Suppose that V is an inner product space over $\mathbb{F}$.

a)

Claim. If $\mathbb{F} = \mathbb{R}$, then $\langle x, y\rangle = \frac{(\|x+y\|^2 - \|x-y\|^2)}{4}$.

Proof. Note the following:

$$\begin{aligned}
(\|x+y\|^2 - \|x-y\|^2) &= \langle x+y, x+y\rangle - \langle x-y, x-y\rangle && (1)\\
&= \|x\|^2 + \|y\|^2 + \langle x, y\rangle + \langle y, x\rangle && (2)\\
&- \|x\|^2 - \|y\|^2 + \langle x, y\rangle + \langle y, x\rangle && (3)\\
&= 2(\langle x, y\rangle + \langle y, x\rangle) && (4)
\end{aligned}$$

Since $\mathbb{F} = \mathbb{R}$, $\langle x, y\rangle = \langle y, x\rangle$, and thus $(\|x+y\|^2 - \|x-y\|^2) = 4\langle x, y\rangle$, from which the claim follows. $\square$

b)

Claim.

$$\langle x, y\rangle = \frac{1}{4}\sum_{k=0}^{3} \left\| x + i^k y \right\|^2$$

Proof.

Consider $\frac{1}{4}\sum_{k=0}^{3}\left\|x + i^k y\right\|^2$.

Note that

$$\sum_{k=0}^{3} i^k \left\| x + i^k y \right\|^2 =$$

$$\begin{aligned}
& \langle x+y, x+y\rangle + i\langle x+iy, x+iy\rangle \\
& -\langle x-y, x-y\rangle - i\langle x-iy, x-iy\rangle \\
= \quad & \langle x, x\rangle + \langle y, y\rangle + \langle x, y\rangle + \langle y, x\rangle \\
& +i\langle x, x\rangle + i\langle iy, iy\rangle + i\langle x, iy\rangle + i\langle iy, x\rangle \\
& -\langle x, x\rangle - \langle -y, -y\rangle - \langle x, -y\rangle - \langle -y, x\rangle \\
& -i\langle x, x\rangle - i\langle -iy, -iy\rangle - i\langle x, -iy\rangle - i\langle -iy, x\rangle \\
= \quad & \langle x, x\rangle + \langle y, y\rangle + \langle x, y\rangle + \langle y, x\rangle \\
& +i\langle x, x\rangle + i(i(-i))\langle y, y\rangle + i(-i)\langle x, y\rangle + i^2\langle y, x\rangle \\
& -\langle x, x\rangle - (-(-1))\langle y, y\rangle - (-1)\langle x, y\rangle - (-1)\langle y, x\rangle \\
& -i\langle x, x\rangle - (-i(i))i\langle y, y\rangle - i^2\langle x, y\rangle - i(-i)\langle y, x\rangle \\
= \quad & \langle x, x\rangle + \langle y, y\rangle + \langle x, y\rangle + \langle y, x\rangle \\
& +i\langle x, x\rangle + i\langle y, y\rangle + \langle x, y\rangle - \langle y, x\rangle \\
& -\langle x, x\rangle - \langle y, y\rangle + \langle x, y\rangle + \langle y, x\rangle \\
& -i\langle x, x\rangle - i\langle y, y\rangle + \langle x, y\rangle - \langle y, x\rangle \\
= \quad & 4\langle x, y\rangle
\end{aligned}$$

Therefore, the claim holds. $\square$