MAT157:

COMPOSITE FUNCTIONS

EXAMPLE

$$f(x) = \sin\left(x^2 + 1\right)$$

$$g(x) = \left(\frac{3x - 2}{x^2 - 6}\right)^{12}$$

$$h(x) = \frac{1}{x^2 + \sin(x)}$$

IF WE WRITE $u(x) = x^2 + 1$;

THEN $f(x) = \sin\left(u(x)\right)$.

IF $\sin(t) = v(t)$,

$$f(x) = v\left(u(x)\right) = v \circ u \,(x).$$

$$g(x) = v \circ u \,(x),$$

IF $u(x) = \dfrac{3x - 2}{x^2 - 6}$,

$$v(t) = t^{12}.$$

IF $u(x) = x^2 + \sin(x)$, $v(t) = \dfrac{1}{t}$,

$$h(x) = v \circ u \,(x).$$

DEFINITION

FUNCTION :: A SET OF ORDERED PAIRS (a,b)

FOR EACH ORDERED PAIR (a,b) IN THE SET WE WRITE $b = f(a)$.

THE SET MUST SATISFY THE PROPERTY THAT IF IT CONTAINS (a,b) AND (a,c), THEN $b = c$.

EXAMPLE $f(x) = x^2$ CORRESPONDS TO

$$\{(a, a^2) \mid a \in \mathbb{R}\}, \text{ IF } x \in \mathbb{R}.$$

DEFINITION
DOMAIN

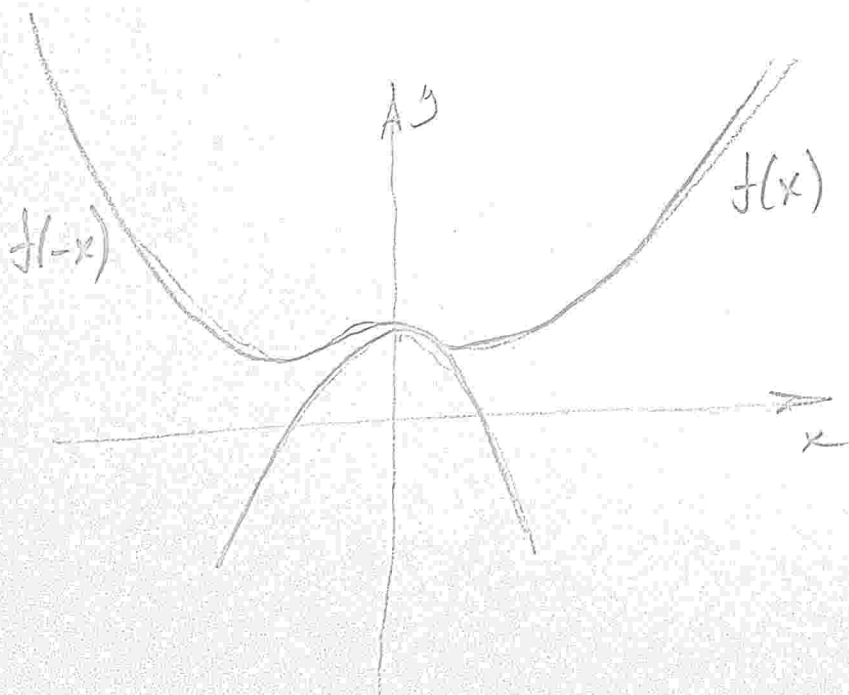THE DOMAIN OF $f(x)$ IS THE SET OF ALL a FOR WHICH $\exists b$ WITH (a,b) IN THE SET.
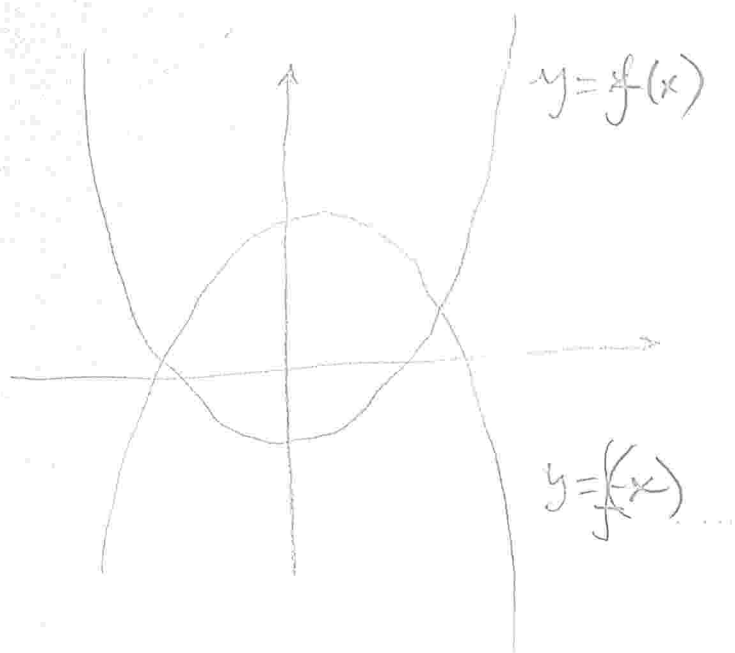
DEFINITION
RANGE
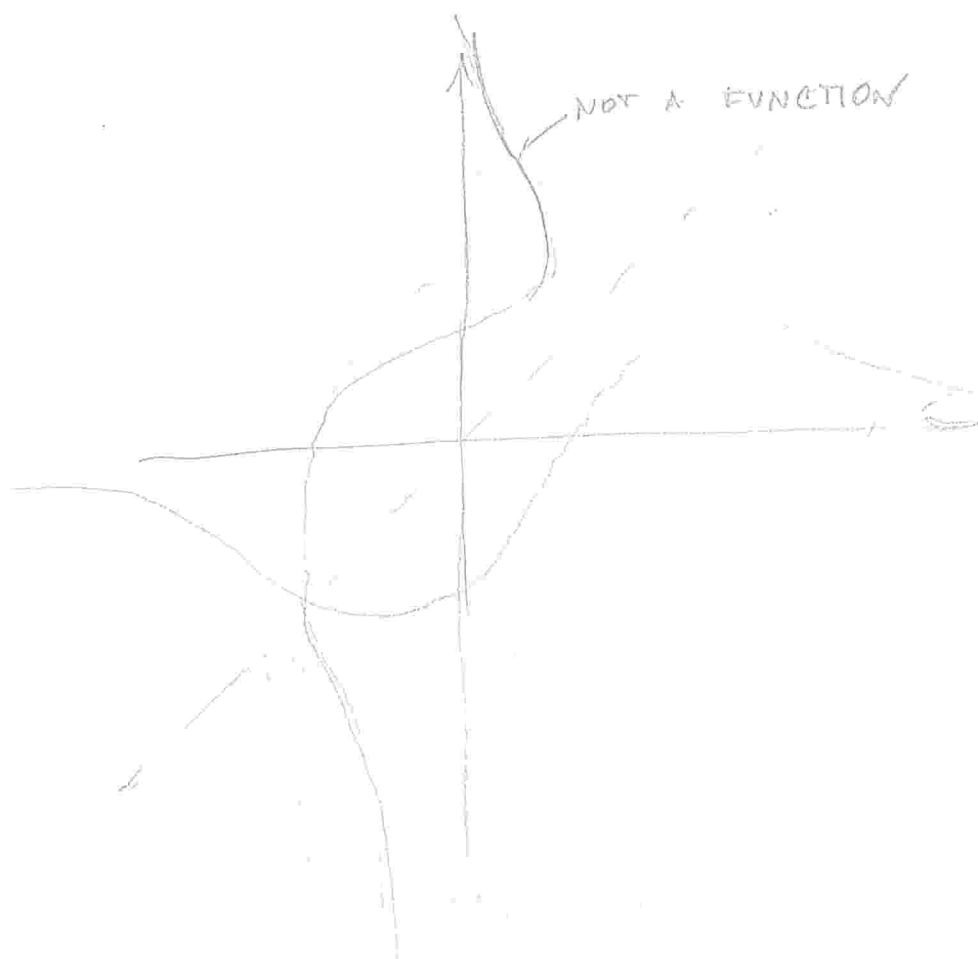
A SET OF ALL b FOR WHICH $\exists a : (a,b)$ IN THE SET. THUS, $b = f(a)$.
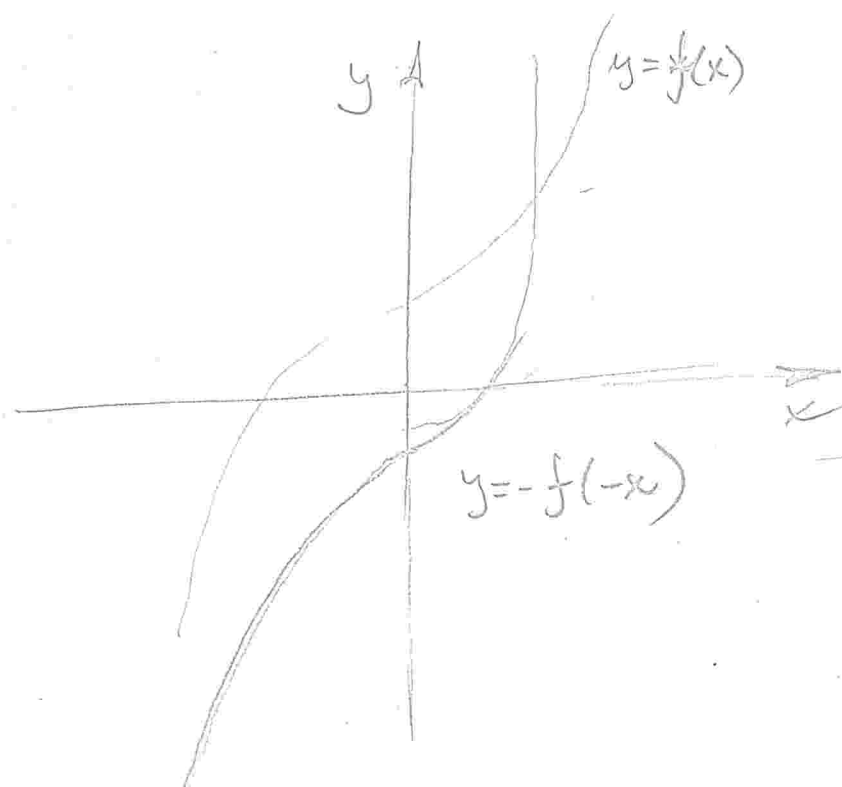

$f(-x)$ $f(x)$ REFLECTION IN THE y-AXIS

$y = f(x)$

$y = f(x)$

y

$y = f(x)$

x

$y = -f(-x)$

NOT A FUNCTION

DEFINITION

EVEN: $f(x)$ $\iff$ $f(x) = f(-x)$

$f(x)$ IS ODD $\iff$ $f(x) = -f(-x)$

E6. $\sin(x) = -\sin(-x)$

$\cos(x) = \cos(-x)$