HW 2

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1.

1.

We have $X_1, \dots X_N \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} * \exp\left\{ \frac{-1}{2\sigma^2} (x_i - \mu)^2 \right\}$$

$\Rightarrow$ Ignoring constants

$$L(\mu, \sigma) = \prod_i \frac{1}{\sigma} \exp\left\{ -\frac{1}{2\sigma^2} (x_i - \mu)^2 \right\}$$

$$\Rightarrow \sigma^{-N} \exp\left\{ \frac{-1}{2\sigma^2} \sum_{i=1}^{N} (x_i - \mu)^2 \right\}$$

$\Rightarrow$ ~~$\sigma$~~ N. Now, $\sum_{i=1}^{N} (x_i - \mu)^2$ can be manipulated as -

$$\sum_{i=1}^{N} (x_i - \bar{x} + \bar{x} - \mu)^2$$

$$\Rightarrow \sum_{i=1}^{N} \left( (x_i - \bar{x})^2 + (\bar{x} - \mu)^2 + 2(x_i - \bar{x}) * (\bar{x} - \mu) \right)$$

$$\Rightarrow N(\bar{x} - \mu)^2 + \sum_{i=1}^{N} \left( (x_i - \bar{x})^2 + 2(x_i - \bar{x}) * (\bar{x} - \mu) \right).$$

$$\rightarrow N(\bar{x} - \mu)^2 + N S^2$$

where $S^2$ is the sample variance.

which gives

$$L(\mu, \sigma) = \sigma^{-N} \exp\left\{ \frac{-1}{2\sigma^2} * N S^2 \right\} \left\{ \frac{-1}{2\sigma^2} N (\bar{x} - \mu)^2 \right\}$$

2.

2.

Given

$$l(\mu,\sigma) = -N\log\sigma - \frac{NS^2}{2\sigma^2} - \frac{N(\bar{X}-\mu)^2}{2\sigma^2}.$$

Setting $\frac{\partial l}{\partial \mu} = 0$

Gives, $0 - 0 \quad -\frac{N}{2\sigma^2} * 2(\bar{X}-\mu) * (-1) = 0$

$$\Rightarrow \quad \frac{N}{\sigma^2}(\bar{X}-\mu) = 0$$

For $\sigma \neq 0$, $\bar{X} = \mu$.

Setting $\frac{\partial l}{\partial \sigma} = 0$

Gives, $-\frac{N}{\sigma} - \frac{NS^2}{2} * (-2\sigma^{-3}) - \frac{N(\bar{X}-\mu)^2}{2} * (-2\sigma^{-3}) = 0$

$$\Rightarrow -\frac{N}{\sigma} + NS^2\sigma^{-3} + N(\bar{X}-\mu)^2\sigma^{-3} = 0$$

$$\Rightarrow \quad \because \bar{X} = \mu, \Rightarrow \hat{\mu} = \bar{X}$$

$$+\frac{N}{\sigma} = NS^2\sigma^{-3}$$

$$\Rightarrow \sigma^2 = S^2 \Rightarrow \hat{\sigma} = S$$

$$\Rightarrow \hat{S} = \sigma$$

3.

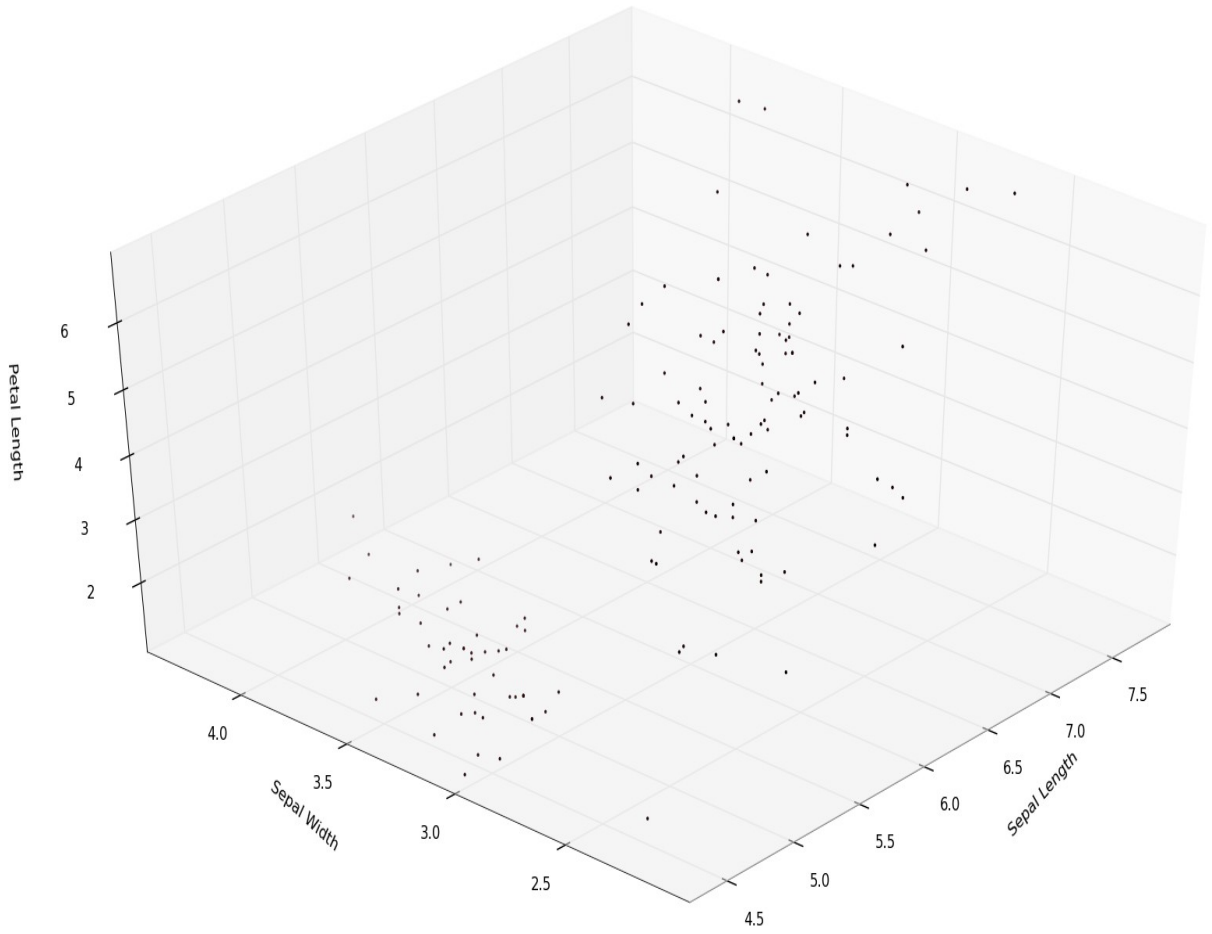
Code Attached

4.

The program on running generates the visualization which can be moved around to see how the clusters separate.

The code provided does the job of splitting clusters color wise, but there seems to be a bug in the matlplotlib extension of python for 3d visualization, which is leading to color loss in the visualization. Similar code base for visualization is given as an example which seems to work on some machines. [http://old.nabble.com/mpl_toolkits-td29400615.html] and other open issues can be found on googling.

The clustering can still be visualized reasonably cleanly.




5.

Cross validation functionality is implemented but not tested thoroughly, in some cases, I ended up getting a singular matrix who's inverse could not be obtained.