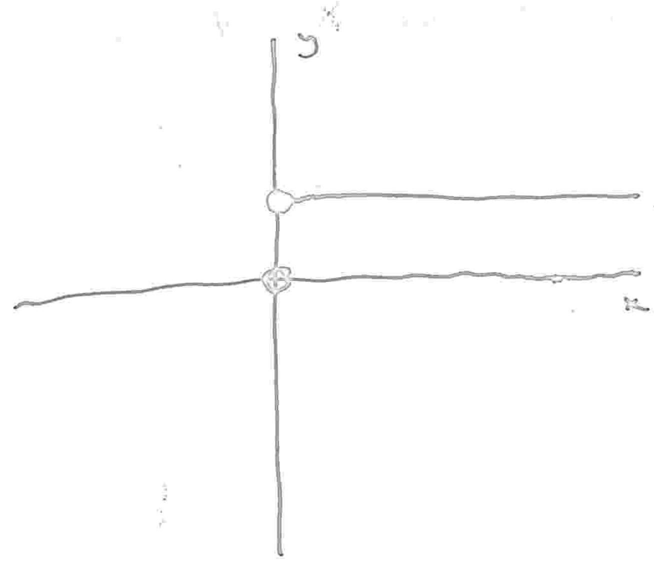
MAT157 2016 11 09

CONTINUITY PROBLEMS

EXAMPLE



$$f(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

$\lim_{x \to 0} f(x)$ DOES NOT EXIST.

Suppose $\lim_{x \to 0} f(x) = L$.

Let $\varepsilon = \frac{1}{2}$. Suppose $\exists \delta > 0$.

Thus, if $0 < |x| < \delta$, then $|f(x) - L| < \frac{1}{2}$.

In particular, if $x = \frac{\delta}{2} < \delta$,

then $f(x) = 1$, $|1 - L| < \frac{1}{2}$.

If $x = -\frac{\delta}{2}$, then $f(x) = 0$. $|0 - L| < \frac{1}{2}$

But $1 = |1 - L + L| \leq |1 - L| + |L| < 1$

$$\lim_{x \to 2} \frac{x+1}{x^2-4} = \lim_{x\to 2} \frac{x+1}{(x+2)(x-4)}.$$

If $x \to 2^+$, $x > 2$ then $x^2 > 4$.

$1 < \delta < 1$, $2 < x < 3$, $\Leftrightarrow$ $4 < x+2 < 5 \Rightarrow$

$$\Rightarrow \frac{x+1}{x+2} > \frac{3}{5}.$$

Given $M > 0$,

let $\delta = \min\left(1, \frac{3}{5M}\right)$.

Then $|f(x)| > M$

Suppose $x < 2$.

Let $\delta = \min\left(1, \frac{1}{2M}\right)$ so $\frac{1}{2-x} > 2M$.

$$\frac{x+1}{x+2} \cdot \frac{1}{2-x} > \frac{1}{2}(M \cdot 2) = M$$

$$\frac{x+1}{x+2} - \frac{1}{2} = \frac{2x+2 - x-2}{2(x+2)}$$

$$= \frac{x}{2(x+2)}$$

f	f'
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec \tan$
$\cos$	$-\sin x$
$\cot$	$-\csc^2$
$\csc x$	$-\csc \cot$

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x \sin x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

Suppose $f(x)$ and $g(x)$ are continuous on $[a, b]$. Let $h(x) = \max(f(x), g(x))$. Is $h(x)$ be continuous at?

Consider $r \in (a, b)$.

Then
$$\left[\begin{array}{l} h(r) = f(r) > g(r) \\ h(r) = g(r) > f(r) \\ h(r) = g(r) = f(r) \end{array}\right.$$

If $f(r) > g(r)$, we can find δ s.t.

$f(x) > g(x)$ on $(r-\delta, r+\delta)$, so

$$\lim_{x \to r} h(x) = \lim_{x \to r} f(x) = f(r) = h(r)$$

Suppose $f(r) = g(r) = h(r)$.

Given $\varepsilon > 0$,

we now find $\delta > 0$ s.t $|x - r| < \delta$

$\Rightarrow \left| f(x) - f(r) \right| < \varepsilon$

AND $\left| g(x) - g(r) \right| < \varepsilon$

$\Rightarrow \left| h(x) - h(r) \right| < \varepsilon$.