MAT157: 20161026

DERIVATIVE II

EXAMPLE

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ x^3, & x < 0 \end{cases}$$

CONSIDER

$$① \quad \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 - 0^2}{h}$$

$$= \lim_{h \to 0^+} \frac{h^2}{h} = \lim_{h \to 0^+} h = 0$$

$$② \quad \lim_{h \to 0^-} \frac{f(h) - f(0)}{h} = \lim_{h \to 0^-} h^2 = 0.$$

③ SINCE ①: AND ②: ARE EQUAL,

$$f'(0) = 0$$

$$f'(x) = \begin{cases} 2x, & x > 0 \\ 0, & x = 0 \\ 3x^2, & x < 0 \end{cases}$$

THEOREM

IF $f(x)$ IS DIFFERENTIABLE AT $x = a$, THEN IT IS CONTINUOUS AT a.

PROOF

SINCE $f(x)$ IS CONTINUOUS,

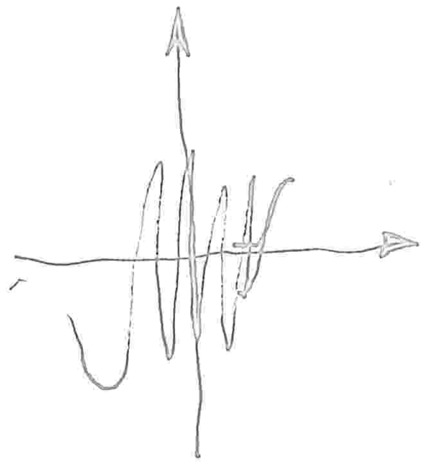
$$\lim_{x \to a} \left(f(x) - f(a) \right) = 0.$$

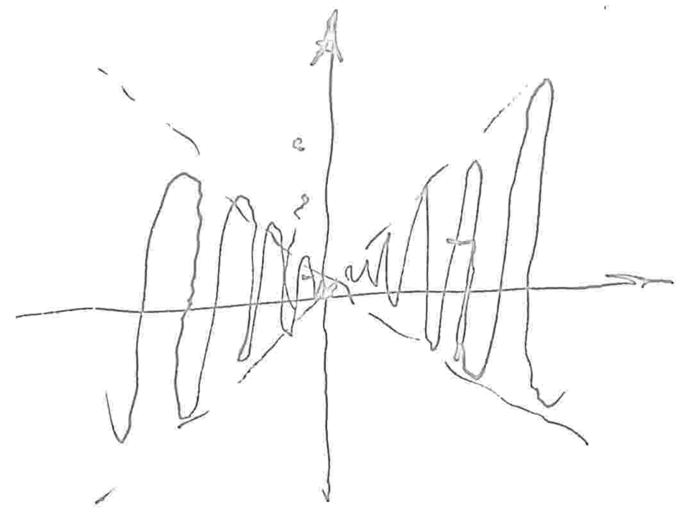
$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot x - a =$$

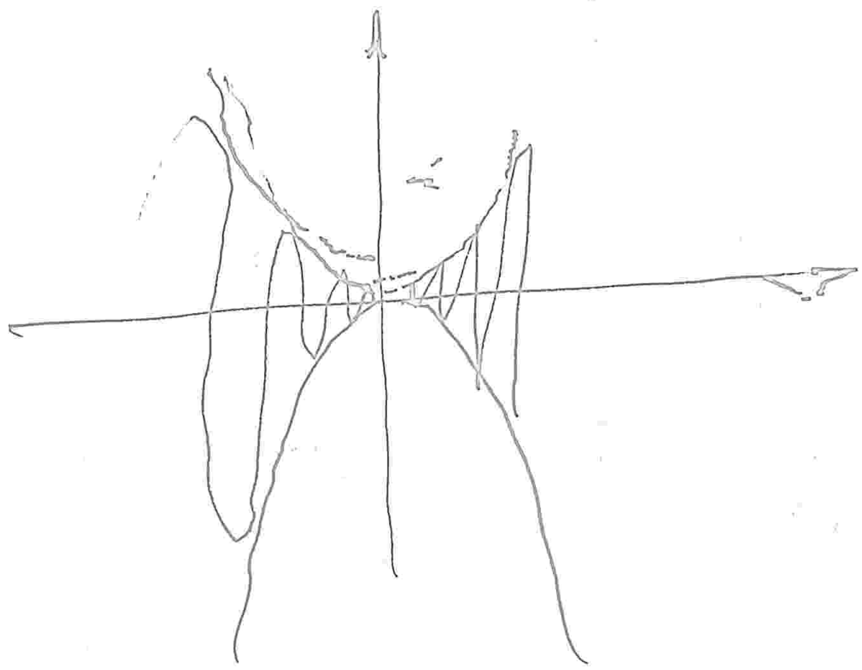
$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim (x - a)$$

$$= f'(a) \cdot 0$$

$$= 0$$







$$\lim_{h \to 0} \left(\frac{h \sin\left(\frac{1}{h}\right) - 0}{h} \right) = \lim_{h \to 0} \sin\left(\frac{1}{h}\right)$$, which

MEANS THAT THE FUNCTION IS CONTINUOUS

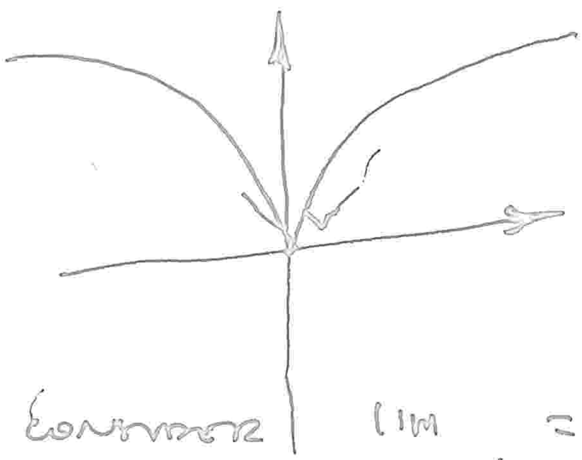
BUT <u>NOT</u> DIFFERENTIAL AT $x=0$

$$\frac{d}{dx}\left(x^2 \sin\left(\frac{1}{x}\right)\right) = \lim_{h \to 0} h \sin\left(\frac{1}{h}\right) = 0$$

SQUEEZED BETWEEN $y = x^2$ & $y = -x^2$,

$f(x)$ has a tangent at the point $(0,0)$.

Let $f(x) = \sqrt{|x|} = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$



Consider $\lim_{h \to 0^+} = \lim_{h \to 0^+} \frac{1}{\sqrt{h}} \longrightarrow \infty$

So $\frac{1}{\sqrt{h}}$ gets bigger than any fixed M.

Thus, given $M > 0$ $\exists\, \delta$:

If $0 < h < \delta$, then $\frac{1}{\sqrt{h}} = M$.

In particular, $\lim_{h \to 0^+} \frac{\sqrt{h} - c}{h}$ does not exist

$\lim_{h \to 0} \frac{\sqrt{h} - 0}{h}$ does not exist.

So $f(x)$ is not

differentiable

$$\lim_{h \to 0} = \frac{f(h) - 0}{h} = \lim_{h \to 0} \frac{\sqrt{-h}}{h} = \lim_{h \to 0^-} -1$$

Thus, $f(x)$ has a 'cusp' at $x=0$.

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ x^3, & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} f' = 2x, & x \geq 0 \\ f' = 3x^2, & x < 0 . \end{cases}$$

$$\lim_{x \to 0^+} \frac{f'(x) - f'(0)}{x} = \lim_{x \to 0^+} \frac{2x - 0}{x} = 2$$

$$\lim_{x \to 0^-} \frac{f'(x) - f'(a)}{x} = \lim_{x \to 0} \frac{3x^2 - 0}{x} = \lim_{x \to 0} 3x = 0$$

$\neq$

If $f(t)$ IS A DISPLACEMENT OF A MOVING OBJECT, $f'(t)$ is a RATE OF CHANGE OF DISPLACEMENT, $f''(t)$ IS A RATE OF CHANGE OF VELOCITY; $f'''(t)$ - jerk.

ALTERNATIVE NOTATION $f''' = f^{(3)}(x)$ $f''''\ldots = f^{(n)}_x$

$$f''(x) = \frac{d^2 f}{dx^2}$$ d two f by d x squared.