MAT157: DIFFERENTIATION II

20161024

$$f'(x) = \underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{\text{"DIFFERENCE QUOTIENT"}}$$

$f'(x)$ :: A FUNCTION WHICH DOMAIN IS THE SET OF x's WHERE f IS DIFFERENTIABLE

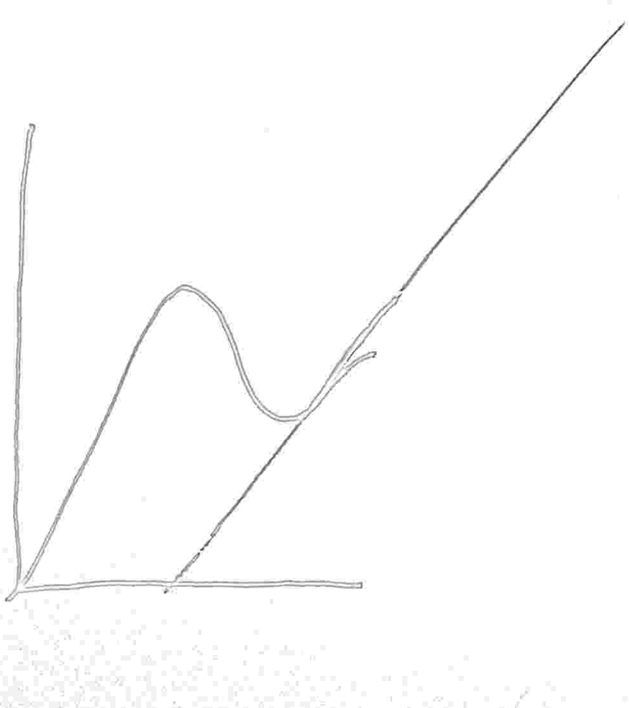
Let t represent time and $f(t)$ a function of time.

CONSIDER A MOTION IN A STRAIGHT LINE.

FIX A STARTING POINT O (ORIGIN) AND LET $f(t)$ BE THE <u>DISPLACEMENT</u> OF THE MOVING OBJECT FROM O AT TIME t.

DISPLACEMENT IS DISTANCE TO THE RIGHT OF O, AN − (DISTANCE) TO THE LEFT.

DISTANCE = |DISPLACEMENT|



$f'(x)$ :: INSTANTENEOUS VELOCITY.

IF $g(t)$ IS ANY QUANTITY, $g'(t)$ IS INTERPRETED AS THE (INSTANTANEOUS) RATE OF CHANGE OF g.

$f(x) = c \in \mathbb{R}$: $\therefore$ $f'(x) = \lim_{h \to 0} \frac{c-c}{h} = \lim_{h \to 0} 0 = 0$

$f(x) = cx + d$ $\therefore$ $f'(x) = c$.

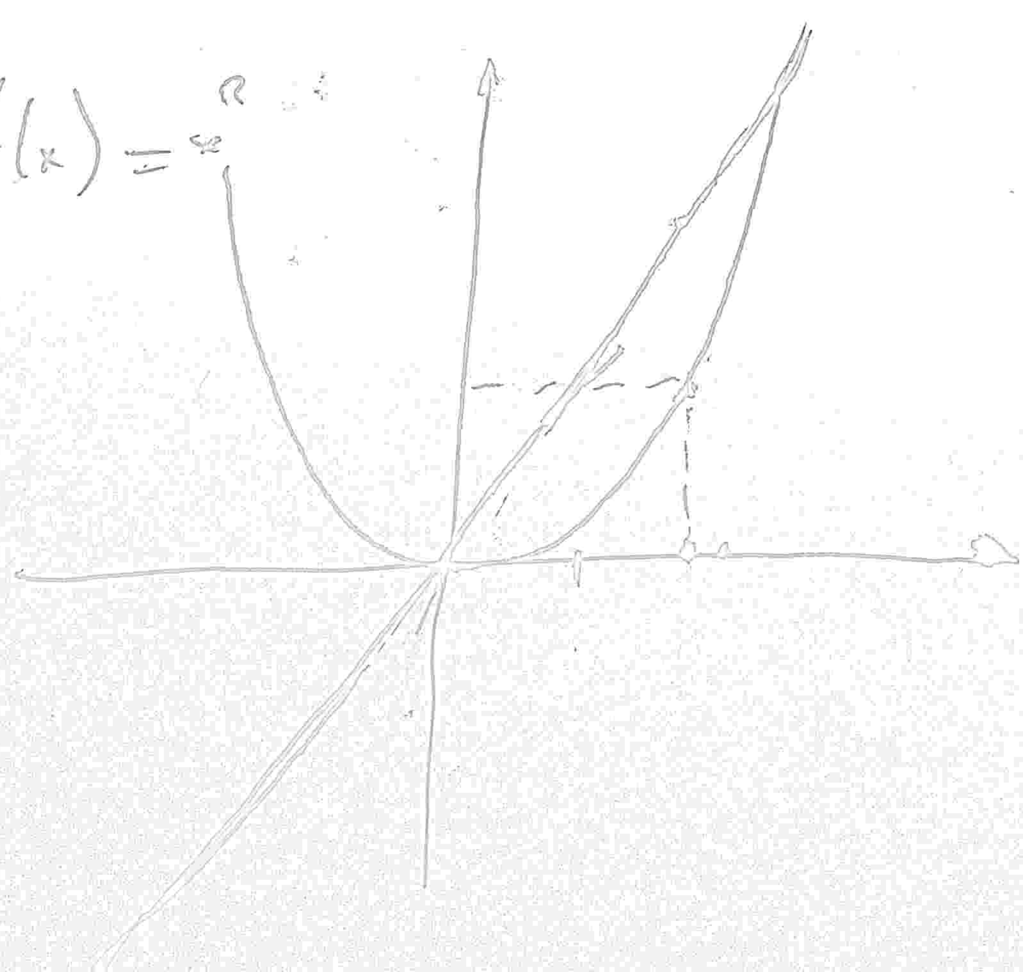
$f(x) = ax^2 + b$ $\quad f'(x) = \frac{a(x+h)^2 + b - ax^2 - b}{h}$

$= \lim_{h \to 0} \frac{2axh + h^2}{h}$

$= \lim_{h \to 0} \left[2ax + h \right]$

$= 2ax$

$f(x) = x^2$



IF $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3hx + h^2 = 3x^2$$

LEIBNIZ NOTATION

$$\frac{dy}{dx} \sim \frac{\Delta y}{\Delta x}; \quad f'(x) = \frac{dy}{dx}$$

$$\frac{d}{dx}\left(|x|\right)$$

$$|x| = \begin{cases} x, & \text{IF } x \geq 0 \\ -x, & \text{IF } x < 0 \end{cases} \quad \Bigg| \quad \frac{d}{dx}\left[|x|\right] = \begin{cases} 1, & \text{IF } x > 0 \\ -1, & \text{IF } x < 0 \end{cases}$$

$$\lim_{h\to 0^+} \frac{|0+h| - |0|}{|h|} = \lim_{h\to 0^+} \frac{h}{h} = 1$$

$$\lim_{h\to 0^-} \frac{|0+h| - |0|}{|h|} = \lim_{h\to 0^-} \frac{-h}{h} = -1$$

So $\lim_{h \to 0} \frac{|0+h| - |0|}{h}$ DOES NOT EXIST.