, g = 1 \\ 0, g = 0 \end{cases}$.

Suppose now $g > 2$. Then a property is generic if it holds on a non-empty open subset of $\mathfrak{m}_g$.

Theorem 1.1 (Enrique's Conjeture)

Fix $n \in \mathbb{Z}^+$.

If $g > 6$, there is no solvable map from the *generic* Riemannian surface of genus g to $\mathbb{P}^1$ of degree n.

Consider now $\{x \in \mathfrak{m}_j \mid \text{ there exists } \phi : x \to \mathbb{P}^1 \text{ of degree } n \text{ such that } \phi \text{ is solvable}\}$.

Remark 1.2. For every Riemann surface of genus g and each x, there exists $\phi : x \to \mathbb{P}^1$ of the degree similar to $\frac{g+1}{2}$.