1 Logarithms

Motivation: simplified multiplication (Napier)

For $x > 0$, write $x = b^{\log_b(x)}$.

Therefore, if $\log_b(x)$ and $\log_b(y)$ exists, then $\log(xy) = \log(x) + \log(y)$.

1.1 Formalisation

Definition 1.1. For $x > 0$, $\log(x) = \int_1^x \frac{1}{t}\,\mathrm{d}t$.

Fix y, and let

$$f(x) = \log(xy) = \int_1^{xy} \frac{1}{t}\,\mathrm{d}t,$$

and therefore

$$f'(x) = \frac{1}{xy}\frac{\mathrm{d}(xy)}{\mathrm{d}t} = \frac{y}{xy} = \frac{1}{x}$$