MAT157: 20161123

MVT

REVIEW:

THEOREM IF $f'(x) > 0$ ON AN INTERVAL, THEN $f(x)$ IS INCREASING ON THE INTERVAL.

IF $f'(x) < 0$ ON AN INTERVAL, THEN $f(x)$ IS DECREASING ON THE INTERVAL.

PROOF SUPPOSE a, b ARE IN THE INTERVAL WITH $a < b$.

FROM MVT, $\exists\ c \in (a, b)$ SUCH THAT

$$f'(c) = \frac{f(b) - f(a)}{b-a}.$$

$$a < b \Rightarrow b-a > 0.$$

BY ASSUMPTION, $f'(c) > 0$

$$\Rightarrow f(b) - f(a) = (b-a) f'(c) > 0$$

$\Rightarrow$ f is increasing.

THE ARGUMENT IS SIMILAR FOR $f'(c) < 0$.

CONVERSE IS NOT TRUE.

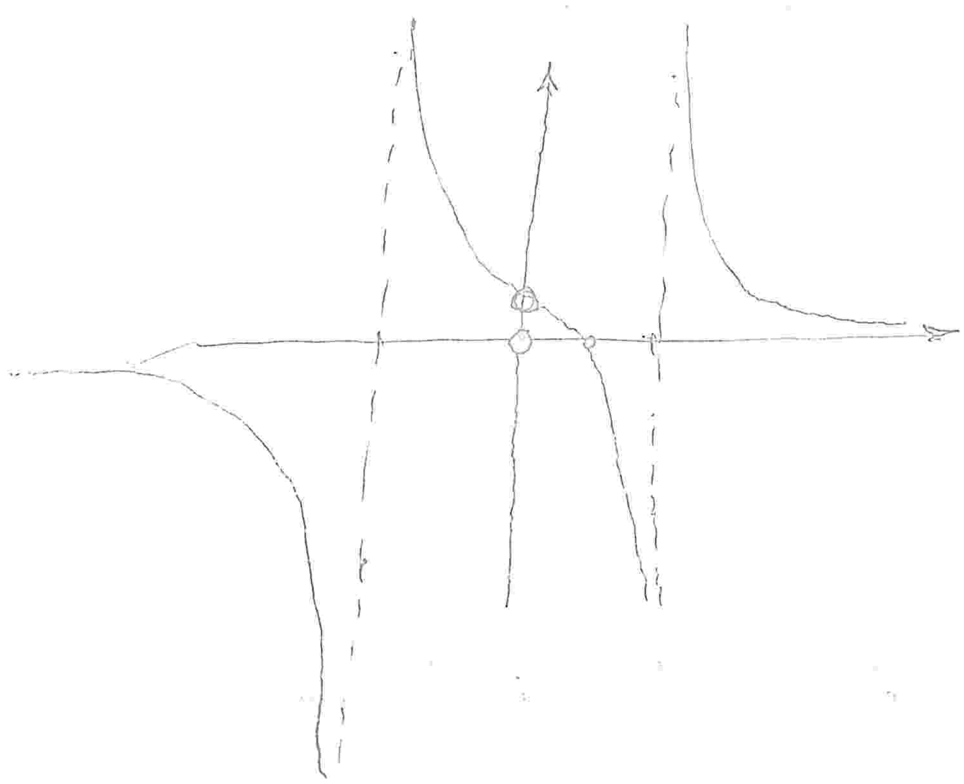
IF $f(x)$ IS INCREASING ON AN INTERVAL AND DIFFERENTIABLE, THEN $f'(x) \geq 0$ ON THE INTERVAL.

SKETCHING THE GRAPHS

$$f(x) = \frac{x^2 - x}{x^3 - 4x} = \frac{x-1}{(x-2)(x+2)}, \quad x \neq 0$$

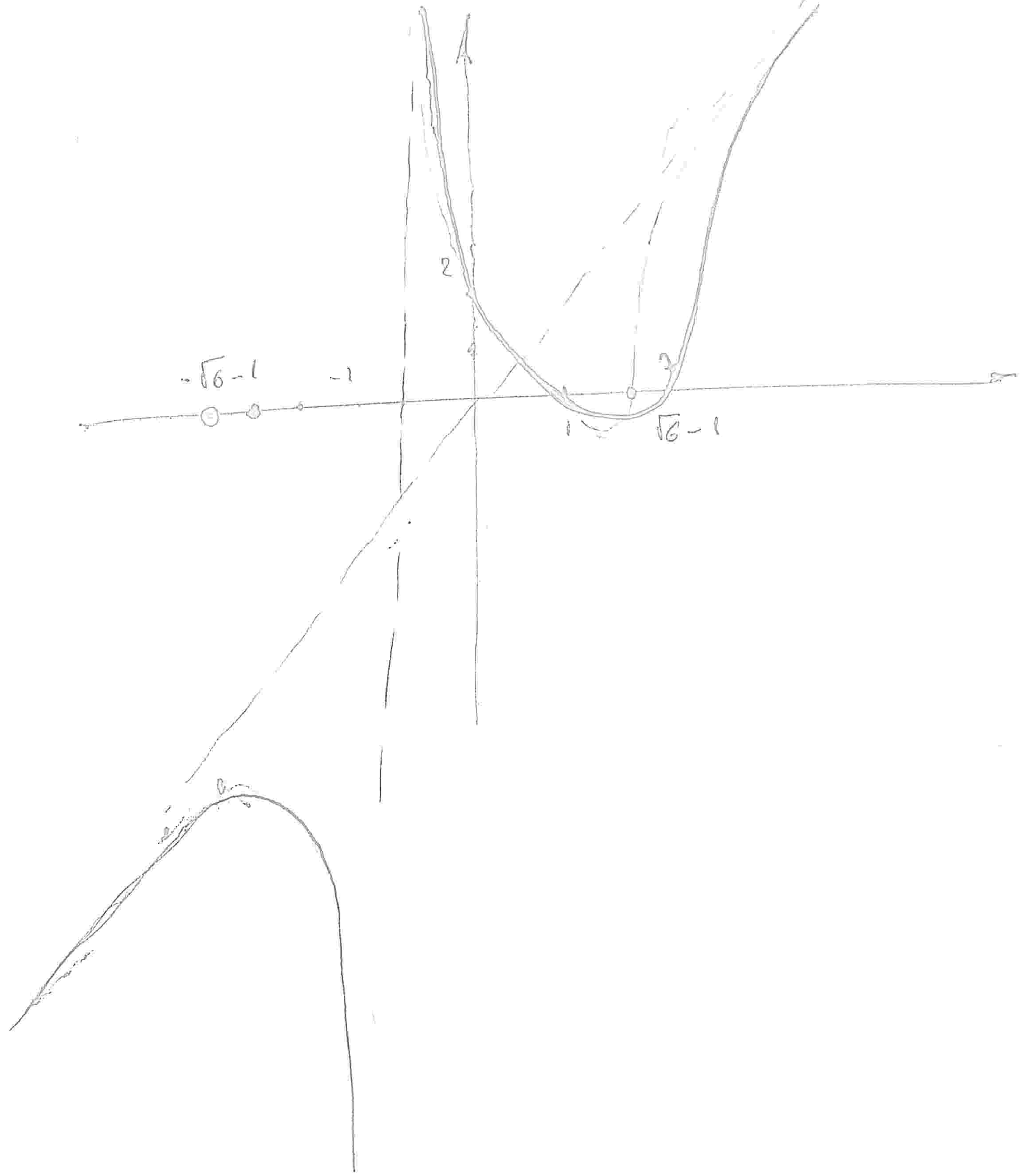
$$f'(x) = \frac{1\,(x^2-4)^2 - 2x\cdot(x-1)}{(x^2-4)^2}$$

$$= 1 - \frac{2x(x-1)}{(x^2-4)^2}, \quad x \neq 0$$



$$g(x) = \frac{x^2 - 3x + 2}{x+1} = x + \frac{2-4x}{x+1} = x - 4 + \frac{6}{x+1}$$

$$g'(x) = 1 - \frac{6}{(x+1)^2}$$



2
1
$-\sqrt{6}-1$
-1
3
1
$\sqrt{6}-1$