STA 100 HW 7 Solutions

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1. (a) $H_0 : Pr\{A\} = 0.5, Pr\{B\} = 0.25, Pr\{C\} = 0.25$

   (b) H_A: At least two of the hypothesize proportions is different than the null.

   (c) The chi-squared test-statistic is:
   $\chi^2_S = \sum_{i=1}^{K} \frac{(o_i-e_i)^2}{e_i} = \frac{(220-227)^2}{227} + \frac{(129-113.5)^2}{113.5} + \frac{(105-113.5)^2}{113.5} = 0.2159 + 2.1167 + 0.6366 = 2.9692$

   (d) At d.f $= k - 1 =$ 3 -1 = 2, we find $Pr\{\chi^2 > 3.22\} = 0.2$. Thus, the range for our p-value is : p-value > 0.2

   (e) Since the p-value is greater than α, we fail to reject H_0.

   (f) We conclude there is enough evidence to support the genetic model.

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2. (a) $H_0 : Pr\{Weekend\} = \frac{2}{7}, Pr\{Weekday\} = \frac{5}{7}$

   (b) $H_A : Pr\{Weekend\} \neq \frac{2}{7}, Pr\{Weekday\} \neq \frac{5}{7}$

   (c) The chi-squared test-statistic is:
   $\chi^2_S = \sum_{i=1}^{K} \frac{(o_i-e_i)^2}{e_i} = \frac{(216-266.286)^2}{266.286} + \frac{(716-665.714)^2}{665.714} = 9.4961 + 3.7985 = 13.2946$

   (d) At d.f $= k - 1 =$ 2 -1 = 1, we find $Pr\{\chi^2 > 3.22\} = 0.001$ and $Pr\{\chi^2 > 15.14\} = 0.0001$
   Thus, the range for our p-value is : $0.0001 <$ p-value < 0.001

   (e) If in reality there were the same proportion of births on the weekend that we would expect by chance, we would observe our data or more extreme with probability between 0.0001 and 0.001

   (f) Since p-value is smaller than α, we reject the null

   (g) We conclude there is not enough evidence to suggest that the same proportion of births occur on the weekend as what we would expect by chance.

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3. (a) $H_0 : Pr\{brown\} = 1/3, Pr\{black\} = 1/3, Pr\{white\} = 1/3$

   (b) H_A: At least two of the hypothesize proportions is different than the null.

   (c) The chi-squared test-statistic is:
   $\chi^2_S = \sum_{i=1}^{K} \frac{(o_i-e_i)^2}{e_i} = \frac{(40-47)^2}{47} + \frac{(59-47)^2}{47} + \frac{(42-47)^2}{47} = 1.0426 + 3.0638 + 0.5319 = 4.6383$

   (d) At d.f $= k - 1 =$ 3 -1 = 2, we find $Pr\{\chi^2 > 4.61\} = 0.1$ and $Pr\{\chi^2 > 5.99\} = 0.05$
   Thus, the range for our p-value is : $0.05 <$ p-value < 0.1

   (e) Since p-value $< \alpha$, we reject H_0 at 10% signficance.

   (f) We conclude that there is evidence to suggest the true proportion of Mongolian Gerbils are not equally likely to be brown, black, or white.

   (g) Since the component of the chi-squared test statisic was largest for the brown hamsters, this is the group that was most different.

4. (a) H_0: Death is independent of treatment (Surgery, Watchful Waiting).

   (b) H_A: Death is dependent on treatment (Surgery, Watchful Waiting)

   (c) The expected counts are:

	Surgery	WW
Died	94.36	94.64
Alive	252.64	253.36

   So that
   $\chi^2_S = \sum_{all\ cells} \frac{(o_i-e_i)^2}{e_i} = \frac{(83-94.364)^2}{94.364} + + \frac{(106-94.636)^2}{94.636} + \frac{(264-252.636)^2}{252.636} + \frac{(242-253.364)^2}{253.364} = 1.3685 + 0.5112 + 1.3646 + 0.5097 = 3.754$

(d) At d.f = (number of rows -1)(number of columns -1) = (2 -1)(2 -1) = 1, we find $Pr\{\chi^2 > 2.71\} = 0.1$ and $Pr\{\chi^2 > 3.84\} = 0.05$
Thus, the range for our p-value is : $0.05 <$ p-value < 0.1

(e) At $\alpha = 0.05$, our p-value is larger than α, so we fail to reject the null.

(f) We support that death was independent of type of treatment at 5% significance.

(g) $\hat{Pr}\{death|surgery\} = \frac{83}{347} = 0.2392$

(h) $\hat{Pr}\{death|WW\} = \frac{106}{348} = 0.3046$

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5. (a) H_0: Pain is independent of treatment (Angioplasty, Bypass Surgery).

(b) H_A: Pain is dependent on treatment (Angioplasty, Bypass Surgery)

(c) The expected counts are: So that

	A	B
Pain	92.32	92.68
No Pain	420.68	422.32

$\chi_S^2 = \sum_{all\ cells} \frac{(o_i - e_i)^2}{e_i} = \frac{(111-92.32)^2}{92.32} + + \frac{(74-92.68)^2}{92.68} + \frac{(402-420.68)^2}{420.68} + \frac{(441-422.32)^2}{422.32} = 3.7797 + 0.8295 + 3.765 + 0.8263 = 9.2005$

(d) At d.f = (number of rows -1)(number of columns -1) = (2 -1)(2 -1) = 1, we find $Pr\{\chi^2 > 6.63\} = 0.01$ and $Pr\{\chi^2 > 10.83\} = 0.001$
Thus, the range for our p-value is : $0.001 <$ p-value < 0.01

(e) If recurrance of pain was truely independent of treatement, we would expect to see our sample data (or more extreme) between 0.1% and 1% of the time.

(f) At $\alpha = 0.01$, our p-value is smaller than α, so we reject the null.

(g) We cannot support that pain was independent of type of treatment at 1% significance.

(h) $\hat{Pr}\{pain|A\} = \frac{111}{513} = 0.2164$

(i) $\hat{Pr}\{pain|B\} = \frac{74}{515} = 0.1437$

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6. (a) TRUE. For example, if $k = 3$, “equally likely” would mean all hypothesized probabilities were 1/3.
