MAT 157: 20161104

CHAIN RULE REVIEW

CHAIN RULE PROOF REVIEW

$g(x)$ IS DIFFERENTIABLE AT $x=a$,

$f(x)$ IS DIFFERENTIABLE AT $x=g(a)$.

DEFINE
$$\varphi(h) = \begin{cases} \dfrac{f\circ g\,(a+h) - f\circ g\,(a)}{g(a+h)-g(a)}, & \text{if } g(a+h)-g(a)\neq 0 \\ f'(g(a)), & \text{if } g(a+h)=g(a) \end{cases}$$

NEED TO SHOW: φ IS CONTINUOUS AT $h=0$.

SUPPOSE $\varepsilon>0$ IS GIVEN.
NOTE THAT BECAUSE f IS DIFFERENTIABLE AT $g(a)$, WE CAN FIND $\delta'>0$ SO THAT IF $|k|<\delta'$, THEN

$$\left| \frac{f(g(a)+k)-f(g(a))}{k} - f'(g(a)) \right| < \varepsilon$$

g IS DIFFERENTIABLE AT $x=a \implies$

g IS CONTINUOUS AT $x=a$. WITH

δ' AS ABOVE, $\delta > 0$ CAN BE

FOUND SO THAT IF $|h| < \delta$,

THEN $|g(a+h) - g(a)| < \delta'$.

IF $|h| < \delta$, THEN $|g(a+h) - g(a)| < \delta'$,

WRITE $k = g(a+h) - g(a)$,

$$g(a+h) = g(a) + k.$$

$|k| < \delta'$, SO

$$\left| \frac{f(g(a)+k) - f(g(a))}{k} - f'(g(a)) \right| < \varepsilon, \text{ THUS}$$

$$\left| \frac{f(g(a+h)) - f(g(a))}{g(a+h) - g(a)} - f'(g(a)) \right| < \varepsilon$$

$\implies$ IF $|h| < \delta$ AND $(g(a+h) - g(a) \neq 0)$,

THEN $|\varphi(h) - \varphi(0)| < \varepsilon$.

IF $g(a+h) - g(a) = 0$,

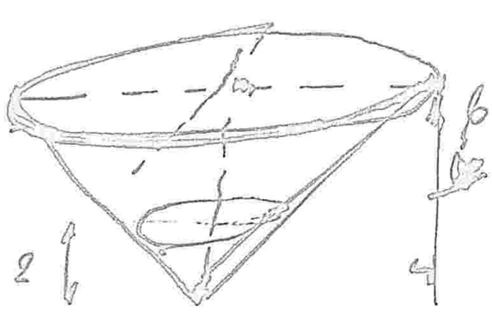
$|\varphi(h) - \varphi(0)| = 0 < \varepsilon$

IF $|h| < \delta$, THEN $|\varphi(h) - \varphi(0)| < \varepsilon$

$$(f \circ g)'(a) = \lim_{h \to 0} \frac{f(g(a+h)) - f(g(a))}{h}$$

$$= \lim_{h \to 0} \varphi(h) \cdot \frac{g(a+h) - g(a)}{h}$$

$$= \varphi(0)\, g'(a) = f'(g(a)) \cdot g'(a)$$



CIRCULAR CONE OF radius ...

$\pi r^2 \cdot h$ = VALUE OF y

$$\frac{dV}{dt} = -10^{-3}\ m^3/s$$

$\frac{y}{x} = 3$ ∴ $V = \frac{\pi}{3} x^2 y$

$$\Rightarrow V = \frac{\pi}{27} y^3 \Rightarrow V' = \frac{\pi}{9} \cdot y^2 \cdot y'$$

$\Rightarrow$ since $V' = -\frac{1}{1000} \frac{m^3}{s} \Rightarrow y' = -\frac{1}{1000} \frac{9}{4\pi} = -\frac{9}{4000\pi}$ m/sec