



EAST WEST UNIVERSITY

Physics Lab
Department of MPS

Course Name :Engineering Physics -1

Course Code :PHY 109 LAB

Section No :04

Group No :01

Experiment No : 01

Name of the Experiment : spring constant and effective mass of a spring.

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Name of the Experiment:

Determination of **spring constant** and **effective mass** of a given spiral spring.

Theory:

If a spring be clamped vertically at one end point, and loaded with a mass at the other end, then, the **time period of vibration** of the spring along a vertical line is given by

$$T = 2\pi \sqrt{\frac{m + m'}{k}} = 2\pi \sqrt{\frac{M}{k}} \dots\dots(1)$$

Where,

$M (= m + m')$ is the **effective mass** of the **oscillating system**(load and spring),

m = mass of **loads** suspended from the spring,

m' = **effective mass** of the spring

k = **spring constant** = $F / x = mg / x$

How the mass of the spring contributes to the **effective mass** of the vibrating system(load and spring) can be shown as follows.

Consider the **kinetic energy** of a spring and its load undergoing simple harmonic oscillation.

At the instant under consideration let the **load m** be moving upwards with **velocity v_o** as shown in the figure.

At this same instant **an element** of dm of the total mass m_o of the spring will also be moving up but with a **velocity v** which is less than v_o .

It is evident that the ratio between v and v_o is just the ratio between y and y_o . (in figure -1)

Hence
$$\frac{v}{v_o} = \frac{y}{y_o},$$

So,
$$v = \frac{y \times v_o}{y_o} \dots\dots\dots(2)$$

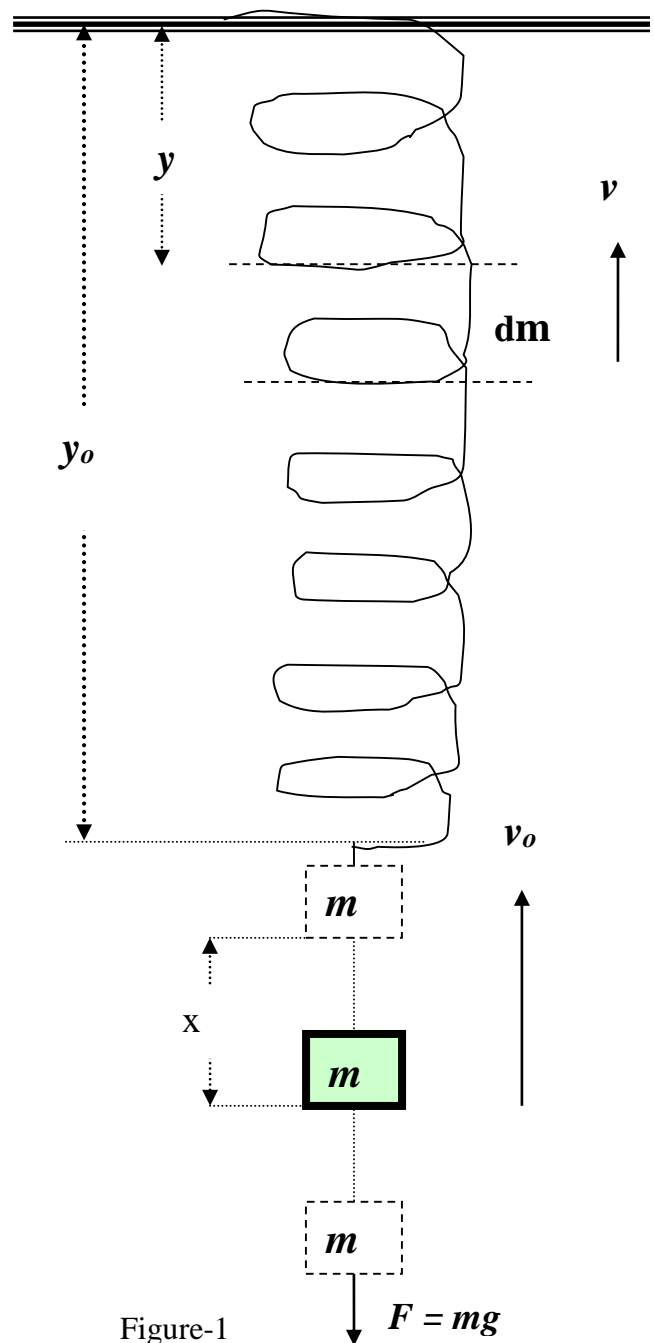


Figure-1

Kinetic energy of small **part of mass** dm of the **spring** will be $= \frac{1}{2} dm v^2$

Therefore **kinetic energy** of full part of the **spring alone** will be $= \int_0^m \frac{1}{2} v^2 dm$

From similarity we can write, $\frac{dm}{m_o} = \frac{dy}{y_o}$, (in figure -1) hence, $dm = \frac{m_o}{y_o} dy \dots\dots\dots(3)$

Where m_o = mass of the spring, dm = mass of an element of the spring
 y_o = length of the spring, dy = length of an element of the spring

Thus the integral becomes (using equation 2 and 3)

$$\int_0^m \frac{1}{2} v^2 dm = \int_0^{y_o} \frac{1}{2} \left(\frac{y \times v_o}{y_o} \right)^2 \frac{m_o}{y_o} dy = \frac{m_o v_o^2}{2 y_o^3} \int_0^{y_o} y^2 dy = \frac{m_o v_o^2}{2 y_o^3} \times \frac{y_o^3}{3} = \frac{1}{2} \left(\frac{m_o}{3} \right) v_o^2$$

Therefore,

Total **kinetic energy (K.E.)** of the system will be

= K.E. of the **load** (suspended mass) + K.E. of the **spring**

$$= \frac{1}{2} m v_o^2 + \frac{1}{2} \left(\frac{m_o}{3} \right) v_o^2 = \frac{1}{2} \left(m + \frac{m_o}{3} \right) v_o^2$$

Hence the **effective mass** of the **vibrating system (load and spring)** = $M = m + \frac{m_o}{3}$.

Comparing with **equation (1)**, $M (= m + m')$; hence $m' = \frac{m_o}{3}$; where,

m' = **effective mass** of the spring, m_o = mass of the spring.

So, the **effective mass** (m') of the spring is **one third mass** ($m_o/3$) of the spring.

Spring Constant:

The **applied force** (weight of the load), $F (=mg)$ on the spring is **proportional** to the **extension** x of the spring within the elastic limit. Therefore

$$F \propto x \text{ Or, } F = k x \quad \text{or,} \quad mg = k x \quad \dots\dots\dots(4),$$

where k is **spring constant**.

Apparatus:

(1) A spiral spring, (2) convenient masses with hanging arrangement, (3) top pan balance, (4) a rigid framework of heavy metal rods, (5) stop watch, (6) meter scale.

Data Sheet:

Table: Data for extensions and time periods for different loads

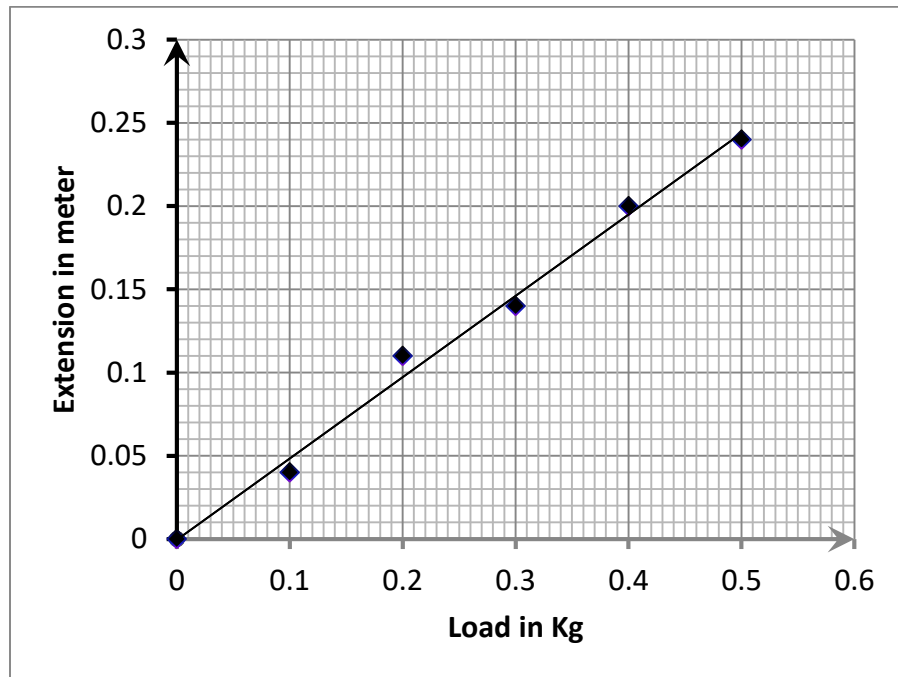
No. of obs.	Load m <i>Kg</i>	Reading of bottom end of the spring on the vertical scale <i>l</i> <i>meter</i>	Extension <i>x</i> <i>meter</i>	Total time for 20 oscillations <i>t / sec</i>	Time period T = (t / 20) <i>sec</i>	T² <i>sec²</i>
1	0 (No load)	<i>l₀ = .48</i>	<i>(l₀ - l₀) = 0</i>			
2	0.05	<i>l₁ = .60</i>	<i>(l₁ - l₀) = .12</i>	13.98	0.699	0.488
3	0.1	<i>l₂ = .72</i>	<i>(l₂ - l₀) = .24</i>	20.58	1.029	1.05
4	0.15	<i>l₃ = .84</i>	<i>(l₃ - l₀) = .36</i>	24.30	1.21	1.46
5	0.2	<i>l₄ = .97</i>	<i>(l₄ - l₀) = .49</i>	28.70	1.435	2.059
6	0.25	<i>l₅ = 1.09</i>	<i>(l₅ - l₀) = .61</i>	32.32	1.616	2.611
7	0.3	<i>l₆ = 1.21</i>	<i>(l₆ - l₀) = .73</i>	35.32	1.766	3.11

Calculation:

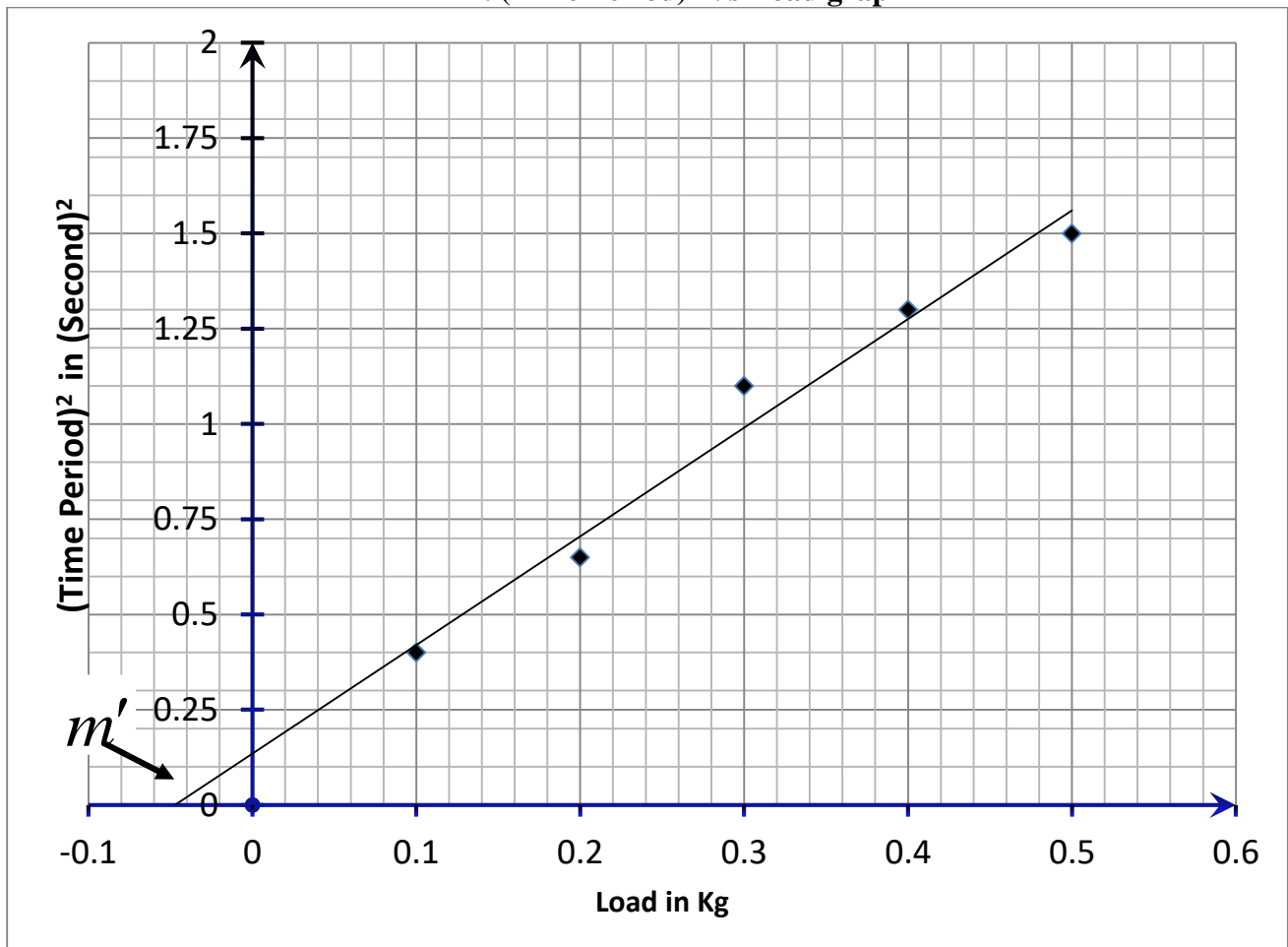
$$\text{Spring Constant, } K = \frac{mg}{x} = \frac{g}{\frac{x}{m}} = \frac{9.8 \frac{m}{s^2}}{\text{SLOPE of Extension vs Load graph}} = \frac{9.8}{2.44} = 4.016 \text{ N/m}$$

Graphs:

1. Extension (x) vs Load (m) Graph:



2. (Time Period)² Vs Load graph



Results:

Spring constant, $K = 4.016N/m$

Discussions:

1. A large number of oscillations should be counted for determining the period of oscillation.
2. The support from which the spring is suspended should be fixed.

Conclusion :

I think if we could do this same experiment in the lab, the measure value would change a little bit and most importantly we could learn how to use these elements.