Example 0.1

Let $u = \cos x$, and thus $\mathrm{d}u = -\sin(x)dx$. Therefore,

$$\int \sin(x)\cos^7(x)\,\mathrm{d}x = -\int u^7\,\mathrm{d}u \tag{1}$$
$$= -\frac{u^8}{8} \tag{2}$$
$$= -\frac{1}{8}\cos^8 \tag{3}$$

Example 0.2

If $m, n \in \mathbb{N}$ are not both even, then $\int \cos^m x \sin^n x\,\mathrm{d}x$ can be solved by using the substitution as above.

If m, n are both even, the equation can be reduced to something more manageable by using double angle formulas.

1 Reduction Formula

Suppose $\int \sin^n x\,\mathrm{d}x = \sin x \sin^{n-1} x\,\mathrm{d}x$ is given. Integrating it by parts, we obtain:

$$\int \sin^n x\,\mathrm{d}x = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x \cos^2 x\,\mathrm{d}x \tag{4}$$
$$= -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2}\,\mathrm{d}x \tag{5}$$

2 Examples

Example 2.1

Note that $\sec\theta\tan\theta + \sec^2\theta = \sec\theta(\tan\theta + \sec\theta)$.

Therefore, $\sec\theta = \frac{\sec\theta(\tan\theta+\sec\theta)}{\tan\theta+\sec\theta)} = \frac{f'}{f}$

$\int \sec\theta\,\mathrm{d}\theta = \log|\sec\theta + \tan\theta|$.

Example 2.2

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,\mathrm{d}x = \int \frac{1}{\cos^2\theta\sin\theta}\,\mathrm{d}\theta \tag{6}$$
$$= -\int \sec^2\theta = -\tan\theta \tag{7}$$

Example 2.3

Consider now $\int x\sqrt{9-x^2}$.

Let $x = 3\sin x$, and $\mathrm{d}x = 3\cos x$.

Therefore,

$$\int x\sqrt{9-x^2} = 27\int \sin\theta\cos^2\theta \tag{8}$$

$$= -9\cos^3\theta \tag{9}$$

$$= -9(1-\frac{x^2}{9})^{3/2} \tag{10}$$