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Results Basin Scale

Overview

This is a document that shows the sampled and full ranges of topographic parameters and then goes into the multiple linear regression and Bayesian model averaging that was used to explain SWE with topographic parameters. Modelled SWE is found for the three study glaciers using the regressions.

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1 Topographic parameters of study sites

1.1 Creating DEM

Topographic parameters can be derived from a digital elevation model (DEM) of the study area. The DEM used in this project was created from imagery taken with the SPOT-5 satellite and it was provided at no cost by the French Space Agency (CNES) through the SPIRIT International Polar Year project (Korona and others, 2009). The DEM has a cell size of 40×40 m. The v2 version of the DEM was used, which employs a set of correlation parameters fit for steeper slopes (E. Berthier personal communication, 2016).

Two DEMs were available for the Donjek Range. The first DEM (GES 08-029) was collected on September 3, 2007 and the second DEM (GES 07-044) was collected on September 13, 2007. Since the DEMs were collected in September, the surface would likely be at a seasonal minimum with minimum snow cover. Therefore, the surface described by the DEM represents the topography below the snow, which means that this study is similar in nature to other snow science investigations that examine ice free basins. A major flaw in using this DEM is that the DEM is from 2007 and there have almost certainly been changes in the end-of-summer glacier surface. However, the SPOT-5 DEM is the best resolution and most current DEM available for the study area.

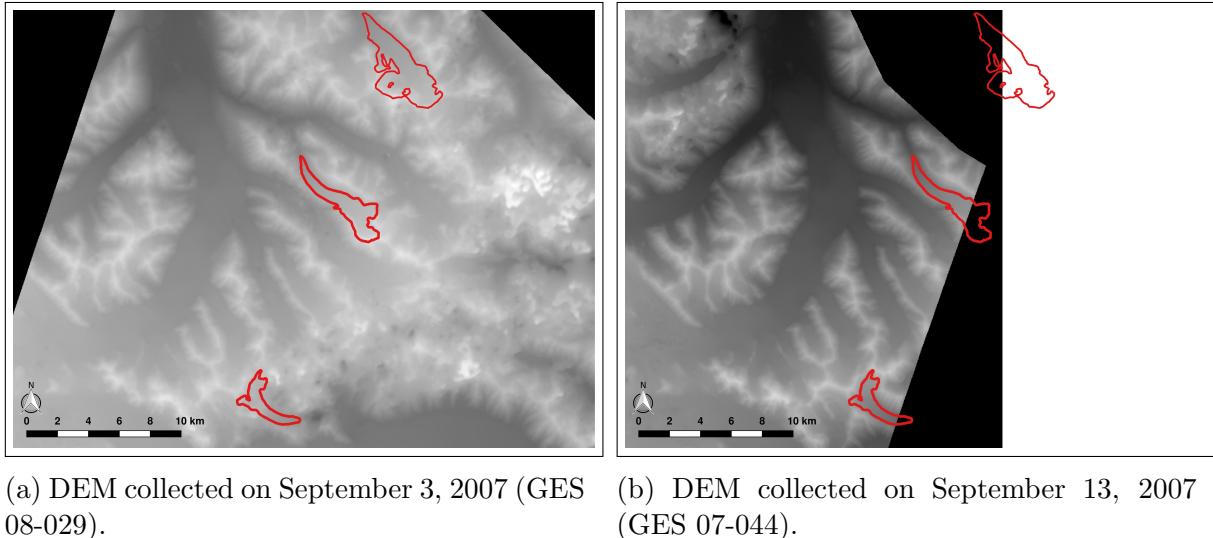


Figure 1: SPOT-5 DEMs available for the Donjek Range. Study glaciers are shown as red outlines. Areas with cloud cover have a distorted DEM as seen on the eastern side of (a).

The GES 08-029 DEM covered all three study glaciers (see Figure 1a) but a large part of Glacier 4 and some areas of Glacier 2 were masked by clouds and/or there was limited contrast in the original stereo image pairs, resulting in incorrect elevation data (E. Berthier personal communication, 2016). The cloudy areas appear as a black regions on the DEM mask (not shown) and as distortion in the DEM, as seen in eastern area of Figure 1a.

The second DEM (GES 07-044) spans only part of the Donjek Range, covering most of Glacier 4 and a large portion of Glacier 2 (see Figure 1b). This DEM had no masked areas over Glacier 2 and 4. It was therefore decided to merge the two DEMs to create a cloud free DEM that spanned all three glaciers.

The merging process was complicated by the fact that there was a horizontal and vertical discrepancy between the two DEMs. The second DEM had generally higher elevation values, as can be seen by the difference map shown in Figure 2a. Although the discrepancy was not consistent throughout the study area, the second DEM was generally higher than the first, as can be seen by the overall purple colour in Figure 2a.

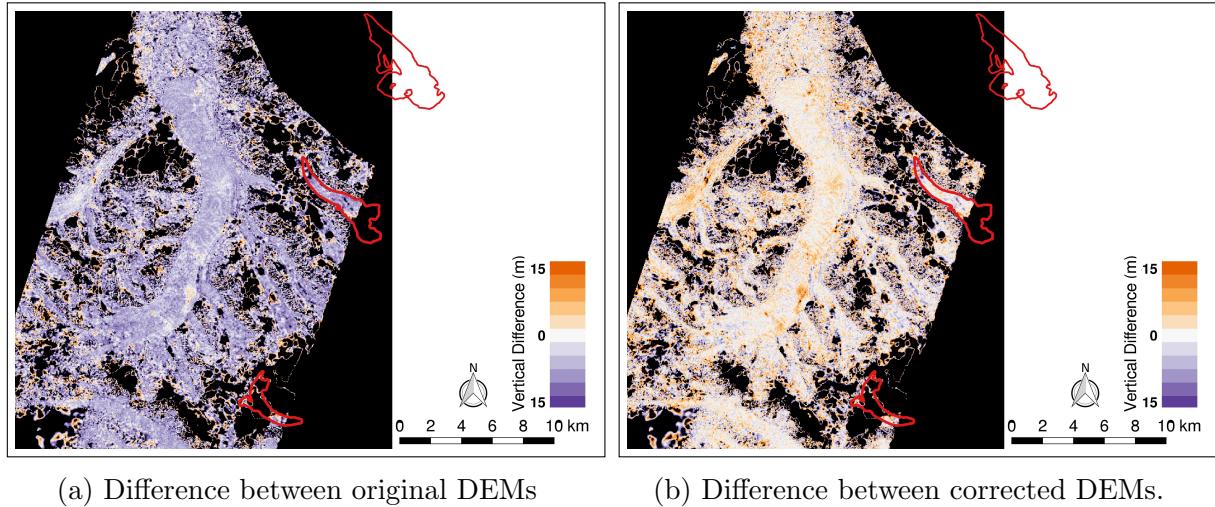


Figure 2: Vertical difference between DEMs in overlapping area. Difference was found by subtracting GES 08-029 from GES 07-044.

The discrepancy was corrected by E. Bertier (2016, personal communication) following an iterative 3D-coregistration algorithm presented by Berthier and others (2007). The GES 07-044 DEM was arbitrarily chosen to be used as the reference DEM. Since there are no accurate ground control points and cells encompass an area with an elevation distribution, there was no way for us to determine whether one DEM was more accurate than the other. Further, the absolute value of the elevation is not necessarily important for the topographic regression, as long as the relative elevations are correct and that they represent the elevation profiles of the glacier. The reference DEM was first shifted vertically by +5.4 m, estimated using ICESat data (Berthier and others, 2010). Then, the mean horizontal and vertical (X, Y, Z) shift between the reference DEM and the GES 08-029 DEM was found by minimizing the standard deviation of the elevation difference between the DEMs. The values of the shift were ~2 m east, ~4 m north, and ~1.9 m vertically. The DEM was then reprojected in the same projection as the reference DEM. The difference map between the two corrected DEMs is shown in figure 2b. Difference values are not uniform but do show both positive and negative values.

Merging of the corrected DEMs was completed in QGIS. First, the rasters were cropped to overlap by a few cells. The crop line was chosen by hand to include as much of the

reference DEM as possible (fewer areas of poor data) but was a relatively small distance from the edge of the DEM (see Figure 3). The second DEM was cropped to follow the same merge line but overlap with the first image by a few cells. This was done to avoid gaps in cell values that arise from cropping across a DEM cell. The merging was completed using the built in QGIS tool ‘Merge’ and in areas where the two DEMs overlapped, the GES 08-029 DEM values were chosen.

The final DEM used for subsequent analysis can be seen in Figure 4. Despite the corrections, there were still discrepancies along the intersection of the DEMs. The contour lines (10 m) that also plotted on this figure show sharp topographic gradients that are aligned with the merge line. However, these discrepancies were not present on the study glaciers and located more than 250 m from the edge of the glacier so this DEM was used as the final version for the Donjek Range. All subsequent topographic analysis and calculations was based on elevation taken from this DEM.

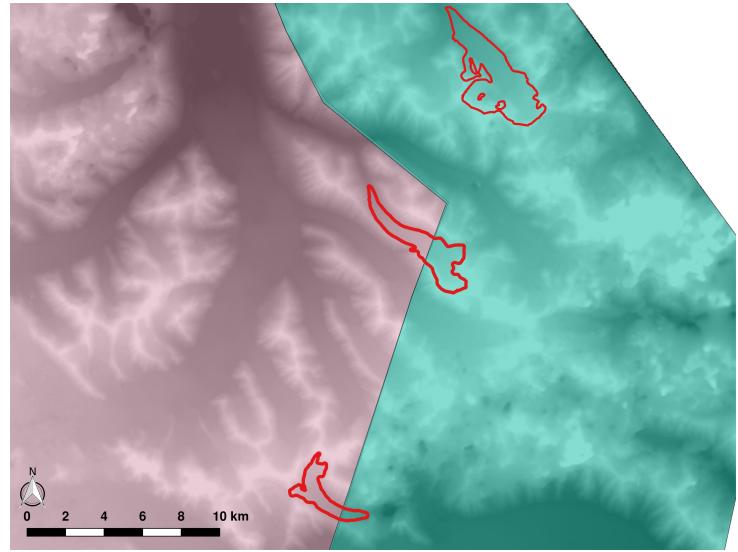


Figure 3: Outlines of the cropped GES 07-044 DEM (pink) and cropped GES 08-029 DEM (blue) used for merging. Note the small overlap of the two DEMs.

1.2 Calculating topographic parameters

Topographic parameters are used to describe local topography that affects snow distribution and act as proxies for physical processes that determine snow deposition and redistribution. Topographic parameters used in snow accumulation studies on glaciers include elevation (z), distance from centreline (d_C), slope (m), tangential (κ_T) and profile (κ_P) curvature, “northness” (N), aspect (α), and Sx , which is a proxy for wind redistribution (Basist and others, 1994; Revuelto and others, 2014; McGrath and others, 2015).

A number of programs were used calculate topographic parameters from the SPOT-5 DEM. Distance from centreline and “northness” were calculated in Matlab. Sx was determined using a executable obtained from Adam Winstral that follows the procedure outlined in Winstral and others (2002). The remaining parameters were calculated using the `r.slope.aspect` module in GRASS GIS software run through QGIS as described in Mitášová and Hofierka (1993) and described in Hofierka and others (2009).

The GRASS module implements a local polynomial approximation method of the form

$$z(x, y) = a \cdot x^2 + b \cdot y^2 + c \cdot x \cdot y + d \cdot x + e \cdot y + f. \quad (1)$$

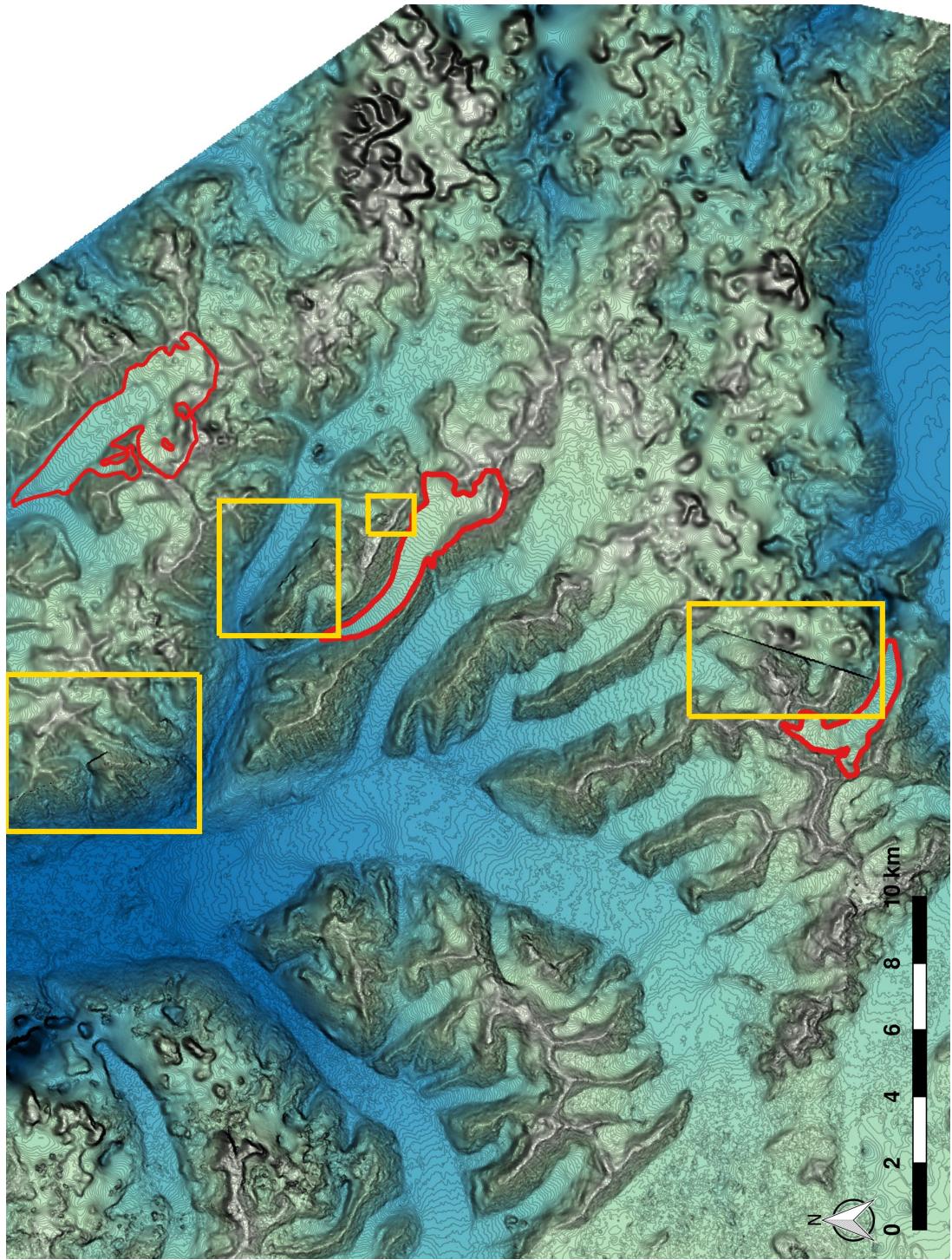


Figure 4: Merged DEM of the Donjek range from two corrected SPOT-5 DEMs with 10 m contour lines. Study glaciers are shown in red. Discrepancies between the DEMs along the merge line can be seen as linear features in the contour lines (appear as sharp topographic boundary) and are emphasized by the yellow boxes. Distorted contours in the eastern regions are a result of errors in the DEM.

This polynomial is fitted to 9 grid points (3×3 array) and the coefficients are determined by weighted least squares. First order partial derivatives are derived using Horn's formula (Horn, 1981; Neteler and Mitášová, 2013). The gradient can be expressed as

$$\nabla \bar{z} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right) = (2 \cdot a \cdot x + c \cdot y + d, 2 \cdot b \cdot y + c \cdot x + e). \quad (2)$$

Since we are interested in the value of the centre cell ($x = y = 0$), the gradient is simply

$$\nabla \bar{z} = (f_x, f_y) = (d, e), \quad (3)$$

where d and e are given by

$$f_x = \frac{(z_7 - z_9) + (2z_4 - 2z_6) + (z_1 - z_3)}{8 \cdot \Delta x}, \quad (4)$$

$$f_y = \frac{(z_7 - z_1) + (2z_8 - 2z_2) + (z_9 - z_3)}{8 \cdot \Delta y}. \quad (5)$$

Here, z_k refers to one of the grid cells surrounding the cell of interest, which is located at row i and column j of the DEM. So $z_3 = z_{i+1,j+1}$, $z_7 = z_{i-1,j-1}$, and so on. Δx and Δy are the grid spacing (resolution) of the DEM. The second order partial derivatives can be written as (Hofierka and others, 2009; Neteler and Mitášová, 2013)

$$f_{xx} = \frac{z_1 - 2z_2 + z_3 + 4z_4 - 8z_5 + 4z_6 + z_7 - 2z_8 + z_9}{6 \cdot (\Delta x)^2}, \quad (6)$$

$$f_{yy} = \frac{z_1 - 4z_2 + z_3 - 2z_4 - 8z_5 - 2z_6 + z_7 + 4z_8 + z_9}{6 \cdot (\Delta y)^2}, \quad (7)$$

$$f_{xy} = \frac{(z_7 - z_9) - (z_1 - z_3)}{4 \cdot \Delta x \Delta y}. \quad (8)$$

The second derivatives are most commonly used to calculate the curvature of a surface. The curvature at a given point on a curve is determined by finding the inverse of the radius of a circle that best fits the curve (Olaya, 2009). Convex surfaces have positive curvature and concave surfaces have negative curvature.

Details about the calculation of topographic parameters are described below.

Elevation values were taken from the (corrected) SPOT-5 DEMs directly.

Distance from centreline was calculated as the minimum distance between the Easting and Northing of the northwest corner of each cell and a centreline that was drawn by hand in QGIS. This was completed in Matlab using the script 'CentrelineDistance.m'.

Slope is taken to be the steepest slope angle and is the magnitude of gradient (Mitášová and Hofierka, 1993). Using the gradient, it can be expressed as (Olaya, 2009)

$$m = \arctan(|\nabla \bar{z}|) = \arctan \left(\sqrt{(f_x)^2 + (f_y)^2} \right) \quad (9)$$

Tangential Curvature (horizontal curvature) represents the curvature in the direction of the contour tangent. The equation for tangential curvature can be written as (Olaya, 2009)

$$\kappa_T = -\frac{e^2 \cdot r - 2 \cdot d \cdot e \cdot s + d^2 \cdot t}{(d^2 + e^2) \cdot \sqrt{1 + d^2 + e^2}} \quad (10)$$

where d and e are as above and

$$r = \frac{\partial^2 z}{\partial x^2} = \frac{z_1 + z_3 + z_4 + z_6 + z_7 + z_9 - 2 \cdot (z_2 + z_5 + z_8)}{3 \cdot \Delta s^2} \quad (11)$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{z_3 + z_7 - z_1 - z_9}{4 \cdot \Delta s^2} \quad (12)$$

$$t = \frac{\partial^2 z}{\partial y^2} = \frac{z_1 + z_2 + z_3 + z_7 + z_8 + z_9 - 2 \cdot (z_4 + z_5 + z_6)}{3 \cdot \Delta s^2} \quad (13)$$

Profile Curvature (vertical curvature) represents the curvature in the direction of the steepest slope (gradient). The equation for profile curvature can be written as (Olaya, 2009)

$$\kappa_P = -\frac{d^2 \cdot r + 2 \cdot d \cdot e \cdot r \cdot s + e^2 \cdot t}{(d^2 + e^2) \cdot \sqrt{(1 + d^2 + e^2)^3}} \quad (14)$$

“Northness” is a solar radiation parameter that has been shown to become increasingly important during the spring (Revuelto and others, 2014). It is also likely that this parameter may be related to sun induced snow metamorphosis and/or sun crusts, both of which affect SWE and snow redistribution (McGrath and others, 2015). “Northness” is defined as the product of the cosine of aspect and sine of slope (Molotch and others, 2005). A value of -1 represents a steep, south facing slope, a value of +1 represents a steep, north facing slope, and a flat surfaces yield 0.

Aspect represents the orientation of the steepest slope, with 0° defined as north and no value given to cells that have no gradient. Using the gradient, it can be expressed as (Olaya, 2009)

$$\alpha = 180 - \arctan\left(\frac{f_y}{f_x}\right) + 90 \cdot \frac{f_x}{|f_x|} \quad (15)$$

Sx represents wind exposure/shelter and is based on selecting a cell within a certain angle and distance from the cell of interest that has the greatest upward slope relative to the cell of interest (Winstral and others, 2002). This can be referred to as the maximum upwind slope. Negative Sx values represent exposure relative to the shelter-defining pixel, which means that the cell of interest was higher than the cell with greatest upward slope. Conversely, positive values represent sheltered cells. To determine Sx values, we use the equation

$$Sx_{A,dmax}(x_i, y_i) = \max \left[\tan^{-1} \left(\frac{z(x_v, y_v) - z(x_i, y_i)}{[(x_v - x_i)^2 + (y_v - y_i)^2]^{1/2}} \right) \right], \quad (16)$$

where A is the azimuth of the search direction, (x_i, y_i) are the coordinates of the cell of interest, and (x_v, y_v) are the set of all cell coordinates located along the search vector defined by (x_i, y_i) , the azimuth (A), and maximum search distance (d_{\max}). Code for this calculation was provided by Adam Winstral (2016, personal communication). As done by McGrath and others (2015), we computed S_x at 5° azimuth increments for d_{\max} distances of 100, 200, and 300 m. These values were then correlated with observed values of SWE and the azimuth and d_{\max} values with the highest correlation were used for subsequent analysis (see Table 1). The code for calculating S_x requires a UTM raster formatted to ASCII in ArcGIS.

Table 1: Values of azimuth (A) and maximum search distance(d_{\max}), which define the S_x that had the highest correlation to observed SWE.

	A (° from North)	d_{\max} (m)	Correlation Coefficient
Glacier 4	85	300	-0.26
Glacier 2	330	300	0.56
Glacier 13	280	200	0.28

1.3 Topographic parameters from QGIS to Matlab

The values of each topographic parameter at the sampling locations was determined in QGIS. The sampling locations were imported to QGIS and the Point Sampling Tool was used to determine the value of the cell of the raster for each topographic parameter at that location. The set of parameters that corresponded to each location was then exported to a .csv file and imported into Matlab with the script 'Import_Topo.m'. Note that selection of S_x values was completed first (see above) and only the S_x data with highest correlation to SWE was exported. After importing to Matlab, the sets of parameter values (x_i) were standardized (x_s) by $x_s = \frac{x_i - \mu}{\sigma}$. The resulting structure is called `topo_sampled` and is used in the MLR to be able to compare the explanatory power of each parameter (Section 2.3). A non-standardized copy of the topographic parameters at the sampling locations was also kept for plotting purposes and is called `topo_sampled_ns`. Both structures are organized as a vector of values corresponding to the vectors in the `SWE` variable.

The raster data for each topographic parameter was also exported from QGIS as a .csv file and then imported into Matlab with the script 'Import_Topo.m'. The values are stored in the structure `topo_full` and are a matrix, where each cell corresponds to one DEM cell. Cells outside of the glacier outline have no value (`NaN`). These values were standardized using the mean and standard deviation of the sampled topographic parameters so that this set of full topographic parameters could be used for modelling SWE using the regression. A copy of the non-standardized values was stored as `topo_full_ns`.

1.4 Parameter correlation

The correlation between topographic parameters at sampling locations on each glacier is shown in Table 2. Correlation values were generally low, with the exception of the correlation between northness and aspect on Glacier 2 and northness and Sx on Glacier 13, which were both larger than 0.7. Since there is little correlation between parameters and the correlations vary between glaciers, the use of a linear regression with these topographic parameters as predictor variables is warranted.

Table 2: Pearson correlation coefficients between topographic parameters at sampled locations. Topographic parameters include distance from centreline (d_C), elevation (z), aspect (α), slope (m), northness (N), profile curvature (κ_P), tangent curvature (κ_T), and wind redistribution (Sx).

	d_C	z	α	m	N	κ_P	κ_T	Sx	
Glacier 4	d_C	1	0.2	-0.28	0.36	0.11	-0.19	-0.13	0.37
	z	0	1	0.27	-0.11	-0.35	0.13	0.04	0.36
	α	0	0	1	-0.29	-0.24	0.19	0.07	0.22
	m	0	0	0	1	0.55	-0.36	-0.09	0.03
	N	0	0	0	0	1	-0.16	-0.03	-0.47
	κ_P	0	0	0	0	0	1	0.41	-0.18
	κ_T	0	0	0	0	0	0	1	-0.28
	Sx	0	0	0	0	0	0	0	1
	d_C	1	0.05	-0.22	0.11	0.24	0.07	0.2	-0.01
	z	0	1	0.56	-0.49	-0.32	0.08	0.03	0.58
	α	0	0	1	-0.39	-0.72	0.19	0.04	0.49
Glacier 2	m	0	0	0	1	0.04	-0.03	-0.02	-0.37
	N	0	0	0	0	1	-0.19	-0.03	-0.11
	κ_P	0	0	0	0	0	1	0.44	-0.23
	κ_T	0	0	0	0	0	0	1	-0.27
	Sx	0	0	0	0	0	0	0	1
Glacier 13	d_C	1	0.13	-0.09	0.06	0.01	0.1	0.03	0.05
	z	0	1	-0.11	-0.01	0.3	0.06	-0	0.31
	α	0	0	1	0.01	-0.59	-0.01	-0.13	-0.47
	m	0	0	0	1	-0.26	0	-0.1	-0
	N	0	0	0	0	1	-0.07	-0.11	0.76
	κ_P	0	0	0	0	0	1	0.36	-0.3
	κ_T	0	0	0	0	0	0	1	-0.47
	Sx	0	0	0	0	0	0	0	1

1.5 Maps of topographic parameters and range of parameters sampled

Maps of the topographic parameters for each glacier are shown below. Additionally, the distribution of topographic parameter values and the sampled distribution for each parameter is also shown. The full range of parameters has highest frequencies for Glacier 13 and lowest for Glacier 4 because Glacier 13 is larger than Glacier 4. Frequency of sampled parameters are comparable between glaciers because the sampling schemes were similar.

Elevation maps in Figure 5 show higher elevations at the head of the glacier and lowest elevation at the terminus for all glaciers. Maximum elevation values are lower for Glacier 4 than the other two glaciers. The elevation distribution differs between glaciers. Glacier 4 and 13 have a larger number of mid-elevations (broader distribution) than Glacier 2 (sharp peaked distribution). The sampled distribution for all glaciers is markedly different than the full distribution as seen in Figure 6. In general, the distribution mean is lower and does not sample the full range of values. This is especially evident for Glacier 13 where a large portion of the higher elevations were not sampled. Unfortunately, high elevation locations are often dangerous to access and could not be sampled.

Distance from centreline is shown in Figure 7. Note that Glacier 13 has two centrelines in the accumulation area because of the confluence of two major arms of the glacier. The sampled distribution generally matches the full distribution. This is likely because the hourglass sampling pattern allowed for locations across the whole width of the glacier to be measured, although this is generally limited to the ablation area. However, the sampling did not allow for the locations at a far distance from the centreline to be measured because these locations are generally in steep, inaccessible terrain.

Slope of the three study glaciers is shown in Figure 9. The majority of the area of the three glaciers has a slope of less than 20° , with only the margins of the accumulation area and a few steps on Glacier 13 having steep slopes. The distribution of slope is similar between glaciers (see Figure 10). All have a peak frequency around 10° and few areas with very steep slopes. The sampled distribution matches the full distribution near this peak but severely under samples steep slope due to dangerous conditions.

Tangential curvature is shown in Figure 11 and profile curvature is shown in Figure 13. The curvature values are generally moderate and centred about 0. The distribution of sampled curvature closely matches the full distribution although high curvature areas were not sampled (see Figure 12 and 14).

Aspect of the glacier surface is shown in Figure 15. Glacier 2 and 13 have similar aspect distributions, with a mean aspect of $\sim 100^\circ$. Glacier 4 has a mean aspect of $\sim 300^\circ$, which is roughly 180° to that of Glacier 2 and 13. The sampled distribution, as shown in Figure 16, for Glacier 4 follows that of the full distribution relatively well. However, the sampled distribution of Glacier 2 and 13 does not match that of the full distribution and there are many aspects that were not sampled.

“Northness” for all study glaciers is shown in Figure 17. The majority of cells have values close to zero, which is likely due to their low slope values. Even for Glacier 4, which is largely south facing and should thus have lower values of “northness”, has a

distribution with a mean close to zero. The low slope values mean that the values of “northness” were determined largely by the aspect, which can be seen by the resemblance between the “northness” map and the aspect map.

Sx maps over the study glaciers are shown in Figure 19 and the wind direction and maximum search distance with the highest correlation to SWE for each glacier are shown in Table 1. For Glacier 4, an approximately east wind and 300 m search distance were most strongly correlated. The correlation was negative, which means that negative values of Sx (exposure) correspond to areas with larger SWE (more snow). This is counter intuitive and perhaps indicates that Sx is not an appropriate topographic parameter to correlate with SWE on Glacier 4. Despite this, Sx was retained in future analysis for consistency between glaciers. For Glacier 2, a north wind with a 300 m search distance was most strongly correlated and for for Glacier 13, a west wind with a 200 m search distance exhibited the strongest correlation. Both of these correlations were positive, so a more positive Sx value (sheltered) corresponded to higher values of SWE (more snow). The correlation values for Glacier 4 and 2 were low (<0.3), which indicates that Sx will likely be an insignificant in modelling snow accumulation. The correlation between Sx and SWE on Glacier 2 is relatively high (0.56).

Full distribution of Sx , as seen in Figure 20, is different for each glacier. Glacier 2 has a mean less than zero, indicating that a large portion of the glacier has exposed topography. Glacier 4 and 13 have positive mean values of Sx , indicating more sheltered topography. The sampled distribution for Glacier 4 and 2 generally follows that of the full distribution, although extreme values are not adequately sampled. For Glacier 13, the sample distribution does not closely match that of the full distribution, particularly for larger, more positive, values of Sx .

Overall, the sampled topographic parameters are representative of the full distribution of parameters. Extreme values are grossly under sampled so that the full range of parameter values was not sampled. This was largely do to dangerous travel conditions and an inability to quickly and accurately measure snow depth in the accumulation area. As a result, extrapolation of regression models will likely result in large errors. This is especially relevant in the accumulation area, which has extreme values for all parameters. Errors in the accumulation area are especially important to acknowledge because this area has the most accumulation and is likely to heavily influence final winter mass balance values. It is also important to note that for a number of parameters (especially elevation) the sampling distribution is off-set from that of the full distribution. Generally, the sampled values do not fully capture the variance in topographic parameters but a regression is still valuable to conduct since topographic regressions are common when estimating winter mass balance. Acknowledging that this methodology is limited and prone to extrapolation errors is paramount to capturing uncertainty in accumulation studies.

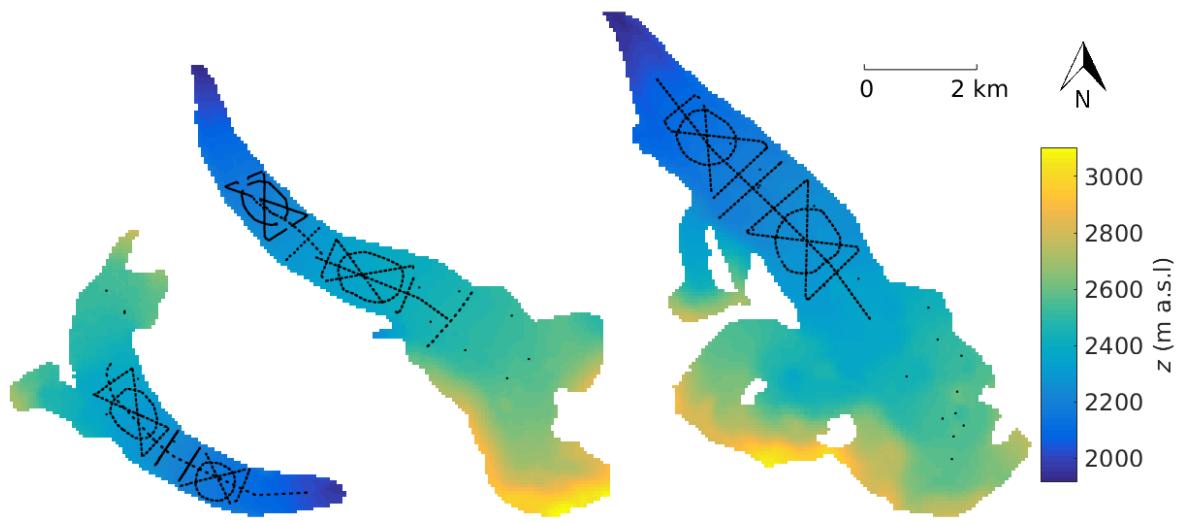


Figure 5: Values of elevation used in the topographic regressions for three study glaciers. This DEM is derived from a SPOT5 satellite image and has a grid size of 40x40 m. Subsequent topographic parameters were derived from this DEM.

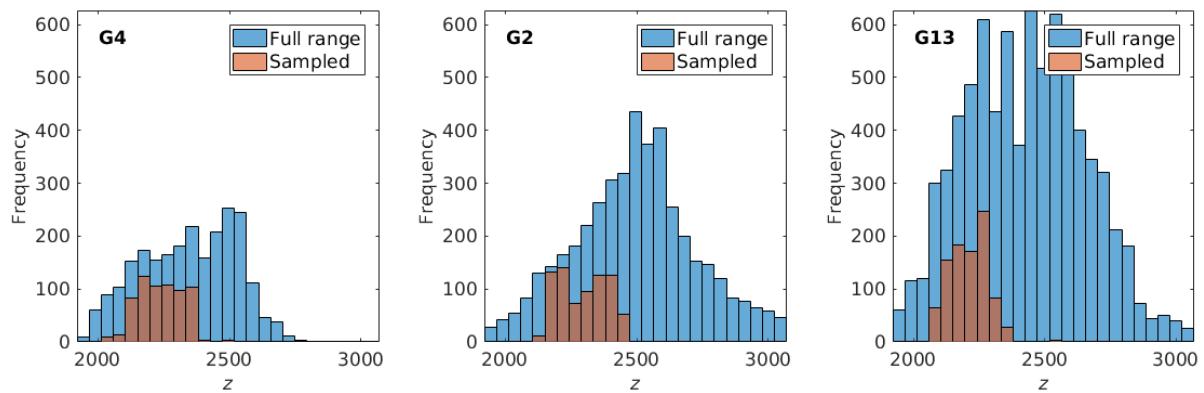


Figure 6: Range of elevation sampled as compared to total range of elevation of study glaciers.

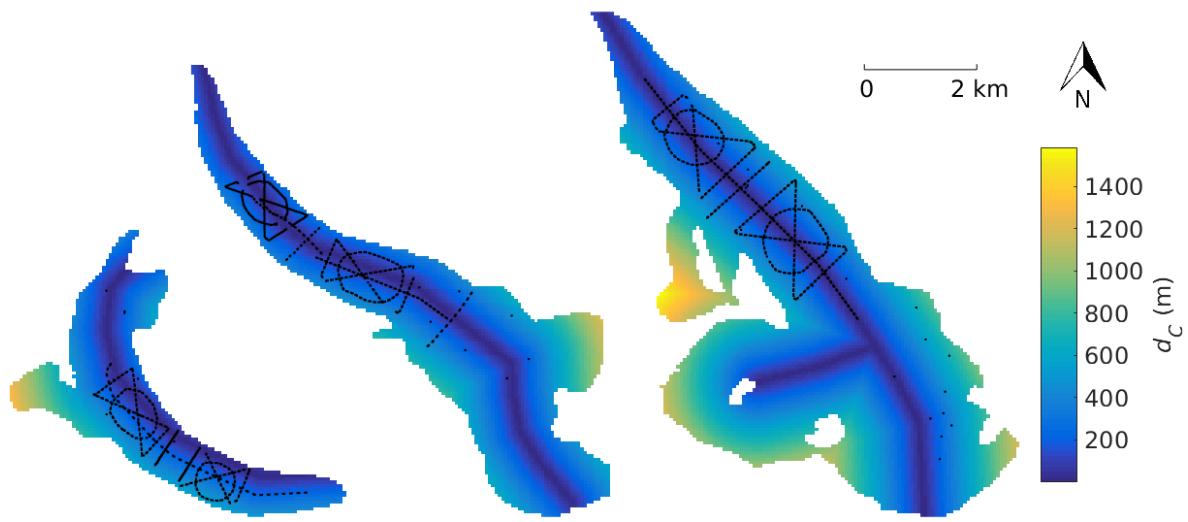


Figure 7: Values of distance from centreline used in the topographic regressions for three study glaciers. Centreline was drawn by hand in QGIS.

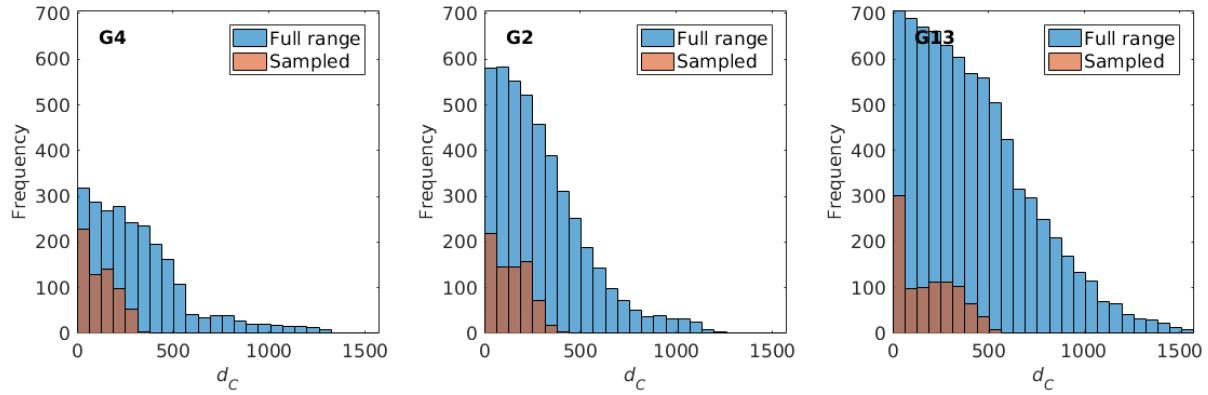


Figure 8: Range of distance from centreline sampled as compared to total range of distance from centreline of study glaciers.

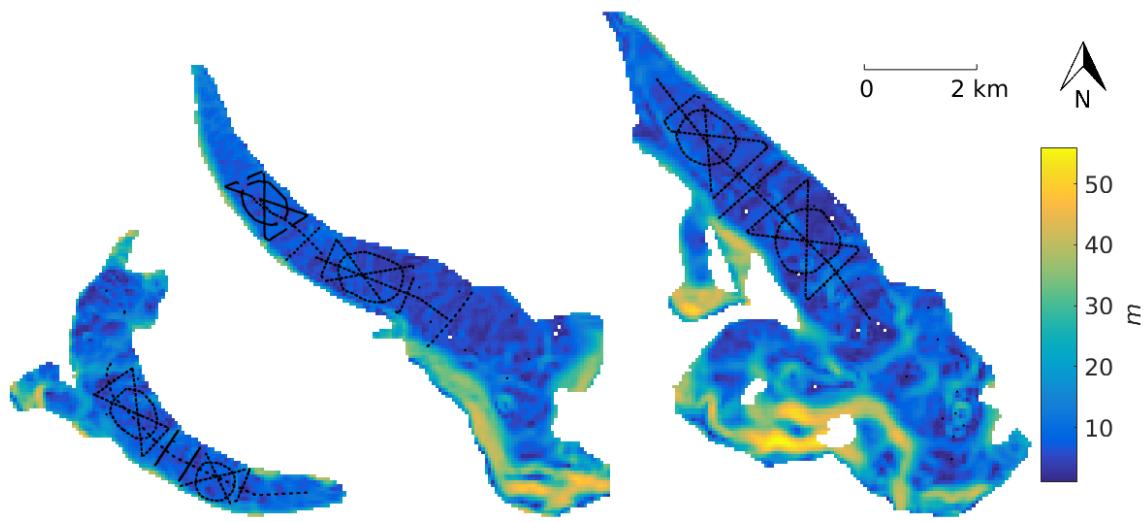


Figure 9: Values of slope used in the topographic regressions for three study glaciers. Values were derived from a SPOT5 satellite derived DEM (grid size of 40x40 m) in QGIS.

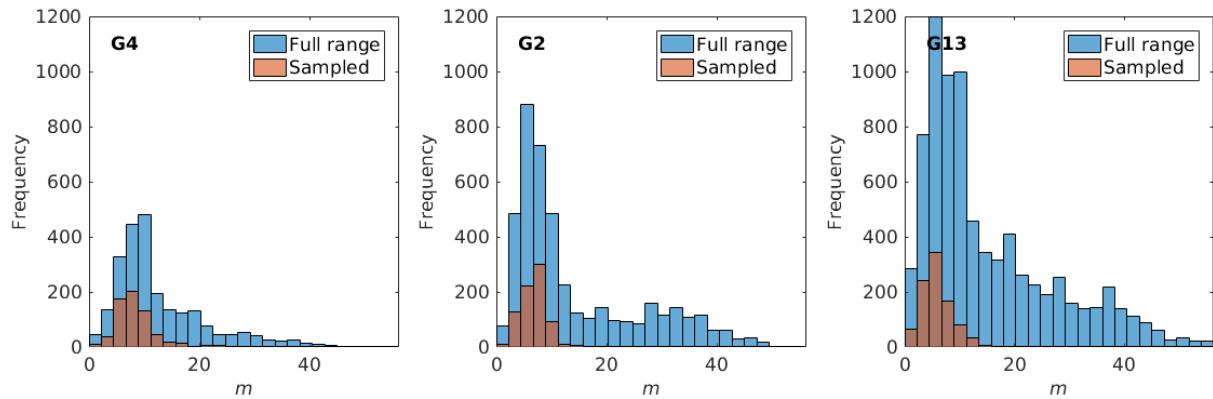


Figure 10: Range of slope sampled as compared to total range of slope of study glaciers.

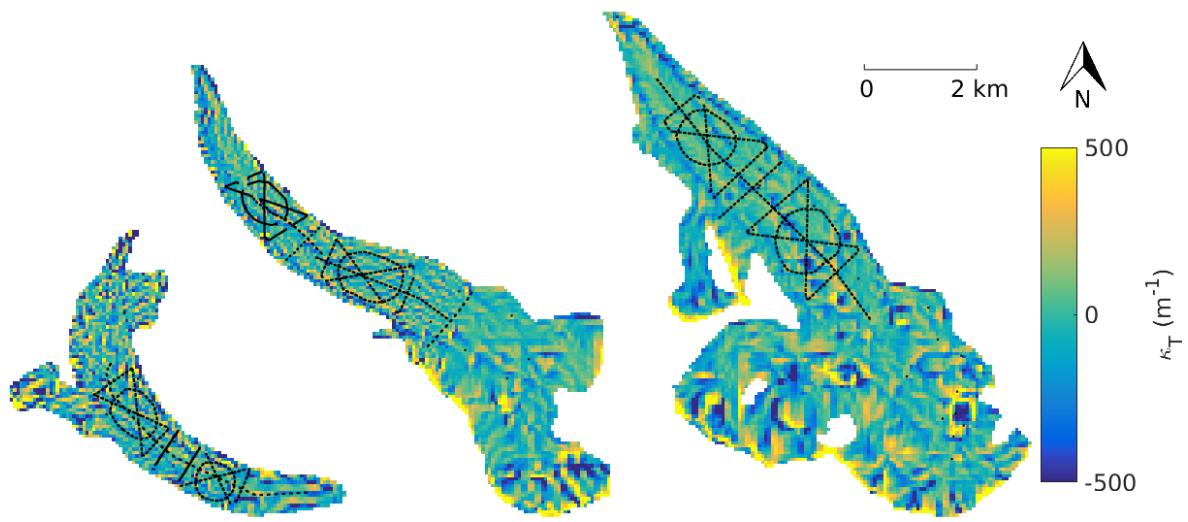


Figure 11: Values of tangential curvature used in the topographic regressions for three study glaciers. Values were derived from a SPOT5 satellite derived DEM (grid size of 40x40 m) in QGIS. Colour axis has been scaled to better resolve values close to zero.

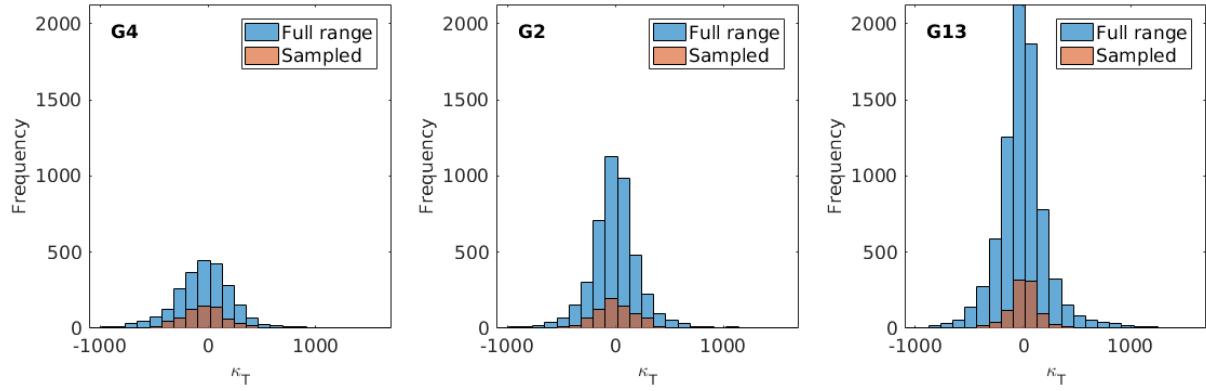


Figure 12: Range of tangential curvature sampled as compared to total range of tangential curvature of study glaciers.

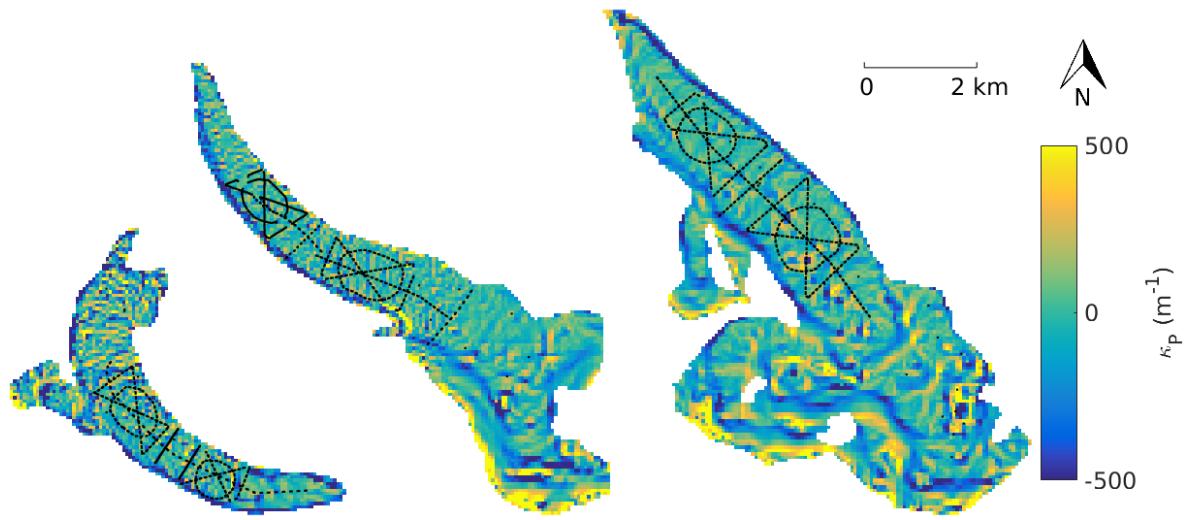


Figure 13: Values of distance from centreline used in the topographic regressions for three study glaciers. Values were derived from a SPOT5 satellite derived DEM (grid size of 40x40 m) in QGIS. Colour axis has been scaled to better resolve values close to zero.

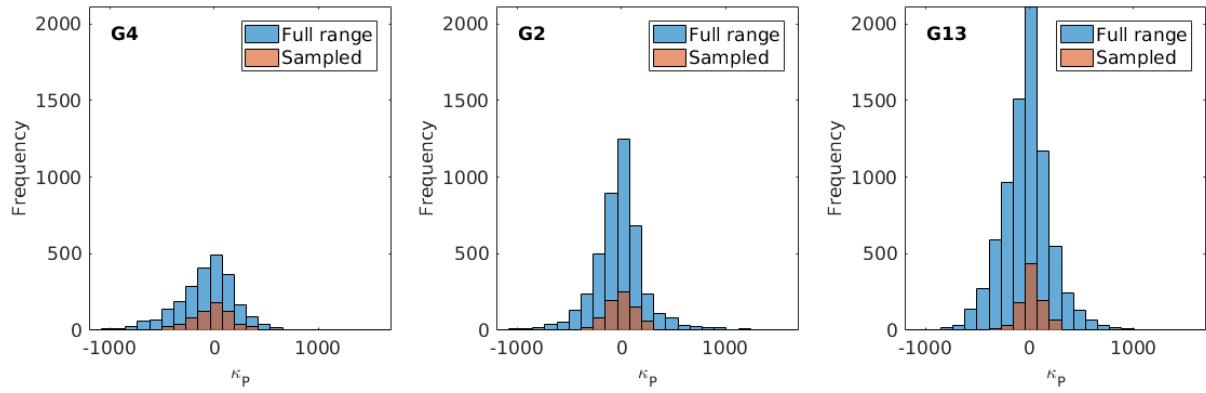


Figure 14: Range of profile curvature sampled as compared to total range of profile curvature of study glaciers.

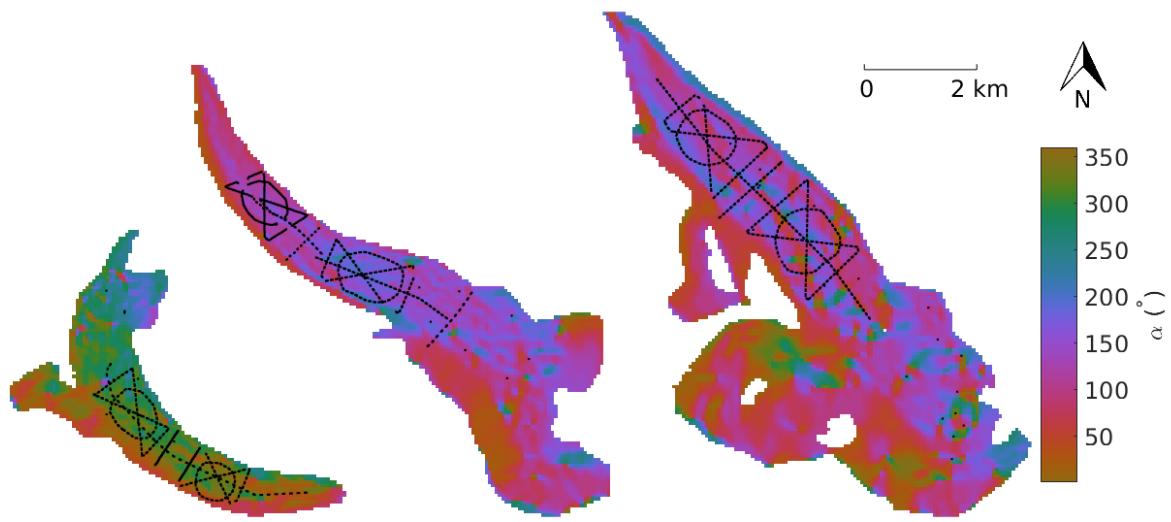


Figure 15: Values of aspect, which represent what direction a slope is facing (0° defined as north), used in the topographic regressions for three study glaciers.

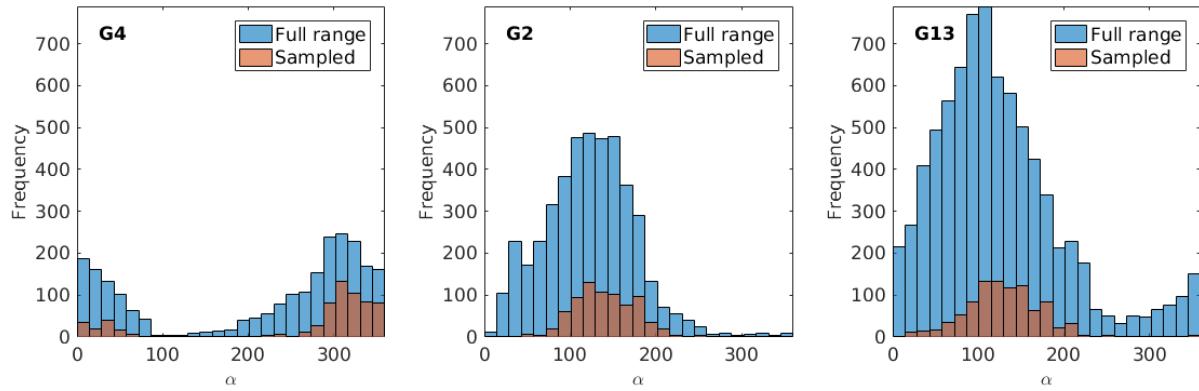


Figure 16: Range of aspect sampled as compared to total range of aspect of study glaciers.

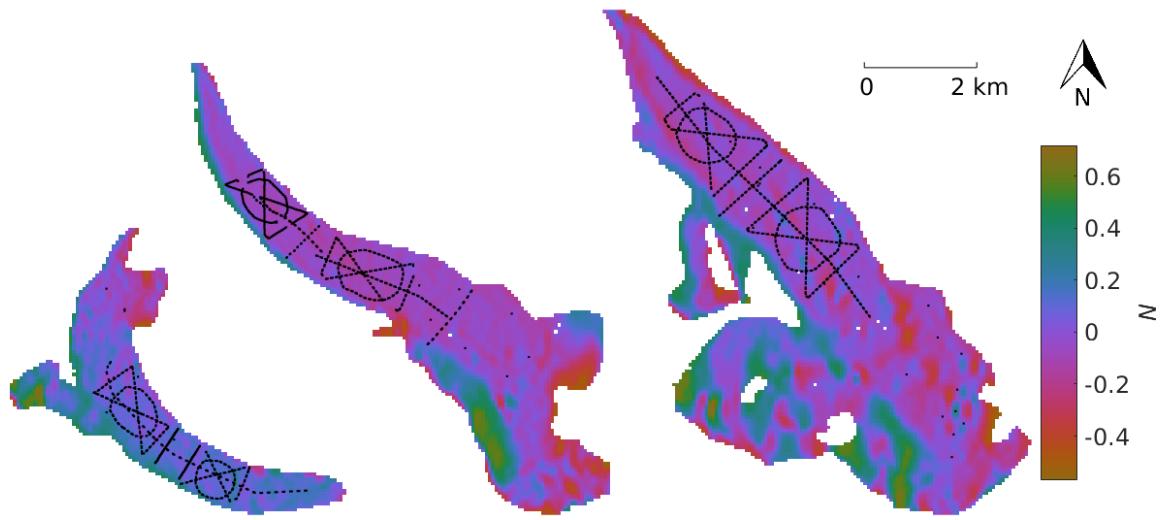


Figure 17: Values of “northness” used in the topographic regressions for three study glaciers. “Northness” is defined as the product of the cosine of aspect and sine of slope. A value of -1 represents a steep, south facing slope, a value of +1 represents a steep, north facing slope, and flat surfaces yield 0. Values were derived from a SPOT5 satellite derived DEM (grid size of 40x40 m) in QGIS.

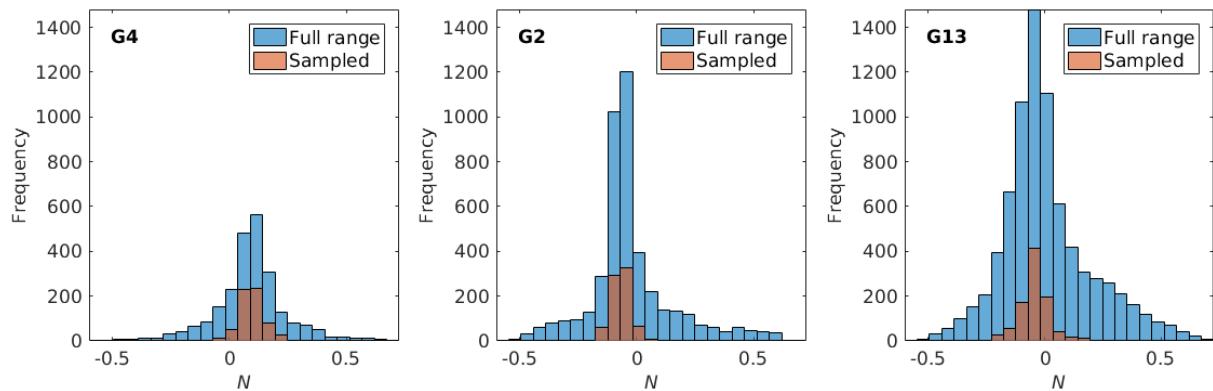


Figure 18: Range of “northness” sampled as compared to total range of “northness” of study glaciers.

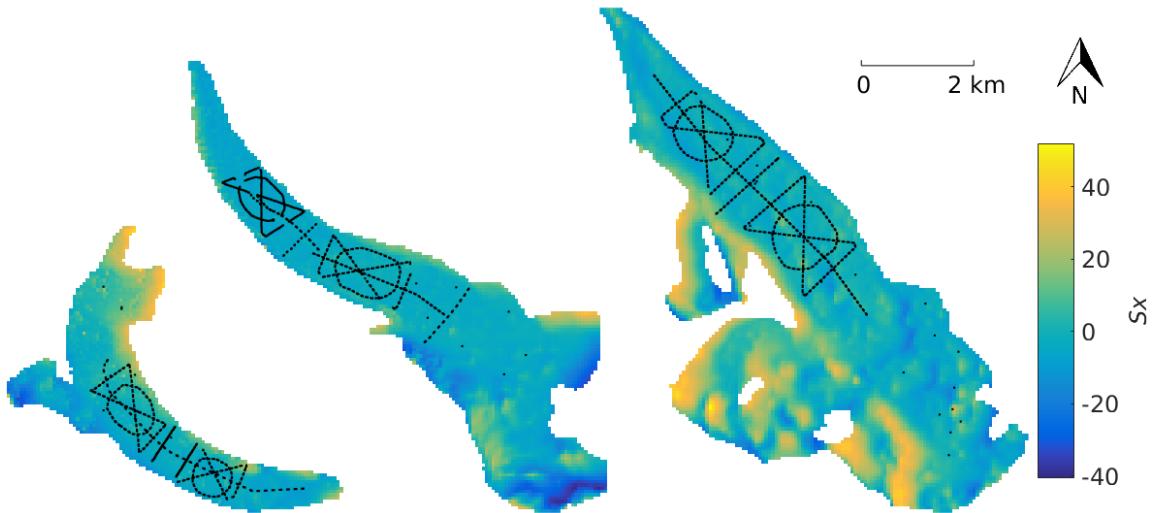


Figure 19: Values of S_x , which is a wind redistribution parameter, used in the topographic regressions for three study glaciers. See section 1.2 and the original paper by Winstral and others (2002) for more details on calculation. See Table 1 for values of best correlated azimuth and maximum search distance for each glacier.

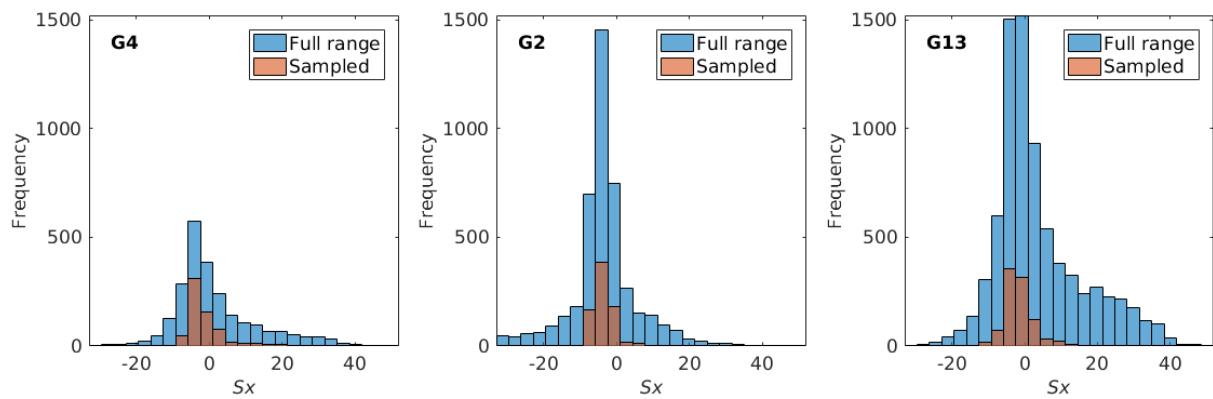


Figure 20: Range of S_x sampled as compared to total range of S_x of study glaciers.

2 Linear Regressions

2.1 Background

Relating snow accumulation and terrain parameters to better predict accumulation within a basin has been employed for decades (e.g. Woo and Marsh, 1978; Molotch and others, 2005; McGrath and others, 2015). The most common type of relation between topographic parameters and accumulation is a linear regression, where the observed snow

water equivalent (SWE) is related to a linear combination of topographic parameters at each measurement location.

A linear regression takes the form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (17)$$

where the matrix \mathbf{X} contains the set of independent regressors \mathbf{x}_i used to explain the dependant variable \mathbf{y} (Davis and Sampson, 1986). The regression coefficients for each regressor is given by $\boldsymbol{\beta}$ and the error or noise of the system is given by $\boldsymbol{\varepsilon}$. Applied to this study, the matrix of independent regressors (\mathbf{X}) contains the topographic parameters at the sampling locations, the dependant variable \mathbf{y} contains the observed SWE, and the $\boldsymbol{\beta}$ values are determined using a fitting model. While there are many types of fitting models, the ones employed in this study are multiple linear regression (MLR) and Bayesian model averaging (BMA).

Once $\boldsymbol{\beta}$ values have been estimated, they can then be used to predict values of the dependant variable in other locations where regressors are known (Davis and Sampson, 1986). Known values of topographic parameters can be multiplied by their respective $\boldsymbol{\beta}$ coefficients and added together to obtain the modelled or predicted value of SWE for that DEM grid cell.

2.2 Density options resulting in various SWE values

Within this analysis, a range of SWE values is used in the topographic regression. These SWE values arise from different choices in density interpolation. Table 3 summarizes these options.

Table 3: Description of density interpolation methods used to calculate SWE used in the topographic regression.

	Density Values Used	Interpolation Method
	Snow pit Federal Sampler	
Option 1	■	Range mean
Option 2		(all glaciers)
Option 3	■	
Option 4		Glacier mean
Option 5	■	Linear elevation regression
Option 6		(individual glacier)
Option 7	■	Inverse distance weighted
Option 8		mean (all glaciers)

2.3 Multiple Linear Regression

2.3.1 Background

Perhaps the most basic and well used method for relating SWE and topographic parameters is a multiple linear regression (MLR) (e.g. Cohen and others, 2013). The best fit line of an MLR is the one described by coefficients that minimize the sum of squares of the vertical deviations of each data point from the line (Davis and Sampson, 1986). Note that if a point falls on the line then the deviation is zero and that the positive and negative deviations from the line do not cancel because the values are first squared and then summed. The residuals are simply the difference between the modelled and observed data.

To reduce the chance of over fitting the data, two additional methods were applied. The first method was model averaging over all possible models that allow for linear combinations of the chosen topographic parameters. An MLR was estimated for each model (total number of models is 2^n , where n is the number of topographic parameters) and the Bayesian information criterion (BIC) value was computed for the model.

BIC is found according to

$$\text{BIC} = -2 \ln L(\hat{\theta}_k | y) + k \ln(n), \quad (18)$$

where the values of $\hat{\theta}_k$, which are the model parameters, maximizes the likelihood function for data y (Burnham and Anderson, 2004). The number of data points is n and the number of regressors is k . BIC values are used to assess the relative predictive success of models while penalizing for overfitting of data. While the absolute BIC value is meaningless, models can be selected or averaged using the relative BIC value, with lower values indicating a better model (Burnham and Anderson, 2004).

In this study, there was no reason to favour any of the models so a weighted average of all models based on their BIC values was conducted. The BIC values for each model (BIC_i) were used to determine the normalized weight of each model (w_i) relative to the best model (lowest BIC value indicated by BIC_{min}) according to (Burnham and Anderson, 2004)

$$w_i = \frac{e^{-0.5(BIC_i - BIC_{min})}}{\sum_{i=1}^R e^{-0.5(BIC_i - BIC_{min})}}. \quad (19)$$

When parameters were excluded from a model, a coefficient of zero was assigned to that parameter. The sum of the weighted coefficients gave the final β values.

The second method used to prevent over fitting was to calculate the MLRs within a cross validation framework. This meant that for each model, a randomly selected portion of the data was used to estimate regression coefficients and these were used to predict values that correspond to the remaining data (Kohavi and others, 1995). The root mean squared error (RMSE) between the modelled and observed data was then calculated. This process was done one thousand times and the set of coefficients that resulted in the lowest RMSE were then chosen for that model.

Together, cross validation allowed for reputable choosing of regression coefficients for each possible model and model averaging with BIC weights allowed for stability of final

coefficient values and favouring of simpler models, which would help to identify main topographic drivers of snow accumulation.

2.3.2 Methods

A multiple linear regression (MLR) between snow water equivalent (SWE) and all linear combinations of the calculated topographic parameters was completed. This methodology for calculating the MLR also involved using cross validation, where a portion of the data is withheld from the MLR fit and these points are then used to assess the ability of the MLR to predict values. The MLR fit with the best prediction ability is then carried forward.

The MLR was completed with the following steps (executed using the function 'MLRcalval.m'):

1. The topographic parameters
2. The `topo_sampled` structure for one glacier as well as the `SWE` structure is passed into the function.
3. A set of initializations is completed. This includes 1) creating a logical matrix to choose all linear combination of topographic parameters, 2) selecting the number of runs, 3) creating a matrix of random numbers for selecting data point in the cross validation procedure, 4) initializing matrices, and 5) converting the input structure to a table.
4. For each linear combination of topographic parameters, one thousand runs of a cross validation MLR are then executed. Two-thirds of the total data is randomly selected (Kohavi and others, 1995) to use for calculating the regression (using the function `regress()` which is a basic regression function with fast execution). The MLR equations is used to predict the SWE using the remaining one-quarter of the topographic parameters. The root-mean-squared error (RMSE) between the predicted and observed SWE values is then calculated and the MLR equation with the lowest RMSE is then chosen for that combination of topographic parameters. The function `fitlm()` is then used to calculate the MLR from the set of data that gave the lowest RMSE. This function is slower but calculates a number of additional values that characterize the fit of the model. One of these values is the Bayesian information criterion (BIC), which allows for model selection among a finite set of models (Burnham and Anderson, 2004). The BIC from the best model for each combination of topographic parameters is saved.
5. A weighted sum of all models found using linear combinations of topographic parameters was then found. The BIC values for each model (BIC_i) were used to determine the normalized weight of each model (w_i) relative to the best model (lowest BIC value BIC_{min}) according to Eq. 19.
6. The percent variance ($var\%$) explained by each parameter was calculated using the equation $var\% = \frac{SSr}{SSt} \times 100$, where SSr is the sum of squares of the residual (fitted topographic parameters) and SSt is the total sum of squares (SWE observations).

The final coefficients and the percent variance explained by each one can be found in the `coeffs_final` table within the function and in the `mlr` structure when run for all density options and glaciers in the main script 'TopoRegression.m'.

7. The residuals of the fit have also been calculated as a separate variable than can be returned when the function is called. The residual was calculated as the difference between the predicted and observed value.

2.3.3 Results

The coefficients determined using MLR for density Option 7 can be seen in Table 4 and can be used to examine the relative importance of topographic parameters in the regression. These coefficients were found for standardized values of topographic parameters so their magnitude is indicative of their importance in predicting SWE. Table 6 shows coefficient values for all density options. The range of percent variance explained by each variable (from all density options) can be seen in Figure 27. The fit between modelled SWE and observed SWE for each glacier for density Option 7 can be seen in Figure 21 and the fit for all density options in Figure 22. The distribution of residuals in then plotted in Figure 23. A map of the predicted values of SWE over the entire glacier using density option 7 can be seen in Figure 24 with basic summary statistics found in Table 5. Modelled SWE using all density options is shown in Figure 28. A map of the difference between maximum and minimum modelled SWE values is shown in Figure 25.

For Glacier 4, the largest coefficients are wind redistribution and slope, which are an order of magnitude larger than the remaining coefficients. Both coefficients are negative, which indicates less snow in 'sheltered' areas and less on steeper slopes. These coefficients account for the largest percentage of variance explained, although this percentage is small ($\sim 7\%$ and $\sim 1\%$, respectively). Overall, the modelled winter balance for Glacier 4 is very poorly explained by the topographic parameters, which results in no correlation between

Table 4: MLR coefficients for topographic regression between measured SWE (found using inverse-distance weighted snowpit-derived densities) and standardized topographic parameters. Topographic parameters include distance from centreline (d_C), elevation (z), aspect (α), slope (m), northness (N), profile curvature (κ_P), tangent curvature (κ_T), and wind redistribution (Sx). Since parameters are standardized, the magnitude of the coefficients is representative of their importance in predicting SWE. The root-mean-squared error (RMSE) between modelled SWE using those coefficients and observed SWE is also given.

	Glacier 4	Glacier 2	Glacier 13
d_C	<-0.001	0.019	0.013
z	0.009	0.098	0.047
α	0.001	-0.002	-0.004
m	-0.021	0.013	<-0.001
N	<-0.001	0.001	0.002
κ_P	-0.002	<0.001	-0.001
κ_T	<-0.001	0.005	-0.001
Sx	-0.045	0.041	0.012
Intercept	0.625	0.267	0.241
RMSE	0.147	0.104	0.089

modelled and observed SWE ($R^2_{avg} = 0.06$) and a comparatively high RMSE value. The modelled values of SWE are largely determined by the intercept value. The map of predicted SWE for the entire glacier shows a relatively uniform SWE distribution over Glacier 4, likely due to the large influence of the intercept on the regression. Areas with steep slopes and high Sx values (sheltered), especially in the accumulation area, have the lowest values of SWE. This regression indicates that the wind effects play a significant role in snow distribution but since the valley in which the glacier sits is steep walled and curved, perhaps having a single cardinal direction for wind is inappropriate. Examining Sx values that assume wind moving up or down glacier and changing direction to follow the valley could allow the Sx parameter to explain more of the variance.

Table 5: Basic summary statistics of modelled SWE using coefficients found using MLR. Values are for coefficients determined using SWE with inverse distance weighted snowpit densities.

	Modelled SWE (m w.e.)		
	Mean	Minimum	Maximum
Glacier 4	0.58	0.10	0.85
Glacier 2	0.55	-0.23	1.76
Glacier 13	0.41	0.03	0.90

underestimate the amount of accumulation. The map of modelled SWE shows the strong dependence on elevation since the values closely resemble the elevation map (Figure 5). The range of predicted SWE is largest for Glacier 2 and it also has the highest SWE (1.76 m w.e) and the lowest SWE (-0.23 m w.e.) values. Both extremes are perhaps unexpected on this glacier and are likely an artefact from extrapolating from the regression, which largely depends on elevation. The southwest region of the accumulation area that has the area of high accumulation is a result of the high elevation and Sx values. The negative values of SWE at the terminus are a result of the low elevation, as the Sx values are close to zero there.

For Glacier 13, the largest coefficient is elevation with distance from centreline and Sx also of the same order of magnitude. All three coefficients are positive, which means that cells at higher elevation, closer to the margins, and more sheltered had higher amounts of SWE. Elevation explains the majority of the variance (24%), while distance from centreline and Sx explain far less (~1% each). Overall, the MLR modelled winter balance is poorly related to observed SWE ($R^2_{avg} = 0.35$) and tends to underestimate the amount of accumulation. The map of modelled SWE displays an increase with elevation, although it is not strong considering the large range of elevation on this glacier.

Qualitatively, there is little variation in the fit between modelled and observed winter balance between the various density options for all glaciers and the residuals display a similar distribution between the density options. The percent variance explained by each parameter also varies little between different density options.

For Glacier 2, the largest coefficient (by an order of magnitude) is elevation. This coefficient is positive, which means that SWE will increase with elevation. Elevation explains the largest amount of variance (~45%) and this is consistent for all density options. Sx and distance from centreline are also significant coefficients in the MLR, explaining ~6% and ~1% of the variance, respectively. Overall, the modelled winter balance is a good estimation of observed SWE ($R^2_{avg} = 0.62$) although it tends to

The effect of density values on the percent variance explained by each topographic parameters in the MLR is shown in Figure 27. The range of percent variance explained by elevation is large for both Glacier 2 and 13 but it does not affect the importance of elevation relative to the other parameters. For these two MLR coefficients, the choice of density interpolation has a large effect on the ability of the model to explain variance in SWE. Using inverse distance weighted, Federal Sampler derived density resulted in the highest percent variance explained (54%). Since the Federal Sampler derived densities are correlated with snow depth, which is correlated with elevation, it is likely that these density values amplify the elevation component of the regression. The range of percent variance explained for all other parameters is small so the MLR coefficients for these parameters are insensitive to the choice of density interpolation. The values for aspect, “northness”, and both curvatures are essentially zero.

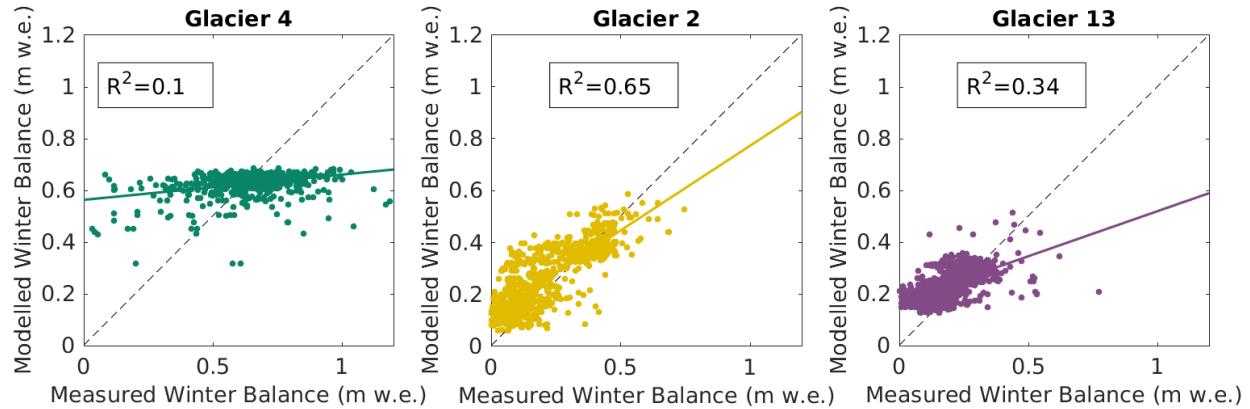


Figure 21: Comparison of predicted (MLR) and observed (original) snow water equivalent (SWE) for three study glaciers. The SWE values were calculated inverse distance weighted snowpit densities (Option 7).

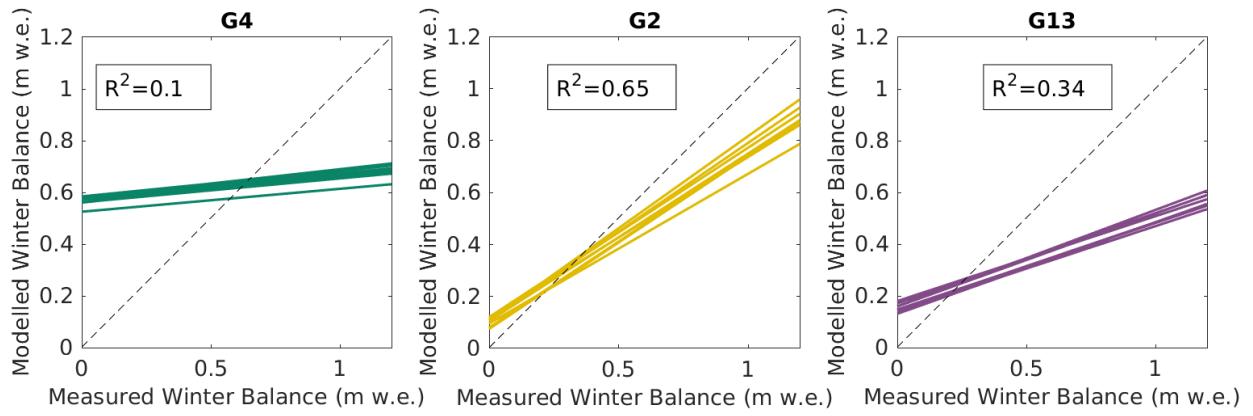


Figure 22: Plot of all linear fits between modelled and observed SWE using eight options for calculating density. Mean R^2 value is shown for each sub-plot and a reference 1:1 line is also provided. See Figure 21 for a plot of the data.

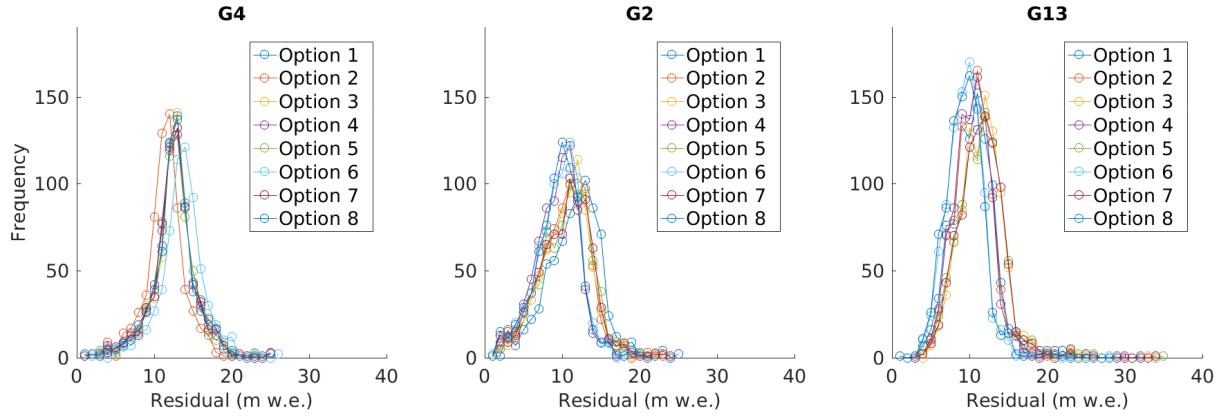


Figure 23: Residuals of SWE predicted using MLR for all options of estimating density.

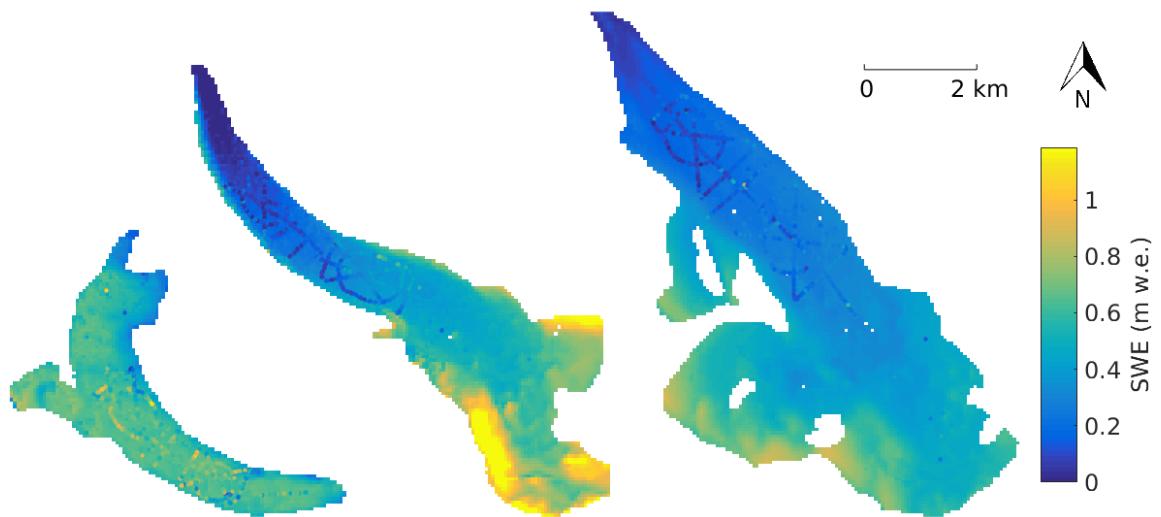


Figure 24: Modelled SWE using coefficients determined using MLR and density interpolated with inverse-distance weighting from snowpits (Option 7). Observed SWE values are overlain on the maps.

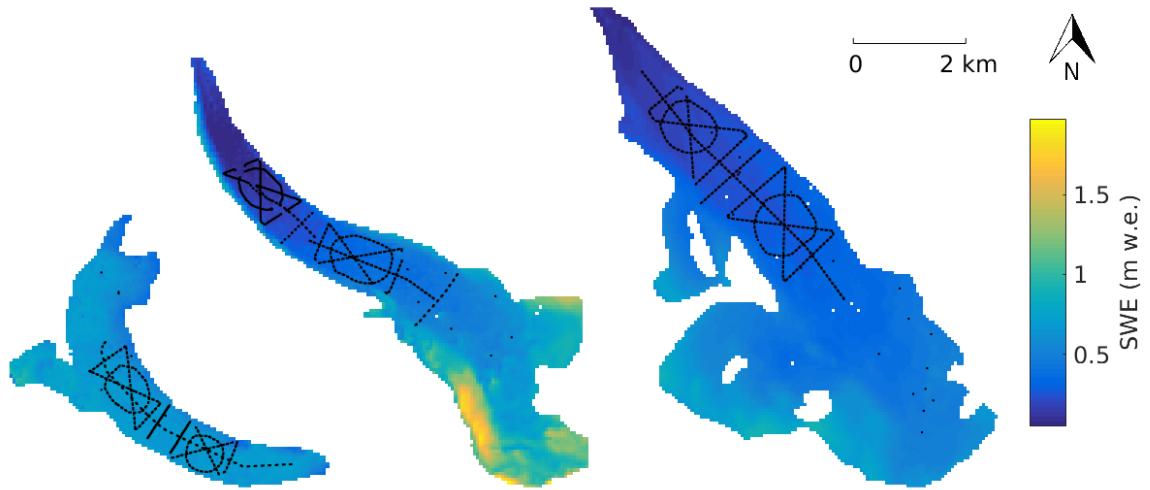


Figure 25: Map of the difference between maximum and minimum SWE values for each DEM cell between all density options using MLR coefficients.

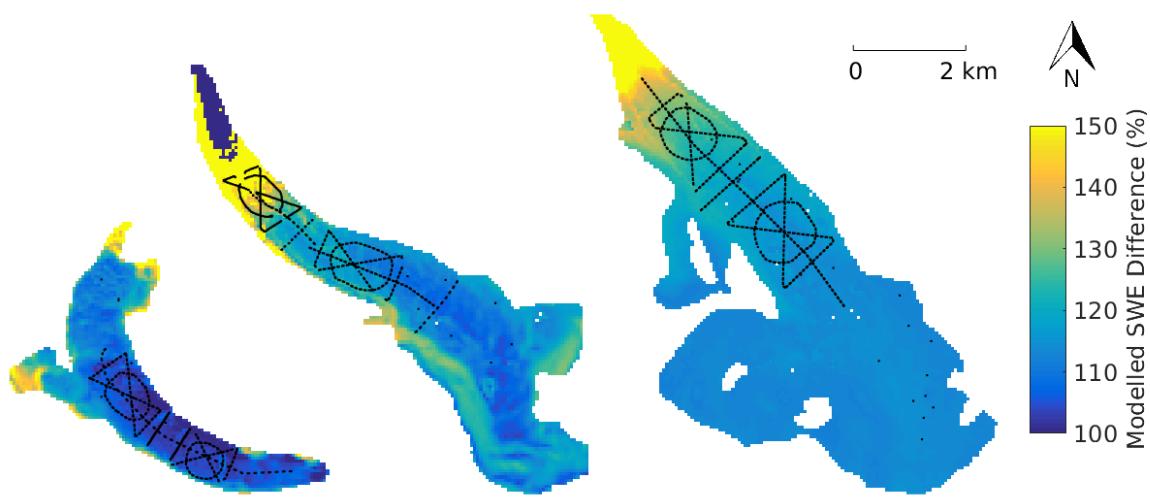


Figure 26: Map of the difference between maximum and minimum SWE values, expressed as a percent of the mean SWE, for each DEM cell between all density options using MLR coefficients. The colours have been scaled to highlight difference in the main part of the glaciers.

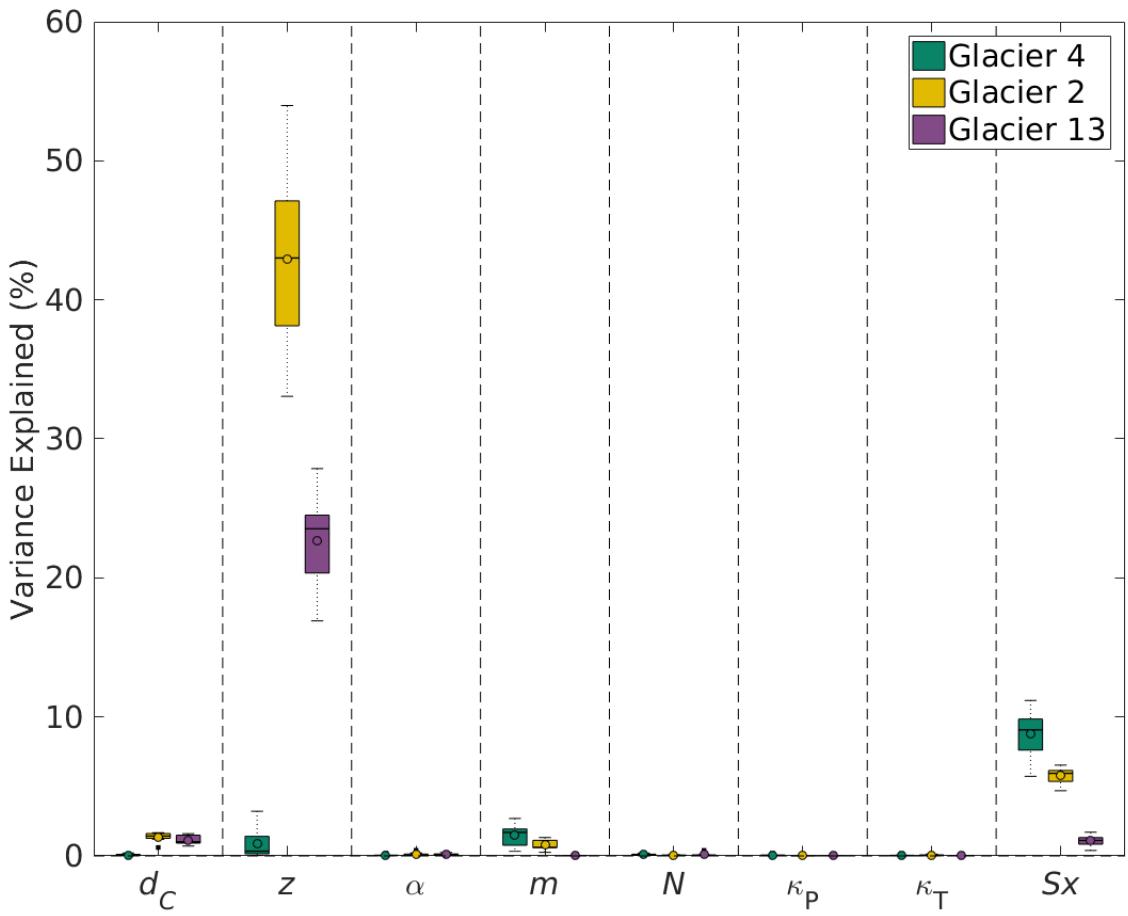


Figure 27: Boxplot showing the range of percent variance explained by each topographic parameter for each option of estimating snow water equivalent (SWE). Topographic parameters include aspect (α), elevation (z), northness (N), profile curvature (κ_P), slope (m), tangent curvature (κ_T), wind redistribution (Sx) and distance from centreline (d_C). Within each box, the mean is shown as a circle, the median as a horizontal line, the interquartile range (IQR) as a coloured box, two times the IQR as dashed lines beyond the box, and outliers as single points.

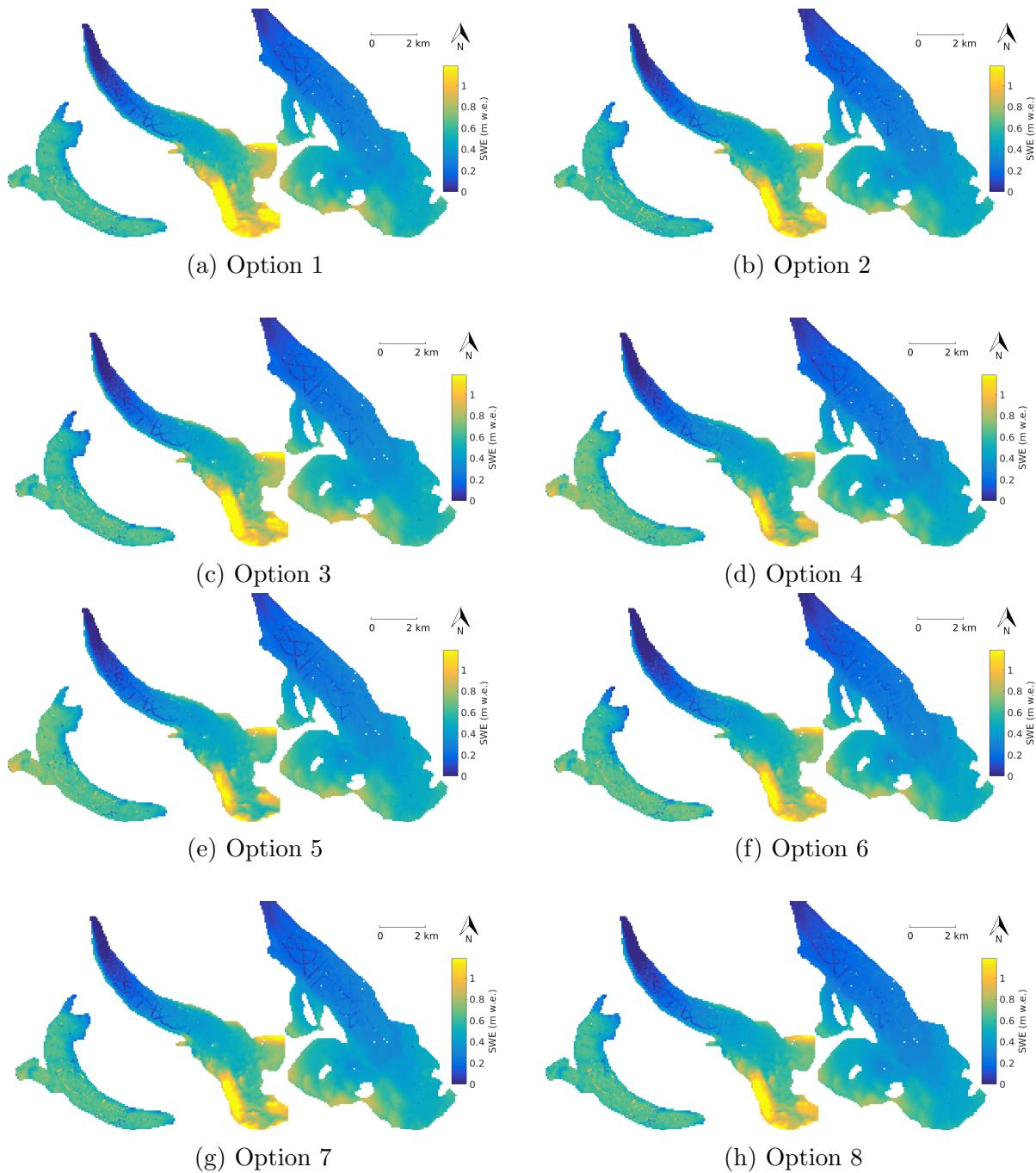


Figure 28: Map of modelled SWE using the MLR coefficient values for all density options. Measured SWE is plotted as overlaid filled circles.

Table 6: Topographic regression coefficients determined using MLR. See Table 3 for description of density options.

	Topographic Parameter	Option 1	Option 2	Option 3	Option 4	Option 5	Option 6	Option 7	Option 8
Glacier 4	d_C	-0.003	-0.001	-0.001	-0.002	-0.002	-0.001	-0.005	-0.005
	z	0.008	0.014	0.004	0.022	0.028	0.000	0.009	0.002
	α	0.001	0.002	0.001	0.001	0.003	0.000	0.001	0.000
	m	-0.020	-0.014	-0.025	-0.012	-0.009	-0.020	-0.021	-0.023
	N	-0.001	-0.004	0.000	-0.006	-0.004	-0.001	0.000	0.000
	κ_P	-0.001	-0.003	0.000	-0.004	-0.004	-0.001	-0.002	-0.001
	κ_T	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Sx	-0.040	-0.044	-0.046	-0.053	-0.048	-0.051	-0.045	-0.038
	Intercept	0.619	0.581	0.627	0.646	0.630	0.648	0.625	0.639
	RMSE	0.146	0.135	0.149	0.151	0.148	0.152	0.147	0.150
Glacier 2	d_C	0.020	0.020	0.020	0.018	0.020	0.017	0.019	0.012
	z	0.113	0.100	0.109	0.089	0.090	0.107	0.098	0.110
	α	-0.004	-0.001	-0.007	-0.001	-0.001	-0.001	-0.001	-0.010
	m	0.019	0.018	0.016	0.011	0.007	0.012	0.013	0.011
	N	0.002	0.000	0.004	0.001	0.000	0.001	0.001	0.005
	κ_P	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
	κ_T	0.001	0.002	0.004	0.000	0.004	0.002	0.005	0.002
	Sx	0.041	0.039	0.040	0.032	0.040	0.033	0.041	0.035
	Intercept	0.268	0.250	0.263	0.225	0.256	0.225	0.267	0.229
	RMSE	0.104	0.097	0.101	0.087	0.102	0.087	0.104	0.088
Glacier 13	d_C	0.013	0.009	0.009	0.013	0.010	0.013	0.009	0.009
	z	0.049	0.047	0.054	0.047	0.044	0.050	0.047	0.047
	α	-0.002	-0.002	-0.003	-0.003	-0.005	-0.001	-0.004	-0.002
	m	-0.002	-0.001	-0.001	0.000	-0.001	-0.001	0.000	0.000
	N	0.003	0.002	0.001	0.006	0.002	0.001	0.002	0.001
	κ_P	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000
	κ_T	0.000	0.000	0.000	0.000	-0.001	0.000	-0.001	-0.001
	Sx	0.011	0.011	0.011	0.006	0.009	0.012	0.012	0.010
	Intercept	0.231	0.213	0.235	0.214	0.246	0.202	0.241	0.207
	RMSE	0.085	0.080	0.087	0.079	0.092	0.074	0.089	0.077

2.4 Bayesian Model Averaging

2.4.1 Background

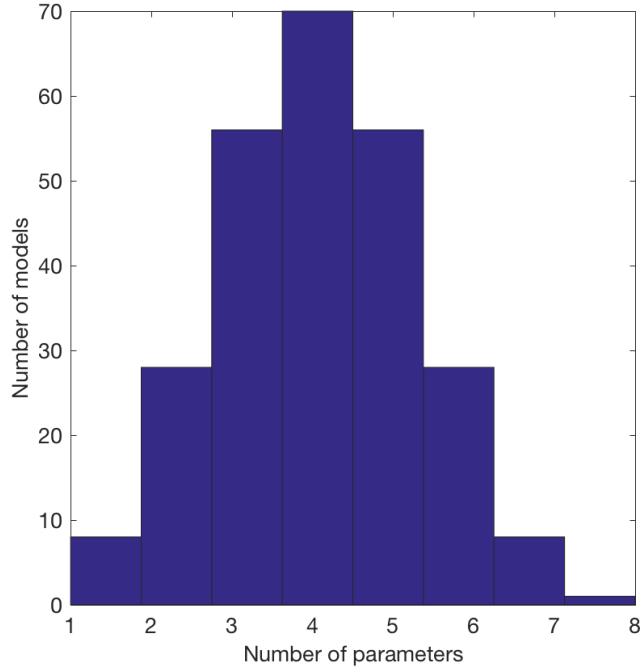


Figure 29: Uniform model prior for eight topographic regressors used in BMA.

a known outcome (i.e. observed data). The term $P(x)$ is an observer determined prior probability distribution (typically just called a *prior*) and it reflects the prior knowledge of the system (Raftery and others, 1997). Choosing an appropriate prior is one of the most challenge components of Bayesian probability theory (Wasserman, 2000). The $P(y)$ term can be obtained by integrating $P(y|x)P(x)$ over all x and is thus a constant for all models that is typically discarded (Wasserman, 2000).

Together then, the posterior model probability is a function of both the model prior, specified by the researcher, as well as the distribution of the observed data — it is the transformation of the prior as a result of considering collected data (Wasserman, 2000). This can be loosely expressed as

$$\text{posterior} \propto \text{prior} \times \text{likelihood} \quad (21)$$

If the prior is uninformative then the posterior will be strongly influenced by the data (Wasserman, 2000). An informative prior will result in a posterior that is a mix of the prior and the data as as the prior becomes more informative, the amount of data needed to transform the distribution increases. If there is a large amount of data then the prior will

Bayesian model averaging (BMA) is a method of estimating all possible linear combinations of predictor variables and then averaging over all models (Raftery and others, 1997; Wasserman, 2000; Raftery and others, 2005). This method is based on Bayesian principals where we wish to determine the probability of an outcome based on an initial “guess” as well as the data provided. Given that the predictive outcome has a probability distribution of x given y , written as $P(x|y)$, we use Bayes’ rule to to write this as

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}. \quad (20)$$

$P(x|y)$ is often called the posterior model probability (PMP). The quantity $P(y|x)$ is the likelihood function, which determines unknown parameters from

have little effect on the posterior. The final value of the desired output is often reported as the posterior distribution mean or the value that maximizes the log-likelihood.

Within BMA, Bayes' theorem is used to find the posterior model probability and this is used as a weight when averaging over all models (Wasserman, 2000). The model weighted posterior distribution for the coefficients β of k number of models after renormalization is given by

$$P(\beta | y, X) = \sum_{i=1}^{2^k} P(M_i | X, y) P(\beta | M_i, y, X), \quad (22)$$

where the responding variable is given by y and the matrix of variables is given by X (Raftery and others, 1997). Here, the model prior is $P(M_i | X, y)$ and the likelihood for the β coefficient is $P(\beta | M_i, y, X)$.

There are a number of different priors to describe model size distribution that have been applied in BMA. A commonly used prior is the ‘uniform’ prior, which assumes a normal model distribution with a total of 2^n models, where n is the number of regressors (Wasserman, 2000). This model prior states that the observer has no knowledge of the system and all models are equally likely. The uniform prior has a prior model probability of the form $P(M_i) = 2^{-n}$, as shown in Figure 29, which is symmetric about the mean $n/2$ (Zeugner and Feldkircher, 2015). This prior inherently favours models of an intermediate size.

Other types of priors include those that are skewed to favour smaller models, ones with equal probability for all model sizes, and ones with varying probability for individual regressors. In this project, a uniform prior was chosen because there was no knowledge of what the model distribution would look like as well as wanting to match the distribution of models sizes in the MLR analysis. This is a common approach in BMA (Wasserman, 2000).

With a small number of regressors, the posterior of all possible models can be determined. As the number of regressors increases, this computation becomes increasingly expensive since the total number of models is 2^n . To overcome this, BMA can use Markov chain Monte Carlo (MCMC) model composition to directly approximate the posterior distribution (Wasserman, 2000). In our study, there are eight regressors so $2^8 = 256$ models. It is possible to visit all models and to obtain an exact solution so an MCMC model composition was not employed.

BMA allows for the calculation of a metric called the posterior inclusion probability (PIP), which is used to evaluate the importance of a regressor in explaining the observed data. PIP is the sum of all posterior model probabilities (PMP) where the variable was included in the mode (Zeugner and Feldkircher, 2015). A higher PIP indicates that the regressor is more important in the regression.

2.4.2 Methods

The BMA process was implemented in R, using the Bayesian model statistics (BMS) package developed by Zeugner and Feldkircher (2015). The function `BMS.R()` computes

the mean value of all β coefficients for topographic parameters as well as the percent variance explained by each parameter using the following steps:

1. The SWE observations and associated topographic parameters are called in the function.
2. A portion of the data (2/3 as suggested by Kohavi and others (1995)) is randomly chosen as the calibration set and saved as a .mat file.
3. The R script is called through the terminal
 - (a) The R script imports the .mat file with SWE and topographic parameter values. It then creates a data frame with the SWE values as the first column and the topographic parameters as the remaining columns.
 - (b) The BMS package is used to complete BMA for the imported values. A uniform model prior was chosen. The mean coefficient value, coefficient standard deviation, PIP, and the posterior probability of a positive coefficient (how probable it is for the sign of the coefficient to be positive) were computed.
 - (c) The coefficients are saved as a .mat file.
4. Values calculated in R are loaded into Matlab and a data table is created with the coefficients.
5. The remaining portion of the topographic parameter values (1/4) are then used to calculate a modelled value of SWE at those locations. These are compared to the observed SWE values and a RMSE value is determined.
6. The above step are completed 1000 times and the coefficients associated with the lowest RMSE are chosen.
7. Percent variance explained by each parameter is the calculated in the same way as for the MLR (see Section 2.3.2).
8. The final table of values includes the coefficients and percent variance explained for all topographic parameters associated with the lowest RMSE. It also includes the intercept and the actual RMSE value. This table is returned from the function.
9. The residuals of the best BMA fit are also calculated and can be return from the function.

2.4.3 Results and Discussion

The coefficients determined using BMA for density Option 7 can be seen in Table 7 and can be used to examine the relative importance of topographic parameters in the regression. These coefficients were found for standardized values of topographic parameters so their magnitude is indicative of their importance in predicting SWE. Table 9 shows coefficient values for all density options. The range of percent variance explained by each variable (from all density options) can be seen in Figure 36. The fit between modelled SWE and observed SWE for each glacier for density option 7 can be seen in Figure 30 and the fit for

all density options in Figure 31. The distribution of residuals is then plotted in Figure 32. A map of the predicted values of SWE over the entire glacier using density option 7 can be seen in Figure 33 with basic summary statistics found in Table 8. Modelled SWE using all density options is shown in Figure 37. A map of the difference between maximum and minimum modelled SWE values is shown in Figure 35.

For Glacier 4, the largest coefficients are wind redistribution (S_x), slope, and elevation, which are an order of magnitude larger than the remaining coefficients. The S_x and slope coefficients are negative, which indicates less snow in ‘sheltered’ areas and less on steeper slopes. The elevation coefficient is positive, indicating that there is more snow at higher elevation. These coefficients account for the largest percentage of variance explained, although this percentage is small ($\sim 7\%$, $\sim 1\%$ and $\sim 1\%$, respectively). Overall, the modelled winter balance for Glacier 4 is very poorly explained by the topographic parameters, which results in no correlation between modelled and observed SWE ($R^2_{avg} = 0.05$) and a comparatively high

RMSE value. The modelled values of SWE are largely determined by the intercept value. The map of predicted SWE for the entire glacier shows a relatively uniform SWE distribution over Glacier 4, likely due to the large influence of the intercept on the regression. The variation of SWE across the width of the glacier is not captured by the regression. Modelled values on glacier right are less than observed and on glacier left the modelled values are greater than observed. This regression indicates that the wind effects play a significant role in snow distribution but since the valley in which the glacier sits is steep walled and curved, perhaps having a single cardinal direction for wind is inappropriate. Examining S_x values that assume wind moving up or down glacier and changing direction to follow the valley could allow the S_x parameter to explain more of the variance. For Glacier 2, the largest coefficient (by an order of magnitude) is elevation. This coefficient is positive, which means that SWE will increase with elevation. Elevation explains the largest amount of variance ($\sim 45\%$). S_x , distance from centreline, and slope are also significant coefficients in BMA, explaining $\sim 6\%$, $\sim 1\%$ and $\sim 1\%$ of the variance, respectively. Overall, the modelled winter balance is a good estimation of observed SWE

Table 7: BMS coefficients for topographic regression between measured SWE and standardized topographic parameters that include distance from centreline (d_C), elevation (z), aspect (α), slope (m), northness (N), profile curvature (κ_P), tangent curvature (κ_T), and wind redistribution (S_x). Since parameters are standardizes, the magnitude of the coefficients is representative of their importance in predicting SWE. The root-mean-squared error (RMSE) between modelled SWE using those coefficients and observed SWE is also given.

	Glacier 4	Glacier 2	Glacier 13
d_C	-0.004	0.020	0.007
z	0.013	0.107	0.047
α	0.002	-0.008	-0.010
m	-0.020	0.011	<-0.001
N	<-0.001	0.004	0.001
κ_P	<0.001	0.003	<-0.001
κ_T	<0.001	0.002	-0.003
S_x	-0.027	0.040	0.007
Intercept	0.619	0.268	0.238
RMSE	0.136	0.098	0.086

$(R^2_{avg} = 0.62)$ although it tends to underestimate the amount of accumulation. The map of modelled SWE shows the strong dependence on elevation since the values closely resemble the elevation map (Figure 5). The range of predicted SWE is largest for Glacier 2 and it also has the highest SWE and the lowest SWE values. Although negative values are inconsistent with accumulation, they do reflect the lack of snow cover over the lower portion of the terminus.

Table 8: Basic summary statistics of modelled SWE using coefficients found using BMA. Values are for coefficients determined using SWE with inverse distance weighted snowpit densities.

	Modelled SWE (m w.e.)		
	Mean	Minimum	Maximum
Glacier 4	0.58	0.26	0.75
Glacier 2	0.57	-0.25	1.76
Glacier 13	0.39	0.02	0.85

underestimate the amount of accumulation. The map of modelled SWE displays an increase with elevation, although it is not strong considering the large range of elevation on this glacier.

Qualitatively, there is little variation in the fit between modelled and observed winter balance between the various density options for all glaciers and the residuals display a similar distribution between the density options. The percent variance explained by each parameter also varies little between different density options.

The effect of density values on the percent variance explained by each topographic parameters from BMA is shown in Figure 36. The range of percent variance explained by elevation is large for both Glacier 2 and 13 but it does not affect the importance of elevation relative to the other parameters. For these two BMA coefficients, the choice of density interpolation has a large effect on the ability of the model to explain variance in SWE. Using elevation regressed Federal Sampler derived density resulted in the highest percent variance explained (59%). Since the Federal Sampler derived densities are correlated with snow depth, which is correlated with elevation, it is likely that these density values amplify the elevation component of the regression. The range of percent variance explained for all other parameters is small so the BMA coefficients for these parameters are insensitive to the choice of density interpolation. The values for “northness” and both curvatures are essentially zero and have negligible ranges.

For Glacier 13, the largest coefficient is elevation with aspect also of the same order of magnitude. The elevation coefficient is positive, which means that cells at higher elevation had higher amounts of SWE. The aspect coefficient was negative, which indicates that larger values of aspect correspond to less snow. Elevation explains the majority of the variance (24%), while aspect explains far less (<1%). Overall, the MLR modelled winter balance is poorly related to observed SWE ($R^2_{avg} = 0.34$) and tends to

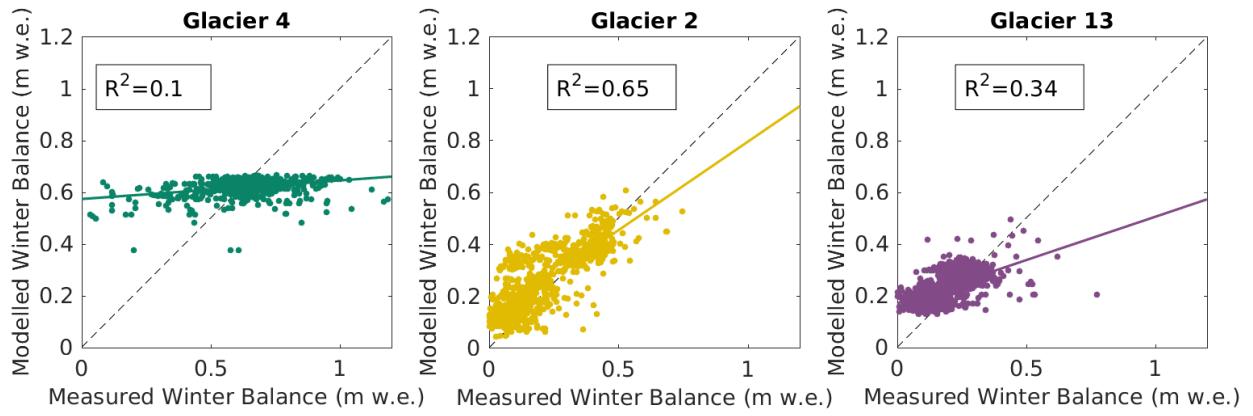


Figure 30: Comparison of predicted (BMA) and observed (original) snow water equivalent (SWE) for three study glaciers. The SWE values were calculated inverse distance weighted snowpit densities (Option 7).

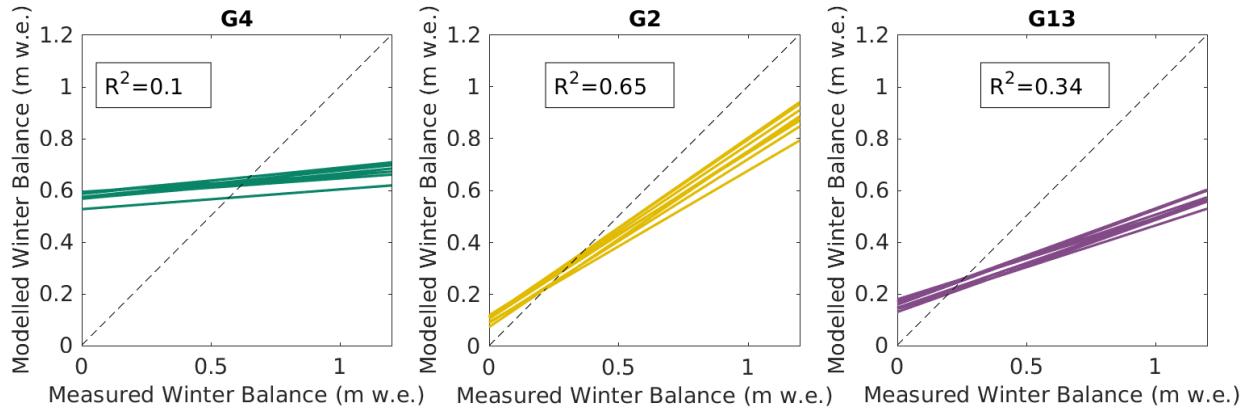


Figure 31: Plot of all linear fits between modelled (BMA) and observed SWE using eight options for calculating density. Mean R^2 value is shown for each sub-plot and a reference 1:1 line is also provided. See Figure 30 for a plot of the data.

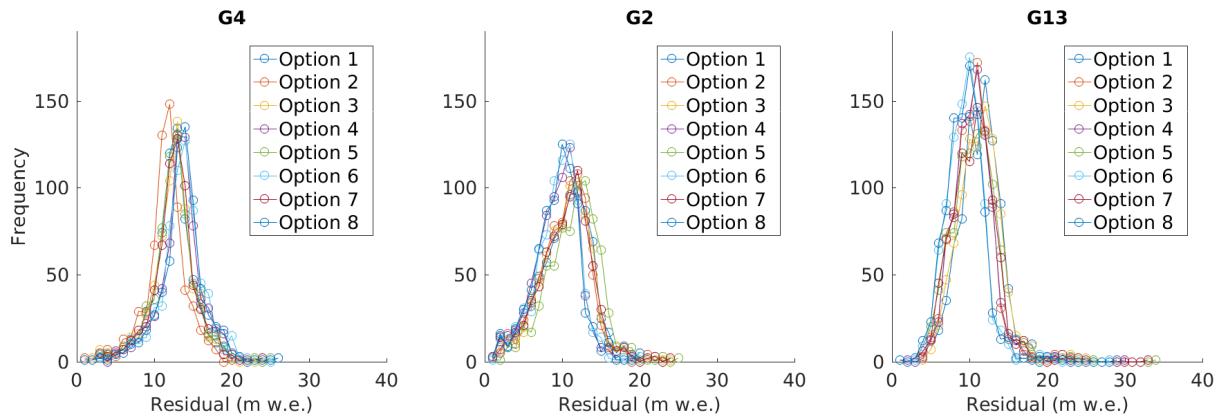


Figure 32: Residuals of SWE predicted using BMA for all options of estimating density.

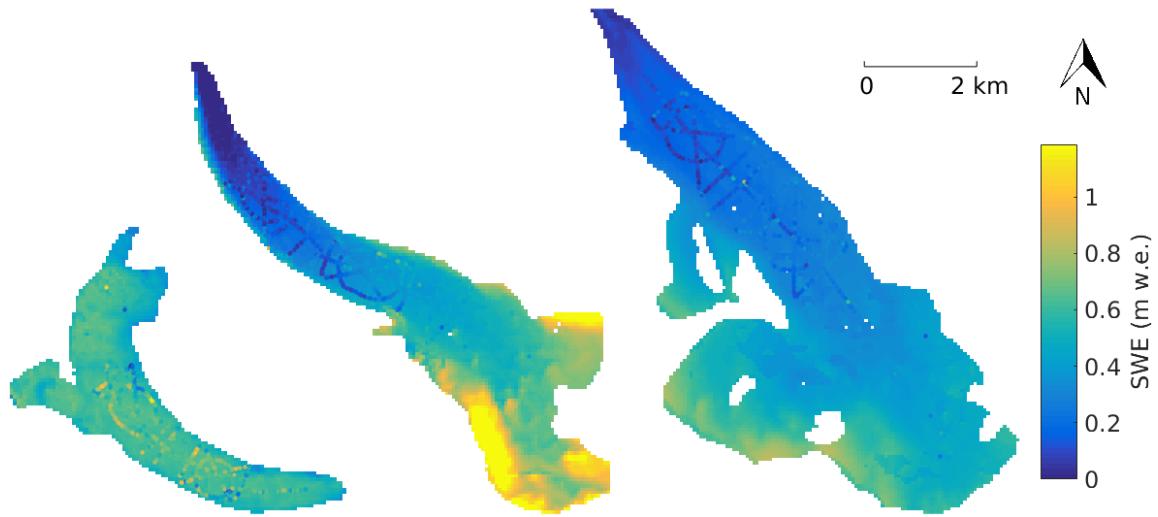


Figure 33: Modelled SWE using coefficients determined using BMA and density interpolated with inverse-distance weighting from snowpits (option 7). Observed SWE values are overlaid on the maps.

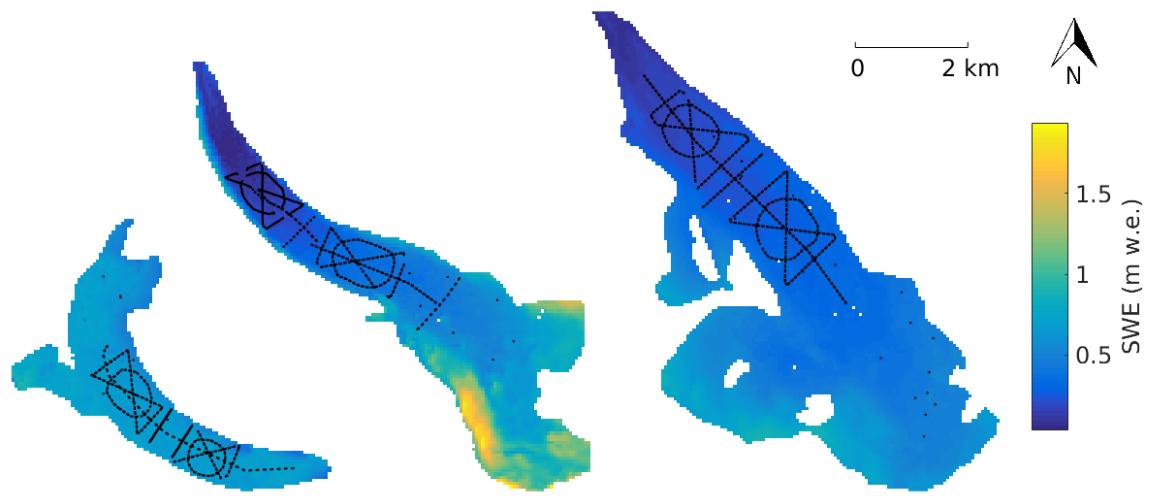


Figure 34: Map of the difference between maximum and minimum SWE values for each DEM cell between all density options using BMA coefficients.

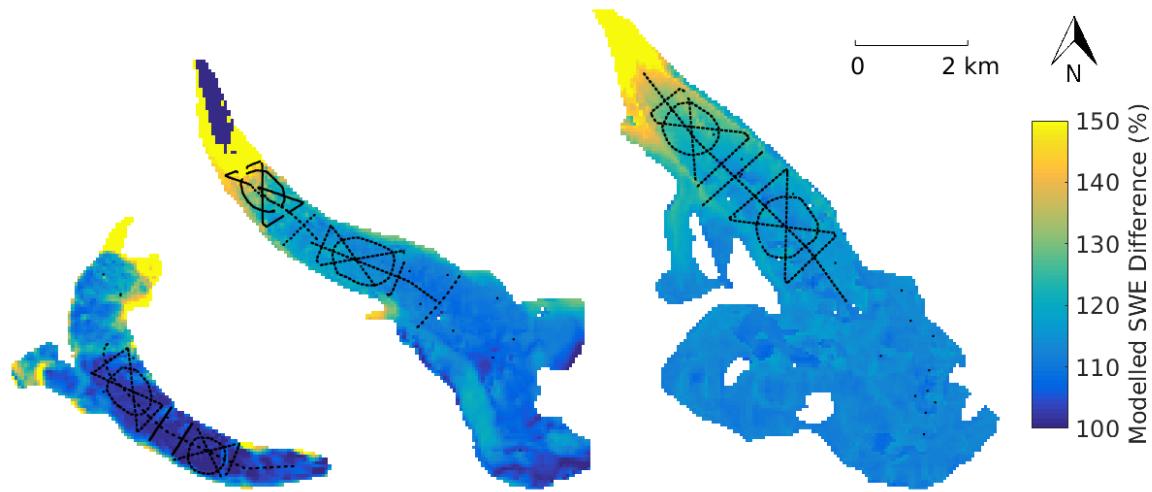


Figure 35: Map of the difference between maximum and minimum SWE values, expressed as a percent of the mean SWE, for each DEM cell between all density options using BMA coefficients. The colours have been scaled to highlight difference in the main part of the glaciers.

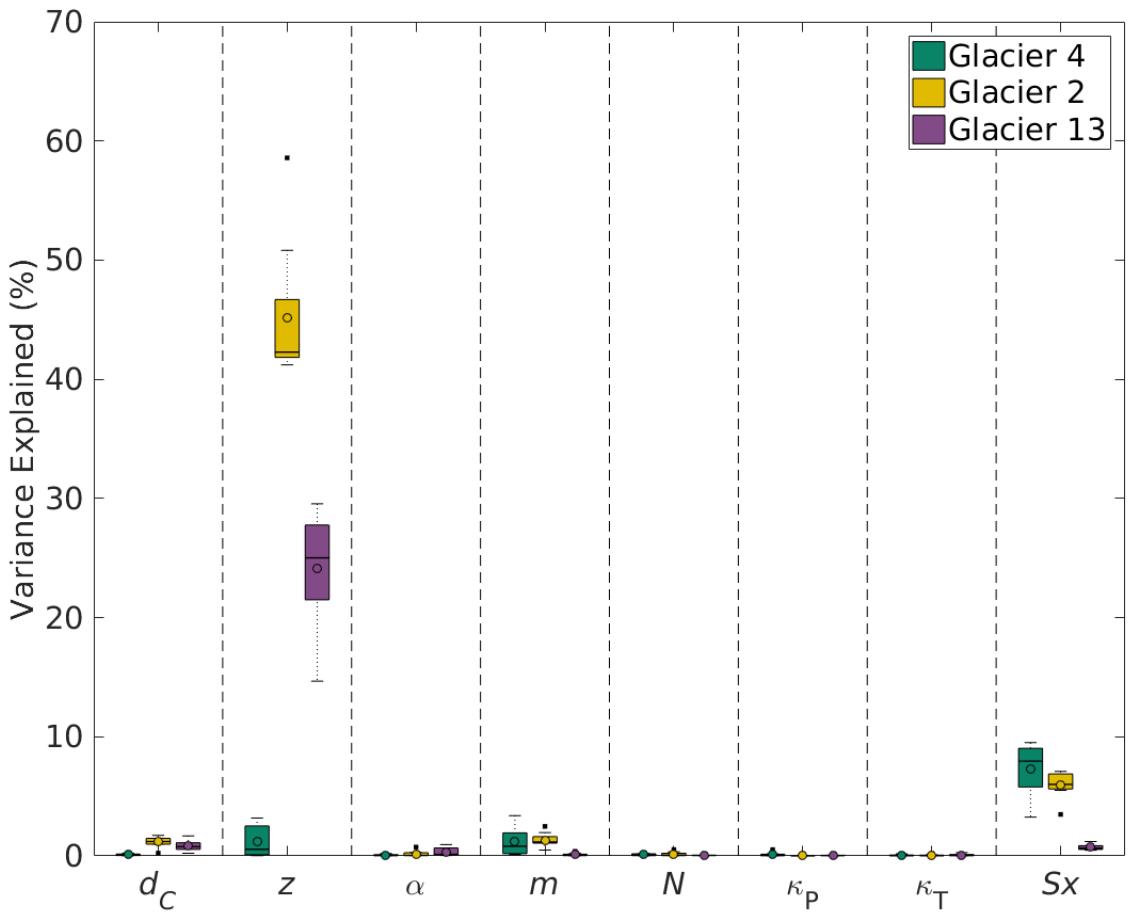


Figure 36: Boxplot showing the range of percent variance explained by each topographic parameter as calculated using BMA for each option of estimating snow water equivalent (SWE). Topographic parameters include distance from centreline (d_C), elevation (z), aspect (α), slope (m), northness (N), profile curvature (κ_P), tangent curvature (κ_T), and wind redistribution (Sx). Within each box, the mean is shown as a circle, the median as a horizontal line, the interquartile range (IQR) as a coloured box, two times the IQR as dashed lines beyond the box, and outliers as single points.

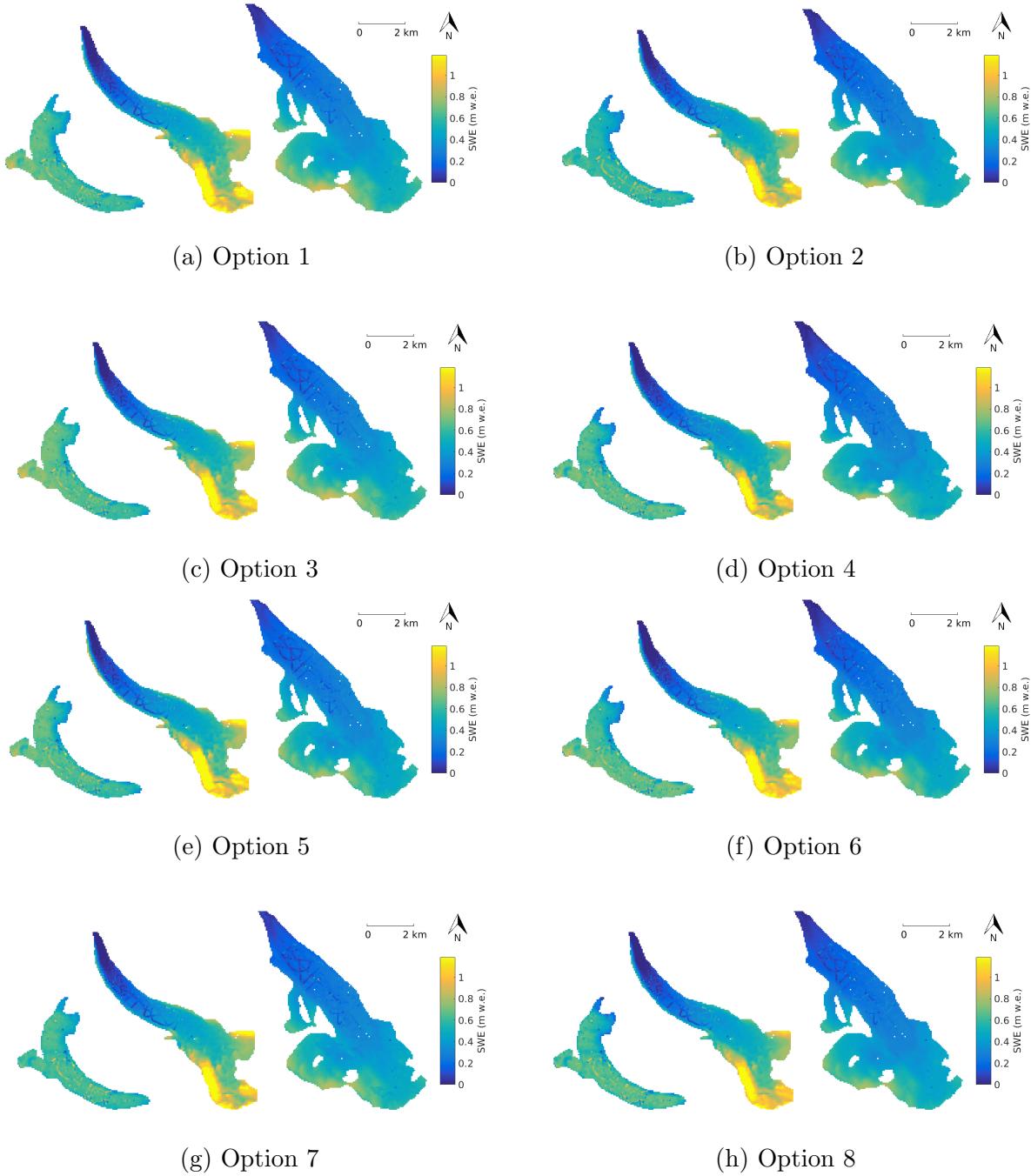


Figure 37: Map of modelled SWE using the BMS coefficient values for all density options. Measured SWE is plotted as overlain filled circles.

Table 9: Topographic regression coefficients determined using BMA. See Table 3 for description of density options.

	Topographic Parameter	Option 1	Option 2	Option 3	Option 4	Option 5	Option 6	Option 7	Option 8
Glacier 4	d_C	-0.004	-0.002	0.000	-0.007	-0.004	-0.004	0.000	0.000
	z	0.021	0.005	0.028	0.007	0.028	0.000	0.012	0.001
	α	0.003	0.003	0.001	0.001	0.001	0.002	0.002	0.001
	m	-0.006	-0.010	-0.008	-0.022	-0.016	-0.005	-0.020	-0.029
	N	-0.006	0.000	0.000	-0.006	-0.003	-0.001	0.001	0.000
	κ_P	0.000	-0.001	0.000	-0.002	-0.011	0.000	0.000	-0.003
	κ_T	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	Sx	-0.046	-0.037	-0.034	-0.047	-0.047	-0.050	-0.027	-0.042
	Intercept	0.622	0.575	0.635	0.650	0.628	0.650	0.619	0.638
	RMSE	0.140	0.132	0.139	0.133	0.140	0.140	0.136	0.146
Glacier 2	d_C	0.020	0.020	0.016	0.016	0.017	0.008	0.020	0.015
	z	0.108	0.101	0.106	0.091	0.100	0.117	0.107	0.107
	α	-0.009	-0.002	-0.004	-0.001	-0.001	-0.013	-0.008	-0.001
	m	0.017	0.017	0.018	0.020	0.025	0.017	0.011	0.016
	N	0.001	0.002	0.001	0.003	0.011	0.005	0.004	0.008
	κ_P	0.001	0.001	0.001	0.000	0.001	0.001	0.003	0.000
	κ_T	0.005	0.001	0.008	0.000	0.000	0.001	0.002	0.001
	Sx	0.044	0.037	0.040	0.037	0.041	0.036	0.040	0.028
	Intercept	0.267	0.251	0.260	0.225	0.260	0.227	0.268	0.232
	RMSE	0.097	0.088	0.094	0.083	0.097	0.084	0.098	0.086
Glacier 13	d_C	0.011	0.010	0.009	0.005	0.014	0.007	0.007	0.008
	z	0.053	0.050	0.052	0.053	0.041	0.051	0.047	0.047
	α	-0.003	-0.003	-0.009	-0.002	-0.007	-0.003	-0.010	-0.002
	m	-0.002	-0.002	-0.001	-0.006	-0.001	0.000	0.000	0.000
	N	0.001	0.001	0.000	0.001	0.000	0.001	0.001	0.002
	κ_P	0.000	-0.002	0.000	0.000	0.000	0.000	0.000	-0.002
	κ_T	-0.002	0.000	-0.003	-0.001	-0.003	-0.001	-0.003	0.000
	Sx	0.008	0.007	0.008	0.008	0.010	0.010	0.007	0.007
	Intercept	0.232	0.216	0.240	0.213	0.242	0.201	0.238	0.206
	RMSE	0.072	0.077	0.080	0.072	0.083	0.071	0.086	0.070

2.5 MLR and BMS comparison

A plot of the ranges of coefficient values resulting from different choices in density interpolation is shown in Figure 38. In general, the range of coefficient values for all glacier is similar between MLR and BMS and the ranges always overlap, although the mean and median values can differ between these two methods. The range of unimportant variables is always small, indicating that for all density options these coefficients are small.

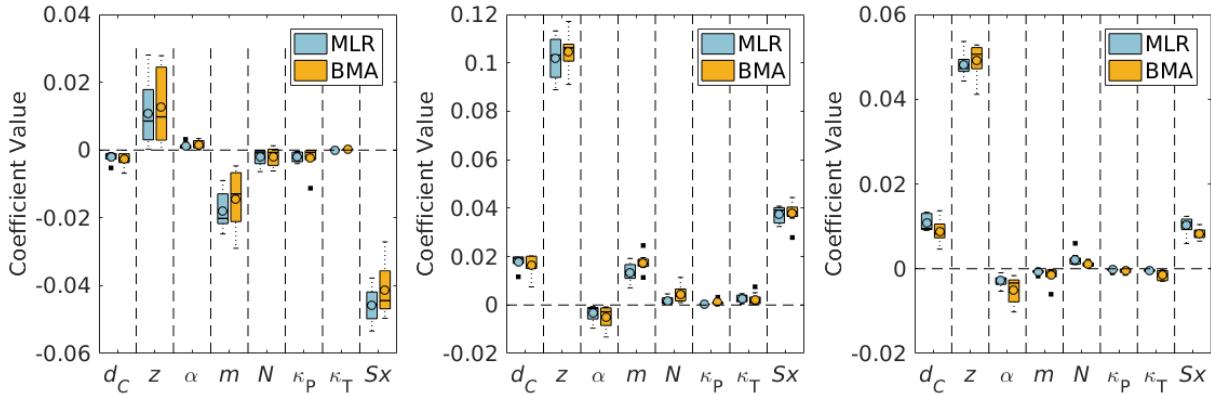


Figure 38: Boxplot showing the range of values of coefficients for each topographic parameter from both MLR and BMA analysis for Glacier 4 (left), Glacier 2 (middle), and Glacier 13 (right). Note the different y axes for the three glaciers. Topographic parameters include distance from centreline (d_C), elevation (z), aspect (α), slope (m), northness (N), profile curvature (κ_P), tangent curvature (κ_T), and wind redistribution (Sx). Within each box, the mean is shown as a circle, the median as a horizontal line, the interquartile range (IQR) as a coloured box, two times the IQR as dashed lines beyond the box, and outliers as single points.

The elevation, slope, and Sx coefficients for Glacier 4 are both the most significant and have the largest range for this glacier. Despite this, the choice of density option would not affect the relative importance of these coefficients (i.e. Sx will always be the most important). For Glacier 4, the choice of density option has a non-negligible impact on modelled SWE values. However, the regression fit is so poor that the fit does not improve regardless of the choice of density option (Figure 22 and 31). The type of regression model used does not appear to have a large impact on the coefficient values. The range is similar between methods, although the mean and median within each method differ for Sx and slope with MLR producing larger coefficients.

The range of coefficient values for Glacier 2 and 13 is generally small for all parameters indicating that the choice of density option does not have a large impact on modelled SWE. Further, the relative importance of the coefficients does not change with choice of density option. The greatest range of coefficient values is for elevation, which is also the most significant coefficient. The type of regression model used does not appear to have a large impact on the coefficient values.

Since the range of coefficient values is so similar between MLR and BMA regression methods, it was decided to calculate a single set of regression coefficients that encompassed both methods. Further, it appears that the choice of density interpolation methods does not alter the relative importance of the coefficients. Therefore, we can generate a set of coefficients and their range of values that gathers information from both the regression method and density interpolation choice. To do this, the mean, minimum, and maximum values of each regression coefficient was calculated. These values are shown in Table 10.

A map of SWE calculated using the mean (Figure 39), minimum (Figure 40), and maximum (Figure 41) coefficient values are shown below. The modelled SWE found using the mean coefficients shows a relatively uniform SWE distribution on Glacier 4 and a strong elevation dependence on Glacier 2 and 13. This is also seen when using the minimum and maximum coefficient values. It is interesting to note that when using the minimum coefficients, there is almost no snow along the edge of the accumulation area on Glacier 4. The difference in SWE using the minimum and maximum coefficients shows that on all three glaciers, the accumulation area is most sensitive to choice of coefficient values. This is likely due to the fact that the most significant coefficients also showed the largest range in values when using different density options.

Table 10: Final regression coefficients. Values are calculated from all coefficients found using MLR and BMA and using all density options.

	Topographic Parameter	Mean	Min	Max
Glacier 4	d_C	-0.0024	-0.0068	0.0000
	z	0.0118	0.0000	0.0280
	α	0.0014	0.0000	0.0034
	m	-0.0162	-0.0290	-0.0047
	N	-0.0020	-0.0064	0.0013
	κ_P	-0.0022	-0.0113	0.0000
	κ_T	0.0001	-0.0003	0.0005
	Sx	-0.0435	-0.0534	-0.0271
Glacier 2	Intercept	0.6270	0.5749	0.6501
	d_C	0.0072	-0.0053	0.0203
	z	0.0578	0.0003	0.1171
	α	-0.0019	-0.0132	0.0031
	m	-0.0002	-0.0247	0.0246
	N	0.0012	-0.0064	0.0114
	κ_P	-0.0005	-0.0040	0.0032
	κ_T	0.0012	-0.0003	0.0075
Glacier 13	Sx	-0.0039	-0.0534	0.0444
	Intercept	0.4377	0.2245	0.6483
	d_C	0.0034	-0.0053	0.0137
	z	0.0301	0.0003	0.0528
	α	-0.0019	-0.0102	0.0031
	m	-0.0097	-0.0247	-0.0002
	N	-0.0005	-0.0064	0.0022
	κ_P	-0.0013	-0.0040	0.0000
	κ_T	-0.0007	-0.0030	0.0004
	Sx	-0.0187	-0.0534	0.0104
	Intercept	0.4251	0.2014	0.6483

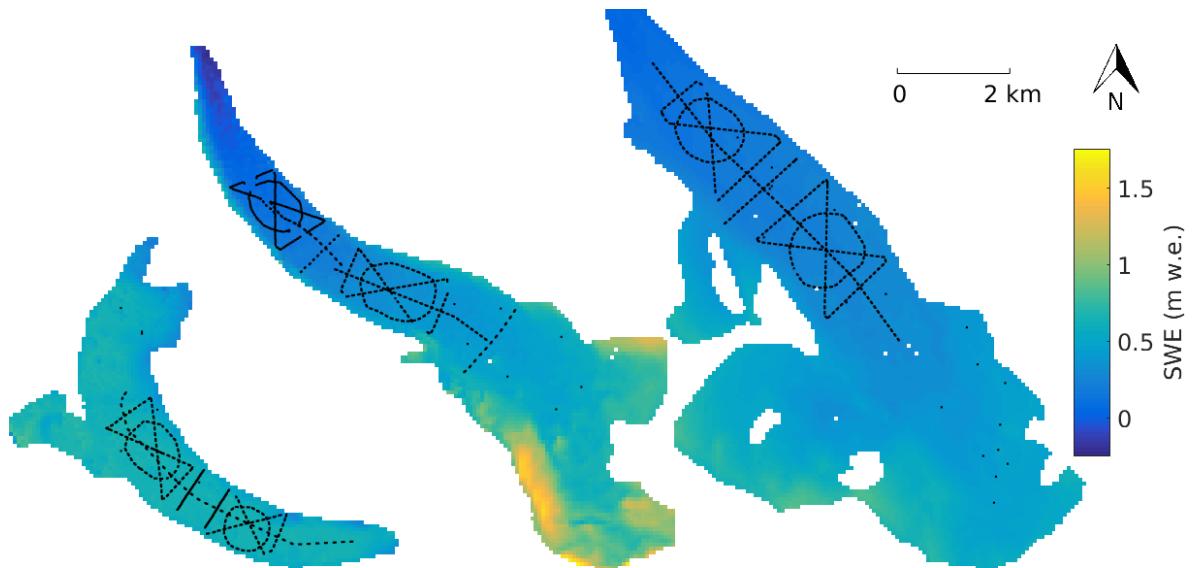


Figure 39: Map of modelled SWE using the mean coefficient values of MLR and BMA using all density options.

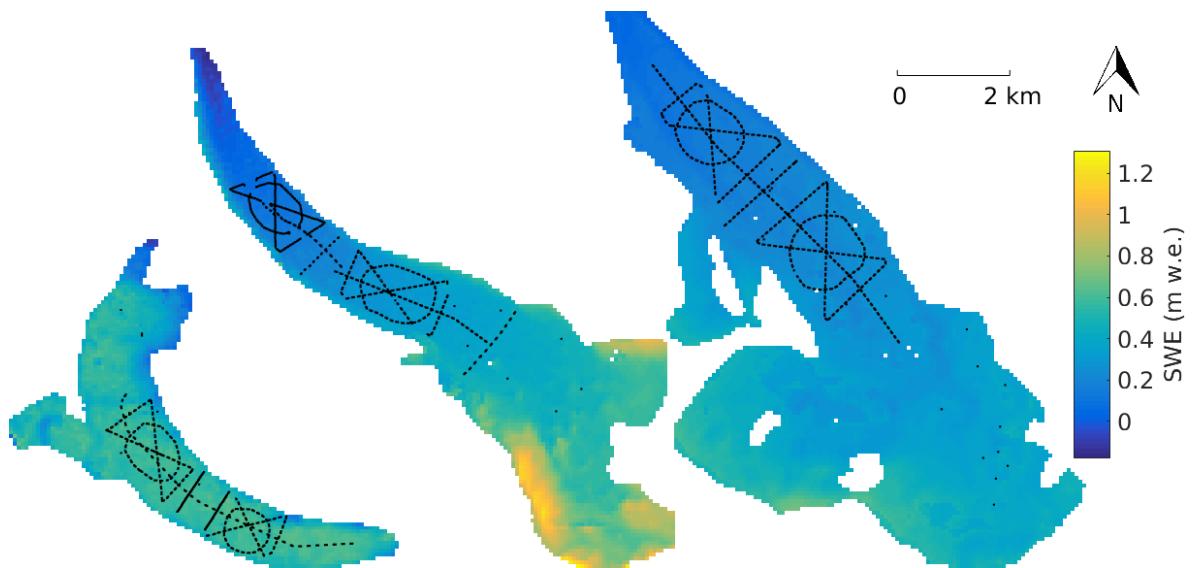


Figure 40: Map of modelled SWE using the minimum coefficient values of MLR and BMA using all density options.

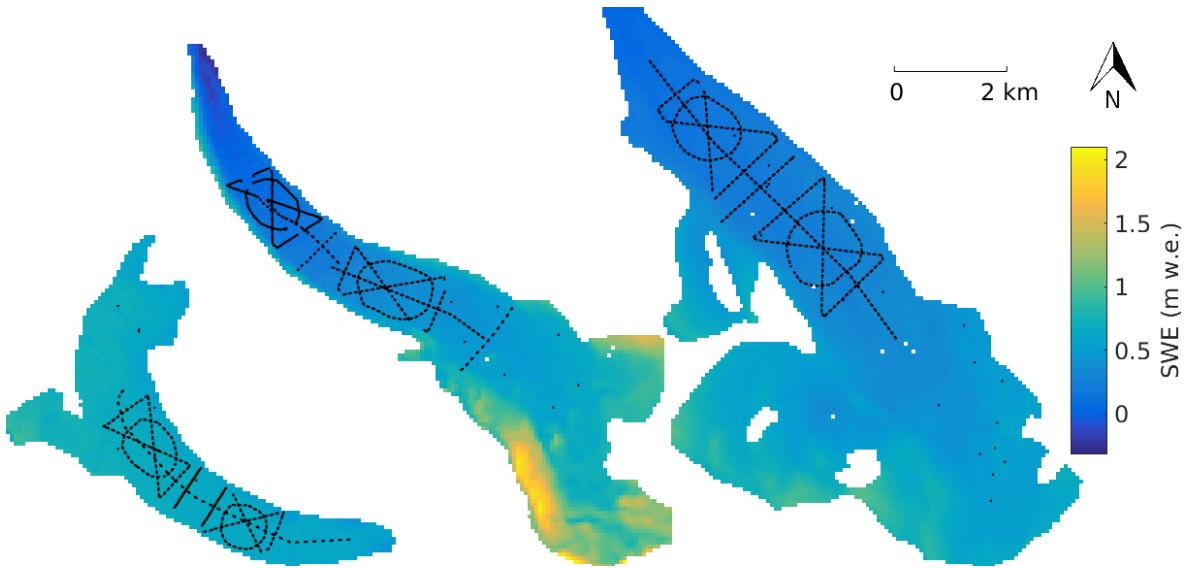


Figure 41: Map of modelled SWE using the maximum coefficient values of MLR and BMA using all density options.

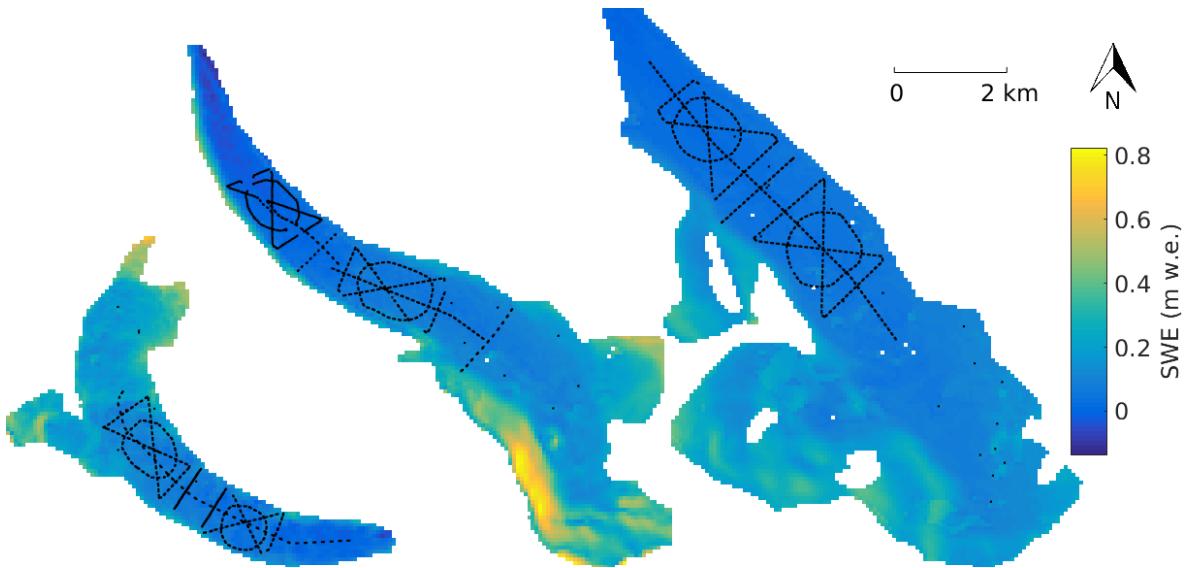


Figure 42: Map of the difference between maximum and minimum SWE values found using the maximum coefficient values of MLR and BMA using all density options.

3 Summary

In this portion of the project, the relation between topographic parameters and SWE at the basin scale was examined. First, a suitable DEM was produced by correcting and merging two SPOT-5 DEMs. Then, a series of topographic parameters was calculated for the study glaciers. The sampled portion of the topographic parameters generally was not

a good representation of the full range of parameters on the study glaciers. The sampling was minimal in the accumulation area and did not sample locations with extreme values of topographic parameters (e.g. high elevation, steep slopes in the accumulation area). This is a major limitation of this study and there is likely a large error induced when extrapolating from relationships between SWE and topographic parameters.

A linear regression between observed SWE (using various density interpolation options) and topographic parameters was then completed. This was done using MLR and BMA. It was found that the choice of density interpolation and the regression method did not have a major impact on the values of topographic coefficients. As a result, a set of mean, minimum, and maximum coefficient values was calculated from all of the ways of completing the regression.

The study glaciers showed varied relationships between SWE and topographic parameters. Glacier 4 had low coefficient values and the modelled SWE was a poor representation of observed SWE. The most significant parameter on Glacier 4 was Sx , which was negatively correlated with SWE. Glacier 2 regression showed that elevation was the strongest predictor of SWE and that the regression was able to explain a large portion of the observed variability. Glacier 13 regression also showed a strong dependence on elevation although less of the observed variable could be explained by this parameter.

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