MAT157: 20161118

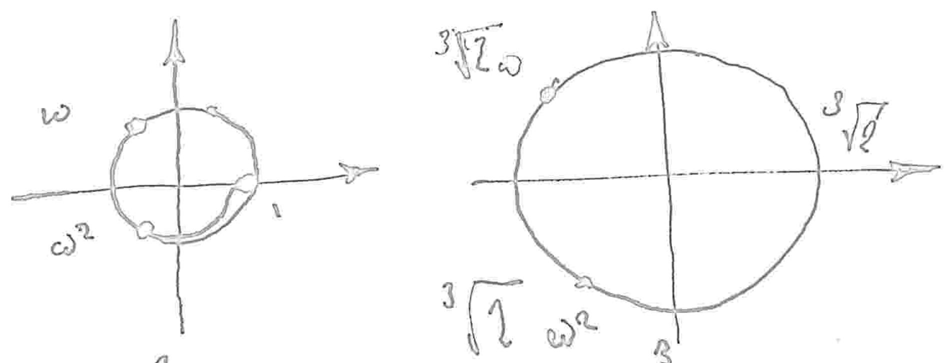
# SIGNIFICANCE OF DIFFERENTIATION

## DIVERSION ON GALOIS THEORY

$a_n x^n + \dots + a_0 = 0$, $a_i \in \mathbb{Q}$.

1. FROM FTA, SUCH AN $x$ EXISTS, AND THUS THE EQ. CAN BE SPLIT ?

$\prod (x - x_i) = 0$




$x^3 - 1 = 0$ $\qquad$ $x^3 - 2 = 0$

TOO MANY SYMMETRIES - NO FORMULA FOR THE ROOTS IN $\cdot, \sqrt[\cdot]{\cdot}, +, -$.

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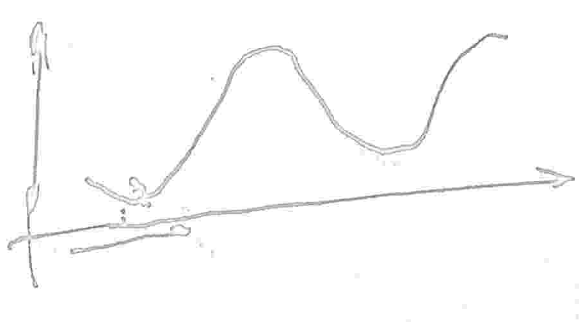
## MAX/MIN PROBLEMS

DEFINITION:

THE FUNCTION $f(x)$ HAS A MAXIMUM AT $x = a$ IF $f(a) \geq f(x)$ $\forall x \in \mathrm{Dom}(f)$.

EXAMPLE: MAX AT THE INFINITE NO. OF POINTS.



THEOREM  SUPPOSE $f(x)$ HAS A MAX (OR MIN) AT $x = a$ AND $f$ IS DIFFERENTIABLE AT $x=a \Rightarrow f'(a) = 0$.

PROOF

$f(x)$ HAS A MAX AT $x=a \Rightarrow$

$f(a) \geq f(x) \Rightarrow f(a) - f(x)$

$$\lim_{x \to a^+} \frac{f(x) - f(a)}{x-a} \leq 0 ;$$

$$\lim_{x \to a^-} \frac{f(x) - f(a)}{x-a} \geq 0$$

$\Rightarrow f'(a) = 0.$

REMARK  THE THEOREM DOES NOT ASSUME THAT $f(x)$ IS DIFFERENTIABLE OR CONTINUOUS AT $x=0$.

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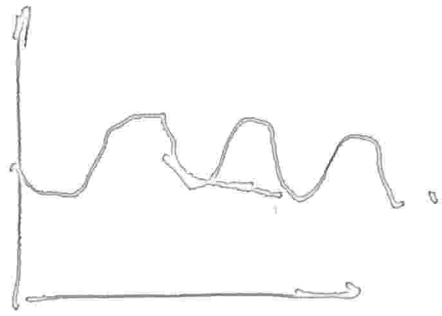

DEFINITION : $x = a$ IS A CRITICAL POINT OF $f(x)$ IF $f'(a) = 0$.

CLASSIFICATION OF POINTS:

1. CRITICAL POINTS

2. POINTS WHERE $f'$ IS NOT [illegible]

3. endpoints.

A max or min occur at one or more of those kinds of points

EXAMPLE

$h(x) = x^2 - 3x + 2$

$h' = 3x^2 - 3 = 3(x^2 - 1)$

Critical points are $x = 1, -1$

$h(1) = 0$

$h(-1) = 4$

$h(5) = -16$

LET $f(x) = x^4 - 4x^2$ on $[0, 5]$.

$f'(x) = 4x^3 - 8x =$

$= 4x(x^2 - 2)$

$= 4x(x - \sqrt{2})(x + \sqrt{2})$

$f'(x) = 0$ at $x = 0$, $\sqrt{2}$

$f(0) = 0$

$f(\sqrt{2}) = -4$ :: pos

$f(5) = 525$

for all

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$g(x) = \sqrt{|x|}$ on $[-2, 4]$

$g(0) =$

$g(-2) = \sqrt{2}$

$g(4)$

$$g'(x) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{on } (0, 4] \\ -\frac{1}{2\sqrt{-x}} & \text{on } [-2, 0) \end{cases}$$

The derivative gives information about the function, but only locally