

1 Methods of Integration

1.1 Integration by Parts

Example 1.1

$$\int \cos^2(x) \, dx = \cos x \sin x - \int -\sin x \sin x \, dx \quad (1)$$

$$= \cos x \sin x + x - \int \cos^2 x, \quad (2)$$

and thus

$$\int \cos^2(x) \, dx = \frac{1}{2}(\cos x \sin x + x). \quad (3)$$

Example 1.2

$$\int \log x \, dx = x \log x - \int x \frac{1}{x} \quad (4)$$

$$= x \log x - x \quad (5)$$

1.2 Reverse Chain Rule

Example 1.3

$$\frac{d}{dx} e^{-x^2} = -2xe^{-x^2}, \text{ and thus } -\int 2xe^{-x^2} = e^{-x^2}.$$

Example 1.4

$$\int x^2 \sin x^3 = \frac{1}{3}(-\cos x^3).$$

Example 1.5

$$\int \cos x \sin^4 x \, dx = \frac{1}{5} \sin^5 x.$$

Theorem 1.6

$$\text{If } \int_a^b f \circ g(x) g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du$$

Example 1.7

$$\int \frac{x}{1+x^2} = \frac{1}{2} \int \frac{1}{u} \, du \quad (6)$$

$$= \frac{1}{2} \log(1+x^2) \quad (7)$$

Example 1.8

$$\int \frac{\log^5}{x} dx = \int u^5 du \quad (8)$$

$$= \frac{1}{6} \int \log \quad (9)$$

Example 1.9

The following example is better solved using integration by parts:

$$\int x e^{2x} dx = \frac{x e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx \quad (10)$$

$$= \left(\frac{x}{2} - \frac{1}{4}\right) e^{2x} \quad (11)$$