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GALOIS GROUPS IN ENUMERATIVE GEOMETRY AND APPLICATIONS

1870 C. JORDAN WROTE A BOOK
ON GALOIS THEORY AND
ENUMERATIVE GEOMETRY

CAYLEY - SALMON

ON A SMOOTH CUBIC
OVER AN AN ALGEBRAICALLY
CLOSED FIELD
THERE ARE 27 LINES IN
A REMARKABLE CONFIGURATION

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f has $\mathbb{Q}$ coefficients.
The lines are defined
over some k of $\mathbb{Q}$.

$G_{al}(k/\mathbb{Q})$ ACTS ON
THE LINES MUST PRESERVE
THIS CONFIGURATION.

Symmetry E_6 = Weyl group of config.

MODERN VIEW

○ BRANCH COVER

○ HAS A MONODROMY GROUP

○ ~~[illegible]~~ GALOIS

GIVEN ANY BRANCHED COVER

$T : X \longrightarrow B$

finite fibres,

X, B IRREDUCIBLE

$\mathbb{P}^{19}$: SPACE OF CUBICS F

SUBGROUP E_6 :

SYMM GR. OF CONFIG.

KNOWN SINCE HERMITE

Gal = Mon

A BRANCHED COVER / GEOMETRIC PROBLEM, GALOIS GROUP IS DEFICIENT IF IT IS NOT THE FULL SYMMETRIC GROUP.

1980 J. HARRIS showed that Gal = E6.

These GALOIS GROUPS ARE AS LARGE AS POSSIBLE GIVEN THE STRUCTURE OF THE PROBLEM.
IT IS DEFICIENT ONLY IF THERE IS SOME ADDITIONAL STRUCTURE.
This structure explains the deficiency.

EXAMPLE OF A COMMON STRUCTURE

SUPPOSE YOU HAVE AN INVOLUTION | B_n signed perms

$$(X,Y) \longleftrightarrow (Y,X)$$

WHICH PRESERVES THE FIBRES of $\pi : X \longrightarrow B$.

In many cases, $G = B_n$; in a few others, $G = D_n$

CLASSICAL SCHUBERT CALCULUS OR ENUMERATIVE GEOMETRY.

ALL PROBLEMS OF DETERMINING THE LINEAR SUBSPACES OF A VECTOR SPACE HAVING PRESCRIBED POSITIONS WITH OTHER FIXED SUBSPACES.

EXAMPLE

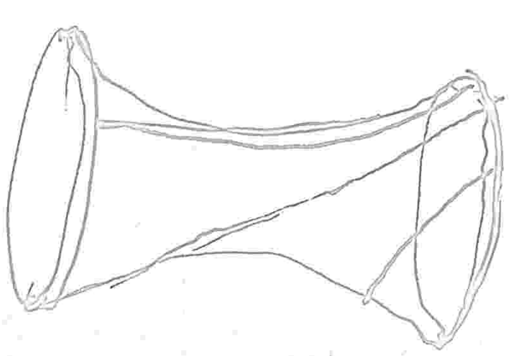
LINES IN $\mathbb{P}^3$ MEET 4 LINES

$l_1, \ldots, l_4$

$l_1, l_2, l_3 \longrightarrow \exists$ unique hyperboloid Q of one sheet (having a quadratic eq.)

PAIRWISE DISJOINT

PIC



The other family is the lines meeting l_1, l_2, l_3

The line l_4 meets Q in 2 points of each gives a line meeting l_1, l_2, l_3, l_4.

Sottile's group studied reality using 6-8 terahertz years of computing.

$\longrightarrow$ now study Galois groups

R. Vakil, Derksen, Sottile's group have found many infinite families of different Schubert problems in every Grassmannian $G(k,n)$ $4 \leq k, n-k$

EXPERIMENTAL

Every Galois group is either

① S_m acting on ordered set partitions $\{1, \dots, m\}$

② Iterated wreath products of ①

- Vakil gave a combinatorial/geometric gr that can determine if Gal ⊃ Alt gr at least alt.

At least alternating:

- All Schub. problems on $G(2, n)$

- simple Schubert problems (White, Williams), ~~the~~ conditions gave 2 are condition 1.

2-transitive:

- all schubert problems on $Gr(3, n)$
- all special Schubert Probs.

$H \mid H_k \cap F_{n-k-1} \neq \{0\}$

White

fully symmetric group
Galois group
fibers
PROBLEM / STRUCTURE OF THE PROB.
Weyl Group / signed permutations
P ?

sheet

monodromy group

Schubert calculus. | S.P. deficient

Grassmanian
Cohomology ring ?! enumerative geometry

Sottile's group & reality

Linear subspaces - -

Algebraic statistics

Alternating group