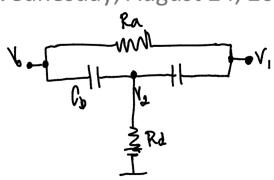
MAE40 HW2

Wednesday, August 24, 2022 5:50 PM



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a) $\frac{(V_2-V_0)SC}{1} + \frac{SC(V_2-V_1)}{2} + \frac{(V_2-0)}{R}2 = 0 \rightarrow \frac{SCR(V_0+V_1)}{2+SCR}$ — 1

$\frac{(V_0-V_2)}{1}SC + \frac{V_0-V_1}{2R} = 0 \rightarrow V_0(1+2SCR) = V_1 + \frac{2(SCR)^2(V_0+V_1)}{2+SCR}$

$$V_0\left[1 + 2SCR - \frac{2S^2R^2C^2}{2+SCR}\right] = V_1\left(\frac{2(SCR)^2(V_0+V_1)}{2+SCR}\right)$$

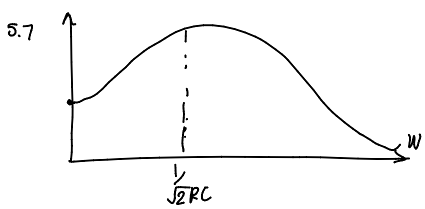
$$V_0\left(\frac{2+5SCR}{\cancel{2+SCR}}\right) = V_1\left(\frac{4S^2R^2C^2+SCR+2}{\cancel{2+SCR}}\right)$$

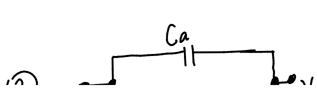
$$\frac{V_0}{V_1} = \frac{4S^2R^2C^2+SCR+2}{2+5SCR} \rightarrow \frac{V_1}{V_0} = \frac{2+5SCR}{4S^2R^2C^2+SCR+2}$$

$$\frac{V_1}{V_0} = \frac{2+\overset{0}{\cancel{j5\omega RC}}}{2-\underset{0}{\cancel{4\omega^2R^2C^2+j\omega RC}}} = \frac{2}{2} = 1 = \left|\frac{V_1}{V_0}\right|$$

$$\omega = \frac{1}{\sqrt{2}RC}, \quad \frac{V_1}{V_0} = \frac{2+j5/\sqrt{2}}{j\sqrt{2}} = \frac{2\sqrt{2}+j5}{j}$$

$$\left|\frac{V_1}{V_0}\right| = 5.7$$





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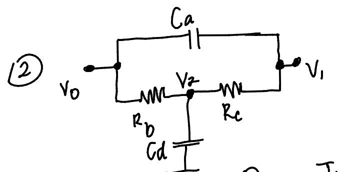


a) $I_0 + I_a + I_b$ — ①
$I_b = I_c + I_d$ — ②
$I_c = -I_a$ — ③

$I_b = \frac{V_0 - V_2}{R_b}$ — ④
$I_a = \frac{V_0 - V_1}{(1/sC_a)}$ — ⑤
$I_c = \frac{V_2 - V_1}{R_c}$ — ⑥
$I_d = \frac{V_2}{(1/sC_d)} \rightarrow sC_d V_2$ — ⑦

$I_a = sC_a(V_0 - V_1) \rightarrow V_1 = \frac{-I_a}{sC_a} + V_0$

$I_b = \frac{V_0 - V_2}{R_b} \rightarrow V_2 = V_0 - I_b R_b$

$I_c = \frac{V_2 - V_1}{R_c} \rightarrow V_2 = V_1 + I_c R_c$

$I_d = sC_d V_2 \rightarrow V_2 = \frac{I_d}{sC_d}$

b) $\frac{V_1(s)}{V_0(s)}$ $\qquad \frac{V_2 - V_1}{R_c} = -sC_a(V_0 - V_1)$

ⓐ — $V_2 = V_1 - sR_cC_a(V_0 - V_1) = \left[1 + sR_cC_a\right]V_1 - sR_cC_aV_0$

ⓑ — $\frac{V_0 - V_2}{R_b} = \frac{V_2 - V_1}{R_c} + sC_d V_2 \rightarrow V_2\left[\frac{1}{R_b} + \frac{1}{R_c} + sC_d\right] = \frac{V_0}{R_b} + \frac{V_1}{R_c}$

$a + b \rightarrow$

$\left[(1 + sR_aC_a)\left(\frac{1}{R_b} + \frac{1}{R_c} + sC_d\right) - \frac{1}{R_c}\right]V_1 = \left[\frac{1}{R_b} + sR_cC_a\left[\frac{1}{R_b} + sR_cC_a\left[\frac{1}{R_b} + \frac{1}{R_c} + sC_d\right]\right]\right]V_0$

$\left[(1 + sR_aC_a)(R_b + R_c + sR_bR_cC_d) - R_b\right] - R_b]V_1 = \left(R_c + sR_cC_a(R_b + R_c + sC_dR_bR_c)\right)$

$$\frac{V_1(s)}{V_0(s)} = \frac{R_c\left[1 + sC_a(R_b + R_c + sR_bR_cC_d)\right]}{(1 + sR_aC_a)(R_b + R_c + sR_bR_cC_d) - R_b}$$

c) $R = 2R$, $C_b = C_c = C$, $R_d = R/2$

T-junction

(a) — $\frac{V_2 - V_0}{(1/sC)} + \frac{V_2}{(R/2)} + \frac{V_2 - V_1}{(1/sC)} = 0 \rightarrow \left[2sC + \frac{2}{R}\right]V_2 = (V_1 + V_0)sC$

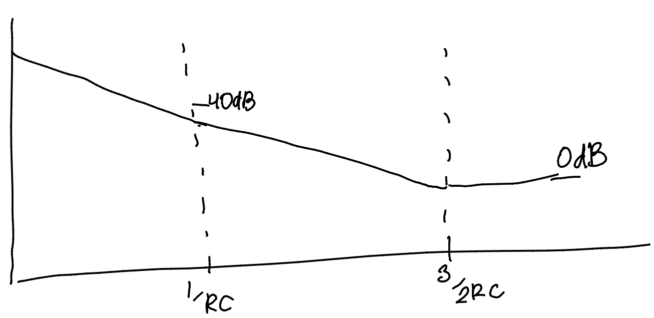
(b) — $\frac{V_0 - V_1}{2R} + \frac{V_0 - V_2}{(1/sC)} = 0 \rightarrow sCV_2 = \left(\frac{1}{2R} + sC\right)V_0 + \frac{1}{2R}V_1$

$(a + b) \rightarrow \frac{\left(2sC + \frac{2}{R}\right)\left(\frac{1}{2R} + sC\right)V_0 + \frac{V_1}{2R}}{sC} = (V_1 + V_0)sC$

$\rightarrow \left(1 + \frac{sRC}{R}\right) \cdot \left[\frac{1 + 2sRC}{R}V_0 + \frac{V_1}{R}\right] = (V_1 + V_0)s^2C^2$

$(1 + sRC - s^2R^2C^2)V_1 = \left[s^2R^2C^2 - (1 + 3sRC + 2s^2R^2C^2)\right]V_0$

$$\frac{V_1(s)}{V_0(s)} = \frac{s^2R^2C^2 + 3sRC + 1}{s^2R^2C^2 - sRC - 1} \rightarrow \left|\frac{V_1(w)}{V_0(w)}\right| = \left|\frac{1 - w^2R^2C^2 + j3wRC}{w^2R^2C^2 - 1 - jwRC}\right| = \left|\frac{\sqrt{(1 - w^2R^2C^2)^2 + 9w^2R^2C^2}}{\sqrt{(1 - w^2R^2C^2)^2 + w^2R^2C^2}}\right|$$



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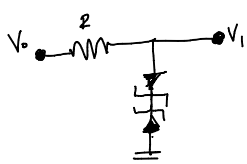


$V_{br} = -7.5V$ $V_d = 0.7V$

a) $V_0(t) = A\sin wt$

$V_1(t) = V_0(t)$ $A < V_d = 0.7V$

for $A < 0.7V$, $V_1(t) = A\sin wt$

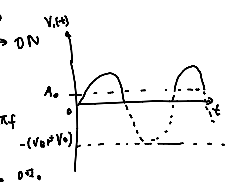
$A_0 = 0.7V$

current through zener diodes...

$I_z = 0$

b) $A \geq A_0$

$Z_1 \rightarrow ON$



- $V_1(t)$
- $w = \frac{2\pi}{T} = 2\pi f$

  $T \propto (1/w)$
- $I_z(t) \begin{cases} A < A_0 & 0 = I_0 \\ A < A_0 & : I_{leakage} \end{cases}$
- R acts as a current limiter across the diode that protects zener diodes from high currents $R = (V_0 - V_1)/I_z \rightarrow R_{max} = (V_0 - V_1)/I_{z\,min}$