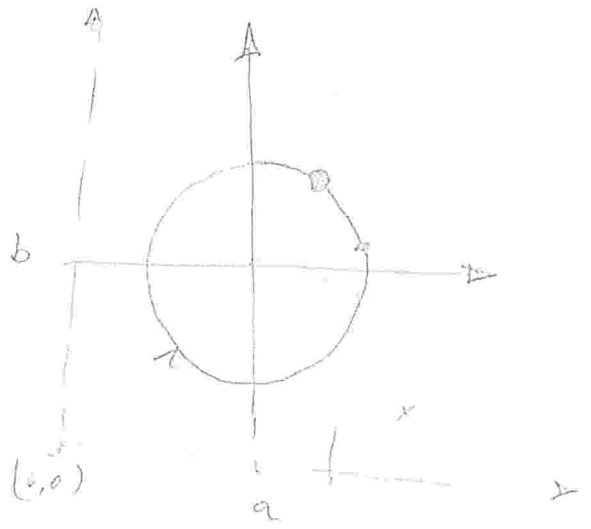
MAT157: ELLIPSES, HYPERBOLAS, LIMITS

2016/10/03

CIRCLE

THE CIRCLE OF RADIUS r AND CENTRE (a,b) IS

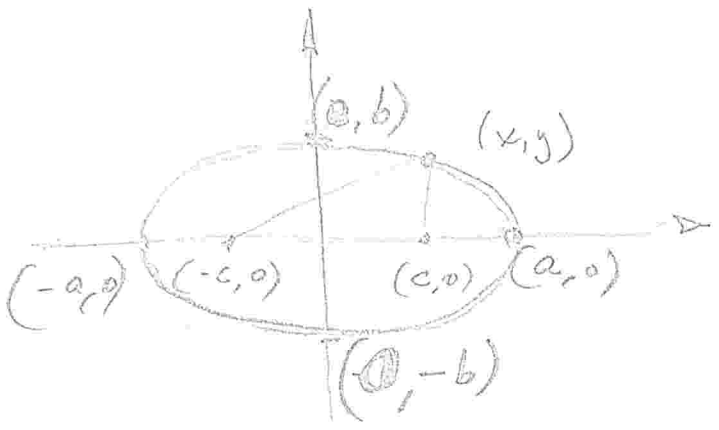
$$(x-a)^2 + (y-b)^2 = r^2$$



ELLIPSE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

THE ELLIPSE IS ALL POINTS (x,y) FOR WHICH THE SUM OF THE DISTANCES TO THE FOCI IS $2a$.

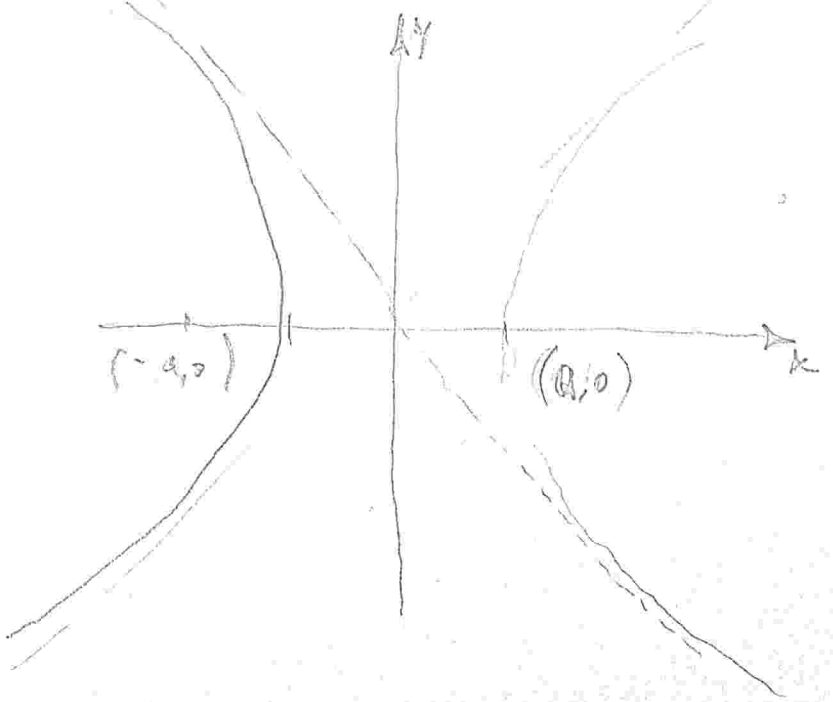


HYPERBOLA

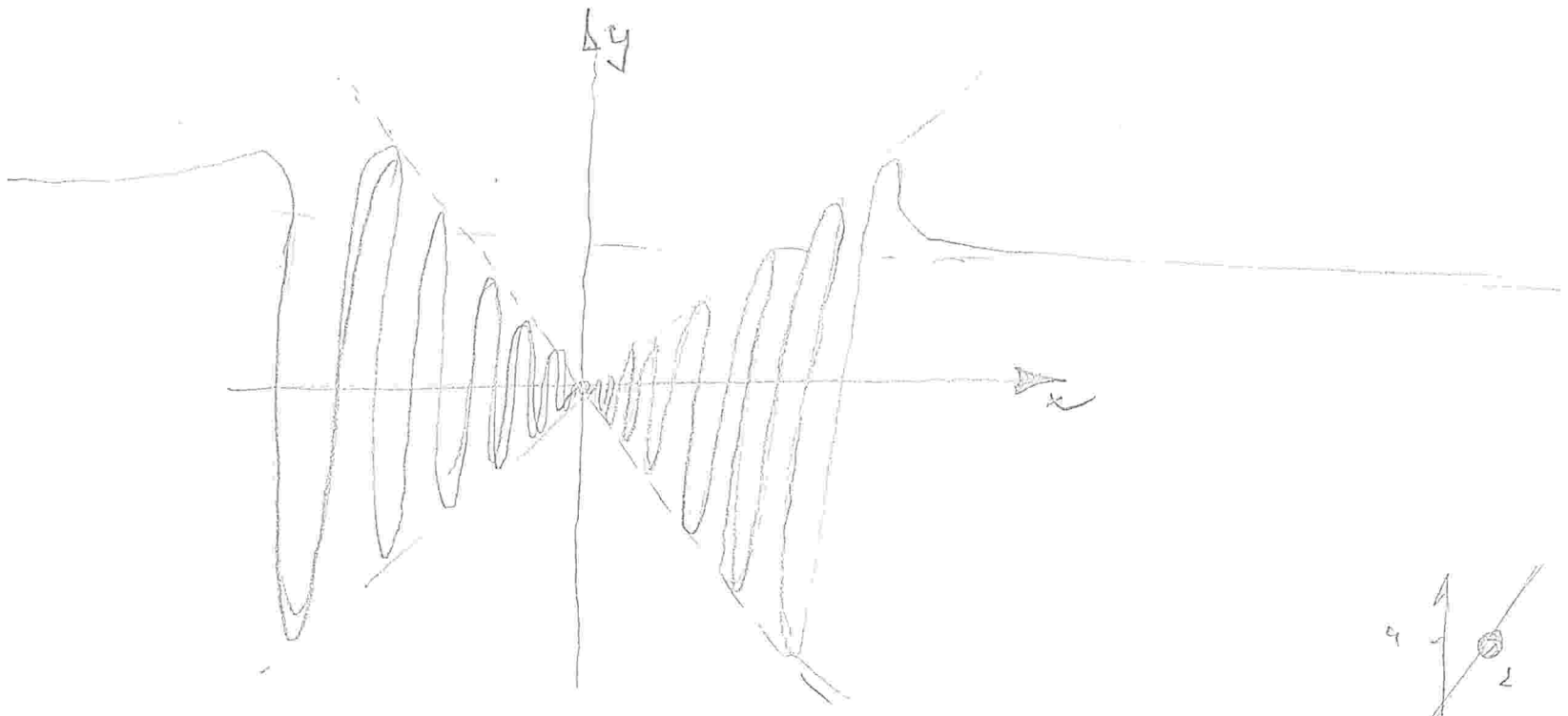
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ASYMPTOTES

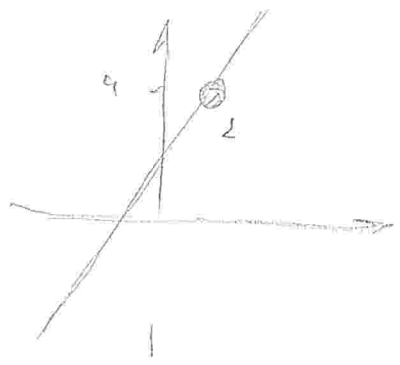
$$y = \pm \frac{b}{a} x$$

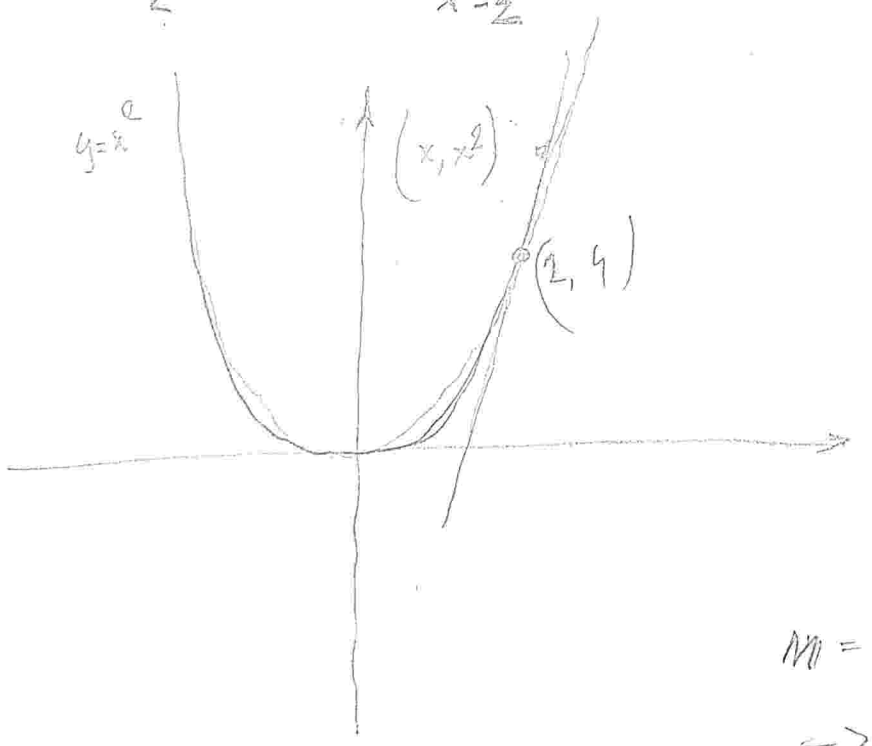


CONSIDER $g(x) = x \sin(1/x)$.



- $h(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x-2)(x+2)}{x-2} = x+2, \quad x \neq 2$





WHAT IS TANGENT TO $y=x^2$ AT THE POINT $(2,4)$?

WHAT IS THE SLOPE?

BUT: THERE IS ONE POINT $(2,4)$, NEED TWO.

CONSIDER THE SLOPE:

$M = \dfrac{x^2-4}{x-2}$, AS $x \to 2$, $m = x+2 = 4$.

$\Rightarrow$ 4 IS THE SLOPE OF THE TANGENT.

DEFINITION.

A FUNCTION $f(x)$ HAS A LIMIT L AS x APPROACHES a, AND WE WRITE $\lim_{x \to a} f(x) = L$,

IF $\exists b, c$ WITH $a \in (b, c)$, i.e. $b < a < c$,

AND $f(x)$ IS DEFINED AT EVERY $x \in (b, c)$ EXCEPT POSSIBLY AT $x = a$:

$$\forall \varepsilon > 0 \; \exists \delta > 0 : |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$