

1 Surfaces in the Space of Surfaces

Key words: totally geodesic moduli space, variety,

2 Planes in Hyperbolic Manifolds

Key words: hyperbolic manifolds and their subspaces (line \rightarrow geodesic), universal cover, orbit, fundamental group

Theorem(Shah, Ratner): The closure of the image plane is a compact, immersed, totally geodesic k -manifold(submanifold) inside n -manifold.

Picture a three hyperbolic space as an interior of a ball. A plane in \mathbb{H}^3 corresponds to a hemisphere.

3 Moduli Space

Key words: moduli space; Riemann surfaces and their genus; complex variety; isometric holomorphic immersion, immersed hyperbolic plane, Teichmüller curve

The Hilbert modular surface is not totally geodesic.

4 Totally Geodesic Subvarieties

Key words: curve embeddings, Fuchsian group, Klein quartic (see Helaman Ferguson)

Almost all subvarieties in the moduli space are *contracted*.

- Covering constructions
- Teichmüller curves

5 Totally Geodesic Varieties in Moduli Space

Key words: polar conic, Hessian, flex, determinant of derivatives, Cayleyan, cover of a curve, bundle, fiber, $SL_2(\mathbb{R})$ orbits, holomorphic 1-forms, quotient elliptic curve, meromorphic quadratic differential, Coxeter diagrams, Prym variety, construction of infinite series of Teichmüller curves

Theorem: There is a primitive, totally geodesic complex surface F (the flex locus) properly immersed into the moduli space $\mathcal{M}_{1,3}$.

See G. Salmon, “Higher Plane Curves” for precursors.