a)

Theorem. $\forall f : \mathbb{R} \to \mathbb{R}$, f *can be uniquely written as a sum of one odd function* f_o *and one even function* f_e.

Proof. Suppose such f_o and f_e exist.

Therefore,

$$\begin{aligned}
& f(x) &&= f_o(x) + f_e(x) && (1)\\
\Rightarrow\quad & f(-x) &&= f_o(-x) + f_e(-x) && (2)\\
& &&= -f_o(x) + f_e(x) && (3)\\
\Rightarrow\quad & \tfrac{f(x)+f(-x)}{2} &&= f_e(x) && (4)\\
\Rightarrow\quad & f_o(x) &&= \tfrac{f(x)-f(-x)}{2} && (5)
\end{aligned}$$

Since f_e and f_o are given in terms of $f \in \mathbb{R}$ and 2^{-1} is unique, f_e and f_o are unique. □

b) Consider the case when $f : \mathbb{F} \to \mathbb{F}$.

If the characteristic of $\mathbb{F}$ is not equal to 2, there is a unique element corresponding to 2^{-1} and hence f_e and f_o are unique.

If the characteristic of $\mathbb{F}$ is equal to 2, from the fact that $1 + 1 = 0$ it follows that $x = -x$ for all $x \in \mathbb{F}$ and hence all such f are both odd and even, and can be written as $f = f + 0$, but not uniquely.