# Notes on Real Analysis

# 1 Foundations

## 1.1 Postulates

### 1.1.1 Numbers

1. **Real Numbers as a Field**

   (a) **Associativity**
   $\forall a, b, c \in \Re : a + (b + c) = (a + b) + c$
   *Exercise 1*:
   Prove that the sums of an arbitrary number of equivalent variables in an immutable sequence are equal up to the placement of parentheses.
   *Exercise 2*:
   Let the immutable sequence written in such a form that there are no two elements not parenthesised be called a *nested* sequence.
   For example, $((a + b) + c) + d$ and $(a + b) + (c + d)$ are both nested sequences.
   How many different nested sequences can be written from a sequence of n letters?

   (b) **Commutativity of Addition**
   $\forall\ a, b \in \Re : a + b = b + a$

   (c) **Commutativity of Multiplication**
   $\forall\ a, b \in \Re : a \times b = b \times a$

   (d) **Existence of an Additive Identity**
   $\exists\ 0 \in \Re\ \forall a \in \Re : a + 0 = a$

   (e) **Existence of a Multiplicative Identity**
   $\exists\ 1 \in \Re\ \forall a \in \Re : a \times 1 = a$

   (f) **Existence of an Additive Inverse**
   $\forall\ a \in \Re\ \exists - a : a + (-a) = 0$

   (g) **Existence of a Multiplicative Inverse**
   $\forall\ a \in \Re\ \exists\ a^{-1} : a \times a^{-1} = 1$

   (h) **Distributivity**
   $\forall\ a, b, c \in \Re : a \times (b + c) = a \times b + a \times c$

2. **Real Numbers as an Ordered Field**

   Let $P$ be the set of positive numbers.

   Let the binary operator $>$ be defined so that $\forall\ a, b \in \Re : a > b \iff a - b \in P$.

   Similarly, $\forall\ a, b \in \Re : a < b \iff b - a \in P$.

   (a) **Trichotomy Law**
   $\forall\ a \in \Re$ one and only one of the following holds:

   - $a = 0$
   - $a \in P$
   - $a \notin P$

   (b)