MAT157: CONVEXITY III & INVERSE FUNCTIONS 20161202

IF f IS DIFFERENTIABLE ON AN INTERVAL AND IF EACH TANGENT LINE TOUCHES THE GRAPH AT ONLY ONE POINT, THEN f BE EITHER CONVEX OR CONCAVE.

PROOF.

EXERCISE

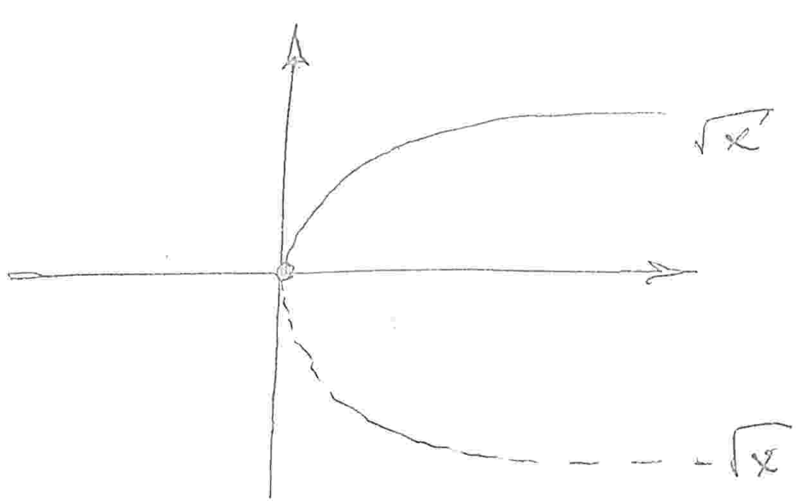
$\sqrt{x}$ IS THE INVERSE OF $x \longmapsto x^2$,

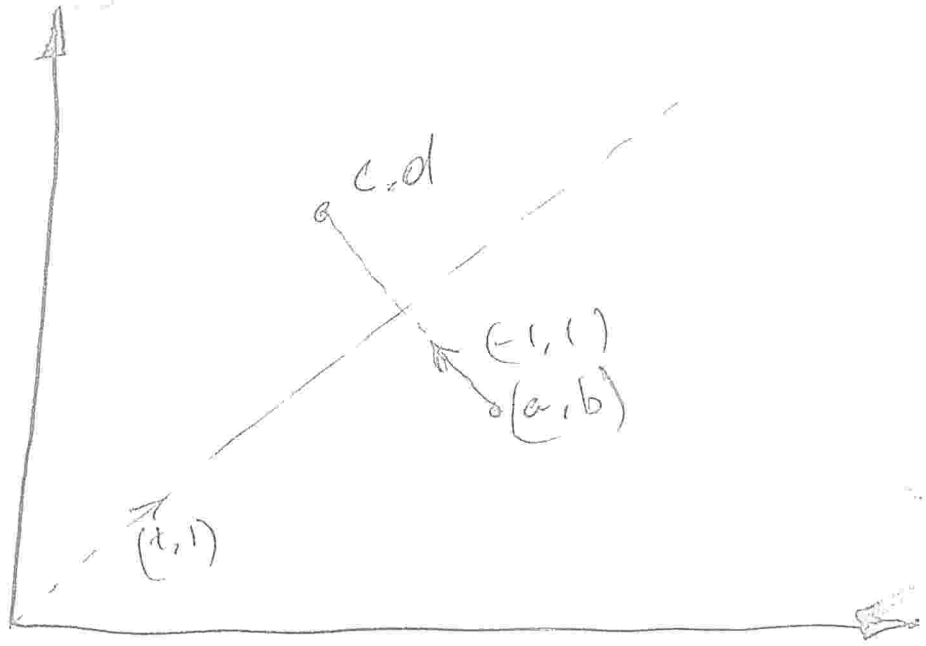
$\sqrt{x} \geq 0$

NOTE:

$\sqrt{x^2} = x$, FOR $x \geq 0$
$(\sqrt{x})^2 = x$, FOR $x \geq 0$

} SAME CONDITION FOR DIFFERENT REASONS





$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} t = \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Let $t = \frac{a-b}{2} \Rightarrow \lambda = \frac{a+b}{2}$;

$$\Rightarrow \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}.$$

<u>DEFINITION</u> IF $f(x)$ IS A FUNCTION, THEN

$$f^{-1} = \{ (f(x), x) : x \in \text{domain}(f) \}.$$

FOR f^{-1} TO BE A FUNCTION, IT NEEDS THE FOLLOWING CONDITIONS

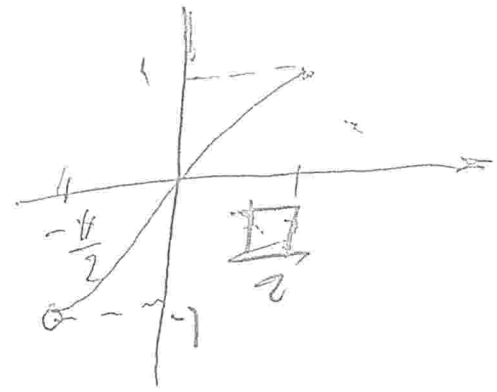
(*) IF $(f(x), x)$ AND $(f(x'), x')$ AND $f(x) = f(x')$, THEN $x = x'$.

EQUIVALENTLY, IF $x \neq x' \in \text{DOMAIN}(f)$, THEN $f(x) \neq f(x')$

DEFINITION f IS INJECTIVE IF $x \neq x' \Rightarrow f(x) \neq f(x')$.

THEOREM $f^{-1}(y)$ IS A FUNCTION IF f IS 1-1

EXAMPLE



THEOREM IF f IS INCREASING ON AN INTERVAL, THEN f IS 1-1.

PROOF: EXERCISE

SUPPOSE f IS CONTINUOUS AND INCREASING

IS f^{-1} CONTINUOUS?

At a point $(a, f(a))$,

GIVEN $\varepsilon > 0$, WANT TO FIND $\delta > 0$:

$g.$ $|y - f(a)| < \delta \Rightarrow$

$|f^{-1}(g) - a| < \varepsilon.$

CONSIDER $f(a+\varepsilon)$, $f(a-\varepsilon)$.

Let $\delta = \min\left(|f(a+\varepsilon) - f(a)|, |f((a-\varepsilon) - f(a)|\right)$.

IF $|y - f(a)| < \varepsilon$, THEN

$$a-\varepsilon < f^{-1}(y) < a+\varepsilon,$$

SINCE f IS INCREASING.