Theorem. *Let V be a vector space over $\mathbb{F}$, and W_1, W_2 two subspaces of V. Suppose that U is a third subspace such that $U \subseteq (W_1 \cup W_2)$. Then $(U \subseteq W_1) \vee (U \subseteq W_2)$.*

Proof. Let u, u' be arbitrary elements of U. Since $U \subseteq (W_1 \cup W_2)$, $u \in W_1$ or $u \in W_2$. To show that $(U \subseteq W_1) \vee (U \subseteq W_2)$, it must be shown that if $u \in W_1$, then $u' \in W_1$, and if $u \in W_2$, then $u' \in W_2$. Without loss of generality, to obtain a contradiction, assume that if $u \in W_1$, then $u' \in W_2$.

Since $u, u' \in U$, $u + u' \in U$ by Additive Closure of a Subspace.

Consider $u - u'$.

$d = (u - u') \in U$ by Additive Closure of a Subspace. Therefore, $(d \in W_1) \vee (d \in W_2)$.

Suppose $d \in W_2$. Therefore, $u' + d = u' + (u - u') = u \in W_2$, which is a contradiction.

Suppose now $d \in W_1$. Therefore, $u - d = u - (u - u') = u' \in W_1$ by Additive Closure of a Subspace, which is a contradiction.

Therefore, $d \notin W_1 \cup W_2$. But then $d \notin U$, which is again a contradiction.

Therefore, $(u \in W_1) \wedge (u' \notin W_2)$. Since $u' \in U$, $u' \in W_1$.

The argument is similar in case $u \in W_2$.

Therefore, $(U \subseteq W_1) \vee (U \subseteq W_2)$.

$\square$