MECEE4998_001_2018_3 - MS PROJECTS IN MECH ENGINEER

A Report On

MATHEMATICAL ANALYSIS OF KINEMATIC MODELS FOR NECK BRACE

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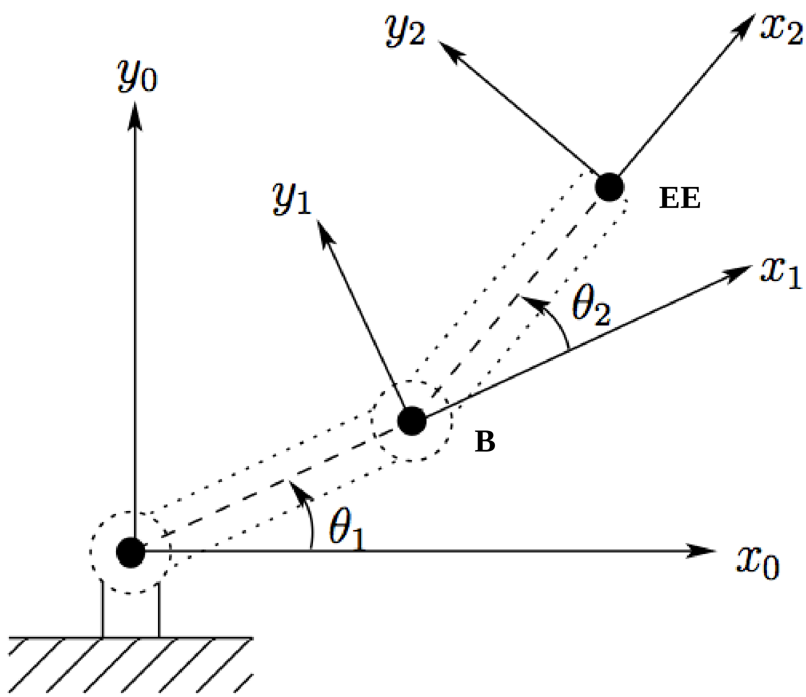
ABSTRACT

People with neurological syndromes diseases such as Amyotrophic Lateral Sclerosis (ALS) and Parkinson’s Disease (PD) suffer from head drop syndrome which makes the neck muscles weak for its movement thus requiring some external device to keep the head in straight position and for rotating it in the desired way. This problem can be solved using cable driven dynamic neck braces to keep the head straight position and provide forces to move the neck in various desired configuration. This report consists of preliminary mathematical analysis required for dynamic neck brace to support the head in a desired configuration. The brace has four links containing the end effectors and kinematic laws are applied to position the end effector. Basic kinematic mathematical models are studied to study how by changing various parameters one can obtain the desired configuration. This report focusses on the different configurations obtained by using different types of link joints and thus helping in deciding a favorable model for the final design.

INTRODUCTION

Many neurological diseases cause neck extensor muscle weakness resulting in dropped head syndrome, which in turn causes paralysis and neck pain due to which the patient is unable to maintain his/her neck in an upright position or move it in a desired orientation. While there is no medicinal cure for the above syndrome, it can be solved by use of some external support which will help keep the neck in upright position and provide appropriate forces to the neck to make it move. Therefore, attempts were made to implement static neck braces for keeping the neck in upright position but when tested on patients, it led to certain uneasiness and discomfort which led to bring up new ideas to design a more patient friendly neck brace. Initially our lab had designed a passive spring-loaded compliant neck brace with adjustable supports to balance the head with the torques provided by three spring actuators. The brace had 3RRS chain between the end-effector and the shoulder support. Later, another 3 DOF mechanism was designed which coupled both the head rotation and translation by optimizing the geometric parameters. The brace could measure the head position and orientation based on a multi-motion capture system. The brace demonstrated nearly 70% of the overall range of the head rotation. Finally, a joystick interface controlled active neck brace was designed. The brace was controlled by three servomotors and the user could control the brace movement with the help of joystick. This design also showed a reasonable accuracy. EMG reduced muscle activation with force and torque data proved that the brace assisted the patient in movement. At present our aim is to implement cable driven 3 DOF parallel spatial manipulator because of its advantages such as large workspace, lighter than rigid links, high payload to weight ratio and fully remote actuation.

PLANAR RR



Forward Kinematics:

The position of B co-ordinate given by (x_B, y_B) could be mathematically stated as:

Rotation$_1$=$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}$

Translation$_1$= $[l_1 \;\; 0]$

(x_B, y_B) = Translation$_1$ x Rotation$_1$

The position of End effector given by (x_{EE}, y_{EE}) could be mathematically stated as:

Rotation$_2$=$\begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) \end{bmatrix}$

Translation$_2$= $[l_2 \;\; 0]$

(x_{EE}, y_{EE}) = Translation$_2$ x Rotation$_2$

Inverse kinematics:

By cosine rule,

$$\theta_2 = \pm \cos^{-1}\left(\frac{-l_1^2 - l_2^2 + x^2 + y^2}{2l_1 l_2}\right)$$

Thus, evaluating θ_1 from geometry of triangles,

$$\theta_1 = tan^{-1}\left(\frac{y}{x}\right) - tan^{-1}\left(\frac{l_2\, sin\, \theta_2}{l_1 + l_2\, cos\, \theta_2}\right)$$

To check for the reachability of the obtained θ_1 and θ_2:

$$l_2(cos\, \theta\, , cos\, \theta_2 - sin\, \theta_1\, sin\, \theta_2) + l_1\, cos\, \theta_1 == x$$
$$l_2(sin\, \theta_1\, cos\, \theta_2 + cos\, \theta_1\, sin\, \theta_2) + l_1\, sin\, \theta_1 == y$$

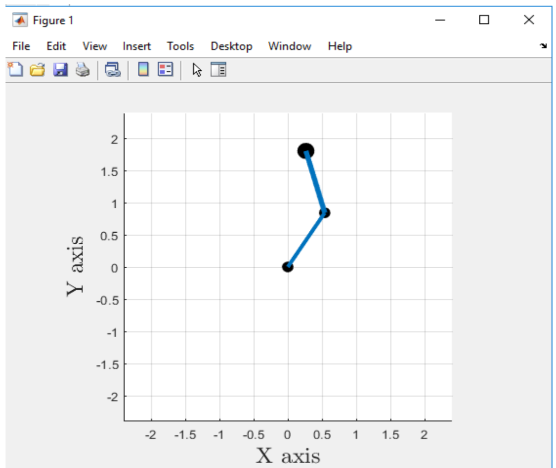
Jacobian:

To evaluate for Jacobian,

$$J = \begin{bmatrix} -l_1\, sin\, \theta_1 - l_2\, sin(\theta_1 + \theta_2) & -l_2\, sin(\theta_1 + \theta_2) \\ l_1\, cos\, \theta_1 + l_2\, cos(\theta_1 + \theta_2) & l_2\, cos(\theta_1 + \theta_2) \end{bmatrix}$$

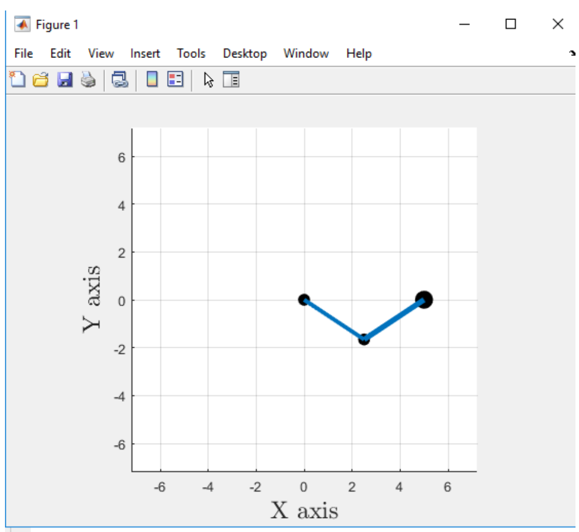
Simulation results:

Forward kinematics:



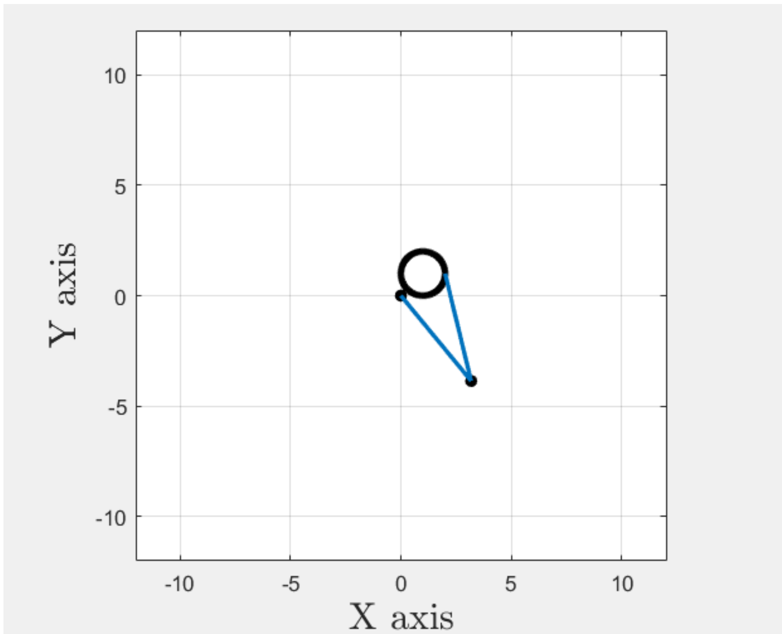

```
>> [xEE,yEE] = ForwardKinematics(robot)

xEE =

    0.2647


yEE =

    1.8027

>> plotRobot(robot)
>>
```

Inverse kinematics:

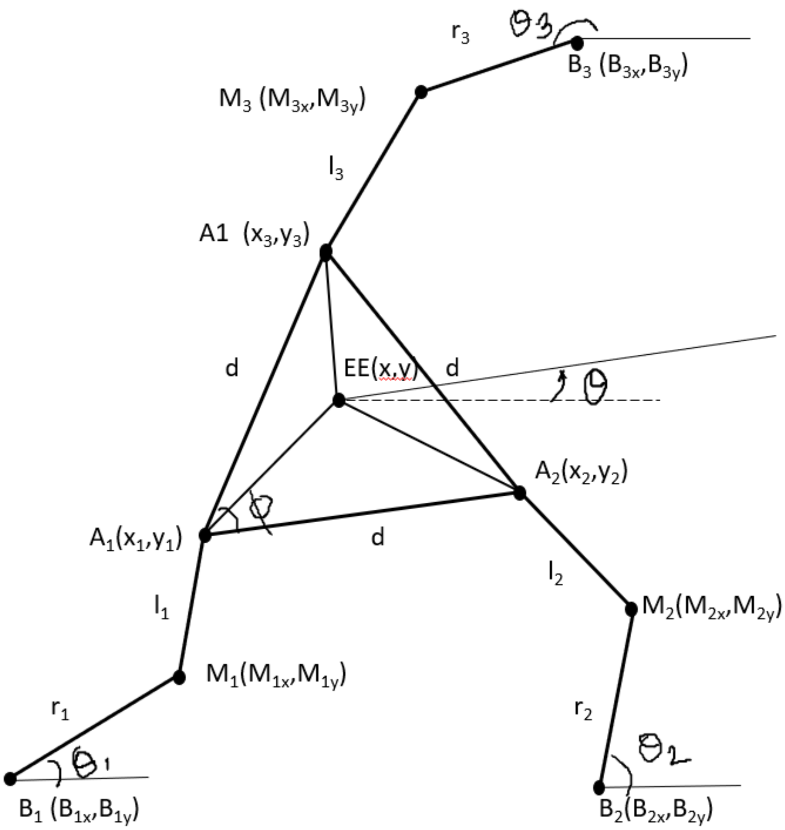



```
    lnkLength: [3 3]

>> [theta1,theta2,reachable] = InverseKinematics(robot)

theta1 =

   -0.5857    0.5857


theta2 =

    1.1714   -1.1714
```

Executing path trajectory



PLANAR 3RRR



Forward Kinematics:

The central points M1, M2 and M3 can be calculated as follows:

$$M_{1x}=B_{1x}+r_1\cos(\theta_1)$$
$$M_{1y}=B_{1y}+r_1\sin(\theta_1)$$
$$M_{2x}=B_{2x}+r_2\cos(\theta_2)$$
$$M_{2y}=B_{2y}+r_2\sin(\theta_2)$$
$$M_{3x}=B_{3x}+r_3\cos(\theta_3)$$
$$M_{3y}=B_{3y}+r_3\cos(\theta_3)$$

Using equation of Circle, we get

$$(x_1-M_{1x})^2 + (y_1-M_{1y})^2 = l_1^2 \quad (1)$$
$$(x_1+d\cos\theta-M_{2x})^2+(y+d\sin\theta-M_{2y})^2=l_2^2 \quad (2)$$
$$(x_1+d\cos(\theta+\Phi)-M_{3x})^2+(y+d\sin(\theta+\Phi)-M_{3y})^2=l_3^2 \quad (3)$$

Simplifying equations (2) and (3) and subtracting (1) from them, the system reduces to following equations:

$$G_1x_1+G_2y_1+G_3=l_2^2-l_1^2 \qquad (5)$$
$$G_4x_1+G_5y_1+G_6=l_3^2-l_1^2 \qquad (6)$$

where,

$$G_1=2M_{1x}-2M_{2x}+2d\cos\theta$$
$$G_2=2M_{1y}-2M_{2y}+2d\sin\theta$$
$$G_3=(M_{2x}^2+M_{2y}^2)-(M_{1x}^2+M_{1y}^2)+d^2-2d(M_{2x}\cos\theta + B_{2y}\sin\theta)$$
$$G_4=2M_{1x}-2M_{3x}+2d\cos(\theta+ \Phi)$$
$$G_5=2M_{1y}-2M_{3y}+2d\sin(\theta+ \Phi)$$
$$G_6=(M_{3x}^2+M_{3y}^2)-(M_{1x}^2+M_{1y}^2)+d^2-2d(M_{2x}\cos(\theta+ \Phi) + B_{2y}\sin(\theta+ \Phi))$$

Solving equations (5) and (6),

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} G_1 & G_2 \\ G_4 & G_5 \end{bmatrix}^{-1} \begin{bmatrix} l_2^2 - l_1^2 - G_3 \\ l_3^2 - l_1^2 - G_6 \end{bmatrix} \qquad (7)$$

Now solving by substituting θ as

$$\theta = \tan^{-1}\left(\frac{2T}{1-T^2}\right)$$

Hence, we obtain an eighth-degree polynomial in T which is evaluated to obtain the values of θ. Filtering the set of values obtained from theta, we decompose the set of eight values of θ to single value of θ.

Hence from θ , x_1 and y_1 could be obtained from equation (7). A2 and A3 could be obtained as follows:

$$x_2=x_1+d\cos\theta$$
$$y_2=y_1+d\sin\theta$$
$$x_3=x_1+d\cos(\theta+ \Phi)$$
$$y_3=y_1+d\sin(\theta+ \Phi)$$

Hence the position of end effector obtained is the centroid of the triangle completed as follows,

$$x = \frac{x_1 + x_2 + x_3}{3} \;, \quad y = \frac{y_1 + y_2 + y_3}{3}$$

Inverse Kinematics:

Let d*y* be the line joining point A3 and opposite side passing through centroid.

$$dy = \sqrt{d^2 + \left(\frac{d}{2}\right)^2}$$

Since in the case of inverse kinematics we know the end effector position (x, y, θ) hence the points A1, A2 and A3 could be evaluated as,

$$A_1(x_1, y_1) = \left(x - \frac{d}{2}, y - \frac{dy}{3}\right)$$
$$A_2(x_2, y_2) = \left(x + \frac{d}{2}, y - \frac{dy}{3}\right)$$
$$A_3(x_3, y_3) = \left(x, y + \frac{2\,dy}{3}\right)$$

The rotation matrix is given by,

$$R = \begin{bmatrix} cos\,\theta & sin\,\theta \\ -\,sin\,\theta & cos\,\theta \end{bmatrix}$$

The transformed A1, A2 and A3 after rotation are as follows:

$$A_1(x_1, y_1) = [x_1 - x \quad y_1 - y] \cdot R$$
$$A_2(x_2, y_2) = [x_2 - x \quad y_2 - y] \cdot R$$
$$A_3(x_3, y_3) = [x_3 - x \quad y_3 - y] \cdot R$$

Thus, the inverse kinematics can be computed as follow:

$$\alpha_n = \pm \cos\left(\frac{-l_n^2 - r_n^2 + x_n^2 + y_n^2}{2 l_n r_n}\right)$$

$$\theta_n = \tan^{-1}\left(\frac{y_n}{x_n}\right) - \tan^{-1}\left(\frac{r_n \sin \alpha_n}{l_n + r_n \cos \alpha_n}\right)$$

where n= 1,2 and 3

Jacobian:

The Jacobian is given by,

$$J = A^{-1}D$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

where

$$a_n = -2(x + b_{xn}\, cos\, \phi) - r_1\, cos\, \theta_n - b_{yn}\, sin\, \phi$$

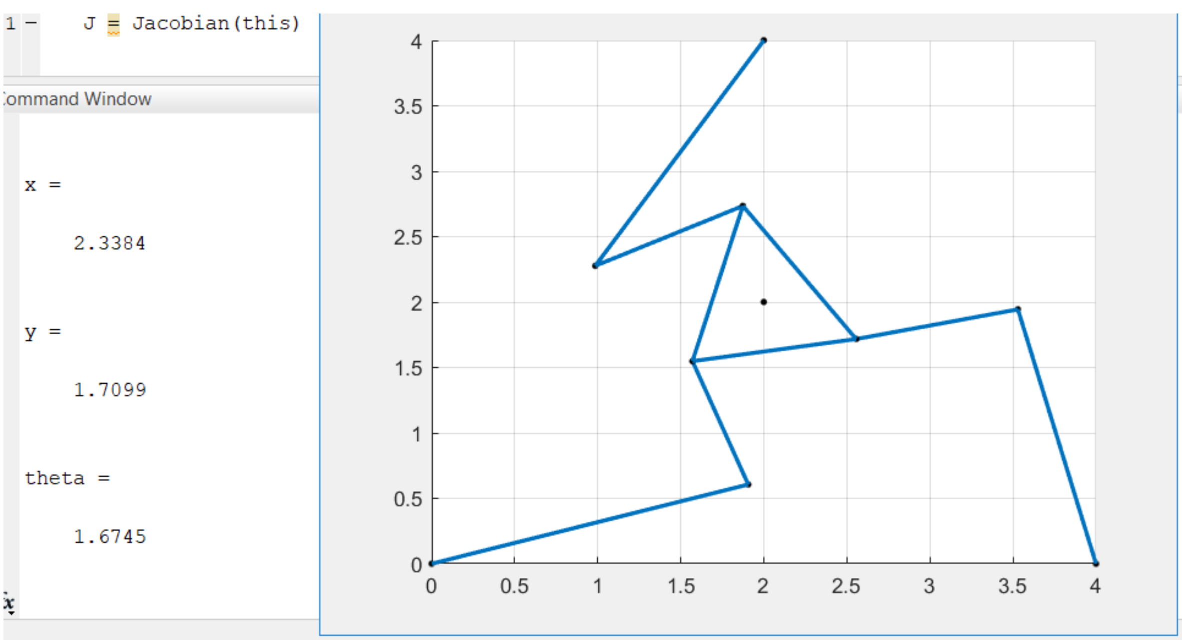
$$b_n = -2(y + b_{xn} \cos \phi) - r_n \sin \theta_n - b_{xn} \sin \phi$$

$$c_n = -2\big(r_n b_{yn}\, cos(\phi - \theta_n) + r_n b_{xn}\, sin(\phi - \theta_n)\big)(x b_{xn} - y b_{xn}) + \sin \phi\big(-x b_{xn} - y b_{yn}\big)$$

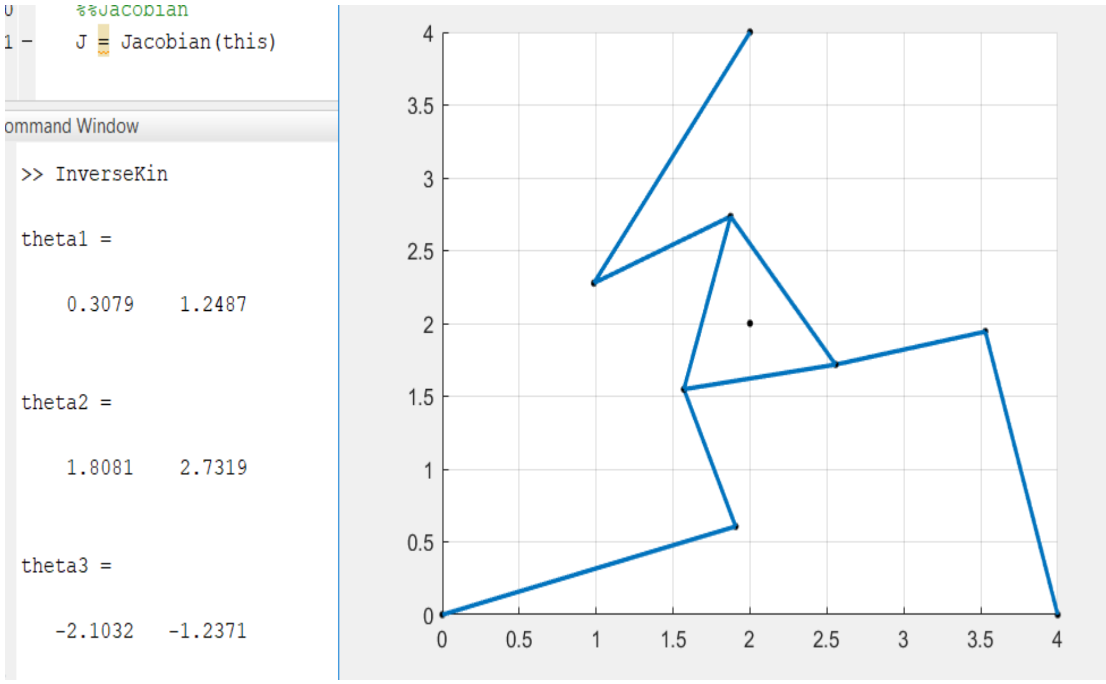
$$d_n = 2\big(r_n \cos \theta_n + r_n \sin \theta n - r_n b_{yn} \cos(\emptyset - \theta_n) - r_n b_{xn} \sin(\phi - \theta_n)\big)$$

Simulation results:

Forward kinematics:



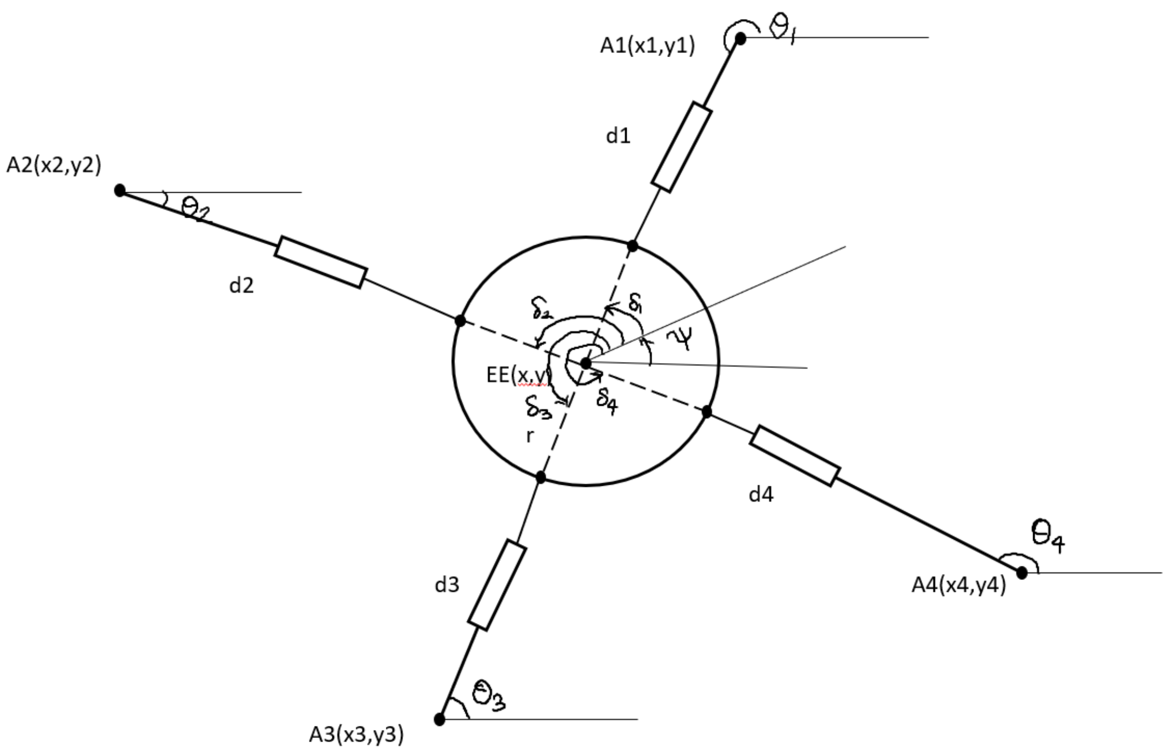
Inverse kinematics:



Jacobian:

```
J =

    0.6254   -2.2201    0.0217
    1.8228    0.1498   -0.0015
    1.9848   -1.0473    0.0966
```

FOUR RPR



Forward kinematics:

The equations from vectoral loci are given by,

$$x_n + d_n \cos\theta_n = x + r\cos(\Psi + \delta_n) \qquad (1)$$

$$y_n + d_n \sin\theta_n = y + r\sin(\Psi + \delta_n) \qquad (2)$$

where n is 1,2,3 and 4.

We need to consider the degrees of freedom for the manipulator to provide a certain minimum number of parameters out of the eight joint parameters to get a certain motion.

Degrees of freedom for 4RPR manipulator are given by:

N= 3(n-1)-2j1-j2 ; where n= number of links

j1= number of lower pairs

j2= number of higher pairs

N= 3(10-1)-(2*12)-0

= 3

Thus, three of the eight joint parameters are required to define the manipulator motion.

Depending upon the known parameters, three equations are selected and solved using fsolve in MATLAB to obtain end effector position EE(x, y, Ψ).
The values of end effector orientation are substituted back in rest five equations to obtain the unknown joint values.

Inverse kinematics:
Equations (1) and (2) could be re-written as:

$$d_n \cos\theta_n = x + r\cos(\Psi + \delta_n) - x_n \tag{3}$$

$$d_n \sin\theta_n = y + r\sin(\Psi + \delta_n) - y_n \tag{4}$$

where n= 1,2,3 and 4

Dividing equation (4) by (3),

$$\theta_n = \tan^{-1}\left(\frac{y + r\sin(\psi + \delta_n) - y_n}{x + r\cos(\Psi + \delta_n) - x_n}\right) \tag{5}$$

where n= 1,2,3 and 4

Squaring both equations (3) and (4) and adding them we get,

$$d_n = \pm\sqrt{(x + r\cos(\psi + \delta_n) - x_n)^2 + (y + r\sin(\psi + \delta_n) - y_n)^2} \tag{6}$$

where n= 1,2,3 and 4

Equation (5) gives joint parameters for revolute joints and (6) gives joint parameters for prismatic joints.

Jacobian:
Jacobian is given by,

$$J = A_P^{-1}B$$

where A_p^{-1} represents pseudo-inverse of A

A is given by,

$$A = \begin{bmatrix} 1 & 0 & -rsin(\psi + \delta_1) \\ 1 & 0 & -r\,sin(\psi + \delta_2) \\ 1 & 0 & -r\,sin(\psi + \delta_3) \\ 1 & 0 & -r\,sin(\psi + \delta_4) \\ 0 & 1 & rcos(\psi + \delta_1) \\ 0 & 1 & rcos(\psi + \delta_2) \\ 0 & 1 & rcos(\psi + \delta_3) \\ 0 & 1 & rcos(\psi + \delta_4) \end{bmatrix}$$

B is given by,

$$B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -d_1 \sin\theta_1 & 0 & 0 & 0 \\ 0 & -d_2 \sin\theta_2 & 0 & 0 \\ 0 & 0 & -d_3 \sin\theta_3 & 0 \\ 0 & 0 & 0 & -d_4 \sin\theta_4 \end{bmatrix}$$

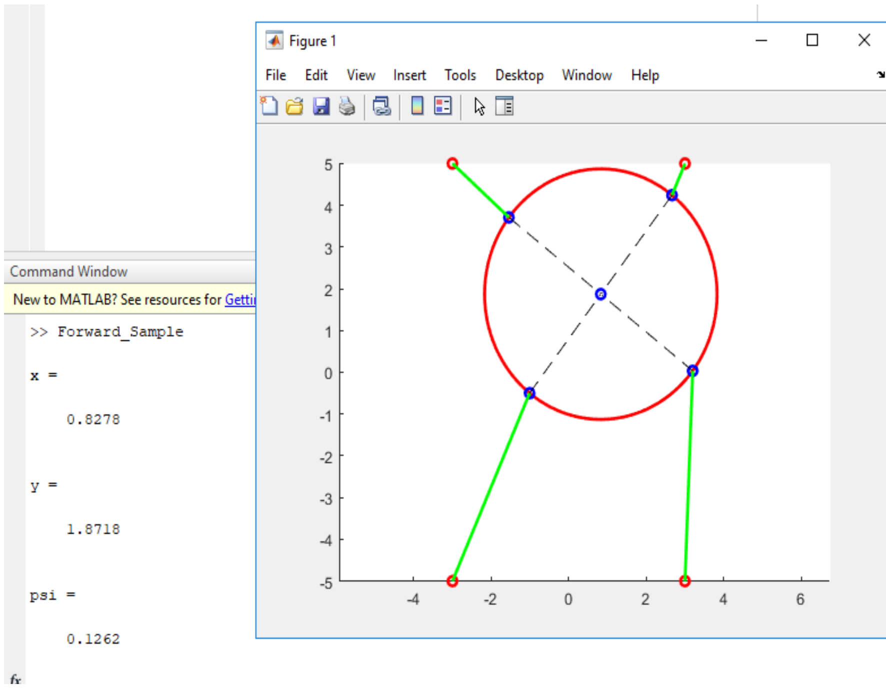
$$B_2 = \begin{bmatrix} cos\,\theta_1 & 0 & 0 & 0 \\ 0 & cos\,\theta_2 & 0 & 0 \\ 0 & 0 & cos\,\theta_3 & 0 \\ 0 & 0 & 0 & cos\,\theta_4 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} d_1\,cos\,\theta_1 & 0 & 0 & 0 \\ 0 & d_2\,cos\,\theta_2 & 0 & 0 \\ 0 & 0 & d_3\,cos\,\theta_3 & 0 \\ 0 & 0 & 0 & d_4\,cos\,\theta_4 \end{bmatrix}$$

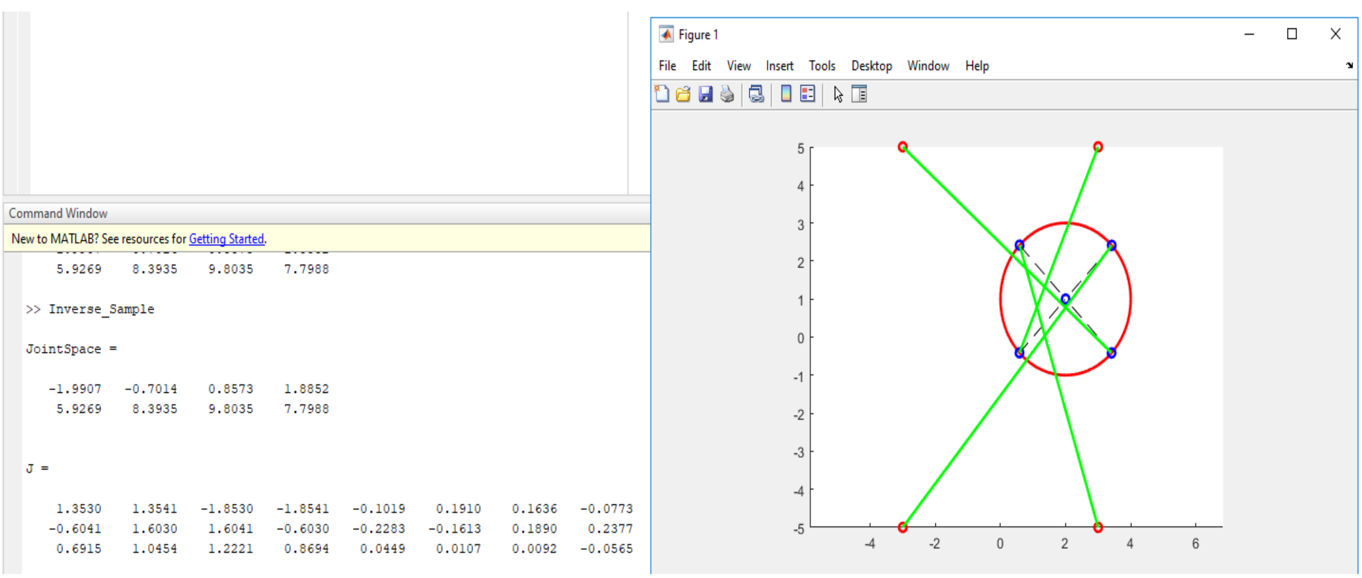
$$B_4 = \begin{bmatrix} sin\,\theta_1 & 0 & 0 & 0 \\ 0 & sin\,\theta_2 & 0 & 0 \\ 0 & 0 & sin\,\theta_3 & \sigma \\ 0 & 0 & 0 & sin\,\theta_4 \end{bmatrix}$$

Simulation results:

Forward Kinematics:



Inverse Kinematics and Jacobian:



CONCLUSION

Various planar kinematic models were studied and analyzed mathematically. Out of which 4RPR stands a flexible option to get into any possible orientation with 3 DOF. This model can be further extended into spatial configuration to build a cable driven neck brace.

FUTURE PLAN

Future plan is to extend the planar 4RPR configuration into a spatial 4RPR configuration. Further we aim to compute force analysis which include the transient and steady state response on the system to study the behavioral model of the system under different end effector positions. As planned with the group, to improve the retractability of the end effector position we plan to inculcate two springs with desired stiffness to obtain the desired motion.