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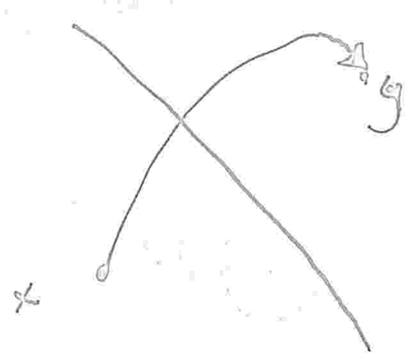
GREG GRIMBERGS

2016 10 04

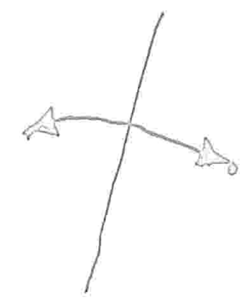
FOLDINGS



$F_v y = y$

$F_v x = y$

REFLECTIONS

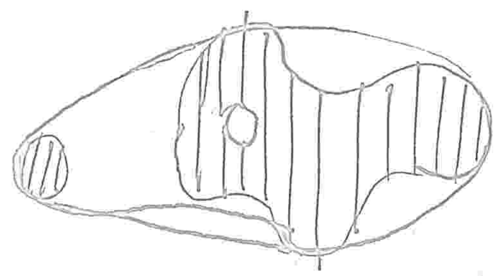


$R x = -y$

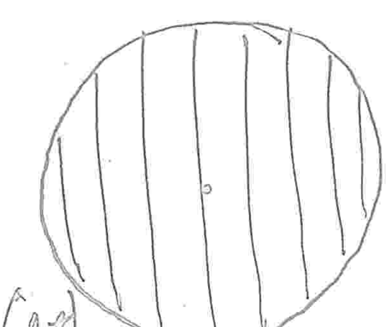
$R y = x$

EXCURSION 1830s JOAP BURGOS

SYMMETRIZATION



A



A^*

$Per(A) \geq Per(A^*)$

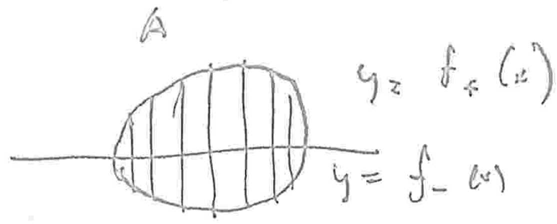
ISOPERIMETRIC INEQUALITY

1835, Steiner

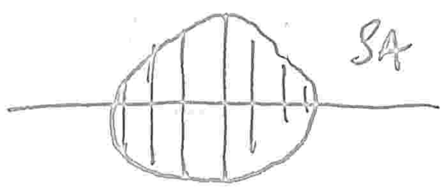
Proof of isoperimetric inequality in the plane

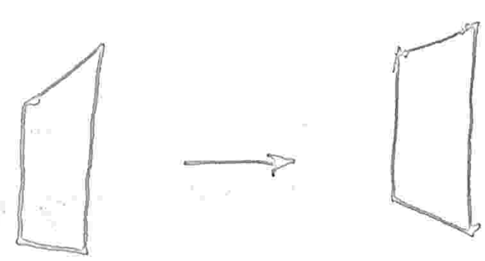
$d = 2$ when A is convex

If convex, polar coord.s can be used



Area (A) = Area $(S_v A)$

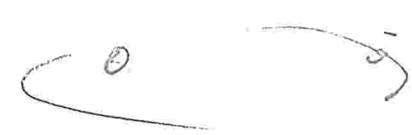




$\mathrm{Per}(A) > \mathrm{Per}(S_v A)$

unless $R_v A = A$ (up to translation)

If A is perimeter-minimizing, then A is symmetric abt a horizontal axis.



So $R_v A = A$ all $v \Rightarrow A$ radial

given convex, A ball

- does not work for higher dim?
- why should a optimizing set exist?

Low-rank modification

$A_n = (S_u S_v)^n A$ (1909)

① $(S_u S_v)^n A \longrightarrow C$

② $S_u C = C$ $\quad$ $R_u C = C$

$S_v C = C$ $\quad$ $R_v C = C$

C symmetrical

$(R_u R_v)^k$, $k \geq 1$

is rotation by an irrational angle:

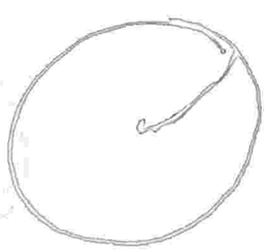
$\mathbb{Q}_{\pi}$: dense on the ball

Q. Given a set of directions,

$G = \underbrace{\{u_1, \dots, u_m\}}_{\text{unit vectors}} \in \mathbb{R}^d$

Look at $\langle G \rangle = \{$finite products of $R_u\}$

dense in the rotation group?



$x \in S^d$ $\quad$ $x_n = R_{u_n} x_{n-1}$.

Need a dense trajectory.

Meeting

Different conditions .

COXETER RULES

(1) G .. CAN'T BE SPLIT INTO $G_1 \perp G_2$

(2) $\dim(G) = d$

(3) G CAN'T GENERATE A FINITE SUBGROUP.

3	$\frac{\pi}{3}$, $\frac{\pi}{4}$, $\frac{\pi}{5}$
5.	$\frac{\pi}{3}$

Foldings

$G = \{u_1, \dots, u_m\} \subset \mathbb{R}^d$

d=2

m=3

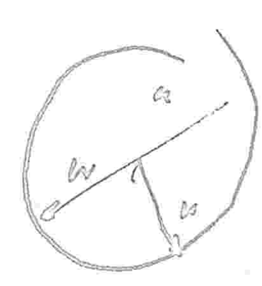
$G = \{u, v, w\}$

Want -

Apply F_u, F_v, F_w

some order to come

to a dense trajectory

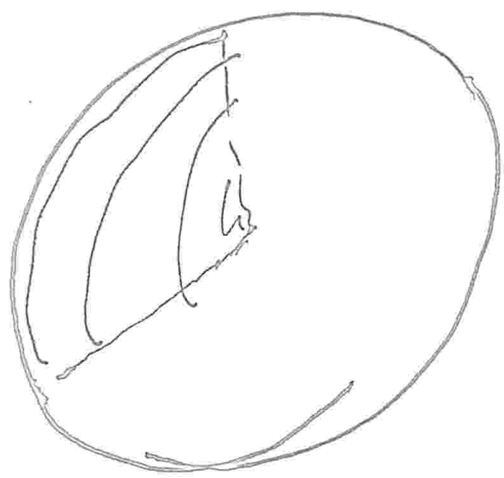


Necessary conditions

(1) GEOMETRIC (2) algebraic

R_u, R_v, R_w

Generate a dense subgroup of $O(3)$



$\bigcup_{u \in G} H_u \supset S^d$

then sufficient