1 Local Limit of Random Sorting

Let S_n be a symmetric group of a list $[n]$, generated by adjacent transposition $(1, i+1)$ for $1 \leq i \leq n-1$.

Let $id = 1, 2, \ldots, n$ and $rev = n, n-1, \ldots, 1$

We can define a sorting nework as a minimal langth path in the Cayley graph of S_n, generated by adjacent transpositions from id to rev.

What is the length of this path? It is $\binom{n}{2}$.

The sorting can be visualised with the end wiring diagram. The exciting result is that the whole diagram can be reconstructed from the crossings of each wiring.

Theorem 1.1 (AHRV, 08)

Let $(S_1, S_2, \ldots, S_{\binom{n}{2}}$ be the random sequence of swaps of a uniformly random sorting network of S_n. Then

$$\frac{1}{\binom{n}{2}} \sum_{i=1}^{n} \delta_{(2\frac{S_i}{n}-1, \frac{1}{n})} \subseteq [-1, 1] \times [0, 1]$$

Moreover, $\mu_n \to$ semicircle law $\times$ Lem$[0, 1]$.

Let $G = [g_{ij}]_{i,j}$ be a matrix, where g_{ij} are iid standard real Gaussians and $i, j \geq 1$.

Define $A = \frac{G - G^T}{\sqrt{2}}$, and let A_m be a top left $m \times m$ conter of A.

Then $\Lambda_m = ((j, \sqrt{2m}\lambda_i^{j+m})$ for $j \in \mathbb{Z}, i \geq 1)$, and $\Lambda_m \subseteq \mathbb{Z} \times \mathbb{R}_{\geq 0}$.

Then the hard edge limit can be proven, as was shown by Forrester and Nordenstam.

Moreover, Edelman-Greene bijection is an explicit bijection between sorting networks of S_n and standard Young tableu of shape $(n-1, n-2, n-3, \ldots, 2, 1)$. In this way, Young tableu can be represented as a bead process.