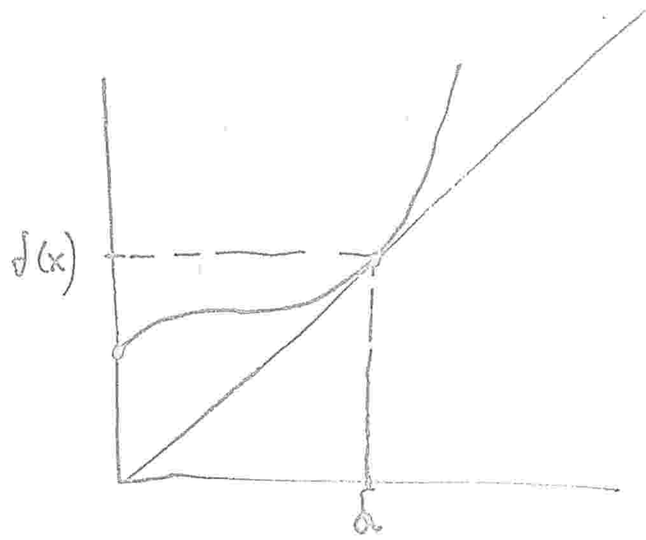
MAT157: DERIVATIVE OF COMPOSITE FUNCTIONS 20161031

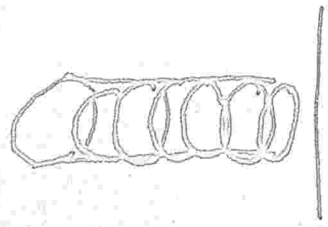


INTERPRET THE DERIVATIVE $f'(a)$ AS A "STRETCH FACTOR", WITH WHICH x CAN BE MAPPED TO y.

COMPOSITE FUNCTIONS:

$$f \circ g(x) = f(g(x))$$

$$(f \circ g)'(a) = f'(g(a)) \circ g'(a)$$



THEOREM

CHAIN RULE

SUPPOSE $g(x)$ IS DIFFERENTIABLE AT $x=a$

AND $f(x)$ IS DIFFERENTIABLE AT $g(a)$.

THEN $(f \circ g)(x)$ IS DIFFERENTIABLE

AT $x = a$ AND $(f \circ g)'(a) = f'(g(a))\,g'(a)$.

EXAMPLE

1) $f(x) = \sqrt{x} \Rightarrow f(x) \cdot f(x) = x$.

$\Rightarrow$ 2) $2 f(x) \cdot f'(x) = (x)' = 1$;

$\Rightarrow$ $f'(x) = \dfrac{1}{2f(x)}$, IF $f(x) \neq 0$.

PROOF

$$\lim_{h \to 0} \frac{(f \circ g)(x+h) - (f \circ g)(x)}{h} =$$

$$= \lim_{h \to 0} \frac{(f \circ g)(x+h)}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h} \Bigg\} \text{WRONG!}$$

A ~~NEW~~ FUNCTION $\Phi(h)$ IS DEFINED.

$$\Phi(h) = \begin{cases} \dfrac{f \circ g(a+h) - (f \circ g)(a)}{g(a+h) - g(a)} \\ \\ f'(g(x)), \quad \text{IF } g(a+h) - g(a) \end{cases}$$

CLAIM: $\Phi(h)$ IS CONTINUOUS AT $h = 0$.

GIVEN $\varepsilon > 0$.

f' DIFFERENTIABLE AT $g(a) \Rightarrow f$ IS CONTINUOUS AT $x = a$.

$\Rightarrow \exists\, \delta'$: $|u - g(a)| < \delta'$, THEN $|f(u) - f(g(a))| < \varepsilon$,

SINCE g IS DIFFERENTIABLE AT a, $\exists\, \delta > 0$:

$|x - a| < \delta \Rightarrow |g(x) - g(a)| < \delta'$.

SUPPOSE $|x - a| < \delta$.

THEN $|g(x) - g(a)| < \delta$

IF $|g(x)-g(a)| = 0$, i.e. $g(x)=g(a)$, THEN

$\Phi(x) = f'(g(a))$

SO $|\Phi(h) - \Phi(0)| < \varepsilon$

IF $|g(x)-g(a)| \neq 0$, $|g(x)-g(a)| < \delta'$, $\varepsilon > 0$

$|f(g(x)) - f(g(a))| < \varepsilon$ thus making

Φ A CONTINUOUS FUNCTION.

$$f \circ g(a) = \lim_{h \to 0} \Phi(h) \frac{g(a+h)-g(a)}{h}$$

$$= \Phi'(a) \cdot g'(a)$$

$$= f'(g(a)) \cdot g'(a)$$

$x^2 + y^2 = L^2 \Rightarrow$

$2x x' + 2y \cdot y' = 0$

$\therefore \quad y' = \frac{-x' \cdot x}{y} = \frac{-3x}{\sqrt{L^2 - x^2}}$