$$\begin{bmatrix} 0 & i & -1 \\ 1+i & 1 & 1+2i \\ 1-i & 2 & 1+i \\ -i & 1-i & 1 \end{bmatrix} \rightsquigarrow \tag{1}$$

$$L_4 \to L_4 + L_1 \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 1+i & 1 & 1+2i \\ 1-i & 2 & 1+i \\ -i & 1 & 0 \end{bmatrix} \tag{2}$$

$$L_2 \to L_2 + L_4 \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 1 & 2 & 1+2i \\ 1-i & 2 & 1+i \\ -i & 1 & 0 \end{bmatrix} \tag{3}$$

$$L_3 \to L_3 - L_4 \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 1 & 2 & 1+2i \\ 1 & 1 & 1+i \\ -i & 1 & 0 \end{bmatrix} \tag{4}$$

$$L_3 \to 2L_3 - L_2 \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 1 & 2 & 1+2i \\ 1 & 0 & 1 \\ -i & 1 & 0 \end{bmatrix} \tag{5}$$

$$L_2 \to \frac{1}{2}(L_2 - L_3) \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 0 & 1 & i \\ 1 & 0 & 1 \\ -i & 1 & 0 \end{bmatrix} \tag{6}$$

$$L_4 \to L_4 + iL_3 - L_2 \mid \rightsquigarrow \begin{bmatrix} 0 & i & -1 \\ 0 & 1 & i \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$L_1 \to L_1 - iL_2 \mid \rightsquigarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & i \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$L_1 \leftrightarrow L_3 \mid \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Since the number of linearly independent columns $\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$ is 2, then the rank of A is 2.

Since columns 1 and 2 are pivotal, then the basis for $\operatorname{Im} L_A$ is $\left\{ \begin{pmatrix} 0 \\ 1+i \\ 1-i \\ -i \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ 2 \\ 1 \end{pmatrix} \right\}$.

Suppose $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \ker L_A$ and $x_3 = \lambda \in \mathbb{R}$. From the matrix in row echelon form above,

$$\begin{cases} x_1 + \lambda = 0 \\ x_2 + i\lambda = 0 \end{cases}$$

Therefore, $\forall (x \in \ker L_A) : x = \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} \lambda$, and hence $\begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix}$ is the basis of $\ker L_A$.