Solutions STA 100 HW 5

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1. (a) The null is: $\mu_N \leq \mu_S$.
   (b) The alternative is: $\mu_N > \mu_S$.
   (c) A Type I Error would mean we concluded there was a significant increase in true average weight gain from the new treatment compared to the old, when in reality there was not.
   (d) A Type II Error would mean we concluded there was a not significant increase in true average weight gain from the new treatment compared to the old, when in reality there was.
   (e) If we wanted to minimize the probability of a Type I Error (α), we would want to use the smallest α (0.01).
   (f) If we wanted to minimize the probability of a Type II Error (α), we would want to use the largest α (0.10).

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2. (a) TRUE. Since $0.002 < 0.01$ (our smallest common value of α), we would reject H_0.
   (b) FALSE. To calculate a p-value we assume the null hypothesis is true, and calculate the probability of observing **our sample data** (or more extreme) if the null was true.
   (c) TRUE. One of the one-sided p-values would certainly be $0.0010 = 0.0020/2$.
   (d) FALSE. The probability of a Type II error is in general not knowable, since it assumes **an** alternative is true, but there are infinitely many alternatives.

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3. (a) Note, $\bar{\bar{y}} = \frac{\sum_{i=1}^{I} n_i \bar{y}_i}{n.} = \frac{32.2+35.7+28.7+29.8}{40} = 3.16$
   $SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2 = (10)(3.22-3.16)^2 + (10)(3.57-3.16)^2 + (10)(2.87-3.16)^2 + (10)(2.98-3.16)^2 = 0.036 + 1.681 + 0.841 + 0.324 = 2.882$
   (b) $SSW = \sum_{i=1}^{I}(n_i - 1)s_i^2 = 9(0.2916) + 9(0.1225) + 9(0.0441) + 9(0.0529) = 2.6244 + 1.1025 + 0.3969 + 0.4761 = 4.5999$
   (c) $d.f.\{B\} = I - 1 = 4 - 1 = 3$
   (d) $d.f.\{W\} = n. - I = 40 - 4 = 36$
   (e) $MSB = \frac{SSB}{d.f.\{B\}} = \frac{2.882}{3} = 0.9607$
   (f) $MSW = \frac{SSW}{d.f.\{W\}} = \frac{4.5999}{36} = 0.1278$
   (g) $F_S = \frac{MSB}{MSW} = \frac{0.9607}{0.1278} = 7.5172$

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4. (a) Mathematically: $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ vs. H_A : At least one μ_i is not equal.
   (b) H_0: The true average GPA for all sororities is equal vs. H_A: The true average GPA is not equal among all sororities.
   (c) $F_S = 7.5172$, with degrees of freedom (3, 36). From the F table at numerator d.f 3, denominator d.f 30 (round down), we find that $Pr\{F > 7.05\} = 0.001$, and $Pr\{F > 9.99\} = 0.0001$. Thus, our p-value is: $0.0001 <$ p-value < 0.001.
   (d) The probability of observing our sample data (or more extreme) if in reality the true average GPA equal for all sororities is between 0.0001 and 0.001.
   (e) Since the p-value is less than α, we reject the null.
   (f) At 5% significance we conclude that at least one of the average GPAs of sororities differs.
   (g) Since the largest difference in sample means is between sorority B and C, and the sample sizes are all the same, the interval for $\mu_B - \mu_C$ is certain **not** to contain zero.
   (h) Since we rejected the null hypothesis, we could have made a Type I error (we rejected the null when in reality the null was false).

(i) A type II error would be we concluded that there was no difference in true average GPA between sororities, when in reality at least one was different.

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5. (a) Note, $\bar{\bar{y}} = \frac{\sum_{i=1}^{I} n_i \bar{y}_i}{n.} = \frac{17.99+25.97+30.03}{21} = 3.5233$
$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2 = (7)(2.57-3.5233)^2 + (7)(3.71-3.5233)^2 + (7)(4.29-3.5233)^2 = 6.3615+0.244+4.1148 = 10.7203$

(b) $SSW = \sum_{i=1}^{I}(n_i - 1)s_i^2 = 6(0.9604) + 6(1.2321) + 6(1.9044) = 5.7624 + 7.3926 + 11.4264 = 24.5814$

(c) $d.f.\{B\} = I - 1 = 3 - 1 = 2$

(d) $d.f.\{W\} = n. - I = 21 - 3 = 18$

(e) $MSB = \frac{SSB}{d.f.\{B\}} = \frac{10.7203}{2} = 5.3601$

(f) $MSW = \frac{SSW}{d.f.\{W\}} = \frac{24.5814}{18} = 1.3656$

(g) $F_S = \frac{MSB}{MSW} = \frac{5.3601}{1.3656} = 3.9251$

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6. (a) Mathematically: $H_0 : \mu_1 = \mu_2 = \mu_3$ vs. H_A : At least one μ_i is not equal.

(b) H_0: The true average number of calls is equal over all shifts vs. H_A: The true average number of calls is not equal over all groups.

(c) $F_S = 3.9251$, with degrees of freedom (2, 18). From the F table at numerator d.f 2, denominator d.f 18, we find that $Pr\{F > 3.55\} = 0.05$, and $Pr\{F > 4.90\} = 0.02$. Thus, our p-value is: $0.02 <$ p-value < 0.05.

(d) Since the p-value is greater than α, we fail to reject the null.

(e) At 1% significance we conclude that there is no significant difference in true average number of calls between the three shifts

(f) A Type I Error would be if we concluded there was a difference in the true average number of calls of at least one of the shifts, when in reality there was not.
