1 Complexity of Random Functions of Many Variables

A random smooth function of many variables can be exponentially complex. Kac-Rice formulae (see Adler Taylor or Azais-Wschebor), and the language of RMT, provide a bais mathematical tool to study the complexity of random functions.

Random smooth functions on the sphere in high dimensions (also known as spherical Spin Glasses) is well understood via a simple modification of the Gaussian Orthogonal Ensemble, i.e. $N \times N$ real symmetric random matrices, where the entries are i.i.d Gaussian.

The general question provides cues to important problems in statistics and machine learning.

1.1 Minimizing Cubics

Consider a random homogeneous polynomial f. What is the minimum value of f on S^{N-1}?

Some algorithms, like a gradient descent, a stochastic gradient descent and Langevin dynamics, can minimize f. Will the allgorithm get to or near to the minimum or stay stuck above it? If it does get stuck, then where?

We know that the minimun of m_N is of order $\sqrt{N}$. A minimization algorithm will probably get stuck at the threshold $-E_\infty\sqrt{N}$, with $E_\infty \approx 1.633$, or slightly above it (see AISTATS 2015 for a stochastic gradient descent approach).

To understand the problem fully, we need some geometric intuition. How does the function look like near its low points?

(see Anna Choromanska, Mikael Henaff, etc)

]](see

1.2