Let $v, w \in M_{n\times 1}(F)$ be non-zero column vectors.

Let $A = vw^t \in M_{n\times n}(F)$.

Claim. $\operatorname{rank} A = 1$.

Proof. Take any $x \in M_{n\times 1}(F)$.

Consider Ax.

$$\begin{aligned} Ax &= (vw^t)x && (1)\\ &= v(w^t x) \mid \text{ Associativity of Matrix Multiplication} && (2) \end{aligned}$$

Note that $w^t x \in M_{1\times 1}$, since $w^t \in M_{1\times n}$, $x \in M_{n\times 1}$.

Thus $v(w^t x)$ corresponds to the product of the vector v and the scalar which is the only entry in $w^t x$. Therefore, $\exists (]w^t x[\in F) : \ v(w^t x) =]w^t x[v$.

Since w is non-zero, then w^t is also non-zero. Thus, $]w^t x[$ is also non-zero for some non-zero x.

Since x is arbitrary and v is non-zero, then $\operatorname{Im} A$ is spanned by the vector v.

Hence, $\operatorname{rank} A = 1$, as required. $\square$