1 Integration Tricks

For rational functions of sin and cos, if all else fails, try the substitution $x = \tan \frac{\theta}{2}$.

Thus, $1 + x^2 = \sec^2(\frac{\theta}{2})$, and hence $\frac{1}{1+x^2} = \cos^2(\frac{\theta}{2})$, and hence $\cos\theta = \frac{1-x^2}{1+x^2}$. Moreover, $\sin^2\theta = \frac{4x^2}{(1+x^2)^2}$, which means that $\sin\theta = \frac{2x}{1+x^2}$.

In this way, this substitution converts any rational function of cos and sin into a rational function of x. In principle, this leads to an integral which always can be integrated. Generally, however, this substitution results in an unwieldy equation, and thus this method should be used as a last resort.

2 Solids of Revolution

Suppose a curve is given in the first quarter of the Cartesian plane, defined over the interval $[a, b]$. Rotate this curve around the x axis to obtain a *solid of revolution*.

Informally, the volume of the obtained solid can be calculated by summing the volumes of very thin slices. Suppose that $\mathrm{d}x$ is the thickness of the slice. Then its volume is $\pi f(x)^2 \,\mathrm{d}x$, and hence $V = \int_a^b \pi f(x)^2 \,\mathrm{d}x$.

Consider a cone, of height h and radius r, with the vertex at the origin. Then $y = f(x) = \frac{r}{h}x$ is the corresponding curve. Thus, $V = \frac{\pi}{3}r^2h$.