MAT240: $\mathbb{C}$ 2019/1920

1. $P \Rightarrow Q$

$\Leftrightarrow$

2. $Q \Leftarrow P$

AXIOMS

F1. COMMUTAT.
F2. ASSOC
F3 NEUTRAL ELEMENT
F4. INVERSE ELEMENT
F5. DISTRIB. LAW

EXAMPLE

ROOM IS MADE OF CHEESE
=> ELVIS IS ALIVE

"P IMPLIES Q",
WHERE P AND Q ARE TRUE/FALSE STATEMENTS
"IF P THEN Q"

$$B(P \Rightarrow Q) = T \text{ IF } \left(B(P) = T \wedge B(Q) = T \right) \vee B(P) = F$$

THEOREM:

Let $\mathbb{F}$ be a field $\forall a, b \in \mathbb{F}$.

$$a \cdot b = 0 \Rightarrow [a = 0 \text{ or } b = 0]$$

PROOF

IF $a = 0$, WE ARE DONE.

ASSUME $a \neq 0$, BY F4, a^{-1} EXISTS

THEN

$$\begin{aligned}
b &= 1 \cdot b && \text{F3} \\
&= (a^{-1} \cdot a) \cdot b && \text{F4} \\
&= a^{-1}(a \cdot b) && \text{F2} \\
&= a^{-1} \cdot 0 && \text{SINCE } a \cdot b = 0 \\
&= 0
\end{aligned}$$

FURTHER PROPERTIES OF FIELDS

1. $-(-a)=a$

$(a^{-1})^{-1}$ IF $a \neq 0$

2. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$;

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ IF $b, d \neq 0$

3. $(-a) \cdot b = -(a \cdot b) = a \cdot (-b)$

4. $(a+b)(a-b) = a^2 - b^2$

REMARK

FROM NOW ON, WE CAN OMIT $\cdot$:

$a \cdot b = ab$

$(\ )$:

$(a+b)+c = a+b+c$

EXAMPLE

$ab + c = (a \cdot b) + c$

ALREADY CONSIDERED FIELDS

$\{a + \sqrt{2}b \mid a, b \in \mathbb{Q}\}$ EXERCISE

$\mathbb{Q}, \mathbb{R}, \mathbb{Z}_p$

'INTERESTING' FIELDS: $f(x) = \frac{p(x)}{q(x)}$, $x \in \mathbb{R}$, p, q are polynomials

$\mathbb{C}$

MOTIVATION

CONSIDER

$Ax^2 + Bx + C = 0$;

$A, B, C \in \mathbb{R}$;

THE EQUATION HAS SOLUTIONS

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

IF $A \neq 0$.

IF $B^2 - 4AC < 0$, $x \notin \mathbb{R}$

DEFINITION

$\mathbb{C} = \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ IS A SET WITH ELEMENTS IN THE FORM

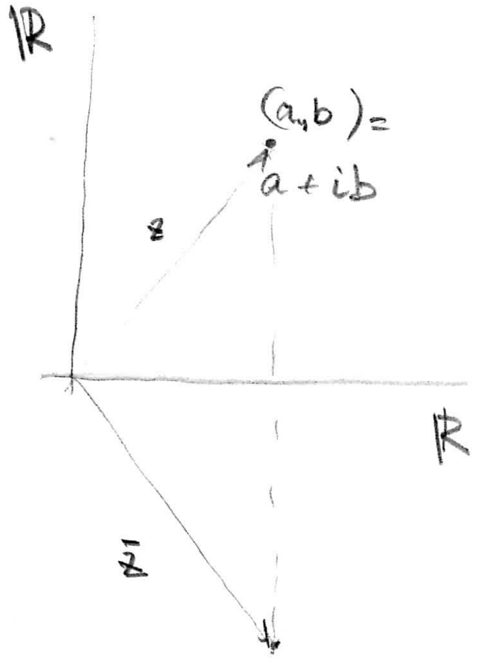
$a + bi$

DEFINE $+$ AND $\cdot$;

$(a+ib) + (c+id) = a+c+i(b+d)$

$(a+ib)(c+id) = ac - bd + i(ad+bc)$

NOTATION FOR $\mathbb{C}$



$z = a + ib$

$a = Re(z)$ "real part"

$b = Im(z)$ "IMAGINARY PART"

z IS CALLED REAL IF $Im(z) = 0$

IMAGINARY IF $Re(z) = 0$

$\bar{z} = a - ib$

IS THE COMPLEX CONJUGATE.

$|z|$ IS THE ABSOLUTE VALUE,

$|z| = \sqrt{a^2 + b^2}$

PROPERTIES

- $\bar{\bar{z}} = z$
- $\overline{z+w} = \bar{z} + \bar{w}$, $\overline{zw} = \bar{z} \cdot \bar{w}$
- $z \in \mathbb{R} \Leftrightarrow \bar{z} = z$
- $z \in \mathbb{C} \setminus \mathbb{R} \Leftrightarrow \bar{z} = -\bar{z}$
- $Re(z) = \dfrac{z + \bar{z}}{2}$

  $Im(z) = \dfrac{z - \bar{z}}{2}$
- i HAS THE MULTIPLICATIVE INVERSE $\frac{1}{i} = -i$
- $z \cdot \bar{z} = |z|^2$
- $z^{-1} = \dfrac{\bar{z}}{z\bar{z}}$