STA 100 Homework 6 Solutions (Book)

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1. (a) The confidence intervals have the form: $(\bar{y}_A - \bar{y}_B) \pm t_{\alpha/(2k)} \sqrt{MS(within)(\frac{1}{n_a} + \frac{1}{n_b})}$
   For $\mu_2 - \mu_1$: $(3.57 - 3.22) \pm (3.1439)\sqrt{0.1278(\frac{1}{10} + \frac{1}{10})}$ or (-0.1526, 0.8526).
   Thus, the lower bound is -0.1526

   (b) By (a), the upper bound is 0.8526.

   (c) For $\mu_2 - \mu_3$: $(3.57 - 2.87) \pm (3.1439)\sqrt{0.1278(\frac{1}{10} + \frac{1}{10})}$ or (0.1974, 1.2026).
   Thus, the lower bound is 0.1974

   (d) By (c), the upper bound is 1.2026.

   (e) For $\mu_2 - \mu_4$: $(3.57 - 2.98) \pm (3.1439)\sqrt{0.1278(\frac{1}{10} + \frac{1}{10})}$ or (0.0874, 1.0926).
   Thus, the lower bound is 0.0874

   (f) By (e), the upper bound is 1.0926.

   (g) The confidence intervals that do not contain zero imply a significant difference, so that we are confident that sorority Bs true average GPA differs significantly from sorority C and D.

   (h) The smallest difference in average GPAs is suggested by the interval for $\mu_2 - \mu_4$. or B and D (since it is the interval that does not contain zero and has the smallest absolute bound).

   (i) We are 99% family-wise (simultaneous/overall) confident that the true average difference in GPA between the sorority B and sorority C is between 0.1974 and 1.2026.

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2. (a) The confidence intervals have the form: $(\bar{y}_A - \bar{y}_B) \pm t_{\alpha/(2k)} \sqrt{MS(within)(\frac{1}{n_a} + \frac{1}{n_b})}$
   For $\mu_1 - \mu_2$: $(17.31 - 11.7) \pm (2.2472)\sqrt{16.8889(\frac{1}{10} + \frac{1}{10})}$ or (1.4799, 9.7401).
   Thus, the lower bound is 1.4799

   (b) By (a), the upper bound is 9.7401.

   (c) For $\mu_1 - \mu_3$: $(17.31 - 17.84) \pm (2.2472)\sqrt{16.8889(\frac{1}{10} + \frac{1}{9})}$ or (-4.7732, 3.7132).
   Thus, the lower bound is -4.7732

   (d) By (c), the upper bound is 3.7132.

   (e) For $\mu_2 - \mu_3$: $(11.7 - 17.84) \pm (2.2472)\sqrt{16.8889(\frac{1}{10} + \frac{1}{9})}$ or (-10.3832, -1.8968).
   Thus, the lower bound is -10.3832

   (f) By (e), the upper bound is -1.8968.

   (g) The confidence intervals that do not contain zero imply a difference between the means. So the confidence intervals comparing the control and the deficient group, and deficient group and the fertilizer group.

   (h) The confidence interval which suggests the largest difference contains the largest value (negative or positive), which would be the interval for comparing the deficient group to the fertilizer group (since it is the interval which does not contain zero and has the largest absolute bound).

   (i) We are 90% family-wise (simultaneous/overall) confident that the true average difference in growth between the control group and the deficient group is between 1.4799 and 9.7401.

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3. (a) $H_0 : \mu_P = \mu_C = \mu_S = \mu_D$

   (b) H_A : At least one of the μ_i's is not equal.

   (c) If we concluded that there was a difference in at least one of the average pain scores, when in reality there was not.

   (d) If we concluded that there was no difference in average pain score, when in reality there was.

(e) No. This would tell us that at least one average was different, not that all averages are different.

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4. (a) The confidence interval is $\tilde{p} \pm Z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$, or $0.2035 \pm (1.96)\sqrt{\frac{0.2035(1-0.2035))}{340+4}}$ or $(0.161, 0.246)$
Thus the lower bound is: 0.161

(b) From (a), the upper bound is: 0.246u

(c) We are 95% confident that the true proportion of infants with an adverse reaction is between 0.161 and 0.246.

(d) Since the confidence interval bounds are less than the value 0.25, it does support the claim that **under** 25% have an adverse reaction.

(e) By the definition of a confidence interval, we would expect $(1-\alpha)100\%$ of them (i.e, 95%) to cover the true mean.

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5. (a) The confidence interval is $\tilde{p} \pm Z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$, or $0.2133 \pm (2.576)\sqrt{\frac{0.2133(1-0.2133))}{71+4}}$ or $(0.0915, 0.3351)$ Thus, the lower bound is 0.0915

(b) The upper bound is 0.3351

(c) We are 99% confident that the true proportion of orangutans with an type B blood is between 0.0915 and 0.3351.

(d) Since the confidence interval contains the value 0.25, it does support the claim that 25% have type B blood.

(e) By the definition of a confidence interval, we would expect 1% not to cover the true proportion.

(f) A random sample was taken, with at least 5 subjects with the trait, and 5 without the trait.

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6. (a) TRUE. If the error is large enough, or we start with a high estimate of p (close to 1), the confidence interval could give nonsensical bounds.

(b) FALSE. We would correct α to $\alpha/4$, which would make $t_{\alpha/2}$ into $t_{\alpha/8}$.

(c) TRUE. If we fail to reject the null we are supporting that all group means are equal, and thus all confidence intervals for a difference between the means should cover 0.

(d) FALSE. The data in this case is categorical, and cannot be normally distributed. We assume that the sample proportion is normally distributed (since it is a mean).
