Let $A \in M_{m\times n}(F)$, $B \in M_{n\times m}(F)$.

Therefore , by definition

$$(AB)_{i,k} = \sum_{j=1}^{n} a_{i,j} b_{j,k}, \text{ for } i,k \in \{1,2,\dots,m\} \tag{1}$$

and

$$(BA)_{j,k} = \sum_{i=1}^{m} b_{j,i} a_{i,k}, \text{ for } j,k \in \{1,2,\dots,n\} \ . \tag{2}$$

Note that by the definition of a trace,

$$\begin{aligned}
\operatorname{tr} AB &= \sum_{i=1}^{m} (AB)_{i,i} &(3)\\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} a_{i,j} b_{j,i} &(4)
\end{aligned}$$

and

$$\begin{aligned}
\operatorname{tr} BA &= \sum_{j=1}^{n} (BA)_{j,j} &(5)\\
&= \sum_{j=1}^{n}\sum_{i=1}^{m} b_{j,i} a_{i,j} &(6)\\
&= \sum_{j=1}^{n}\sum_{i=1}^{m} a_{i,j} b_{j,i} &(7)\\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} a_{i,j} b_{j,i} &(8)\\
&= \operatorname{tr} AB, &(9)
\end{aligned}$$

since

$$\begin{aligned}
&(a_{1,1}b_{1,1} + a_{1,2}b_{2,1} + \cdots + a_{1,n}b_{n,1}) +\\
+&(a_{2,1}b_{1,2} + a_{2,2}b_{2,2} + \cdots + a_{2,n}b_{n,2}) + \dots\\
+&(a_{m,1}b_{1,m} + a_{m,2}b_{2,m} + \cdots + a_{m,n}b_{n,m})\\
=&(a_{1,1}b_{1,1} + a_{2,1}b_{1,2} + \cdots + a_{m,1}b_{1,m}) +\\
+&(a_{1,2}b_{2,1} + a_{2,2}b_{2,2} + \cdots + a_{m,2}b_{2,m}) + \dots\\
+&(a_{1,n}b_{n,1} + a_{2,n}b_{n,2} + \cdots + a_{m,n}b_{n,m})
\end{aligned}$$

Therefore, $\operatorname{tr}(AB) = \operatorname{tr}(BA)$.