Solutions STA 100 HW 2

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1. Let S_1 = Species 1, S_2 = Species 2, and I = Infected.
   (a) $Pr\{S_1\} = \frac{54}{109} = 0.4954$
   (b) $Pr\{I\} = \frac{58}{109} = 0.5321$
   (c) $Pr\{S_1 \cdot and \cdot I\} = \frac{38}{109} = 0.3486$
   (d) $Pr\{S_2 \cdot and \cdot I^C\} = \frac{35}{109} = 0.3211$

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2. (a) $Pr\{I|S_1\} = \frac{Pr\{I \cdot and \cdot S_1\}}{Pr\{S_1\}} = \frac{38}{54} = 0.7037$
   (b) $Pr\{I|S_2\} = \frac{Pr\{I \cdot and \cdot S_2\}}{Pr\{S_2\}} = \frac{20}{55} = 0.3636$
   (c) $Pr\{S_1|I\} = \frac{Pr\{I \cdot and \cdot S_1\}}{Pr\{I\}} = \frac{38}{58} = 0.6552$
   (d) $Pr\{S_2|I\} = \frac{Pr\{I \cdot and \cdot S_2\}}{Pr\{I\}} = \frac{20}{58} = 0.3448$
   (e) Since $Pr\{I|S_1\} = 0.7037 \neq Pr\{I\} = 0.5321$, they are not independent (they are dependent.)

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3. (a) $Pr\{+ \cdot and \cdot D\} = Pr\{+|D\}Pr\{D\} = (0.95)(0.04) = 0.038$
   (b) $Pr\{+\} = Pr\{+ \cdot and \cdot D\} + Pr\{+ \cdot and \cdot D^C\} = Pr\{+|D\}Pr\{D\} + Pr\{+|D^C\}Pr\{D^C\}$
   $= (0.95)(0.04) + (1 - 0.99)(1 - 0.04) = 0.0476$
   (c) $Pr\{D|+\} = \frac{Pr\{+ \cdot and \cdot D\}}{Pr\{+\}} = \frac{0.038}{0.0476} = 0.7983$
   (d) $Pr\{D^c|-\} = \frac{Pr\{- \cdot and \cdot D^c\}}{Pr\{-\}} = \frac{Pr\{-|D^c\}Pr\{D^c\}}{Pr\{- \cdot and \cdot D^c\ + Pr\{- \cdot and \cdot D\}} = \frac{Pr\{-|D^c\}Pr\{D^c\}}{Pr\{-|D^c\}Pr\{D^c\} + Pr\{-|D\}Pr\{D\}}$
   $= \frac{(0.99)(1-0.04)}{(0.99)(1-0.04)+(1-0.95)(0.04)} = 0.9979$

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4. Let R = Symptom relief, D = Drug group.
   (a) $Pr\{R \text{ or } D\} = Pr\{R\} + Pr\{D\} - Pr\{R \text{ and } D\} = \frac{45+32}{114} + \frac{45+12}{114} - \frac{45}{114} = \frac{77+57-45}{114} = \frac{89}{114} = 0.7807$
   (b) $Pr\{R|D\} = \frac{Pr\{R \text{ and } D\}}{Pr\{D\}} = \frac{45}{57} = 0.7895$
   (c) $Pr\{R|D^C\} = \frac{Pr\{R \text{ and } D^C\}}{Pr\{D^C\}} = \frac{32}{57} = 0.5614$
   (d) $Pr\{R\} = \frac{77}{114} = 0.6754$
   (e) The difference is: $Pr\{R|D\} - Pr\{R|D^C\} = 0.7895 - 0.5614 = 0.2281$

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5. (a) $Pr\{B \text{ or } G\} = Pr\{B\} + Pr\{G\} - Pr\{B \text{ and } G\} = 0.85 + 0.1 - 0.08 = 0.87$
   (b) $Pr\{G|B\} = \frac{Pr\{B \text{ and } G\}}{Pr\{B\}} = \frac{0.08}{0.85} = 0.0941$
   (c) $Pr\{G|B^C\} = \frac{Pr\{B^C \text{ and } G\}}{Pr\{B^C\}} = \frac{Pr\{G\}-Pr\{B \text{ and } G\}}{1-Pr\{B\}} = \frac{0.1-0.08}{1-0.85} = \frac{0.02}{0.15} = 0.1333$
   (d) $Pr\{B|G\} = \frac{Pr\{B \text{ and } G\}}{Pr\{G\}} = \frac{0.08}{0.1} = 0.8$
   (e) No, since $Pr\{B|G\} = 0.8 \neq Pr\{B\} = 0.85$

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6. (a) FALSE: The union will always include the intersection, so that the intersection cannot be larger.
   (b) TRUE: The union will always include the probability of A, B, or both, so it must be larger than just the probability of A alone.

(c) FALSE: It will always be larger, since it is the intersection divided by a number smaller than one.

(d) FALSE: If two events are indpendent, it means either (i) $Pr\{A \text{ and } B\} = Pr\{A\}Pr\{B\}$, or (ii) $Pr\{A|B\} = Pr\{A\}$
