1 Ricci Flow and the Topology of 3-Manifolds

The following result was proved in XIX century:

Theorem 1.1

Any closed orientable surface N is diffeomorphic to a connected sum $T^2\#\cdots\#T^2$. Moreover, N has a Riemannian metric g with a constant Gaussian curvature $K\in\{-1,0,1\}$.

1.1 Topology and Geometry of 3-Manifolds

Key words: quotient manifold, Poincare dodecahedral space, flat manifolds, Thurston geometry, homogeneous Riemannian 3-manifolds, fundamental Lie groups and their quotients, geometric manifold, connected sum of manifolds, prime manifolds

Definition 1.2. A smooth 3-manifold M is geometrizable if it admits a Riemannian metric g such that (M,g) is geometric.

It was proven by Kneeser and Milnor that decomposition of every nontrivial 3-manifold into prime manifolds is always possible, and the corresponding primes are unique up to order. However, there are prime manifolds which are not geometrisable.

1.2 Ricci Flow

Intuitively, Ricci flow connects the change of a manifold with time to its Ricci curvature.

Key Words: deformation retraction, homotopy equivalence, fibration, stabiliser, Sobolev space