Suppose $\lambda + 1 = 0$.
Thus,

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ -1 & 1 & 2 & -1 & 1 \\ 1 & 1 & 0 & 1 & 3 \\ 1 & 1 & 0 & 1 & 3 \end{array}\right] = \tag{1}$$

$$\begin{matrix} L_4 \to L_1 - L_4 \\ L_3 \to L_1 - L_3 \end{matrix} \quad | = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ -1 & 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{2}$$

$$L_2 \to \frac{1}{2}L_2 + L_1 \ | = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{3}$$

$$L_1 \to L_1 - L_2 \ | = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{4}$$

$$\tag{5}$$

Let $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ be such that $Ax = b$ for the augmented matrix $(A|b)$ above ,
and let $\alpha$ be equal to $x_3$ and $\beta$ to $x_4$.
Therefore, from the matrix in row echelon form above,

$$\begin{cases} x_1 - \alpha + \beta = 1 \\ x_2 + \alpha = 2 \end{cases}$$

Therefore, the solution set is generated by the solutions in the form

$$x = \begin{pmatrix} 1 + \alpha - \beta \\ 2 - \alpha \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Suppose $\lambda = 0$.
Thus,

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ -1 & 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1 & 3 \end{array}\right] = \tag{6}$$

$$L_4 \to L_1 - L_4 \mid \; = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ -1 & 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{7}$$

$$L_2 \to L_2 + L_1 \mid \; = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{8}$$

$$L_2 \to \frac{1}{2}(L_2 - L_3) \mid \; = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{9}$$

$$L_3 \leftrightarrow L_2 \mid \; = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{10}$$

Let $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ be such that $Ax = b$ for the augmented matrix $(A|b)$ above , and let $\mu$ be equal to $x_4$.

Therefore, from the matrix in row echelon form above,

$$\begin{cases} x_1 + \mu = 3 \\ x_2 + \mu = 3 \\ x_3 = \frac{1}{2} \end{cases}$$

Therefore, the solution set is generated by the solutions in the form

$$x = \begin{pmatrix} 3 \\ 3 \\ \frac{1}{2} \\ 0 \end{pmatrix} - \mu \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

Suppose now $\lambda \neq 0$.

$$\left[\begin{array}{cccc|c} 1 & -\lambda & 0 & 1 & 3 \\ -1 & 1 & 2 & \lambda & 1 \\ -\lambda & 1 & 0 & 1 & 3 \\ 1 & \lambda^2 & 0 & 1 & 3 \end{array}\right] = \tag{11}$$

$$L_1 \to \lambda L_1 \mid = \left[\begin{array}{cccc|c} \lambda & -\lambda^2 & 0 & \lambda & 3\lambda \\ -1 & 1 & 2 & \lambda & 1 \\ -\lambda & 1 & 0 & 1 & 3 \\ 1 & \lambda^2 & 0 & 1 & 3 \end{array}\right] \tag{12}$$

$$L_3 \to \lambda L_1 + L_3 \mid = \left[\begin{array}{cccc|c} \lambda & -\lambda^2 & 0 & \lambda & 3\lambda \\ -1 & 1 & 2 & \lambda & 1 \\ 0 & 1-\lambda^2 & 0 & 1+\lambda & 3(1+\lambda) \\ 1 & \lambda^2 & 0 & 1 & 3 \end{array}\right] \tag{13}$$

$$L_2 \to \lambda L_4 + L_2 \mid = \left[\begin{array}{cccc|c} \lambda & -\lambda^2 & 0 & \lambda & 3\lambda \\ 0 & 1+\lambda^2 & 2 & 1+\lambda & 4 \\ 0 & 1-\lambda^2 & 0 & 1+\lambda & 3(1+\lambda) \\ 1 & \lambda^2 & 0 & 1 & 3 \end{array}\right] \tag{14}$$

$$L_1 \to \frac{1}{\lambda(1+\lambda)}(-\frac{1}{\lambda}L_1 + L_4) \mid = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 1+\lambda^2 & 2 & 1+\lambda & 4 \\ 0 & 1-\lambda^2 & 0 & 1+\lambda & 3(1+\lambda) \\ 1 & \lambda^2 & 0 & 1 & 3 \end{array}\right] \tag{15}$$

$$\begin{array}{l} L_2 \to L_2 - (1+\lambda^2)L_1 \\ L_3 \to L_3 - (1-\lambda^2)L_1 \\ L_4 \to L_4 - \lambda^2 L_1 \end{array} \mid = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1+\lambda & 4 \\ 0 & 0 & 0 & 1+\lambda & 3(1+\lambda) \\ 1 & 0 & 0 & 1 & 3 \end{array}\right] \tag{16}$$

$$L_2 \to \frac{1}{2}(L_2 - L_3) \mid = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2}(1-3\lambda) \\ 0 & 0 & 0 & 1+\lambda & 3(1+\lambda) \\ 1 & 0 & 0 & 1 & 3 \end{array}\right] \tag{17}$$

$$L_3 \to \frac{1}{1+\lambda}L_3 \mid = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2}(1-3\lambda) \\ 0 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1 & 3 \end{array}\right] \tag{18}$$

$$L_4 \to L_4 - L_3 \mid = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2}(1-3\lambda) \\ 0 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right] \tag{19}$$

$$L_4 \uparrow L_1 \mid = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2}(1-3\lambda) \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \tag{20}$$

$$\tag{21}$$

Let $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ be such that $Ax = b$ for the augmented matrix $(A|b)$ above.

Therefore, from the matrix in row echelon form above,

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = \frac{1}{2}(1 - 3\lambda) \\ x_4 = 3 \end{cases}$$

Thus, for a specific $\lambda$ there is only one solution given by the following formula:

$$x = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \\ 3 \end{pmatrix} - \frac{3\lambda}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$