MAT 157: 20061028

PROPERTIES OF DERIVATIVES

THEOREM IF $f(x)$ AND $g(x)$ ARE DIFFERENTIABLE AT $x=a$, THEN SO IS $f(x)+g(x)$, AND

$$(f+g)'(a) = f'(a) + g'(a).$$

PROOF.

$$(f+g)'(a) = \lim_{h\to 0} \frac{(f+g)(a+h)-(f+g)(a)}{h}$$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)+g(a+h)-g(a)}{h}$$

$$= f'(a) + g'(a).$$

THEOREM IF $f(x)$ AND $g(x)$ ARE DIFFERENTIABLE AT $x=a$, THEN SO IS $f(x)\cdot g(x)$,

AND $$(f\cdot g)'(a) = f'(a)\,g(a) + g'(a)\,f(a).$$

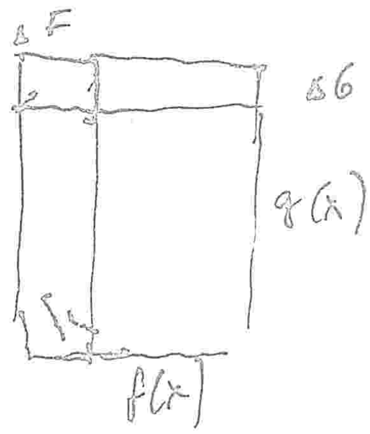
PROOF $(f\cdot g)'(a) = \lim_{h\to 0} \dfrac{(f\cdot g)(a+h) - (f\cdot g)(a)}{h}$

$$= \lim_{h\to 0} \frac{f(a+h)g(a+h) - f(a)\,g(a+h) + f(a)\cdot g(a+h) - f(a)(g(a))}{h}$$

$$= f'(a)\,g(a) + f(a)\cdot g'(a),$$

SINCE g IS DIFFERENTIABLE

THUS



THEOREM

$$\frac{d}{dx}\, x^n = n\, x^{n-1}$$

PROOF

THE STATEMENT HOLDS FOR $n=1$.

SUPPOSE THE CLAIM HOLDS FOR k.

$$\Rightarrow \quad \frac{d}{dx}\left(x^{k+1}\right) = \frac{d}{dx}\left(x^{k}\cdot x\right) = n x^{n-1}\cdot x + x^{n} =$$

$$= (n+1)\, x^{n}.$$

THEOREM $\frac{d}{dx}\left[\frac{1}{g(x)}\right] =$

$$= \lim_{x \to 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h}$$

$\Rightarrow$: NEED TO SHOW $g(x+h) \neq 0$

AND $g(x) \neq 0$.

~~[illegible]~~

SUPPOSE $g(x)$ IS DIFFERENTIABLE AT

D AND $g(x) \neq 0$.

DIFFERENTIABLE $\Rightarrow$ CONTINUOUS

$g(x) = 0$ AND g IS CONTINUOUS

AT $x \Rightarrow g \neq 0$ ON AN

INTERVAL AROUND x.

$$\frac{d}{dx}\left(x^{-n}\right) = \frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-nx^{n-1}}{x^{2n}}$$

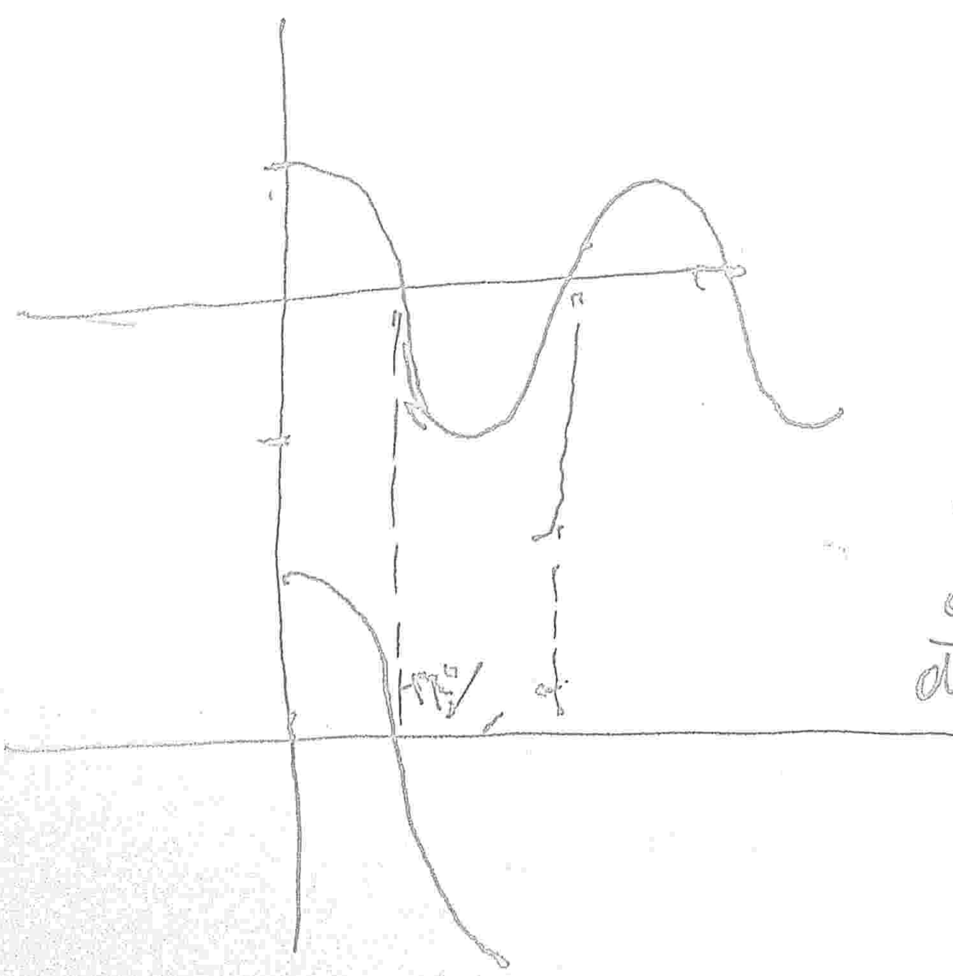
$$= -nx^{-n-1}.$$

If p is a polynomial,

i.e.

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

$$\Rightarrow \sum p'(x) = \sum_{i=0}^{n-1} n-i \; a_i x^{n-1-i}$$



$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$d(\tan(x)) = d\left(\sin(x) \cdot \frac{1}{\cos(x}\right)$$

$$= \frac{\cos^2(x) + \sin^2 x}{\cos^2(x)} = \sec^2(x)\ .$$