A NONEMPTY SUBSET W OF A VECTOR SPACE V IS A SUBSPACE IF IT IS CLOSED UNDER T AND O. THAT IS, V, W & W => V+W & W V & W, a & F => QUEW

## EXAMPLES

ALL THESE ARE SUBSPACES OF MIXIN(F).

## EXAMPLE

ooV= Ch, W=IRh.

W IS NOT A SUB-SPACE DEV, LIEWED AS A VECTOR SPACE OVER (

W IS A SUBSPACE OF V, VIEWED AS A VEGTOR SPACE OVER R.

## THEOREM

IF W, W2 ARE EVBSPACES OF V, THEN WINWE IS A SUBSPACE OF V.

PROOF

WINNE FR, HARE DEW, AND OEWE.

IF V, WE WINNE, MEN V, WEWI. SO VINEWA AND VINEWS, SO VINEWS.

MENCE VANE WINW,

Similar FOR SCAURE MUSTIPHION:

EXAMPLES

O V = Mnxn (F).

We = { UPPER TRIANGLAN }

W2 = { LOWER ARIENTALER }

WINWE E PLAGONACT

D V= Mnxn(F)

WI= { SYMMETTELC' MATRICES &

W2= { Skew symmetrice MATRICES 3

Willing = (0) PROVIDED THAT LANG IN F.

NOTE: FITH-0, EVETER SYREER

MORE GENERALLY, FROM SIMILAR PROLIMONT.
IF WW. XEA ARE WBSPACES OF V, THEN
NX = {VGV VE WX HXEAD IS A SUBSTICE
OV J F (R, R)
Wn = {f: R >  R   f(n) = 0}, 162
ne 2 Wn = { f: R -> R   f(n)= 0 for all ne 2
1 V ANY VECTOR EPACE, S C V SUBSET
LET WORL, REA, AU SUBSTANCES SE Was.
WE IS A SUBSPACE
THEN SCIN EXERCISE: W.

-> KNOTHER CONSTRUCTION IS STAM (S) 0

SUBSTRICTS OF TOWNAMING S.

FOR ANY EUBSPACES WI, W2 CV, THE SUM  WI+W2 = { WI+W2   W16 W1, W2 E W2 }  PROOF:  (1) WI+W2 = Ø BECAUSE O+O = O & (W1+We)  (2) V, W & W, +V2, WRITE V=V1+V2 VIC W1,  W = W1+We, W1 & W1	SUM OF WBSPACES  FOR ANY ENBSPACES WI, W2 CV, THE SUM  WITH W2 = { WITH W1   W1 G W1, W2 G W2 }  PRODE:  (1) WITHW2 + Ø BECAUSE O+0=0 G (WITHWE  (2) V, W G W, + V2, WELTE V=VI+V2 VIC WI,  W= WITHW2, W1 G W1  TO V+W= (VI+W1) + (W2+V2), G WITHW2  3) SIMILARY, UG W1+W2, QG F THON QN G W = 6	ONLY IF W, C	We er Wo	< W.	
FOR ANY EUBSPACES WI, W2 CV, THE SUM  WI+W2 = { W1+W2   W1 GW1, W2 GW2 }  PROOF:  (1) W1+W2 = Ø BECAUSE O+O=O G (W1+We)  (2) V, W GW, +V2, WRITE V=V1+V2 V1 CW1,  W= W1+W2, W1 GW1  TO V+W= (V1+W1) + (W2+V2), W1+W2  3) SIMILARIY, VG W, +W2, QG F THON QN GW + W1	FOR ANY EUBSPACES WI, W2 CV, THE SUM  WI+W2 = { W1+W2   W, GW1, W2 GW2 }  PROOF:  (1) W1+W2 = Ø BECAUSE O+O=O G (W1+We)  (2) V, W GW, +V2, WRITE V=V1+V2 VIC W1,  W= W1+We, W1 GW1  -> V+W= (V1+W1) + (W2+V2), W1+W2  3) SIMILARUX, VG W, +W2, QG F THON QN GW - W	The second section of the second seco			gio adel se il Cinadina III e il III
WITH W2 = { WI+W1   W, 6 W1, W2 ∈ W2 }  PROOF:  (1) WI+W2 ≠ Ø BECAUSE O+O=O ∈ (W1+We)  (2) V, W & W, +V2, WRITE V=V1+V2 V1 € W1,  W= W1+We, W1 € W1  -> U+W- (V1+W1) + (W2+V2), EN W1+W2  3) SIMILARY, UE W, +W2, Q ∈ F FRON QW & W+ W1	WITH W2 = { WI+WI   W, 6 WI, W2 ∈ W2 }  PROOF:  (1) WI+W2 ≠ Ø BECAUSE O+O=O ∈ (W1+We)  (2) V, W & W, +V2, WRITE V=VI+V2 VIC WI,  W= WI+We, W, EWI  -> U+W= (V1+WI) + (W2+V2), E W1+W2  3) SIMILARY, UE W, +W2, Q ∈ F FRON QW & W+ W	fum of WBSPA	CES	aglaciones i company and an anticompany of the form of the company	
WI+W2 = { WI+W2   W, 6 W1, W2 ∈ W2 }  PROOF:  (1) WI+W2 = Ø BECAUSE O+O=O ∈ (W1+We)  (2) V, W & W, +V2, WRITE V=V1+V2 V1 € W1,  W = W1+We, W1 € W1  -> V+W= (V1+W1) + (W2+V2), E W1+W2  3) SIMILARIY, UE W, +W2, Q ∈ F FRON QW & W + W1	WI+W2 = { WI+W2   W, 6 W1, W2 ∈ W2 }  PROOF:  (1) WI+W2 ≠ Ø BECAUSE O+O=O ∈ (W1+We)  (2) V, W & W, +V2, WRITE V=V1+V2 V1 € W1,  W = W1+We, W1 € W1  -> V+W= (V1+W1) + (W2+V2), E W1+W2  3) SIMILARY, V & W, +W2, Q & F FRON QN & W + W	FOR ANY EUBS	SPACES W1, W2	CV, TUG S	VM
PROOF:  (1) WI+W2 + & BECAUSE 0+0=0 & WI+We  (2) V, W & W, +V2, WRITE V=VI+V2 VIC WI,  W= WI+WE, WIEW,  -> U+W- (VI+WI) + (W2+V2), EN WI+WZ  3) SIMILARY, UE W, +W2, QE F FRON ON & W+ 10	PROOF:  (1) WI+We & BECAUSE 0+0=0 & WI+We  (2) V, W & W, +V2, WRITE V=VI+V2 VIC WI,  W= WI+We, W_1 & W,  TO V+W- (VI+WI) + (WE+V2), EN WI+WE  3) SIMILARY, UE W, +WE, as F FRON ON & W-10	W1+W2 = 4	[ w1+w2   w, 6	: Wi, wre V	Vol
2) V, W & W, + V2, WRITE V=V1+V2 VIC WY,  W = W1+W2, W1 & W1.  TO V+W= (V1+W1) + (W2+V2), W W1+W2  3) SIMILARY, UE W, +W2, QE F THON ON & W + W	2) V, W & W, + V2, WRITE V=V1+V2 VIC WY,  W = W1+WE, W1 & W1.  TO V+W= (V1+W1) + (W2+V2), WWW W1+W2  3) SIMILARY, UE W, +W2, QEF THON ON & W + W	Proof:			to the convenients.
2) V, W & W, + V2, WRITE V=V1+V2 VIC WY,  W = W1+W2, W1 & W1.  TO V+W= (V1+W1) + (W2+V2), W W1+W2  3) SIMILARY, UE W, +W2, QE F THON ON & W + W	2) V, W & W, + V2, WRITE V=V1+V2 VIC WY,  W = W1+WE, W1 & W1.  TO V+W = (V1+W1) + (W2+V2), WWW W1+W2  3) SIMILARY, UE W, +W2, QE F THON ON & W + W	( WI+W2 7	f Ø BECAUS	E 0+0=0	6 (WI+We
38 MILARY, VEW, TWE, REF THON ON GW-	38 MILARY, VEW, TWE, REF THON ON GW.	2) V, W &	W, + 1/2, WK	TE V=V, 4	Va VIC WA.
DOIMILARY, VEW, TWE, a & F THON ON 6 W-	DOIMILARY, VEW, TWE, a & F THON ON 6 W.			$N = M^{4+1}$	We, WyEWA,
DOIMILARY, VEW, TWE, a & F THON ON 6 W= 4	DOIMILARY, VEW, TWE, a & F THON ON 6 W.	31	/= ( /+ W( ) + (	We+Ve),	= WI+WZ
EXAMPLE:	EXAMPLE:	DOMILARIT, U	'EW, TWE, a	e F How	an e Wal
		EXAMPLE!			