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Using mixture design and neural networks to build stock selection decision support systems

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Abstract There are three disadvantages of weighted scoring stock selection models. First, they cannot identify the relations between weights of stock-picking concepts and performances of portfolios. Second, they cannot systematically discover the optimal combination for weights of concepts to optimize the performances. Third, they are unable to meet various investors' preferences. This study aimed to more efficiently construct weighted scoring stock selection models to overcome these disadvantages. Since the weights of stock-picking concepts in a weighted scoring stock selection model can be regarded as components in a mixture, we used the simplex-centroid mixture design to obtain the experimental sets of weights. These sets of weights are simulated with US stock market historical data to obtain their performances. Performance prediction models were built with the simulated performance data set and artificial neural networks. Furthermore, the optimization models to reflect investors' preferences were built up, and the performance prediction models were employed as the kernel of the optimization models, so that the optimal solutions can now be solved with optimization techniques. The empirical values of the performances of the optimal weighting combinations generated by the optimization

models showed that they can meet various investors' preferences and outperform those of S&P's 500 not only during the training period but also during the testing period.

Keywords Neural networks · Design of experiments · Multifactor · Weighted scoring · Stock selection

1 Introduction

It is suggested that a stock's price rises or falls simultaneously following changes of all related factors in the market according to the efficient market hypothesis [9]. Thus, there should be no excess returns besides the risk premium. However, previous studies [1, 13, 14, 24] found that the return of investment portfolios can be increased by adopting the appropriate factors in stock selection because the stock markets may not be semi-strong-form efficient or weak-form efficient in some periods. The size effects of Banz [4] identified that return rates of small-size firms' stocks tend to outperform those of large-size firms' stocks. The momentum effect observed by Jegadeesh and Titman [16] suggested that, empirically, there is a tendency for rising asset prices to rise further and falling asset prices to keep falling. For instance, it was shown that stocks with a strong past performance continue to outperform stocks with poor past performance during the next period. The value effect of Fama and French [10] and Rosenberg et al. [27] indicates that return rates of value stocks were much higher than those of growth stocks.

An increasing number of recent literature [11, 20, 24, 26, 32] focuses on building multifactor stock selection models by combining the effects mentioned above to enhance investment return. In these studies, the scoring approach may be a

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relatively simple but effective way of building multifactor stock selection models. For instance, the steps of two-factor (*B/P* and ROE) scoring approach consists of:

- Step 1. *Factor scoring* Stocks are sorted by book value-to-price ratios (*B/P*). The stock with the greatest (or smallest) value of *B/P* gets a grade value of 100 (or 0). The interpolation method is applied to the rest of stocks. Stocks are also sorted by return on equity (ROE). The stock with the greatest (or smallest) value of ROE gets a grade value of 100 (or 0). The interpolation method is also applied to the rest of stocks.
- Step 2. *Overall scoring* The overall score is obtained by adding up the score of *B/P* and that of ROE. Thus, a stock with a higher overall score implies that its firm is profitable but its price is relatively cheap, i.e., undervalued, while a stock with a lower score implies that it is overvalued. The stock with the highest overall score is regarded as the best stock and vice versa.

Though this scoring approach is useful, it does not take the weightings of factors (B/P and ROE) into consideration. A weighted scoring approach is able to improve and to further optimize the performance of multifactor stock selection models. The steps of the weighted scoring approach consist of:

- Step 1. *Factor screening* Factors identified as useful in scoring the stocks are chosen.
- Step 2. Factor scoring Stocks are sorted by each factor. If we expect that the greater the factor, the better the performance of the stock, the stock with the greatest (or smallest) value of the factor gets a grade value of 100 (or 0). The interpolation method is applied to the rest of the stocks. On the other hand, if we expect that the smaller the factor, the better the performance of stock, the stock with the greatest (or smallest) value of the factor gets a grade value of 0 (or 100). The interpolation method is also applied to the rest of the stocks.
- Step 3. Weight assigning Each of the factors chosen is assigned with a certain weight. The summation of the weights of all the chosen factors should be 100 %
- Step 4. *Overall weighted scoring* The overall weighted score of each stock can be obtained by its factor scores and the factor weights. The stock with the highest overall weighted score is regarded as the best stock and vice versa.

A lot of previous researches applied this weighted scoring approach in constructing their multifactor stock

selection model to maximize the investment return. However, this approach sets up the factor weights subjectively or uses a simple average. It therefore has two disadvantages: First, it cannot identify the relations between the weights of stock-picking factors and the performances of portfolios. Second, it cannot systematically discover the optimal combination for weights of factors to optimize the performances.

Furthermore, as investors' acceptable level of investment risk, financial capabilities, and investment preferences vary, so naturally, their emphases on performances of portfolios are different as well. Investors with higher-risk tolerance may adopt return as the most important performance indicator, while investors with lower-risk tolerance may choose risk as the most important performance indicator. The stock selection models which only seek to maximize return are apparently unable to meet various investors' preferences.

In this paper, we apply the mixture design in constructing the stock selection models to overcome the disadvantages of the methodologies employed by early literature mentioned above. A mixture design is a kind of design of experiments (DOEs). In a mixture design, the independent factors are proportions of different components of a blend. For example, if you attempt to optimize the mixture of concrete, the factors of experiment may be the proportions of cement, sand, gravel, and water in the concrete. The fact that the proportions of the different factors must sum to 100 % complicates the design and analysis of mixture experiments. The objective of the mixture of concrete may be to minimize cost, and its constraint may be to satisfy the requirements of the strength of concrete.

In the process of building the optimal stock selection model, if the weight of the stock-picking factor is regarded as the component in the mixture design, the combination of weights may be regarded as the mixture formula in the mixture design. The objective of this mixture is the investors' expectations, such as return; the constraints of this mixture are the investors' requirements, such as risk and liquidity. Thus, to build the optimal stock selection models, in this study the mixture design is employed to discover the optimal combinations of weights, which can optimize the investors' expectations and satisfy the investors' requirements.

This study aims to build a stock selection decision support system to generate the optimal stock selection models to meet various investors' preferences. The inputs of the system include:

 Objective: the performance of the portfolio that needs to be optimized as the forms:
 maximize performance or minimize performance



 Constraints: the performance of the portfolio that must be satisfied as the forms:
 performance ≥ lower limit, or performance ≤ upper limit.

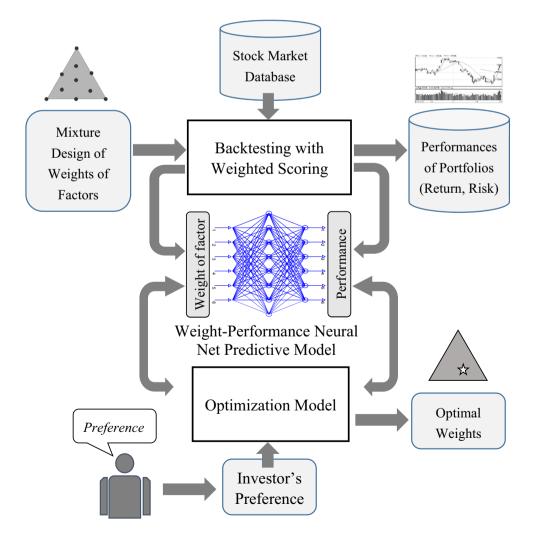
The performances comprise annualized return rates, excess return rates, systematic risk, etc. The outputs of the system are the optimal weights of stock-picking concepts, which include large *B/P* and large ROE. Thus, the system is able to produce customized weights of stock-picking concepts to form the portfolios whose characteristics may meet various specific investors' preferences.

The procedure of building the stock selection decision support system in this paper is as follows (refer to Fig. 1):

Step 1. Generate weighting combinations of stockpicking concepts with a mixture design A set of weighted combinations (x) of stock-picking concepts is generated with a mixture experimental design.

Fig. 1 Diagram of stock selection decision support system

- Step 2. Simulate weighting combinations of stock-picking concepts through backtesting The investment performances (y) of the weighting combinations of stock-picking concepts are obtained by conducting a backtest through stock market historical data. These results can be collected and matched as (x, y) to build up the data set.
- Step 3. Build and analyze the performance prediction model with neural networks The performance prediction model, y = f(x), can be built with neural networks through the data set. The relations between the weights and the performances can be obtained by analyzing the predictive model.
- Step 4. Seek the optimal weighting combinations of stock-picking concepts through optimization A set of various investors' preferences is employed to build up the optimization models. Their optimal weighting combinations can be obtained by optimization techniques.





Step 5. Validate the optimal weighting combinations of stock-picking concepts through backtesting The empirical performances of the optimal weighting combinations are obtained by conducting a backtest through the stock market's historical data. These results can be used to validate whether the optimal weighting combinations can meet various investors' preferences.

The remainder of the paper is structured as follows: Sect. 2 reviews previous literature. Section 3 describes generating weighting combinations of stock-picking concepts with mixture design and simulating them through backtesting. Section 4 introduces building and analyzing the performance prediction model with neural networks. Sections 5 and 6 present seeking and validating the optimal weighting combinations of stock-picking concepts through backtesting. Section 7 concludes the paper.

2 Literature review

2.1 Multifactor stock selection models

Hart et al. [11] examined the profitability of various stock selection strategies in 32 emerging markets over the period from 1985 to 1999. The empirical results found that the value, momentum, and earnings revisions strategies are the most successful and generate significant excess returns, in contrast to strategies based on size, liquidity, and mean reversion. Thus, the effectiveness of the strategies can be enhanced by selecting stocks based on multiple characteristics.

Mohanram [17] combined traditional fundamentals such as earnings and cash flows with measures appropriate for growth firms such as earning stability and growth stability to create a G-SCORE index for long-short strategies. They concluded that investors could use a modified fundamental analysis strategy to identify mispriced stocks and earn substantial abnormal returns.

Noma [22] combined traditional fundamentals such as return on assets, cash flow from operations and operation margins as a F-SCORE index. The F-SCORE was applied and demonstrated that the mean return could be increased by 7.8 % through a hedging strategy that buys high F-SCORE firms and that shorts low F-SCORE firms. In addition, an investment strategy that buys high book-to-price ratio firms with a high F-SCORE and shorts low book-to-price firms with a low F-SCORE earns a 17.6 % annual return. The empirical result also reveals that the F-SCORE can predict future earnings.

In addition, some advanced methodologies have opted to construct more effective multifactor stock selection models. For example, some studies applied neural networks [5, 6, 8, 23, 25], some employed regression trees [26, 28, 30], and others adopt hybrid approaches [7, 15, 21, 29]. The related methodologies used and their empirical results were compared and reviewed in the studies of Atsalakis and Valavanis [2] and Bahrammirzaee [3].

2.2 Design of experiments

The design of experiments is one branch derived from mathematical statistics. It is often used to investigate the effects of some factors on some objects. The design of experiments is thus a discipline that has a very broad application across all the natural and social sciences and engineering [18, 19]. Furthermore, in a mixture experiment, the independent factors are proportions of different components of a blend. It, therefore, has several limitations as follows:

$$x_i \ge 0, \quad i = 1, 2, ..., q$$
 (1)

$$\sum_{i=1}^{q} x_i = x_1 + x_2 + \dots + x_q = 1$$
 (2)

where q stands for the number of independent factors.

The use of a Cartesian coordinate system is not appropriate due to the fact that the components of mixture designs are subjected to the constraint that they must sum to one. A simplex coordinate system (see Fig. 2) is frequently employed instead. Furthermore, there are standard mixture designs for fitting standard models, such as a simplex-centroid design, which is widely applied among various forms of mixture experiments. There are 2^q-1 experiments for a mixture with q components in a simplex-centroid design, including.

1-component designs: It consists of different combinations of 1 and 0:

$$(1, 0, 0, \ldots, 0), (0, 1, 0, \ldots, 0), \ldots, (0, 0, 0, \ldots, 1).$$

2-component designs: It consists of different combination of 1/2 and 0:

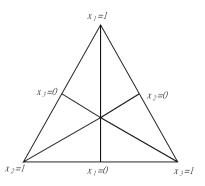


Fig. 2 Simplex coordinate system



$$(1/2, 1/2, 0, 0, \ldots, 0), (1/2, 0, 1/2, 0, \ldots, 0), \ldots, (1/2, 0, 0, \ldots, 1/2).$$

3-component designs: It consists of different combination of 1/3 and 0:

$$(1/3, 1/3, 1/3, 0, 0, ..., 0), (1/3, 1/3, 0, 1/3, 0, ..., 0), ..., (1/3, 1/3, 0, ..., 1/3).$$

q-component design: (1/q, 1/q, ..., 1/q).

Plots of 3- and 4-component mixture design in the simplex coordinate system are displayed in Figs. 3 and 4.

After conducting all the experiments in the mixture design, response data can be collected and matched as pairs, proportions of components and responses, to build up the data set. Since the measured response is assumed to depend only on the relative proportions of the components in the mixture, the response prediction models can be built up using the data set and modeling tools such as regression analysis and neural networks.

After building the response prediction models, we could then maximize or minimize some responses and/or satisfy some responses through adjusting the proportions of components by optimization tools such as mathematical programming techniques. Detailed modeling tools and optimization tools can be found in the literatures [18, 19].

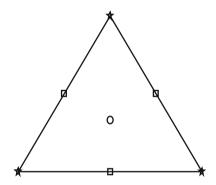


Fig. 3 Simplex-centroid design with 3 components

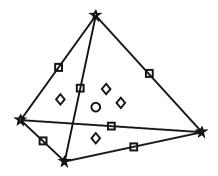


Fig. 4 Simplex-centroid design with 4 components

2.3 Artificial neural networks

In a stock market, the relations between the weights of stock-picking concepts and portfolio performances are often nonlinear. The greatest advantage of artificial neural networks is their native nonlinear system characteristic, which makes them able to build various nonlinear models. A multilayered perceptron (MLP) may be the most popular paradigm of artificial neural networks. MLPs adjust their weights and biases by learning rules so as to construct accurate nonlinear models between the input variables and the output variables. Detailed algorithms can be found in the literature [12].

Although MLP neural networks can build accurate nonlinear models, they are black-box models, which make users unable to easily understand the effects of each input variable on each output variable. Hence, they lack explanatory ability. To overcome this limitation, many approaches have been proposed. The most simple but rather effective approach may be the weight approach. The effect (or sensitivity) of the *i*-th factor (input variable) on the *j*-th response (output variable) can be estimated by the following formula [31]:

$$S_{ij} = \sum_{k} W_{kj} \cdot W_{ik} \tag{3}$$

where W_{ik} is the connection weight between the *i*-th unit in the input layer and the *k*-th unit in the hidden layer; W_{kj} is the connection weight between the *k*-th unit in the hidden layer and the *j*-th unit in the output layer.

3 Mixture design and backtesting of weighting combinations of concepts

3.1 Performance indicators

Two types of indicators, return and risk, are frequently used to evaluate investment performances of portfolios. The return indicators include:

Annualized return rate (ARR)

$$ARR = \left(\left(1 + R \right)^{\frac{1}{i}} \right) - 1 \tag{4}$$

where R is the accumulated return rates; t is the duration in years.

• Excess return rate α

It can be estimated from the regression coefficient value α in the following regression equation. If the excess return rate is >0, it signals that the portfolio return outperforms the market return.

$$R_{\rm i} - R_{\rm f} = \alpha_i + \beta_i (R_{\rm m} - R_{\rm f}) \tag{5}$$



where $R_{\rm f}$, risk-free return rate; $R_{\rm m}$, market return rate; $R_{\rm i}$, investment portfolio return rate.

Absolute winning rate

$$Win_{abs} = \frac{n_1}{N}$$
 (6)

where n_1 is the number of portfolio holding periods when the portfolio return rate is above 0; N is the total number of portfolio holding periods.

• Relative winning rate

$$Win_{rel} = \frac{n_2}{N} \tag{7}$$

where n_2 is the number of portfolio holding periods when the portfolio return rate beats the market return rate.

The risk indicators include:

Systematic risk β
 It can be estimated from the regression coefficient value β in the above regression Eq. (5). The larger the coefficient value is, the higher the systematic risk of the

Total risk σ

portfolio is.

It can be estimated from the standard deviation of the return rate of the portfolio. It signals the volatility of the return rate during a certain period. The larger the volatility of the return rate, the higher the total risk of the portfolio.

3.2 Stock-picking factors

Stock-picking factors often used in the previous literature are adopted in this paper. They include:

- *Value factors* The return rates of undervalued stocks tend to outperform those of overvalued stocks. Earning-to-price ratios (*E/P*), book value-to-price ratios (*B/P*), sales-to-price ratios (*S/P*) are often used to evaluate whether the firm's stock is undervalued or not. The larger the above ratios are, the higher the possibility that the stock is undervalued.
- Growth factors The return rates of stocks of profitable firms tend to outperform those of non-profitable firms. Return on equity (ROE) is often used to evaluate a firm's profitability. The larger the ROE is, the more profitable the firm is.
- Momentum factors The return rates of stocks may appear as reversion in the short term, momentum in the midterm, and reversion in the long term. Reversion indicates that if the return rate of the stock is currently high, it will become lower in the future. Momentum indicates that if the return rate of the stock is currently

- high, it will continue to go higher in the future. Since the rebalance period of the portfolio in this study is onequarter, which may be regarded as a midterm, momentum may be adopted as a stock-picking concept.
- Scale factors The three-factor model of Fama and French (1998) pointed out that there is negative relationship between the scale (market capitalization) of a firm and the return rate of the firm's stock. Thus, the return rates of small-size firms' stocks tend to outperform those of large-size firms' stocks. Moreover, it is expected that the smaller the size of the firm, the lower the liquidity and the higher the risk of the firm's stock.
- Risk factors Beta (β) measures the fluctuation in stock returns relative to benchmarks (market); that is, systematic risks. If a stock's β is >1, its fluctuation in return is greater than the benchmark, and vice versa. The beta value of a stock might be persistent; that is, stocks with currently large (small) β would typically have large (small) β in the near future.

In this study, six factors have been chosen to conduct the stock selection: book value-to-price ratio, sales-to-price ratio, return on equity, return rate in the last quarter, market capitalization, and systematic risk.

3.3 Stock-picking concepts

It is expected that the greater the book value-to-price ratio, sales-to-price ratio, return on equity, and return rate in the last quarter, the higher the return rates of the stocks. Hence, the stock with the greatest (or smallest) values of these factors gets a grade value of 100 (or 0).

It is expected that the smaller the size of the firm, the higher the return rate of the firm's stock. However, to some investors, the return rate is not the only important performance indicator due the investors' expectations on high liquidity and low risk. We thus suggest that the stock with the greatest (or smallest) market capitalization gets a grade value of 100 (or 0).

According to classic theory, it is expected that the larger the systematic risk, the higher the return rate of stock. However, since most investors tend to be risk averse, they may prefer investing in stocks demonstrating characteristics of low systematic risk. Due to the characteristic of persistence of systematic risk, we suggest that the stock with the smallest (or greatest) systematic risk gets a grade value of 100 (or 0).

To sum up, we adopt six stock-picking concepts in this paper: large book value-to-price ratio (B/P), large sales-to-price ratio (S/P), large return on equity (ROE), large return rate in the last quarter, large market capitalization, and small systematic risk. The main duty of the first four concepts is to



select the stocks with the characteristics of high return rates. The main duty of the large market capitalization concept is to select the stocks with the characteristics of high liquidity and low risk. Finally, the main duty of the small systematic risk concept is to select the stocks with characteristics of low risk in the next holding period.

3.4 Experimental design

In this study, to systematically collect experimental data, a design of experiments is employed. Since the weights of stock-picking concepts in a stock selection model must sum to one, a mixture experimental design is adopted. The weight of stock-picking concepts is regarded as the components of a mixture. As aforementioned, there are six factors adopted in this study, so in total, the number of experiments required to be conducted is up to $2^6 - 1 = 63$ in the simplex-centroid mixture design. Therefore, it comprises 63 different weighting combinations. There are seven levels of weights for each stock-picking concept: 1, 1/2, 1/3, 1/4, 1/5, 1/6, and 0.

3.5 Experimental implementation

The performances of the 63 weighting combinations of stock-picking concepts with a weighted scoring approach were obtained by simulating them with Standard and Poor's Compustat US database. Sorted by a weighted scoring approach, the stocks with the top 10 % overall weighted scores are selected to form the investment portfolio. The holding and rebalancing period of the investment portfolio in this study is one-quarter. The backtest period includes 80 quarters, from 1990/Q4 to 2010/Q3. Through the backtesting, each weighting combination can get 80 quarterly return rates of the portfolio. Six performance indicators can be computed based on these quarterly return rates: the annualized return rate, the excess return rate, the absolute winning rate, the relative winning rate, total risk, and systematic risk. Except for the annualized return rate, the period units of the other five indicators are quarters.

4 Building the performance predictive model with neural networks

4.1 Dividing data set with moving time-frame method

Since neural networks are applied to construct the model, they must be trained, and to train the neural networks, the data set must first be divided into a training set and a testing set. There are certain relations between the weights of stock-picking concepts and the performance indicators of portfolios. For instance, the *B/P* and the annualized return rate may frequently have a positive relation, but this might depend on time. Hence, it is necessary to take time factors into account in dividing the data into training data and testing data.

The holding and rebalancing period of the investment portfolio in this study is one-quarter. The total backtest period covers 80 quarters (20 years). Each quarter starts from the end of March, June, September, and December. For example, the first quarter starts from the end of September and goes to the end of December in 1990. The last quarter starts from the end of June 2010 and goes to the end of September 2010. Hence, a moving time-frame approach is employed to divide the backtest period into four periods, and each period has 20 quarters (5 years) as given in Table 1.

4.2 Normalization of performance indicators

There are a lot of factors possibly affecting the stock investment return. They can be divided mainly into two types: characteristics of the individual firm and trends of the stock market. The multifactor weighted scoring approach can cover the characteristics of each firm's stock by weighting its factors. However, it cannot reflect the overall market tendency. When the stock market is in the down (up) tendency, even the stocks of the firms with good (poor) characteristics have low (high) return rates. Hence, the weighted scoring approach can only affect the relative return rates of firms' stocks but cannot affect the absolute return rates. Therefore, a precise performance prediction model cannot be built up if the dependent variables, performance indicators such as annualized return rate, are not normalized beforehand.

Each of the performance indicators adopted in this study is all normalized into the same scale during the same time frame of 5 years (20 quarters) as aforementioned. Since the sigmoidal transfer function is applied to construct the neural network prediction models, it is advisable that the output values avoid the saturation regions of the sigmoidal function, so the scale has a range of 0.2–0.8. For example, when one weighting combination has the maximum (or minimum) annualized return rate during one 5-year time frame among the 63 weighting combinations designed by the simplex-centroid design, its normalized annualized return rate is to be the maximum of 0.8 (or minimum of 0.2). The linear interpolation method is applied to the rest of the weighting combinations.

4.3 Building performance predictive model with neural networks and cross-validation

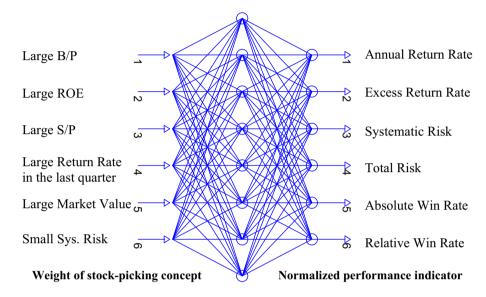
The cross-validation methodology is employed to evaluate the performance of the prediction model built with neural



Table 1 Moving time frame of the backtest period

| Time frame | The beginning time of the 1st holding period | The beginning time of the 20th holding period | The length of period | | |
|----------------|--|---|-----------------------|--|--|
| The 1st period | September 1990 | June 1995 | 20 quarters (5 years) | | |
| The 2nd period | September 1995 | June 2000 | 20 quarters (5 years) | | |
| The 3rd period | September 2000 | June 2005 | 20 quarters (5 years) | | |
| The 4th period | September 2005 | June 2010 | 20 quarters (5 years) | | |

Fig. 5 Performance predictive model built with neural networks



networks, as shown in Fig. 5. That is, when the data set of one of the four time frames is adopted as the training data set, the rest are adopted as the testing data set. The correlation coefficients between actual values and values predicted by the neural networks are displayed in Table 2. These coefficients imply that the predictability of most models during the period of this study is unstable. This is consistent with the empirical findings of previous literature; it is difficult to predict future investment performances. Nonetheless, the coefficients in Table 2 provide some intriguing results.

First, the predictability of the excess return rates is greater than that of the annualized return rates. Normally, there are two methodologies that can increase portfolio return: stock selection and market timing. Excess return rates are attributed to stock picking. On the other hand, annualized return rates are both affected by stock picking and market timing. Therefore, the predictability of the excess return rates is greater than that of the annualized return rate because individual firm's fundamental analysis is only effective in stock picking but not useful in market timing.

Second, the predictability of total risk is greater than that of systematic risk. Stock market risk can be normally divided into systematic and non-systematic risk.

Systematic risks are highly affected by market timing. On the other hand, non-systematic risks are mainly affected by stock picking. Thus, the predictability of total risk is greater than that of systematic risk because individual firm's fundamental analysis is only effective in stock picking.

Third, the predictability of the relative winning rates is greater than that of the absolute winning rates. The relative winning rates are only affected by stock picking. The absolute winning rates, however, are affected by both stock picking and market timing. Again, the reason is that individual firm's fundamental analysis is only effective in stock picking.

4.4 Effects of weights of stock-picking concepts on performance indicators

Since a cross-validation methodology is employed to evaluate the performances of the prediction models and there are four time frames, four neural network-based prediction models were built. Equation 3, as aforementioned, is employed to explore the effects of weights of each stock-picking concept on performance indicators of portfolios implied in the four neural network-based prediction models. The results are given in Table 3.



Table 2 Correlation coefficients of the predictive models in testing periods

| Backtest period | Training period Testing period | 1st 2nd | 1st 3rd | 1st 4th | 2nd 3rd | 2nd 4th | 3rd 4th | The average value |
|---|-----------------------------------|------------|------------|------------|------------|------------|------------|-------------------|
| Investment performance indicator (normalized) | Annual return | -0.224 | 0.819 | 0.265 | -0.548 | 0.100 | 0.469 | 0.147 |
| | Excess return | 0.141 | 0.911 | 0.400 | 0.000 | 0.173 | 0.424 | 0.342 |
| | Systematic risk | 0.742 | -0.100 | -0.300 | 0.100 | -0.224 | 0.200 | 0.070 |
| | Total risk | 0.742 | 0.557 | 0.141 | 0.469 | 0.000 | 0.332 | 0.373 |
| | Abs. win rate | -0.200 | 0.529 | -0.200 | 0.000 | 0.374 | -0.173 | 0.055 |
| | Rel. win rate | 0.436 | 0.346 | 0.141 | 0.000 | 0.000 | 0.458 | 0.230 |
| The average value | | 0.273 | 0.510 | 0.075 | 0.004 | 0.071 | 0.285 | |

Table 3 Effects of weights of stock-picking concepts on performance indicator

| Performance indicator | Training period | Stock-picking concept | | | | | | | | | |
|------------------------|-----------------|-----------------------|-------|-------|------------------|--------------|-------|--|--|--|--|
| | | B/P | ROE | S/P | Quarterly return | Market value | Beta | | | | |
| Annualized return rate | The 1st period | 8.4 | 7.1 | 3.3 | -11.3 | -21.2 | 5.2 | | | | |
| | The 2nd period | -7.1 | 9.5 | -4.5 | 9.4 | 4.1 | -3.6 | | | | |
| | The 3rd period | 14.7 | 2.6 | 16 | -9.3 | -11.9 | -1.3 | | | | |
| | The 4th period | -7.6 | 5.5 | 11.5 | 6.7 | 13.6 | -18.4 | | | | |
| | Average | 2.1 | 6.2 | 6.6 | -1.1 | -3.9 | -4.5 | | | | |
| Excess return rate | The 1st period | 19.9 | 1.9 | 19.8 | -9.3 | -9.2 | -10.8 | | | | |
| | The 2nd period | -1.3 | 12.2 | -13.4 | 5.3 | 1.3 | -2.1 | | | | |
| | The 3rd period | 13.5 | 2.3 | 15.5 | -7.8 | -11.5 | 0.7 | | | | |
| | The 4th period | -0.9 | -3.4 | 2.5 | 3.6 | -7.8 | -12 | | | | |
| | Average | 7.8 | 3.3 | 6.1 | -2.1 | -6.8 | -6.1 | | | | |
| Systematic risk | The 1st period | -15.9 | 12.3 | -4.1 | 3.3 | 3.5 | 5.8 | | | | |
| | The 2nd period | -18.2 | 4.1 | -2.8 | 8.2 | 3.8 | 3.2 | | | | |
| | The 3rd period | -5 | -15.8 | -0.9 | -2.4 | 7.9 | 16.7 | | | | |
| | The 4th period | -4.4 | 2.7 | 0.9 | 4.8 | 0.8 | -18.6 | | | | |
| | Average | -10.9 | 0.8 | -1.7 | 3.5 | 4.0 | 1.8 | | | | |
| Total risk | The 1st period | -7.3 | 12.4 | 1.4 | 0.1 | -20.3 | 11.1 | | | | |
| | The 2nd period | -16.9 | 0.4 | -8.6 | 16.4 | 1 | 4.8 | | | | |
| | The 3rd period | -2.7 | -11.7 | 1.8 | 10.9 | 3.4 | 14.7 | | | | |
| | The 4th period | -6 | 8.8 | 6.1 | 8.8 | 1.3 | -10.6 | | | | |
| | Average | -8.2 | 2.5 | 0.2 | 9.1 | -3.7 | 5.0 | | | | |
| Absolute winning rate | The 1st period | 16.1 | -14.6 | 15 | -6 | -0.9 | -1.5 | | | | |
| | The 2nd period | -1.4 | 11 | -17.6 | 6.5 | 13.3 | -11.4 | | | | |
| | The 3rd period | 15 | 4.4 | 9 | -9 | -7.8 | -8.4 | | | | |
| | The 4th period | 9.5 | 9.3 | 1.2 | 9.9 | -5.7 | -9.6 | | | | |
| | Average | 9.8 | 2.5 | 1.9 | 0.4 | -0.3 | -7.7 | | | | |
| Relative winning rat | The 1st period | 1.3 | 15 | -0.4 | -0.7 | 0.2 | 0.1 | | | | |
| | The 2nd period | -9.9 | 12.9 | -8.4 | 0.2 | -7.8 | 5.5 | | | | |
| | The 3rd period | 3.5 | 13.8 | 9.2 | -11.5 | 0.6 | -11.8 | | | | |
| | The 4th period | -6.7 | -5.8 | 10.5 | -8.5 | 14.8 | 1.9 | | | | |
| | Average | -3.0 | 9.0 | 2.7 | -5.1 | 2.0 | -1.1 | | | | |

 Annualized return rate It appears that only the weight of the large ROE concept has positive effects on annualized return rates in all the four neural network models, which are, respectively, built with the four different time frames. The larger the weight of the large ROE concept, the higher the annualized return rate.



- Meanwhile, the effects of the rest are unstable in the four time frames.
- Excess return rate The effects of the weighting of each stock-picking concept on the excess return rates in the first time frame and third time frame are similar to each other, but those in the second time frame are rather unique. Broadly speaking, the weights of the large B/P, large S/P, and large ROE concepts have positive effects on excess return; it appears that profitable firms' cheap stocks have high excess return. Contrarily, the weights of the large market capitalization and small systematic risk concepts have negative effects on excess returns; it appears that large-size firms' stocks with low betas have low excess returns. These results are consistent with the earlier studies.
- Systematic risk The results illustrate that only the weight of the large B/P concept has negative effects on systematic risk in all the four time frames. The larger the weight of the large B/P concept, the lower the systematic risk. Meanwhile, the effects of the rest are unstable in the four time frames.
- Total risk The results illustrate that only the weight of the large B/P concept has negative effects and the large return rate in the last quarter concept has positive effects on total risk.
- Absolute winning rate The results illustrate that only the weight of the large B/P concept has positive effects and the small systematic risk concept has negative effects on the absolute winning rate.
- Relative winning rate The results illustrate that only the weight of the large ROE concept has positive effects on the relative winning rate.

5 Generating the optimal weighting combinations of stock-picking concepts

In this section, we attempt to build the optimization models to reflect investors' specific preferences on investment performances and obtain the optimal weights of the stockpicking concepts of these models. The performance prediction model whose training data were collected from the first time frame (1991–1995) has been adopted as the kernel of the optimization models.

5.1 Model 1: maximizing objective without constrain

To meet various investors' preferences, there are six optimization models in total:

- Maximizing annualized return rates,
- Maximizing excess return rates,

- Minimizing systematic risk,
- Minimizing total risk,
- Maximizing absolute winning rates,
- Maximizing relative winning rates.

Their optimal weighting combinations of stock-picking concepts and predictive normalized performance indicators are presented in Table 4. These results are also displayed in Figs. 6 and 7.

- Maximizing annualized return rate Maximization of annualized return rates can be obtained with weights of a large ROE concept of 0.7 and a small systematic risk concept of 0.3. The predictive normalized annualized return rate obtained from the prediction model reached a high level of 0.85. However, the predictive normalized systematic and total risk also reached higher levels simultaneously.
- Maximizing excess return rate The highest excess return rate can be obtained by combining large B/P, large ROE, and small systematic risk concepts. The predictive normalized excess return rate reached a high level of 0.80, but the predictive normalized systematic and total risk did not reach higher levels simultaneously. This discovery is meaningful. When investors attempt to maximize their annualized return rate, they must tolerate higher risk, but when they attempt to maximize their excess return rate, they do not need to tolerate higher risk.
- *Minimizing the systematic risk* The lowest systematic risk can be obtained with weights of a large *B/P* concept of 0.4 and a large return rate in the last quarter concept of 0.6. The predictive normalized systematic risk could be lowered to <0.25, but the annualized return rate would also become lower simultaneously.
- Minimizing the total risk The lowest total risk can be obtained with weights of a large B/P concept of 0.1 and a large market capitalization concept of 0.9. The predictive normalized total risk could be lowered to <0.2. However, the annualized return rate, excess return rate, and absolute winning rate would also become much lower simultaneously.
- *Maximizing absolute winning rate* The highest absolute winning rate can be obtained with the weights of a large *B/P* concept of 0.6 and a large market capitalization concept of 0.4. The predictive normalized absolute winning rate could be higher than 0.6 without increasing systematic or total risk simultaneously.
- Maximizing the relative winning rate The highest relative winning rate can be obtained with the weights of a large ROE concept of 0.7 and a small systematic risk concept of 0.3. The predictive normalized relative winning rate could be higher than 0.8. However, systematic and total risk would also increase simultaneously.



| Optimization model | Weights of the stock-picking concept | | | | | | | Predictive normalized performance | | | | | |
|--------------------------|--------------------------------------|-------|-------|------------------|-----------------|-------|---------------|-----------------------------------|--------------|---------------|-------------|-------------|--|
| | B/P | ROE | S/P | Quart. return | Market value | Beta | Annual return | Excess return | Sys. risk | Total risk | Abs. win | Rel. win | |
| Maximize annual return | 0.000 | 0.704 | 0.000 | 0.000 | 0.000 | 0.296 | 0.854 | 0.486 | 0.774 | 0.683 | 0.531 | 0.823 | |
| Maximize excess return | 0.400 | 0.269 | 0.000 | 0.000 | 0.000 | 0.330 | 0.691 | 0.802 | 0.412 | 0.523 | 0.600 | 0.525 | |
| Minimize systematic risk | 0.393 | 0.000 | 0.000 | 0.607 | 0.000 | 0.000 | 0.402 | 0.685 | 0.223 | 0.326 | 0.563 | 0.468 | |
| Minimize total risk | 0.112 | 0.000 | 0.000 | 0.000 | 0.888 | 0.000 | 0.256 | 0.261 | 0.342 | 0.169 | 0.401 | 0.331 | |
| Maximize absolute win | 0.639 | 0.000 | 0.000 | 0.000 | 0.361 | 0.000 | 0.606 | 0.704 | 0.370 | 0.437 | 0.648 | 0.628 | |
| Maximize relative win | 0.000 | 0.723 | 0.000 | 0.000 | 0.000 | 0.277 | 0.853 | 0.481 | 0.773 | 0.682 | 0.533 | 0.823 | |

Fig. 6 Optimal weights of the stock-picking concepts of Model 1

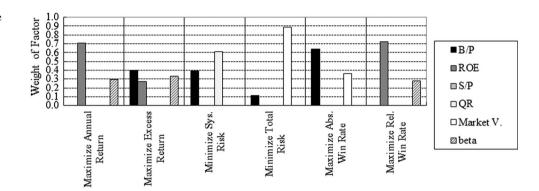
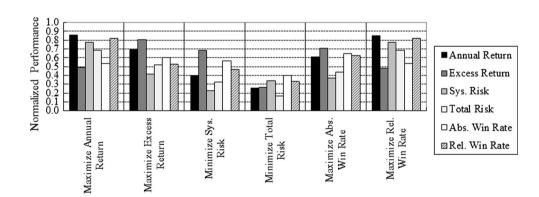


Fig. 7 Predictive normalized performance indicators of Model 1



5.2 Model 2: maximizing return with risk constrain

Investors in general attempt to maximize return and tolerate certain levels of risk. To meet their preferences, an optimization model is built up:

Maximize Annualized return rate
Subjected to Total risk \leq specified upper limit

Since the performance prediction model can only provide predictive normalized performance indicators, the above-specified upper limit is in the range between 0.0 and 1.0. Hence, ten specified upper limits, 0.1, 0.2,..., 0.9, and 1.0, are set up. Their optimal weighting combinations of

stock-picking concepts and predictive normalized performance indicators are presented in Figs. 8 and 9. The findings from the two figures are noteworthy.

- Weightings of stock-picking concepts When lower total risk is required, large-size firms' stocks are chosen. When moderate total risk is tolerable, stocks with large ROE and large B/P are chosen. When high total risk is tolerable, stocks with large ROE and small systematic risk are chosen.
- Predictive normalized performance indicators Predictive normalized annualized return rate becomes higher as the specified upper limit of risk constraint is increased. However, risk also increases simultaneously.



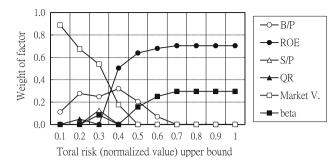


Fig. 8 Optimal weights of the stock-picking concepts of Model 2

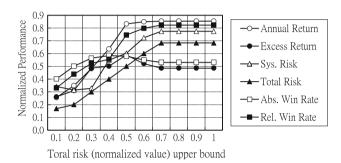


Fig. 9 Predictive normalized performance indicators of Model 2

6 Verifying the optimal weighting combinations of stock-picking concepts

To meet investors' specific preferences on investment performances, we built up optimization models and obtained their optimal solutions, or weighting combinations. In this section, the empirical performances of the optimal weighting combinations were obtained by simulating them with Standard and Poor's Compustat US database.

Since the performance prediction model whose training data set consists of the backtesting performances during the first time frame (1991–1995) was adopted as the kernel of the optimization models, it can be expected that the empirical performance of the optimal weighting combinations may well meet the objectives and constraints of the optimization models in the first time frame. To evaluate the generalization of the optimal solutions, in this section, these optimal solutions would be validated by conducting backtesting not only with the training period data (1991–1995, totally 5 years) but also with the testing period data (1996–2010, total 15 years). The performances of Standard and Poor's 500 were employed as the benchmark.

6.1 Model 1: maximizing objective without constrain

The empirical results of the optimal weighting combinations in Sect. 5.1 are displayed in Table 5. Empirical findings in the training period and the testing period are noteworthy.

6.1.1 Training period (1991–1995)

- Maximizing the annualized return rates The annualized return rate (28.7 %) obtained by the optimization model is higher than that of Standard and Poor's 500 (17.6 %) during the same period.
- Maximizing the excess return rates The quarterly excess return rate (2.6 %) is significantly larger than zero by using the quarterly returns of S&P 500 as the benchmark.
- *Minimizing the systematic risk* The systematic risk (0.78) is significantly smaller than 1.0 by using the quarterly returns of S&P 500 as the benchmark.
- Minimize the total risk σ The total risk. The quarterly total risk (4.5 %) is almost the same as that of S&P 500 (4.6 %), but it is much lower than those of the other five optimization models.
- Maximizing the absolute winning rate The absolute winning rate (80 %) is the same as that of S&P 500 during the same period.
- *Maximizing the relative winning rate* The relative winning rate (80 %) is >50 % by using the quarterly returns of S&P 500 as the benchmark.

Thus, the results of the six optimization models met their goals well in the training period.

6.1.2 Testing period (1996–2010)

- Maximizing the annualized return rates The annualized return rate (12.6 %) obtained by the optimization model is greater than that of Standard and Poor's 500 (7.0 %) during the same period.
- Maximizing the excess return rates The quarterly excess return rate (1.5 %) is significantly >0 by using the quarterly returns of S&P 500 as the benchmark.
- Minimizing the systematic risk The systematic risk (1.08) is >1.0 by using the quarterly returns of S&P 500 as the benchmark.
- *Minimize the total risk* The total risk. The quarterly total risk (9.8 %) is greater than that of S&P's 500 (9.0 %), but it is lower than those of the other five optimization models.
- Maximizing the absolute winning rate The absolute winning rate (68.3 %) is greater than that of S&P's 500 (66.5 %) during the same period.
- Maximizing the relative winning rate The relative winning rate (62.0 %) is >50 % by using the quarterly returns of S&P 500 as the benchmark.



Table 5 Backtesting performances of Model 1

| Optimization model | Training period (the 1st term) | | | | | | | Testing period (the 2nd to the 4th term) | | | | | |
|--------------------------|--------------------------------|-----------|--------------|---------------|-------------|-------------|--------------|--|--------------|---------------|-------------|-------------|--|
| | Ann. ret. | Exc. ret. | Sys. risk | Total risk | Abs. win | Rel. win | Ann. ret. | Exc. ret. | Sys. risk | Total risk | Abs. win | Rel. win | |
| Maximize annual return | 28.7 | 1.1 | 1.39 | 8.4 | 85.0 | 80.0 | 12.6 | 1.37 | 1.00 | 10.7 | 60 | 55 | |
| Maximize excess return | 26.7 | 2.6 | 0.88 | 6.6 | 85.0 | 60.0 | 12.6 | 1.50 | 0.84 | 10.7 | 72 | 55 | |
| Minimize systematic risk | 20.3 | 1.7 | 0.78 | 5.9 | 80.0 | 55.0 | 12.4 | 1.32 | 1.08 | 12.5 | 62 | 52 | |
| Minimize total risk | 17.2 | 0.3 | 0.93 | 4.5 | 85.0 | 55.0 | 8.0 | 0.39 | 1.05 | 9.8 | 67 | 50 | |
| Maximize absolute win | 25.9 | 2.0 | 0.99 | 6.1 | 80.0 | 65.0 | 11.7 | 1.16 | 1.15 | 13.1 | 68 | 52 | |
| Maximize relative win | 27.8 | 0.9 | 1.38 | 8.3 | 85.0 | 80.0 | 12.5 | 1.35 | 1.01 | 10.8 | 60 | 62 | |

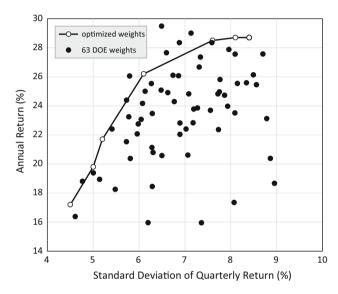


Fig. 10 Empirical values of the performances of Model 2 during the training period

To sum up, the empirical results of the six optimization models used during the testing period outperformed those of S&P's 500 in the same period except those of minimizing the systematic risk and total risk. In addition, in the case of minimizing total risk, although the risk is greater than that of S&P's 500, it is smaller than those of the other five optimization models. Thus, the model is still effectively able to lower the total risk.

6.2 Model 2: maximizing return with risk constrain

The empirical results of the optimal weighting combinations in Sect. 5.2 are displayed in Figs. 10 and 11, whose horizontal axes are the standard deviations of the quarterly return rates, while the vertical axes are the annual return rates. The scattered black dots in the figures comprise the performances of the 63 weighting combinations generated by the design of experiments. The white dots on the broken line stand for the performances of the ten weighting

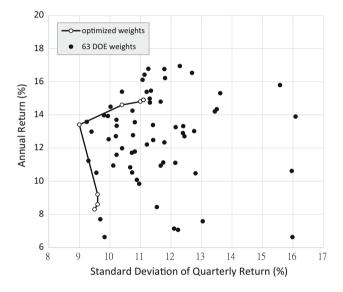


Fig. 11 Empirical values of the performances of Model 2 during the testing period

combinations generated by the optimization models under ten different specified upper limits of the risk constraint. Hence, there should be ten dots on the broken line in the two figures, but only seven dots appear. That is because the optimal weighting combinations are the same when the upper limit of the normalized total risk is beyond 0.7 (0.7, 0.8, 0.9, 1.0). Thus, there are four dots overlapping on the upper right end of the broken line.

These results provide important implications as follows:

- Training period (1991–1995) The empirical values for the performances of the optimized weighting combinations lie on the efficient frontier. Thus, the goal of maximizing the return under various levels of risk constraint during the training period can be obtained by employing optimization models.
- Testing period (1996–2010) The empirical values for the performances of the optimized weighting combinations also nearly lie on the efficient frontier. This



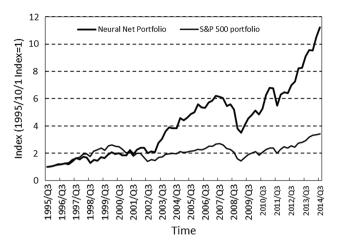


Fig. 12 Accumulative return of the maximizing annualized return rates model

proves that the return maximization, under various levels of risk constraint during the testing period, can also be obtained by the optimization models. A few empirical values produced by the weighting combinations generated by the simplex-centroid mixture design are beyond the broken line generated by the optimization models. However, which of them would lie beyond the broken line or on the efficient frontier is unpredictable beforehand. On the other hand, the kernel of the optimization model is the performance prediction model whose training data set consists of the backtesting performances during the first time frame (1991–1995). Therefore, it is possible for investors to obtain the optimal weighting combinations by the optimization model at the end of 1995 and employ them in the testing period (1996–2010). Hence, the optimization models are useful for investors to search for the optimal investment strategies to meet their specific preferences.

6.3 The accumulative return of the maximizing annualized return rates model

To evaluate the accumulative return during the testing period, we backtested the maximizing annualized return rates model generated by neural networks and optimization techniques with the training data set consisting of 1990/Q4–1995/Q3 stock market data in Sect. 6.1. To show the robustness of the model, we backtested the model not only with the data recorded during the original testing period (1995/Q4–2010/Q3) but also with the data recorded during 2010/Q4–2014/Q3 of the original testing period. We set the index equal to 1 on the start date of the 1995/Q4. The results are shown in Fig. 12 and illustrate that the model outperforms the S&P 500 except for the period

during 1998/Q1–2000/Q4. One possible explanation is that there was a dot com bubble during 1997–2000. In this speculative bubble period, the market did not follow the principle of value investing advocated by fundamental analysis. Therefore, the model trained with the data set collected from the normal market cannot exert its effect on stock picking.

7 Conclusions

The important insights of the empirical evidence from the performance prediction models and optimization models in this study include:

- According to the evaluation of performance prediction models based on neural networks, excess return rates can be predicted more precisely than annualized return rates, total risk than systematic risk, and relative winning rate than absolute winning rate. These may be attributed to the reason that an individual firm's fundamental analysis is only effective for stock picking but not useful for market timing.
- 2. According to the analysis of performance prediction models based on neural networks, profitable firms' stocks with relatively cheap prices have higher return rates. The stocks with relatively cheap prices have higher absolute winning rates, lower systematic risks, and lower total risks. Profitable firms' stocks have higher relative winning rates.
- 3. The empirical values of the performances of the optimal weighting combinations generated by the optimization models showed that they can meet various investors' preferences and outperform those of S&P's 500 not only during the training period but also during the testing period. Thus, the optimization models are useful for investors to search for the optimal investment strategies to meet their specific preferences.

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