

Problem

Let S denote all subsets of some set of elements U .

For $A, B \in S$, define $A \Delta B = \{x \in U \mid (x \in A) \text{ XOR } (x \in B)\}$.

Note that δ is commutative and associative since XOR is commutative and associative.

Let $n \in \mathbb{Z}^+$ and let $A_1, \dots, A_n \in S$.

Formally prove (using induction) that, for all $x \in U$, $x \in A_1 \Delta A_2 \Delta \dots \Delta A_n$ if and only if $\{i \in \{1, \dots, n\} \mid x \in A_i\}$ is odd.