

1     Let  $F = \mathbb{Z}_5$ .

$$\left[ \begin{array}{ccc|cc} 1 & 4 & 3 & 1 & 2 \\ 0 & 2 & 4 & 2 & 4 \\ 4 & 1 & 3 & 3 & 1 \end{array} \right] = \quad (1)$$

$$L_3 \rightarrow L_3 + L_1 \mid = \left[ \begin{array}{ccc|cc} 1 & 4 & 3 & 1 & 2 \\ 0 & 2 & 4 & 2 & 4 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (2)$$

$$L_2 \rightarrow L_2 + L_1 \mid = \left[ \begin{array}{ccc|cc} 1 & 4 & 3 & 1 & 2 \\ 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (3)$$

$$L_1 \rightarrow L_1 - L_2 \mid = \left[ \begin{array}{ccc|cc} 0 & 3 & 1 & 3 & 1 \\ 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (4)$$

$$L_1 \rightarrow L_1 - L_3 \mid = \left[ \begin{array}{ccc|cc} 0 & 3 & 0 & 4 & 3 \\ 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (5)$$

$$L_1 \rightarrow 2L_1 \mid = \left[ \begin{array}{ccc|cc} 0 & 1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (6)$$

$$L_2 \rightarrow L_2 - L_1 \mid = \left[ \begin{array}{ccc|cc} 0 & 1 & 0 & 3 & 1 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (7)$$

$$L_2 \rightarrow L_2 - 2L_3 \mid = \left[ \begin{array}{ccc|cc} 0 & 1 & 0 & 3 & 1 \\ 1 & 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (8)$$

$$\text{switch } L_1 \text{ and } L_2 \mid = \left[ \begin{array}{ccc|cc} 1 & 0 & 0 & 2 & 4 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (9)$$

2     Therefore, in the first case,  $x_1 = 2, x_2 = 3, x_3 = 4$ ,

3     while in the second case  $x_1 = 4, x_2 = 1, x_3 = 3$ .