Optimal snow-survey design for the estimation of winter balance on alpine glaciers

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ABSTRACT. Efficient collection of snow depth and density data is critical to a successful snow measurement campaign and to accurately estimate glacier winter balance. Extensive, high resolution and accurate snow accumulation measurements on glaciers are almost impossible to achieve so surveys need to optimize the extent and spacing of snow measurements to obtain reliable estimates of winter balance. To address this need, we estimate winter balance and root mean squared error (RMSE) using synthetic and real data from subsets of extensive surveys on three glaciers in the St. Elias Mountains, Yukon. We generate six different survey designs, which encompass possible snow sampling patterns and various numbers of measurement locations. We then use linear regression with topographic parameters to interpolate measurements. Analysis of both synthetic and real data indicates that an 'midline & transect' and 'hourglass' sampling patterns result in the most efficient and accurate estimate of winter balance, while the midline pattern results in poor estimates of winter balance for all glaciers. RMSE decreases with increased sample size, with no further reduction after about 30 measurement locations. This study highlights the ability for future winter balance and snow survey studies to optimize snow data collection within a glacierized basin.

Keywords: glacier; alpine; snow survey design; optimize; St. Elias Mountains; snow probing

INTRODUCTION

Estimates of basin-wide seasonal snow accumulation are critical for monitoring glacier mass balance and for predicting the availability and timing of surface runoff, especially in mountainous regions. The net accumulation of snow on a glacier over a winter season is known as the winter surface mass balance, or "winter balance" (WB) and is typically reported in meters of water equivalent (m w.e.) (**?**). Winter balance accounts for half of the seasonally resolved mass balance, initializes ablation conditions and affects energy and mass exchange between the land and atmosphere (e.g. **??**).

Optimal survey designs for snow depth are central to accurately estimating snow distribution and mass balance from *in situ* measurements. Measuring snow depth and travelling between measurement locations is both time consuming and can disturb the snow, so care must be taken to choose a survey design that avoids bias, allows for the greatest variability to be measured and minimizes distance travelled (e.g. **??**). Moreover, the period of seasonal maximum snow depth can be brief, further motivating the need for time-efficient survey designs.

There are a number of different spatial sampling patterns that have been employed to obtain point measurements of snow depth, including pure random (e.g. **?**), linear random (e.g. **?**), nested (e.g. **?**), gridded random (e.g. **???**) and gridded (e.g. **???**). Sampling patterns that incorporate randomness are favourable because they limit sampling bias

by varying sample spacing and direction. However, they are less efficient than designs that incorporate grids. Grid-based designs minimize travel distance but measurements are biased by regularly spaced intervals and linear orientations which can result in an under-representation of snow-depth variability (**?**).

Snow surveys on glaciers are conducted to estimate winter balance, and multi-year sampling programs are often established to monitor changes in winter balance with time. An optimized survey design requires (1) a sampling scheme that captures spatial variability and minimizes travel distance and (2) knowledge of the minimum number of measurement locations needed to estimate WB to the desired precision. Optimization of winter-balance survey design is rarely investigated because the locations of snow-depth measurements are often dictated by field resources and logistics. Few studies have investigated the number of measurement locations needed to effectively sample the winter-balance distribution (c.f. **??**). The sampling patterns used for most winter-balance programs do not include randomness, and measurements are typically made along the glacier centreline (e.g. **?**) to capture the expected orographic effects (e.g. **?**). However, centreline surveys are known to underestimate winter balance, so transverse profiles are often added to improve the reliability of the sampling scheme (e.g. **?**). An hourglass with an inscribed circle (personal communication from C. Parr, 2016) is an alternative sampling scheme that captures changes in winter
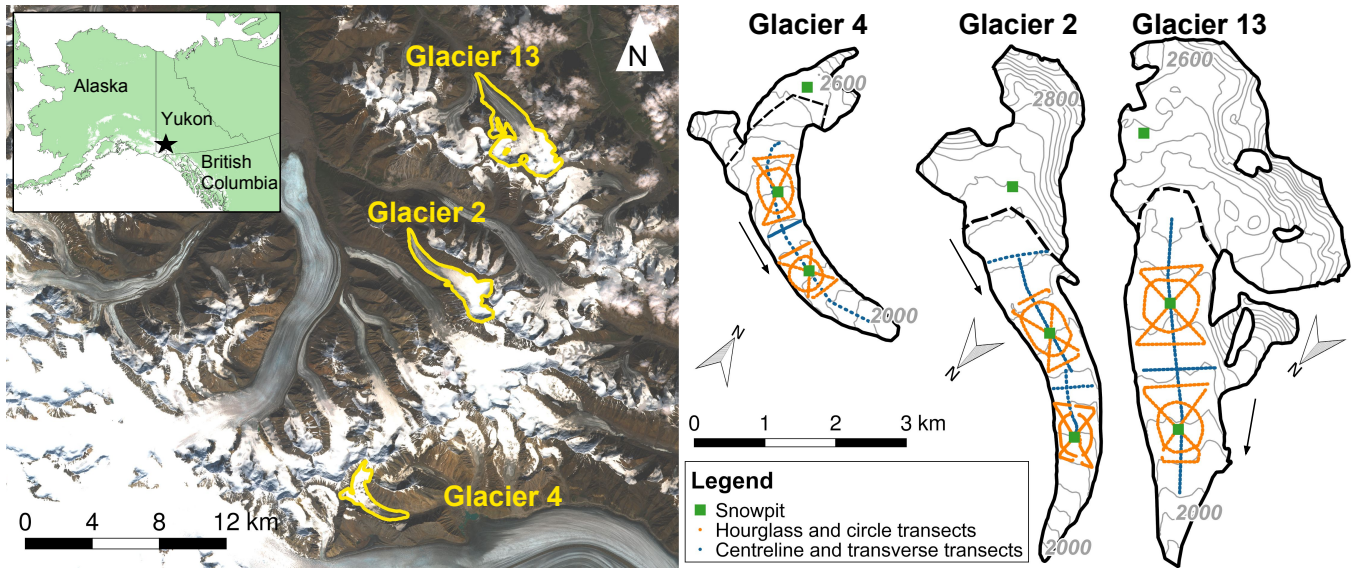
Fig. 1: Study area location and sampling schemes for Glaciers 4, 2 and 13. (Left) Study region in the Donjek Range of the St. Elias Mountains of Yukon, Canada (inset). Imagery from Landsat8 (29 August 2013, data available from the U.S. Geological Survey). (Right) Details of the spatial sampling schemes, with centreline and transverse profiles (blue dots), hourglass and circle designs (orange dots) and locations of snow-density measurements (green squares). Arrows indicate ice-flow directions. Approximate location of ELA on each glacier is shown as a black dashed line. Contour lines in increments of 50 m are shown in grey.

balance with elevation while avoiding the centreline bias, and is easy to travel. To our knowledge, no study has yet compared the performance of these different sampling patterns in estimating the spatially distributed winter balance.

The goal of this work is to determine the optimal survey design for estimating distributed and glacier-wide (integrated) winter balance. We consider both the spatial sampling scheme and the number of measurement locations in defining the optimal survey design. For three alpine glaciers in the St. Elias Mountains of Yukon, Canada, we explore the effects of survey design on: (1) the precision in estimated glacier-wide (integrated) winter balance using synthetic data, and (2) the accuracy in estimated winter balance at locations distributed across the glacier using real data.

STUDY SITE

We investigate winter-balance survey design for three unnamed glaciers in the Donjek Range of southwest Yukon, Canada. Situated on the northern flanks of the St. Elias Mountains, which rise sharply from the Pacific Ocean, the Donjek Range experiences a continental climate. Monitoring of snow distribution and glacier mass balance in the St. Elias Mountains began in the 1950s and 1960s with a series of research programs, including Project "Snow Cornice" and the Icefield Ranges Research Project (**??**). More recent studies have focused on studies of individual alpine glaciers (e.g. **??**) as well as regional glacier mass balance and dynamics (e.g. **???**).

Glacier 4, Glacier 2 and Glacier 13 (labelling adopted from **?**) are small alpine glaciers (3.8–12.6 km^2) with simple geometries. The elevation of these glaciers ranges from 1900 to 3100 m a.s.l. and ELAs are located at $\sim$2500 m. The glaciers are generally oriented southeast-northwest in steep-walled valleys. We suspect that all three glaciers are polythermal, based on a targeted study of Glacier 2 (**?**) and related theoretical modelling (**?**). A detailed analysis of winter-balance estimation on these three glaciers is presented by **?**.

METHODS

We aim to determine the optimal sampling pattern and number of measurement locations needed to obtain accurate estimates of WB for small alpine glaciers. We estimate distributed WB using both synthetic- and real values of gridcell WB. Real data are subsets of all gridcell values of WB, which is derived from direct measurements of snow depth and density, chosen based on different survey designs. A linear regression is used to interpolate gridcell values resulting in an estimated WB. Synthetic data are generated by sampling a synthetic distribution of WB, which is based on field measurements, using various survey designs. Uncertainty in gridcell WB is incorporated by introducing noise into the synthetic distribution of WB. A linear regression of synthetic data on topographic parameters is then used to estimate WB. A set of estimated WB is generated by repeating this process with different levels of noise. We then directly compare the estimated WB and the synthetic distribution of WB. Three study glaciers, with
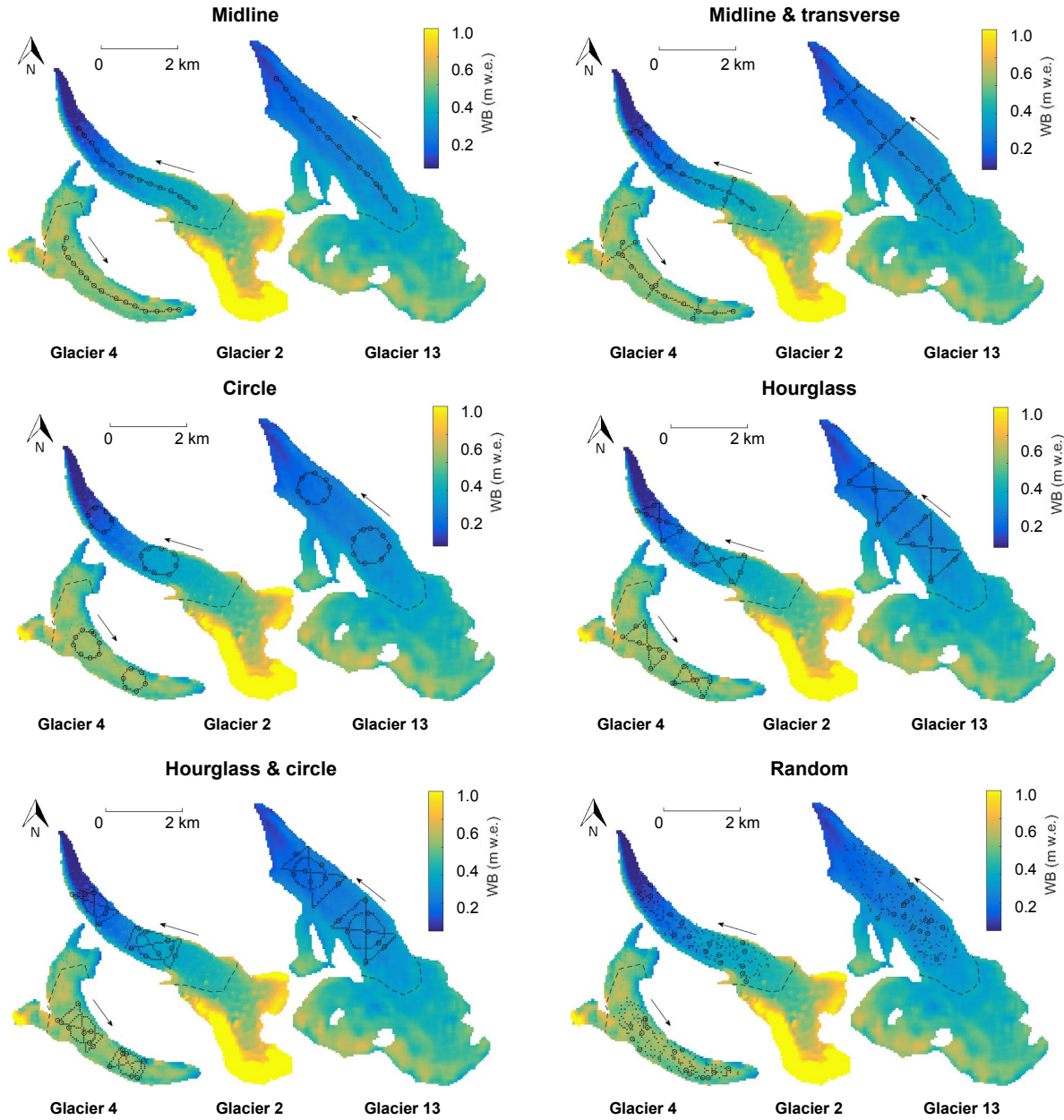
Fig. 2: Synthetic sampling patterns on the three study glaciers. All synthetic sampling locations (small black dots) and a subset of 15 synthetic sampling locations (open circles) are shown for each sampling scheme. The random scheme shows a total of 200 synthetic sampling locations. Sampling patterns are overlain on the synthetic distribution of winter balance (WB) in colour.

differing spatial patterns of WB, are used to determine the applicability of our conclusions between glaciers.

Here we describe the process of collecting WB data in the field and interpolating/extrapolating to obtain the synthetic distribution of WB on the three study glaciers. Subsets of the real data are used to investigate the effect of various survey designs on the error associated with estimating gridcell-scale values of WB. Then, we detail the methods used to obtain synthetic data and describe the various sampling patterns and number of measurement locations investigated in this study. A comparison of the synthetic distribution of WB and WB estimated using different survey designs with synthetic data is presented.

Field data

Point-scale values of WB are found by obtaining direct measurements of snow depth and density (Figure **??**). Snow depth was measured using a 3.2 m graduated aluminium avalanche probe. Measurement locations followed linear and curvilinear transects, which were similar between study glaciers, with a sample spacing of 10–60 m. Spacing was constrained by protocols for safe glacier travel. Each
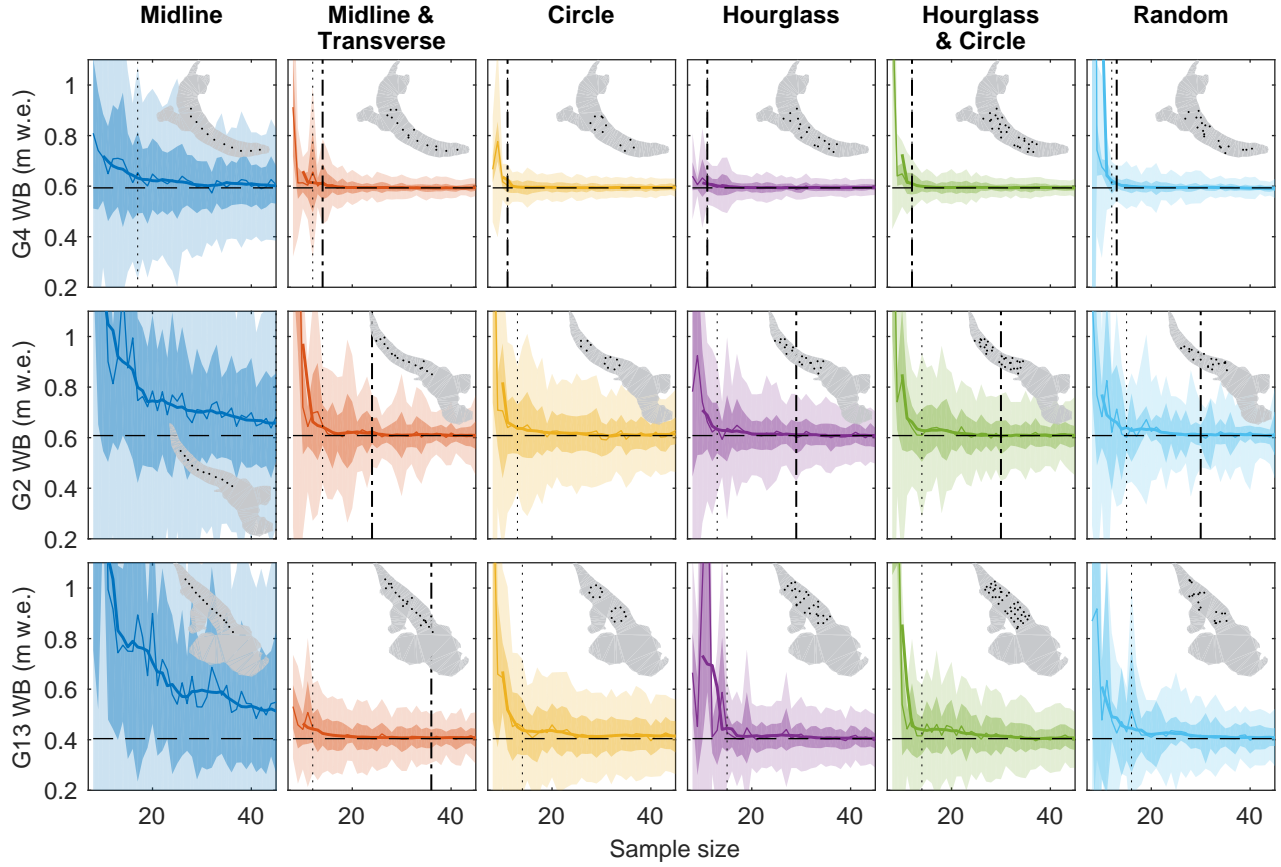
Fig. 3: Glacier-wide winter balance (WB) estimated with various survey designs for Glacier 4 (top row), Glacier 2 (middle row) and Glacier 13 (bottom row). Sampling patterns are shown in columns. Insets show an example of a survey design corresponding to the panel. Mean WB from 100 runs of a linear regression of synthetic WB on topographic parameters for various patterns and sample sizes (solid coloured line) with second-order exponential fit (solid black line). Standard deviation in glacier-wide WB due to low noise (dark shading) and high noise (light shading). Glacier-wide WB of validation data (black dashed line). Sample size when fitted mean is within 0.05 m w.e. of true mean (black dotted line) and when standard deviation due to high noise is less than 0.15 m w.e. (black dotted-dashed line) are shown.

Table 1: Ranking of survey designs from 1 to 6 (top row) for Glaciers 4, 2 and 13 (GL 4, Gl 2, Gl 13, left column) based on three different metrics: precision of estimated glacier-wide winter balance using synthetic data (top three rows), accuracy of estimated winter balance at real measurement locations distributed across the glacier (middle three rows) and travel distance required to perform survey (bottom three rows). Sampling patterns are: midline (M), midline and transverse (MT), circle (C), hourglass (H), hourglass and circle (HC) and random (R). Colours follow figures. Values in parentheses represent number of samples required to achieve a precision of 5% (top three rows), number of samples required to achieve an accuracy of 25% (middle three rows) or travel distance (km). Travel distances given for random sampling scheme are averages.

Metric		**1**		**2**		**3**		**4**		**5**		**6**	
	G4	C,H	(8)			MT,HC	(9)			R	(10)	M	(14)
Convergence	**G2**	C,H	(10)	HC	(11)	MT,HC	(11)			R	(12)	M	(43)
	G13	MT	(9)	HC	(11)	C,H	(12)			R	(13)	M	(101)
	G4	C,H	(8)			HC	(9)	R	(10)	MT	(13)	M	(−)
Variability	**G2**	MT	(21)	H	(26)	HC,R	(29)			C	(47)	M	(−)
	G13	MT	(34)	H	(44)	R	(49)	HC	(50)	C	(96)	M	(−)
Distance (km)	**G4**	C	(4.8)	M	(6.3)	H	(6.9)	R	(∼7.9)	MT	(8.3)	HC	(11.1)
	G2	M	(4.3)	C	(5.7)	H	(8.4)	MT	(8.6)	R	(∼9.2)	HC	(12.5)
	G13	C	(7.0)	M	(8.0)	H	(10.6)	MT	(11.0)	R	(∼11.3)	HC	(16.8)

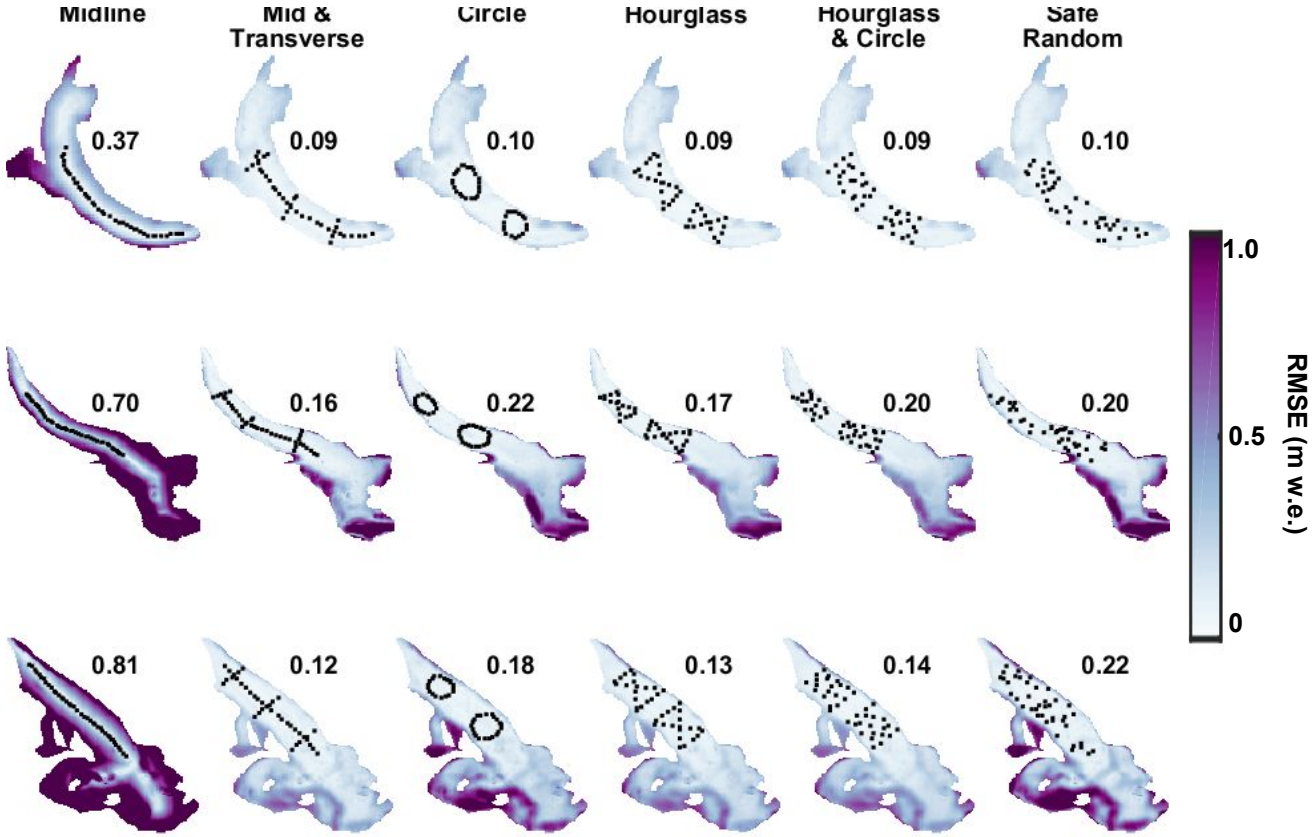Fig. 4: Root-mean-squared-error (RMSE) between validation WB and WB estimated with various sample patterns ($n = 40$) for 100 runs of high noise linear regression. Sampling patterns are shown in columns for Glacier 4 (top row), Glacier 2 (middle row) and Glacier 13 (bottom row). RMSE for the entire glacier is displayed. Synthetic sampling locations are shown (black dots).

observer made 3–4 depth measurements within ∼1 m at each transect measurement location. We restricted snow-depth sampling to the ablation area, where the clear distinction between snow and ice ensured that only snow from the current accumulation season was measured. In total, we collected more than 9000 snow-depth measurements throughout the study area. Snow density was measured using a wedge cutter in three snow pits that spanned a large portion of the elevation on each glacier. A mean density was then calculated for each glacier and this value was used to convert snow depth at all measurement locations to values of point-scale WB. Mean density was $348\pm13\,\mathrm{kg\,m^{-3}}$ on Glacier 4, $333\pm26\,\mathrm{kg\,m^{-3}}$ on Glacier 2 and $349\pm38\,\mathrm{kg\,m^{-3}}$ on Glacier 13. All point-scale values of WB located within a common DEM gridcell (40×40 m) were averaged to obtain values of gridcell WB.

Real data

survey designs are derived from all measurement locations for each study glacier (Figure **??**). We investigate numerous survey designs that are unique combinations of six different sampling patterns and the number of measurement locations. Midline and midline & transverse transects are the most common survey designs used in WB studies (e.g. **??**). The midline survey aims to capture changes in WB with elevation and transverse transects provide observation of lateral variations in WB. Hourglass and inscribed circle allow for sampling in multiple directions and are easy to travel (personal communication from C. Parr, 2016). We use hourglass and circle patterns separately and combined. Finally, a random pattern of measurement locations is obtained by selecting random gridcells from the full data set. For all sampling patterns we estimate WB using n measurement locations, where n ranges from a minimum of eight (constrained by using all seven topographic parameters for interpolation) to a maximum determined by the number of gridcells within a sampling pattern (ranges between 57–228 gridcells). The measurement locations are evenly distributed within the sampling pattern.

A linear regression is then used to interpolate the real values of WB for each glacier. Real data is regressed on elevation, slope, aspect, distance from glacier centreline, "northness", curvature and a wind redistribution parameter. The linear regression calculates regression coefficients that minimize the sum of squares of the vertical deviations
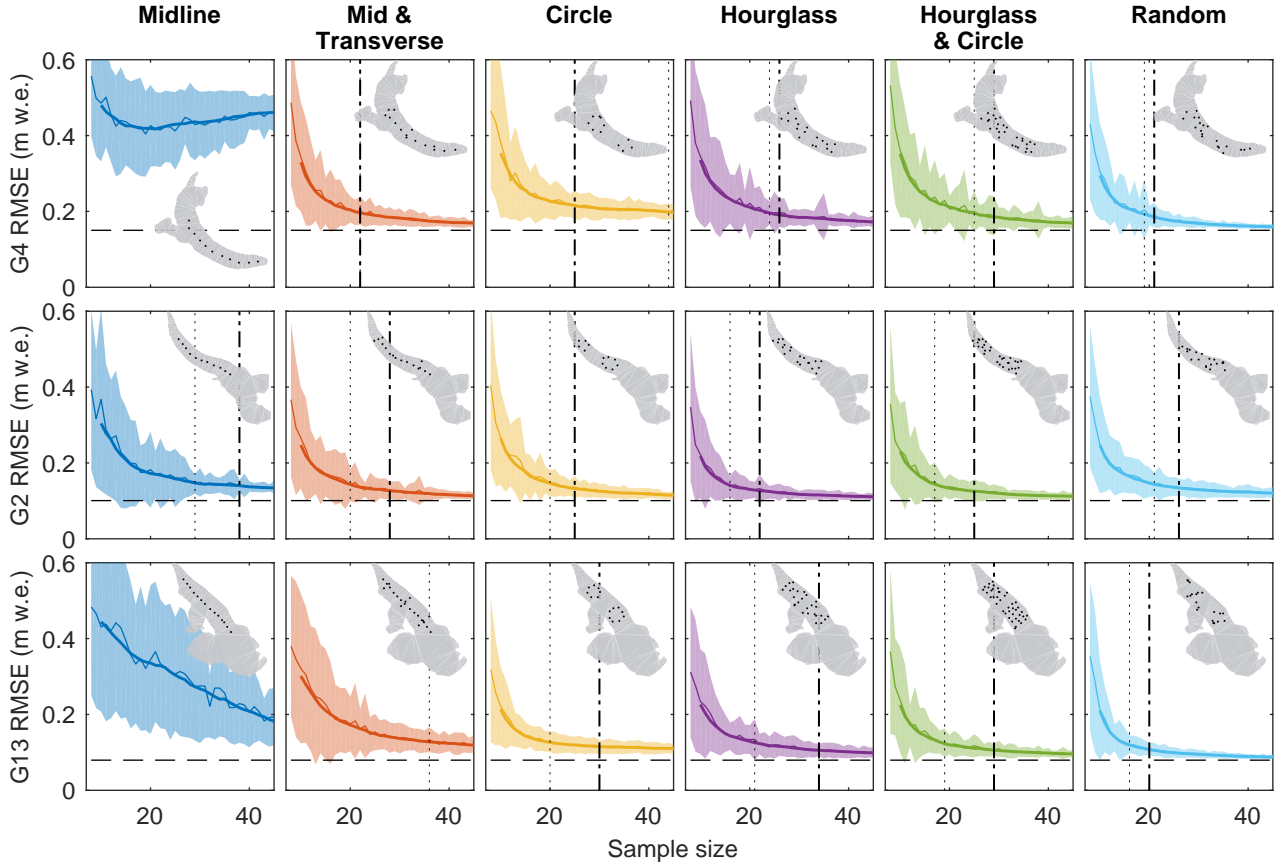
Fig. 5: Root-mean-squared-error (RMSE) between WB estimated with various survey designs of real data and field values of WB at all measurement locations for Glacier 4 (top row), Glacier 2 (middle row) and Glacier 13 (bottom row). Sampling patterns are shown in columns. Insets show an example of a survey design corresponding to the panel. Mean RMSE from 100 runs of a linear regression of real WB on topographic parameters for various patterns and sample sizes (solid coloured line) with second-order exponential fit (solid black line). Standard deviation in RMSE due to low noise (dark shading) and high noise (light shading). RMSE of estimated WB when all measurement locations are used in the linear regression (black dashed line). Sample size when fitted RMSE mean is within 0.05 m w.e. of true RMSE mean (black dotted line) and when standard deviation due to high noise is less than 0.01 m w.e. (black dotted-dashed line) are shown.

of each datum from the regression line (**?**). The resulting regression coefficients are then applied to the topographic parameters associated with each gridcell to obtain an estimate of distributed WB. Results are presented as a RMSE value, which is calculated by taking the square root of the mean difference between WB estimated using various survey designs and real data at all measurement locations.

Synthetic data

We obtain synthetic data using various survey designs from the synthetic distribution of WB to test which survey design best estimates WB. Gridcell values of WB that fall within each survey design are extracted from the synthetic distribution of WB distribution and then used to estimate a new WB distribution. The WB distribution derived from various survey designs is directly compared to the synthetic distribution of WB distribution to estimate spatially resolved error.

The synthetic distribution of WB is obtained by computing a linear regression of gridcell values of WB on topographic parameters for each glacier (**?**). Topographic parameters are derived from a SPIRIT SPOT-5 DEM (**?**) and include commonly applied topographic parameters (e.g. **?**) such as elevation, slope, aspect, distance from glacier centreline, "northness", curvature and a wind redistribution parameter. A linear regression, along with cross-validation and model averaging, is used to obtain a set of regression coefficients for the standardized topographic parameters on each glacier (**?**). Distributed WB is found by multiplying fitted regression coefficients by corresponding topographic parameters for each gridcell. The distributed WB calculated using all available WB data is hereafter referred to as the synthetic distribution of WB.

Synthetic data are obtained for all of the survey designs presented above. Midline, midline & transects, circle, hourglass, hourglass & circle and random sampling patterns are used (Figure **??**) and values of n from

eight to the maximum number of observation with each sampling pattern are investigated. All sampling patterns are restricted to the ablation area, where terrain is accessible and direct measurements of snow depth are easy to obtain. The random pattern selects measurements locations within the area that encompasses the original survey designs.

To simulate the process of measuring WB, we first obtain WB values from the synthetic distribution of WB distribution at selected measurement locations for each sampling pattern. Then, we add a low or high level of noise to the WB data. Low noise is defined by a normal distribution that is centred at zero and has a standard deviation equal to the mean standard deviation of WB data from a series of high-density gridcell-scale surveys on each glacier (**?**). High noise is defined in the same way as low noise but the standard deviation of the normal distribution is three times larger, which is approximately equal to the standard deviation of the WB probability distribution that accounts for uncertainty due in grid-scale WB, density assignment, and interpolation for the three study glaciers (**?**). For each gridcell value of WB, a random number from the high or low noise distribution is added to obtain a synthetic observation of WB.

As above, a linear regression is calculated between synthetic data and topographic parameters and an estimated WB is then calculated. The process of adding random noise to gridcell-scale values of WB to obtain synthetic observations and fitting a regression to then estimate distributed WB is repeated 100 times to create a population of possible WB estimates. Each repetition uses a different set of random noise resulting in a range of WB values from which a mean WB and RMSE is then calculated. RMSE is found by taking the square root of the mean difference between all gridcells in the synthetic distribution of WB distribution and the WB distribution derived with the survey design.

Performance metrics

To quantify the performance of each sampling pattern, we use two performance metrics. The first metric is 'convergence', which is defined as the number of measurement locations (n) needed to obtain a mean WB within 5% of the synthetic glacier-wide WB or the RMSE for WB estimated using all real data points. The relation between WB or RMSE and n is smoothed (...describe it here). Convergence is a metric that describes the minimum number of measurement locations required to have an accurate estimate of WB. The second metric is 'variability' (or 'sensitivity'?), which is defined as the number of measurement locations required to have the standard deviation of WB or RMSE due to high noise within 5% of the synthetic glacier-wide WB or the RMSE for WB estimated using all real data points. Again, the curve is smoothed. Variability is a metric of how sensitive the WB estimate is to uncertainty in grid-scale variability and interpolation. The total distance required to obtain measurements along a sampling pattern from a central location is also presented. An efficient survey design would have lower n values for convergence and variability metrics as well as a short travel distance.

RESULTS

Synthetic data

Based on synthetic data, the most effective sampling patterns are midline & transect and hourglass. Midline & transect and hourglass quickly converge to the synthetic glacier-wide value of WB, have comparatively low standard deviations due to noise and involve the shortest travel distances (Figure **??** and Table **??**). Hourglass requires slightly fewer measurements to converge to the synthetic WB and is slightly shorter than midline & transect but midline & transect has slightly lower variability. If using midline & transect or hourglass sampling patterns, a sampling size of ~35 is sufficient to estimate glacier-wide WB on all glaciers with no marked improvement in accuracy or precision at larger sample sizes. With all sampling patterns, WB is significantly overestimated at low sample size.

The remaining sampling patterns are less effective for estimating WB. Circle and hourglass & circle also require small sample sizes for convergence but circle has exceptionally high variability on Glaciers 2 and 13 and hourglass & circle requires a substantially greater travel distance. Notably, hourglass & circle is not a substantial improvement on the hourglass sampling pattern given the considerable increase in travel distance required to sample the along the hourglass & circle. By far the worst sampling pattern is the midline. WB values are highly sensitive to noise and even with a large number of measurement locations the validation values of WB are not achieved.

Although the convergence patterns are similar between glaciers, the standard deviation due to noise and the sample size needed to accurately estimate WB are smallest for Glacier 4 and greatest for Glacier 13 for all sampling patterns. RMSE is also larger on Glaciers 2 and 13 than on Glacier 4, especially in the accumulation area (Figure **??**). When midline pattern is used, the errors are exceptionally high in areas that do not fall along the midline for all glaciers.

Real data

The RMSE rapidly decreases with an increased sample size for all sampling patterns on all three glaciers (Figure**??**). The sampling patterns trends are similar to those of the synthetic data. Midline & transverse, hourglass, circle, hourglass & circle and random all perform well. However, the higher travelling distance of hourglass & circle and random make these patterns less efficient. Again, the worst sampling pattern is the midline, which results in exceptionally high values of RMSE for all three glaciers. There is no marked decrease in RMSE for sample sizes greater than ~ 30 for all sampling patterns. None of the sampling patterns result in the RMSE reaching that of the full data for all three glaciers.

DISCUSSION

The optimal survey design for our study region is the midline & transverse or hourglass pattern with a sample size of ~ 30. Based on both synthetic and real data, both designs

result in low error, is least sensitive to noise and involves relatively small travelling distance. A sample size greater than 30 does not significantly improve the accuracy of WB estimated and does not decrease error. This surprisingly low number of measurement locations indicates that high-resolution sampling is not required to accurately estimate WB. Instead, taking measurements throughout the study basin should be prioritized when choosing a survey design. If the goal is to estimate WB then field resources should be more strongly allocated to distributing measurement locations to capture basin-scale spatial trends in WB since error is greatest in areas far from sampling locations and in areas with extreme values of topographic parameters.

The most effective survey designs capture the dominant WB-elevation trend but also have measurement locations in gridcells that span the range of other topographic parameters. Despite the fact that the synthetic distribution of WB on Glaciers 2 and 13 is largely controlled by elevation, measurements locations that are not along the midline are needed to constrain the regression. When the midline pattern is used, the topographic parameters (excluding elevation) fall within a narrow range so the regression becomes sensitive to noise. Further, the accuracy of estimated WB does not improve at large sample sizes along a midline pattern because the sampling locations do not capture relationships between WB and the remaining topographic parameters. However, if there are a large portion of measurement locations far from the midline then the elevation trend may be less strong resulting in larger errors. Random and hourglass & circle patterns may have larger errors on Glaciers 2 and 13 as a result of the higher proportion of measurement locations far from the midline.

On Glacier 4, the standard deviation of WB due to noise and the number of measurement locations needed to estimate WB is smaller than Glaciers 2 and 13. This trend is likely a result of the small influence of topographic parameters in the distribution of synthetic distribution of WB. The synthetic distribution of WB is derived from a linear regression between field measurements of WB and topographic parameters that explains little of the observed variance (R^2 =0.07). As a result, glacier-wide WB is approximately equal to the mean of WB field measurements. This mostly uniform distribution of snow can therefore be described with few measurement locations, regardless of where the measurements are obtained.

We find that, on average, WB is over-estimated with small sample sizes (< 20), especially on Glaciers 2 and 13 with $n < 10$. We hypothesize that over-estimation results from an exaggeration of the elevation regression coefficient, which is the dominant explanatory parameter for Glaciers 2 and 13. At small sample sizes, the elevation trend over-shadows the relationships with other topographic parameters and is sensitive to noise, which results in large WB values in the accumulation area. Our finds are inconsistent with **?**, who found that using only midline probings underestimates WB.

Further work into the optimization of snow survey survey design is needed. The most obvious limitation of our study is the restriction of survey design to accessible locations in the ablation area. Lack of measurements in the accumulation area is a sub-optimal survey design. However,

direct measurements of snow depth in the accumulation area are extremely costly because a large amount of time is needed to dig a snow pit, especially in regions with high accumulation. Given the importance of spatially distributed measurements though, there is likely an optimal balance between costly measurements in the accumulation area and less costly measurements in the ablation area. Use of more efficient snow-depth measurement methods, such a ground-penetrating radar (GPR) and lidar, in the accumulation area can also be investigated. Additional extensions to our work include an investigation of the size and placement of hourglass patterns and to optimize measurement locations based on factors such as topographic parameters.

CONCLUSION

From an analysis of synthetic and real WB data, we find that 'midline & transect' and 'hourglass' sampling patterns with a sample size of approximately 30 measurement locations is the optimal survey design for estimating glacier-wide WB. Since a relatively low sample size is needed and errors are greatest in the accumulation area, we recommend that field resources be allocated such that measurements locations are distributed throughout the sample basin rather than obtaining high-resolution WB measurements. A midline sampling pattern results in poor estimates of WB and high sensitivity to noise. We find that WB is over-estimated with very small sample sizes ($n < 10$) and along the midline so care should be made to adequately sample the basin to capture changes in elevation as well as transverse variation in WB.

REFERENCES