A REPORT

ON

A STUDY ON REAL LIFE APPLICATION OF DATA ENVELOPEMENT ANALYSIS

BY

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BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

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By

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With warm regards,

G.Ashwin and Apurva Mittal

ABSTRACT

In this project we have used Data Envelopment Analysis (DEA), which is a non-parametric method in optimization and economics to estimate production frontiers of decision making units (DMUs). It is a linear programming methodology to measure the efficiency of DMUs when we have multiple inputs and outputs. It talks about various models in DEA including Charnes, Cooper and Rhodes model (CCR)[2], Banker Charnes Cooper model (BCC)[1] and Slack based measure model (SBM)[4]. The first model takes into account the technical efficiencies and mix efficiencies of the DMUs with constant returns to scale. The second model is an extension of the first, which also includes scale efficiencies and DMUs with variable returns to scale. The last model further is invariant to the units of measure of various inputs and outputs.

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1. INTRODUCTION

Data Envelopment Analysis (DEA) is a linear programming technique which was introduced in 1978 by Charnes, Cooper and Rhodes[2] based on the ideas of Farrell[3] who was working on measuring the efficiency of firms (also called decision making units). DEA is used to measure the efficiencies of firms with multiple outputs and multiple inputs. In microeconomic theory, a production function is used to depict a firm's input and output combination. Using this function, one can express the maximum output which can be obtained given certain combinations of inputs or in other words one can build a production technology frontier.

Originally built as a performance measurement tool for certain government organizations and non-profit organizations, DEA has been expanded and developed in the last decade and is being used in a plethora of fields that include education,agriculture, benchmarking,regulation, environment, manufacturing, economy, health, services, information technology, tourism government,operations,insurance, public policy, human resources, retail, banking and marketing.

The reason DEA became prevalent is because of the flexibility of its method that readily integrates the existence of multiple outputs and multiple inputs without any causal assumption of a functional form. Instead, DEA involves general distribution and production assumptions only. Nevertheless, if these assumptions are too frail, inefficiency levels may be steadily underestimated in small samples. For a given set of outputs and inputs of various DMU's, DEA builds its own functional form and hence it avoids the risk of misspecification of frontier technology. In addition, DEA does assume that all DMU's use the same technology but instead assesses the efficiency of each DMU relative to its peer.

In this report we discuss the foundations of basic DEA models, certain properties of geometric DEA models, which might be valuable in the exercise of efficiency measurement and decision making and examples which explain how the different constructions of DEA models work. The working of the Charnes, Cooper and Rhodes (CCR)[2] model along with Banker, Charnes and Cooper (BCC)[1] model is provided in brief along with a small discussion that includes certain examples based on these models. This is followed by a discussion on Slack Based Measure(SBM)[4] and the reason for using it.

2. PRODUCTION FUNCTION

Generally the production function of the fully efficient firm is unknown and in Data Envelopment Analysis the efficient isoquant is estimated by sampling the data. Thus a non-parametric piecewise linear convex isoquant is constructed using piecewise interpolation. In this constructed isoquant no observed point can lie to the left or below of the efficient frontier as seen in Figure 1 for an input oriented model.

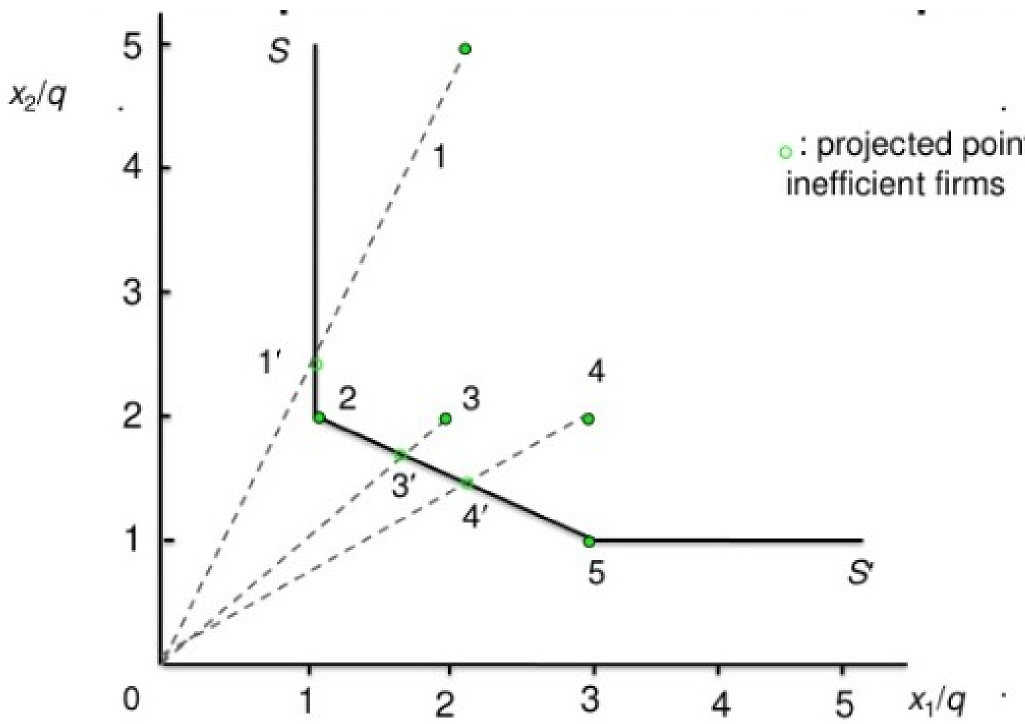

Figure 1: Piecewise Linear Convex Isoquant

3. EFFICIENCY MEASURES

Since many years there are majorly 2 efficiency measures used to estimate frontiers:

1. Data Envelopment Analysis
2. Stochastic frontiers

DEA is a mathematical model whereas stochastic frontier analysis is a method of economic modeling. Data Envelopment Analysis is a non parametric data analysis technique that is it makes no assumption about the form of the production technology and has no parameter to depend upon. The frontier is build by enveloping the input- output vectors observed of different frontiers. Thus it fits into a piecewise linear frontier using linear programming techniques. Stochastic frontier approach is a parametric approach in which a hypothetical functional form is taken and the data is used to econometrically estimate the parameters of that function using the set of DMUs.

According to Farrell in 1957[3] who drew on the work of Debreu and Koopmans proposed that the firm efficiency which could include multiple inputs and outputs consists of two components:

1. Technical Efficiency: It is the ability of the firm to get maximum output from given number of inputs (Output Oriented) or use minimum amount of inputs for given number of outputs (Input Oriented).
2. Allocative Efficiency: It is the ability of the firm to produce goods up to the point where the last unit gives a marginal benefit to customers equal to marginal cost of production. Thus the firm has to use its inputs to maximum proportions, given their respective prices.

3.1 INPUT-ORIENTED MEASURES

In input oriented measures the inputs are reduced by their maximum limit with given outputs. Farrell could explain this by an example using the DMU with two inputs and one output assuming that the DMU has constant returns to scale. Let us assume that the isoquant of a fully efficient firm is known (this is not the case in real life and the isoquant is estimated using DEA) and is represented by function Y=1 in Figure 2. Let the given firm use inputs corresponding to the point P, to produce one unit of output, then the technical inefficiency of the DMU is given by the distance QP, that is the amount of inputs (x1 and x2) that can be radially reduced without changing the output y. Thus QP/OP gives by what percentage can the inputs be reduced. Thus the technical efficiency is given by the ratio,

TE(I) = OQ/OP,

The point Q which lies on the efficient isoquant is said to be technically efficient.

Also there is a linear iso-cost line which interests the isoquant at Q’ through which allocative efficiency is calculated. Thus allocative efficiency (AE) is given by the ratio:

AE(I) = OR/OQ,

Thus total economic efficiency is given by:

EE(I) = OR/OP = (OQ/OP)*(OR/OQ) = TE(I)*AE(I)

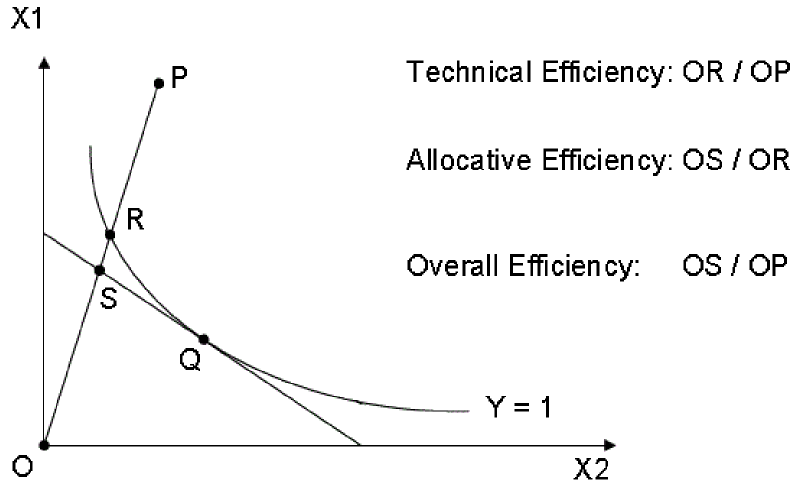

Figure 2: Input oriented efficiencies

3.2 OUTPUT ORIENTED MEASURES

In input oriented measures, the output quantities are fixed and the input quantities are proportionally reduced. On the other hand, if the output quantities are proportionally expanded without varying the input quantities, it is known as an output oriented measure. Using an example that involves one input and one output, the difference between input and output oriented measures can be explained. In the figure given below, there is an inefficient firm depicted by point P. The output oriented measure of technical efficiency is equal to CP/CD whereas the input oriented measure is given by AB/AP.

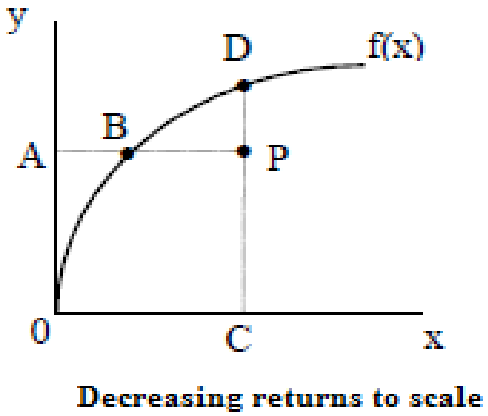

Figure 3- Input and Output Oriented Technical Efficiency

In order to explain output oriented measures better, another scenario can be considered that involves one input(x1) and two outputs (y1 and y2). In the figure below, given a production possibility curve (PPC), an inefficient firm is shown by the point A. It is clearly visible that the point is inside the PPC, as it acts as an upper bound of all possible productions.

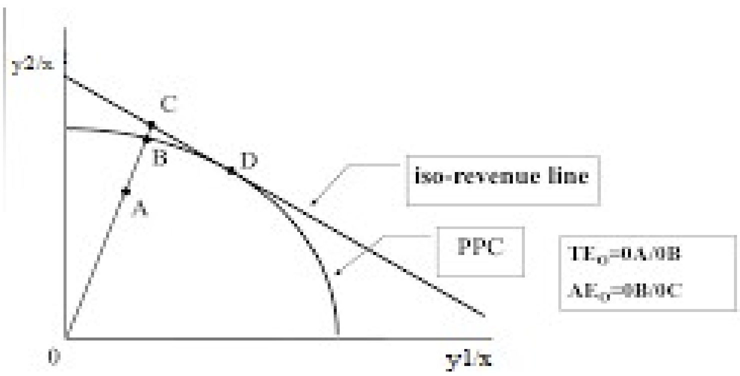

Figure 4- Output oriented firm

The technical inefficiency, which is the extent by which outputs can be increased without changing the input value, is represented by the distance AB. To measure output efficiency, we use the ratio

$TE_0 = 0A/0B$

By making use of the iso-revenue line, the allocative efficiency is defined by

$AE_0 = 0B/0C$

Using both the technical and allocative efficiency, the total economic efficiency is defined by

$EE_0 = TE_0 \text{ x } AE_0 = (0A/0B) \text{ x } (0B/0C) = 0A/0C$

It is to be noted that all these efficiency measures are measured along a line from the origin (radial measures) , keeping the proportions constant and are hence units invariant which means that even if the units are changed(example- seconds to hour), the value of efficiency measure will remain unaffected.

4. DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis is a non parametric technique mathematical approach to estimate frontiers. Farrell in 1957[3] suggested a linear convex isoquant approach to estimate frontiers. Then Boles in 1966 and Afriat in 1978 gave a mathematical programming approach to it by using linear programming. Charnes, Cooper and Rhodes in 1978[2] gave the term data envelopment analysis and thus proposed the input oriented constant returns to scale model. Banker, Cooper and Charnes in 1984 [1] gave the variable returns to scale model. Further in order to combine input oriented and output oriented efficiencies Slack based measure was introduced by K. Tone in 2001 [4] in which inputs decreased and outputs increased simultaneously.

4.1 THE CRS MODEL

The Constant returns to scale model (CRS) is also known as Charnes, Cooper and Rhodes model (CCR)[2]. Let us assume our data is on K inputs and M outputs on N DMUs. The i^{th} DMU is represented by vectors x_i and y_i. Thus the K x N input matrix X and the M x N output matrix Y represent the data of all N DMUs. The ratio form is used to represent the DEA. The ratio $u'y_i/v'x_i$ where u is M x 1 vector of output weights and v is K x 1 vector of input. Therefore to find optimum weights the following mathematical model is given:

$\max_{u,v} (u'y_i/v'x_i)$,

$st\ u'y_i/v'x_i \leq 1, j = 1,2...,N,$

$u,v \geq 0.$

In order to avoid fractional programming this problem is converted to following multiplier form of linear programming problem.

$max_{u,v} (\mu'y_i),$

$st\ v'x_i = 1,$

$\mu'y_i - v'x_i \leq 1, j = 1,2...,N,$

$\mu,v \geq 0.$

Thus finally equivalent dual form is:

$min_{\theta,\lambda} \theta,$

$st\ -y_i + Y\lambda \geq 0,$

$\theta x_i - X\lambda \geq 0,$

$\lambda \geq 0$

where θ is a sclar and λ is a N x 1 vector.

Since there are lesser number of constraints in dual problem than primal and also solution obtained by dual is more straightforward we prefer dual over primal. θ obtained from dual is directly the technical or radial efficiency. If it's value is 1 it implies that the point lies on the efficient frontier and also the DMU is technically efficient.

Slacks: Slacks or mix inefficiency come into play when the section of frontier runs parallel to the axes. See Figure 5. Point A is technically inefficient. When the inputs are reduced in same proportion according to their weights we get to the point A'. But at A' input x_2 can further be reduced without decreasing the output thus coming to point C. This is the difference between technical efficiency and CCR efficiency. Point A' is technically efficient but not CCR efficient but point C is both technically and CCR efficient. Thus we define two kinds of slack, output slack (OS) and input slack (IS),

$OS = -y_i + Y\lambda$

IS = θx_i - $X\lambda$

Thus if θ = 1, it is known as technically efficient but if θ = 1 and slacks are 0 then it is known as CCR eficient.

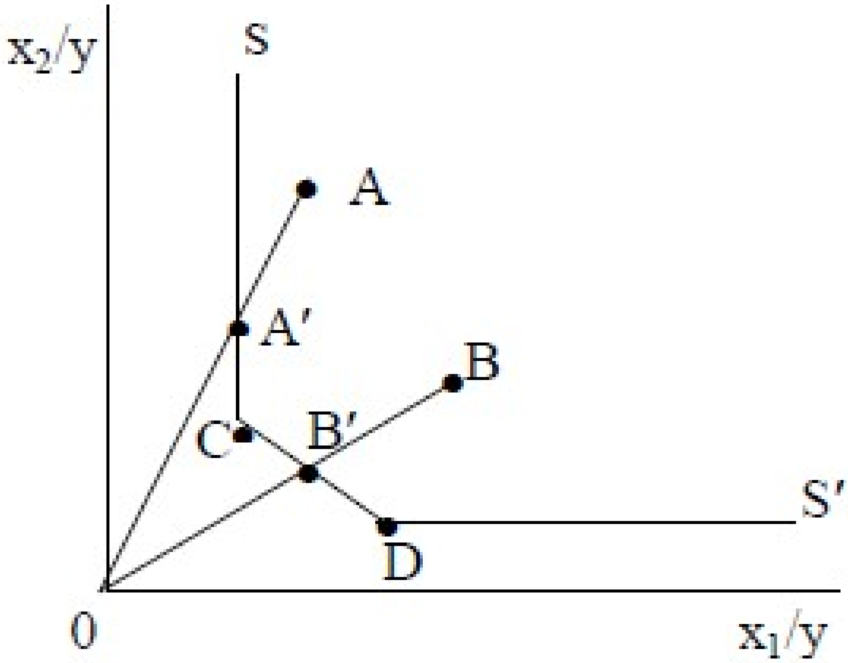

Figure 5: Slacks

Example 1: Let us observe 5 DMUs with two inputs and one output. Refer Table I.

Table I: Example for CRS DEA

DMU	y	x_1	x_2	x_1/y	x_2/y
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

The dual problem of each DMU is solved with 3 constraints using TORA. Thus solution for one DMU is shown in Figure 6. Thus the graphical representation is obtained in Figure 7. Finally Table 2 gives a tabulated solution to the problem.


Title: DMU1
Final Iteration No.: 8
Objective Value (Min) =0.50

Next Iteration | All Iterations | Write to Printer

Variable	Value	Obj Coeff	Obj Val Contrib
x1: THETA	0.50	1.00	0.50
x2: LAMDA1	0.00	0.00	0.00
x3: LAMDA2	0.50	0.00	0.00
x4: LAMDA3	0.00	0.00	0.00
x5: LAMDA4	0.00	0.00	0.00
x6: LAMDA5	0.00	0.00	0.00

Constraint	RHS	Slack-/Surplus+
1 (>)	1.00	0.00
2 (>)	0.00	0.00
3 (>)	0.00	0.50+

Sensitivity Analysis

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: THETA	1.00	0.00	infinity	0.00
x2: LAMDA1	0.00	-0.50	infinity	-0.50
x3: LAMDA2	0.00	-1.00	1.00	0.00
x4: LAMDA3	0.00	-1.50	infinity	-1.50
x5: LAMDA4	0.00	-1.00	infinity	-1.00
x6: LAMDA5	0.00	-2.00	infinity	-2.00

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (>)	1.00	0.00	infinity	0.50
2 (>)	0.00	-0.20	infinity	0.50
3 (>)	0.00	-infinity	0.50	0.00

Figure 6: TORA solution of DMU 1

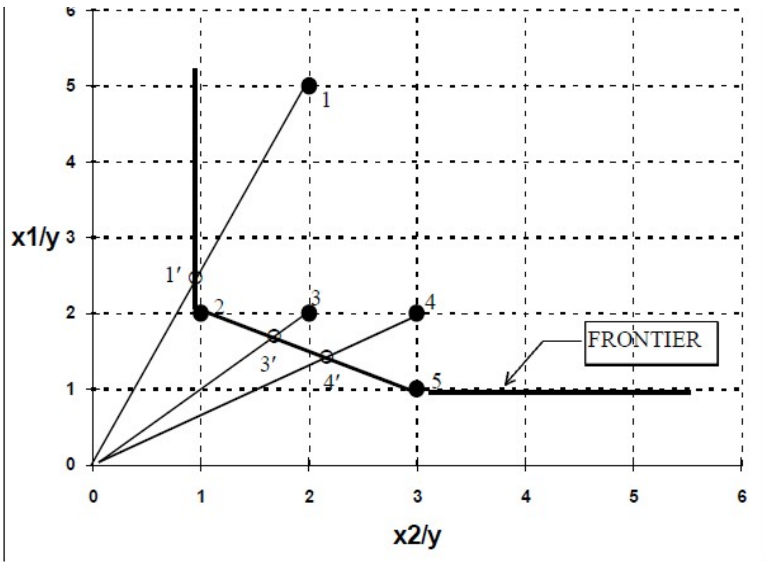

Figure 7: CRS DEA example

Table II: Solution of CRS DEA example

DMU	θ	λ_1	λ_2	λ_3	λ_4	λ_5	IS_1	IS_2	OS
1	0.5	-	0.5	-	-	-	-	0.5	-
2	1.0	-	1.0	-	-	-	-	-	-
3	0.833	-	1.0	-	-	0.5	-	-	-
4	0.714	-	0.214	-	-	0.286	-	-	-
5	1.0	-	-	-	-	1.0	-	-	-

4.2 THE VRS MODEL

The Constant Returns to Scale (CRS) model does not work properly when factors like financial constraints and imperfect competition come into play. In order to be able to calculate the efficiencies of the firms even when they are not operating at an optimal scale, Banker, Charnes and Cooper[1] came up with an extension of the CRS model giving rise to the VRS model. Calculating technical efficiency with the VRS model do not give scale efficiencies unlike the CRS model where not all firms are at optimum scale.

In order to convert the CRS model to VRS model, we add the convexity constraint $N1'\lambda=1$

$$
\begin{aligned}
&\min_{\theta,\lambda} \theta, \\
&\text{st} \quad -y_i + Y\lambda \geq 0, \\
&\qquad \theta x_i - X\lambda \geq 0, \\
&\qquad N1'\lambda=1 \\
&\qquad \lambda \geq 0,
\end{aligned}
$$

where N1 is an Nx1 vector of ones. Using this approach a convex isoquant is obtained which envelops the data points more precisely than the conical hull formed by CRS and hence always results in a technical efficiency higher than that of the CRS model.

By conducting both VRS and CRS DEA on the same data, we can obtain two technical efficiency scores, one due to pure technical inefficiency and one due to scale inefficiency. If a difference is obtained between both the calculations, it shows that the DMU is scale inefficient and the value of the scale inefficiency can be calculated by taking the difference between the VRS and CRS technical efficiency.

The figure below tries to illustrate this where there is one-output, one-input case and the CRS and VRS frontiers are shown. The distance PP_C is the input oriented technical inefficiency under CRS while the distance PP_V is the technical inefficiency under VRS. P_CP_V is the difference between these two quantities and it is the scale inefficiency.

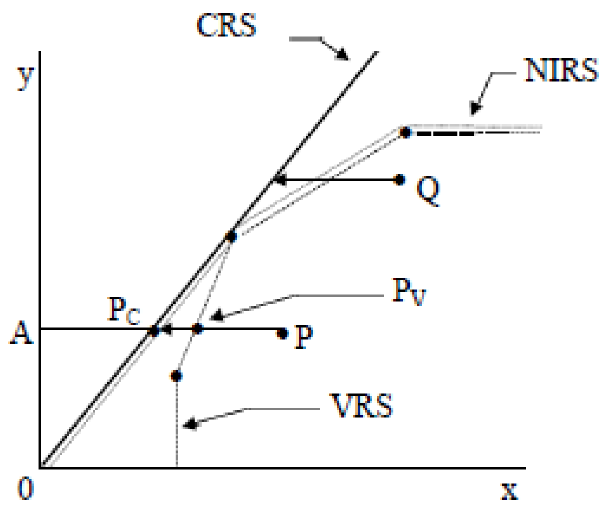

Figure 8: Scale economies in DEA

All the ratio efficiency measures can be expressed as

$TE_{I,CRS} = AP_C/AP$

$TE_{I,VRS} = AP_V/AP$

$SE_I = AP_C/AP_V$

We also note that

$TE_{I,CRS} = TE_{I,VRS} \times SE_I$

because

$AP_C/AP = (AP_V/AP) \times (AP_C/AP_V)$.

All these measures lie between zero and one.

Even though this increases the efficiency of the DEA calculation, one limitation of this process is the inability to figure out whether the DMU is under increasing returns to scale or if it is acting under decreasing returns to scale. A slight change in the DEA model by imposing a non-increasing returns to scale condition takes care of this issue. This is done $N1'\lambda=1$ substituting

with $N1'\lambda \leq 1$, that gives

$\min_{\theta,\lambda} \theta,$

st $\quad -y_i + Y\lambda \geq 0,$

$\theta x_i - X\lambda \geq 0,$

$N1'\lambda \leq 1$

$\lambda \geq 0,$

The nature of these scale inefficiencies can be decided by seeing if NIRS and VRS technical efficiencies are equal. Increasing returns to scale exists when they are unequal and decreasing returns to scale exists when they are equal.

Example: Let us look into five DMU's with single input and single output. The data is given in the table below-

Table III: Example for VRS DEA

DMU	y	x
1	1	2
2	2	4
3	3	3
4	5	5
5	5	6

The dual problem of each DMU is solved with 3 constraints using TORA. Thus the graphical representation is obtained in Figure --. Finally Table -- gives a tabulated solution to the problem.

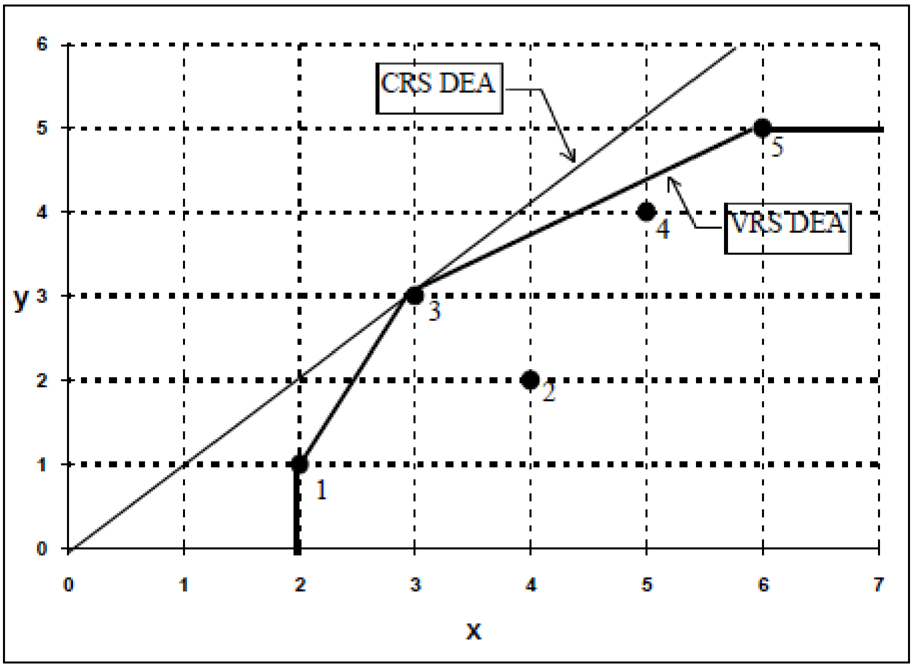

Figure 9: Example of VRS DEA

Table IV: Solution of VRS DEA Example

DMU	CRS TE	VRS TE	SCALE	
1	0.500	1.000	0.500	irs
2	0.500	0.625	0.800	irs
3	1.000	1.000	1.000	-
4	0.800	0.900	0.889	drs
5	0.833	1.000	0.833	drs
mean	**0.727**	**0.905**	**0.804**	

4.3 THE SBM MODEL

The Slack based measure model or the SBM model[4] is said to be dimension free and units invariant since it is invariant w.r.t. the units of measurement of all outputs and inputs. Also the slack based measure is monotonically decreasing w.r.t. output slacks and input slacks.

The formula is given by:

$$x\,io - s\,i - \frac{¿}{x\,io}$$

$$y\,ro - s\,r + \frac{¿}{y\,ro}$$

$$\frac{1}{s}\sum_{r=1}^{s} ¿ \qquad -1$$

$$\frac{1}{m}\sum_{i=1}^{m} ¿¿$$

$$\rho = ¿$$

Here the first term gives men proportional reduction rates of the input and second term gives the mean proportional rate of output expansion. Thus we require to minimize the expression which can be rewritten as:

$$\text{[SBM-Min]} \quad \rho_o^{\min} = \min_{\lambda, s^-, s^+} \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$

$$\text{subject to}$$

$$x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \; (i = 1, \ldots, m)$$

$$y_{ro} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ \; (r = 1, \ldots, s)$$

$$\lambda_j \geq 0\,(\forall j),\; s_i^- \geq 0(\forall i),\; s_r^+ \geq 0(\forall r).$$

Thus the SBM model can be converted into linear programming by making denominator 1 and introducing a positive scalar variable t.

$$
\text{[SBM-Min-LP]} \quad \tau^* = \min_{t,\Lambda,S^-,S^+} t - \frac{1}{m}\sum_{i=1}^{m} \frac{S_i^-}{x_{io}}
$$

subject to

$$
1 = t + \frac{1}{s}\sum_{r=1}^{s} \frac{S_r^+}{y_{ro}}
$$

$$
tx_{io} = \sum_{j=1}^{n} x_{ij}\Lambda_j + S_i^- \ (i = 1,\ldots,m)
$$

$$
ty_{ro} = \sum_{j=1}^{n} y_{rj}\Lambda_j - S_r^+ \ (r = 1,\ldots,s)
$$

$$
\Lambda_j \geq 0\ (\forall j),\ S_i^- \geq 0 (\forall i),\ S_r^+ \geq 0 (\forall r),\ t > 0.
$$

A DMU is said to be SBM efficient if and only if τ^* = 1 thus making slacks 0. Therefore we can also conclude that a DMU is SBM efficient if and only if it is CCR efficient[5].

5. RESEARCH DESIGN

We know apply these models to find the efficiencies of all the departments in the three campuses of BITS Pilani (Pilani, Goa and Hyderabad campuses).

5.1 Selection of DMUs

The two factors that affect the selection of DMUs are – homogeneity and number of DMUs. The DMUs must be homogeneous. They must perform the similar tasks and should have same objectives. The inputs and outputs of DMUs should be identical, except for differences in intensity or magnitude.

We select the departments of BITS Pilani, Goa and Hyderabad as our DMUs since they are homogenous as they perform similar tasks. All the departments use staff and sanctioned money for teaching and research purposes. There are a total of 34 DMUs, 12 departments of Pilani, 10 of Goa and 12 of Hyderabad. All the data is collected from Annual Report of 2014 and thus is secondary data.

5.2 Inputs

5.2.1. *Academic Staff:* This is the main human resource used by all departments for teaching and research purposes. This includes Professors, Associate Professors, Assistant Professors, Lectures and Visiting Facility. Now different weights are given to different kinds of academic staff to combine to give a single input. Weights given are 1 to Professors, 0.75 to Associate Professors, 0.5 to Assistant Professors and 0.25 to the rest. Therefore, Academic Staff =

(Prof + 0.75*Associate Prof + 0.5*Assistant Prof + 0.25*Lectures + 0.25*Visiting)/(1+0.75+0.5+0.25+0.25)

5.2.2 *Sanctioned Money:* This includes the money sanctioned for the ongoing projects in 2014 and the completed projects in 2014. The weights assigned are 1 and 1.5 respectively and are calculated the same way as in Academic staff.

5.3 Outputs

5.3.1 *Total Students*: The main function of every department is teaching. Total students represents the quantity and quality of teaching. It is directly related to the staff. Each department has total number of undergraduate students on campus, no. of students placed and Ph.D. students. Weights given are 0.25,0.75 and 0.5 respectively.

5.3.2 *Progress:* It is related to research. Research is one of the primal activities of a Department. We have to consider several parameters together which are necessary to measure progress in research field. This includes number of publications, number of patents, number of conference attended and number of books published. The weights given are 1, 0.25, 0.6 and 0.7 respectively.

5.3.3 *Awards:* Awards are also a way to measure the success of a particular department.

5.4 Choice of DEA model

In our study there are two inputs “Academic staff” and “Sanctioned Money” and three outputs “Total studets”, “Progress” and “Awards”. We have to choose between constant returns to scale (CRS) and variable returns to scale (VRS) models. CRS assumes that there is no significant relation between scales of operations and efficiency. That is, large departments are just as small ones in converting inputs to outputs. On the other hand, VRS means that a rise in input is expected to result in disproportionate rise in outputs. It is preferred when a significant correlation between DMU size and efficiency is found in a large sample. The CRS efficiency score represents technical efficiency (TE), which measures inefficiencies due to input/output configuration and as well as size of operations. On the other hand, VRS efficiency score represents pure technical efficiency (PTE) which is a measure of efficiency without scale efficiency (SE). To maintain the simplicity we have used CRS model.

6. OVERALL PERFORMANCE AND ASSESSMENT OF DEPARTMENTS

In order to assess the departments, 2 inputs and 3 outputs have been considered. We have carried out the analysis using CRS assumptions using output orientation. The inputs of Pilani, Goa and Hyderabad are displayed in table 1, 2 and 3 respectively whereas the outputs from these respective campuses are shown in tables 4, 5 and 6

Table 1 – Pilani Inputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	Enl&eee	Humanities	Maths	Mech	Pharma	Physics
Prof	2	4	1	4	5	1	5	0	3	4	3	2
Associate Prof	5	1	1	1	3	1	3	5	3	5	1	5
Assistant Prof	12	10	9	12	7	5	9	15	11	12	8	15
Lectures	2	1	7	0	9	1	18	2	0	7	4	0
Visiting Fac	1	0	1	0	0	0	0	0	0	2	0	0
Sanc Money ongoing	364.078	316.8	235.87	490.73	87.45	142.5	149.7	49.257	136.64	382.538	173.25	17
Sanc comp	746.3219	105.46	38.7	178.458	Nil	23.6	Nil	6.25	21.762	71.36	22	203.42

Table 2 – Goa Inputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	Enl&eee	Humanities	Maths	Mech	Pharma	Physics
Prof	0		1	2	1	0	2	1	0	0		1
Associate Prof	5		2	3	3	1	1	0	1	2		3
Assistant Prof	11		4	12	4	5	8	8	10	7		12
Lectures	0		4	0	8	2	12	3	2	4		0
Visiting Fac	1		0	0	0	0	0	3	0	0		1
Sanc Money ongoing proj	240.40419		473.314	532.6387	62.345	17.5	40	16.614	60.1936	69.98		148.025
Sanc comp	17		30	6803.0563	0	0	0	11.58	0	49.97		13.55

Table 3- Hyderabad Inputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	Enl&eee	Humanities	Maths	Mech	Pharma	Physics
Prof	2	2	1	3	0	1	2	0	1	1	2	1
Associate Prof	2	2	3	1	1	1	0	0	2	5	0	1
Assistant Prof	7	5	9	11	6	5	7	5	12	5	8	12
Lectures	0	2	1	0	6	1	8	1	0	6	0	0
Visiting Fac	0	0	0	0	4	3	4	2	1	0	0	0
Sanc Money ongoing	530.28	46.22	230.89	258.9	116.91	0	2	13.28	38.94	89.5	851.622	111.96
Sanc comp	110.57	0	12	0	12.5	0	0	0	0	0	0	0

Table 4 – Pilani outputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	Enl&eee	Humanitie	Maths	Mech	Pharma	Physics
No. of Students on	246	241	243	225	447	286	628	80	263	326	111	256
No. of phd	37	12	7	48	10	9	15	12	7	23	26	25
No. Students empl	32	42	48	31	91	44	136	4	46	75	22	39
Publications	26	20	11	45	16	2	39	17	25	49	47	28
Patents	1	0	0	0	0	0	0	0	0	0	0	0
Conference/Papers	52	30	69	29	10	6	0	54	12	7	15	19
Books Pub	5	1	2	0	0	0	6	2	3	1	1	0
Awards	26	7	12	11	1	0	7	3	3	6	10	2

Table 5- Goa Outputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	EnI&eee	Humanities	Maths	Mech	Pharma	Physics
No. of Students on campus	174		231	162	342	209	651	142	199	312		166
No. of phd	35		2	21	14	11	17	10	7	7		12
No. Students employed/higher studies	28		41	33	74	29	139	8	35	69		28
Publications	29		5	40	8	13	20	7	16	20		8
Patents	1		0	3	0	0	1	0	0	0		0
Conference/Papers	23		8	31	20	10	17	5	14	17		6
Books Pub	1		0	0	0	2	0	5	0	5		0
Awards	5		2	6	1	1	0	4	1	5		0

Table 6 – Hyderabad Outputs

Departments	Bio	Civil	Chemical	Chem	CS	Eco	EnI&eee	Humaniti	Maths	Mech	Pharma	Physics
No. of Students on campus	119	154	146	122	403	182	375	79	164	225	40	132
No. of phd	41	12	7	21	0	3	28	2	2	21	39	10
No. of students placed	0	30	36	0	99	1	116	43	1	49	8	0
Publications	22	9	10	28	11	14	9	4	33	17	64	18
Patents	0	0	0	0	0	0	0	0	0	0	0	0
Conference/Papers	28	17	11	24	26	7	37	7	12	18	17	14
Books Pub	1	0	1	0	0	0	0	1	0	0	0	0
Awards	2	1	7	2	2	1	7	0	2	3	1	0

After combining all the data and calculating efficiencies, we see from table 7 that of the 34 departments only 10 departments are technically efficient, namely - Bio (Pilani), Chemical (Pilani), Pharmacy (Pilani), E&I (Goa), Mechanical (Goa), E&I (Hyderabad), Pharmacy (Hyderabad), Civil (Hyderabad), Economics (Hyderabad) and Humanities (Hyderabad).

Table 7 – Technical efficiencies

Pilani	theta		Goa	theta		Hyd	theta
Bio	1		Bio	0.6713		Bio	0.5376
Civil	0.5596		Chemical	0.3681		Chemical	0.6252
Chemical	1		Chem	0.5767		CS	0.7348
Chem	0.7835		CS	0.4903		E&I	1
CS	0.3267		Eco	0.9924		Math	0.8774
Eco	0.3158		E&I	1		Pharma	1
E&I	0.571		Hum	0.6744		Civil	1
Hum	0.8757		Math	0.6323		Chemistry	0.6271
Math	0.4964		Mech	1		Eco	1
Mech	0.4964		Phy	0.2007		Hum	1
Pharma	1					Mech	0.5775
Phy	0.4363					Phy	0.4964
Avg Efficiency	0.655117			0.66062			0.789667

All other departments are not technically efficient since their technical efficiencies are less than 1. The average efficiency of Pilani is .655117, average efficiency of Goa is .66062 and for Hyderabad the average efficiency is 0.789667. Thus making Hyderabad the most efficient, then Goa and least Pilani. The lowest technical efficiency across campuses is Physics Goa with efficiency of 0.2007 which indicates overall poor performance of the department. This is majorly because extremely low output in terms of progress and placements. There have been no placements, books published, patents made and awards given. On the other hand Bio Pilani is a technically efficient DMU which efficiency 1 which is majorly because of low inputs in terms of Academic staff and high outputs especially in terms of conferences attended and paper published which are 52 which is maximum among all the departments. We can also see Pharmacy of Pilani has an efficiency of 1 which is because there is less amount of sanctioned money which is an input and large number of publications which is 47, second highest after Mechanical Pilani among all the departments. Also, efficiency of Eco Pilani is extremely low that is 0.3158 because of extremely low outputs with only 2 publications, no patents, no books published and no awards given.

7. CONCLUSION

We have discussed 3 basic Data Envelopment Analysis models including the CCR model, BCC model and the SBM model. It helps us to link two different approaches of efficiency namely the radial and non radial efficiency. Also we have discussed about allocative and overall efficiencies when cost and profits are of interest. With different returns to scale efficiencies we have identified where additional shortcomings can appear when even the size of our data can effect the efficiencies.

Also, we have applied the CCR model to find the efficiencies of all the departments of 3 campuses of BITS Pilani (Pilani, Goa, Hyderabad).

8. REFERENCES

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9. APPENDIX

```
clear

clc

I1=[5,4,3.3,4.3,5.2,1.8,6.5,4.7,4.3,6.4,3.5,5.3,3.8,2.2,4.1,2.9,1.5,3.9,2.6,2.5,2.4,3.8,2.8,3.3,2.5
,3.4,3.3,2,2.6,3.7,2.1,1.3,3.5,3.1];

I2=[5.9342,2.0891,1.17578,3.0337,0.6041,1.2476,0.5988,0.1145,0.73852,1.9583,0.869,1.288
5,1.0636,2.0733,42.9489,0.5537,0.07,0.16,0.135936,0.24078,0.38132,0.6734,2.7845,0.99556
,0.54264,0.008,0.15576,3.4065,0,1.0356,0,0.05312,0.358,0.44784];
```

```
O1=[70.17,60.167,60,71.833,109.83,66.83,157.5,20.67,49.333,90.833,38.833,68.167,55.167,52.5,44.5,86.66,51.33,488.66,31,47.33,78.5,41,40.33,39.83,100.17,115.17,28.667,28.833,41.66,30.83,32.167,28.5,64.33,27];

O2=[23.902,15.176,21.098,24.47,8.63,2.196,16.94,19.92,13.451,20.941,22.235,15.45,16.941,3.84,23.168,7.843,8,11.941,5.569,9.569,13.22,4.549,15.490,6.573,10.431,12.235,16.08,29.098,7.53,16.63,7.1373,8.2,10.902,10.353];

O3=[26,7,12,11,1,0,7,3,3,6,10,2,5,2,6,1,1,0,4,1,5,0,2,7,2,7,2,1,1,2,1,0,3,0];

for i = 1:34

   f=[1;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];

   A=[0,-O1(1),-O1(2),-O1(3),-O1(4),-O1(5),-O1(6),-O1(7),-O1(8),-O1(9),-O1(10),-O1(11),-O1(12),-O1(13),-O1(14),-O1(15),-O1(16),-O1(17),-O1(18),-O1(19),-O1(20),-O1(21),-O1(22),-O1(23),-O1(24),-O1(25),-O1(26),-O1(27),-O1(28),-O1(29),-O1(30),-O1(31),-O1(32),-O1(33),-O1(34);0,-O2(1),-O2(2),-O2(3),-O2(4),-O2(5),-O2(6),-O2(7),-O2(8),-O2(9),-O2(10),-O2(11),-O2(12),-O2(13),-O2(14),-O2(15),-O2(16),-O2(17),-O2(18),-O2(19),O2(20),-O2(21),-O2(22),-O2(23),-O2(24),-O2(25),-O2(26),-O2(27),-O2(28),-O2(29),-O2(30),-O2(31),-O2(32),-O2(33),-O2(34);0,-O3(1),-O3(2),-O3(3),-O3(4),-O3(5),-O3(6),-O3(7),-O3(8),-O3(9),-O3(10),-O3(11),-O3(12),-O3(13),-O3(14),-O3(15),-O3(16),-O3(17),-O3(18),-O3(19),-O3(20),-O3(21),-O3(22),-O3(23),-O3(24),-O3(25),-O3(26),-O3(27),-O3(28),-O3(29),-O3(30),-O3(31),-O3(32),-O3(33),-O3(34);-I1(i),I1(1),I1(2),I1(3),I1(4),I1(5),I1(6),I1(7),I1(8),I1(9),I1(10),I1(11),I1(12),I1(13),I1(14),I1(15),I1(16),I1(17),I1(18),I1(19),I1(20),I1(21),I1(22),I1(23),I1(24),I1(25),I1(26),I1(27),I1(28),I1(29),I1(30),I1(31),I1(32),I1(33),I1(34);
-I2(i),I2(1),I2(2),I2(3),I2(4),I2(5),I2(6),I2(7),I2(8),I2(9),I2(10),I2(11),I2(12),I2(13),I2(14),I2(15),I2(16),I2(17),I2(18),I2(19),I2(20),I2(21),I2(22),I2(23),I2(24),I2(25),I2(26),I2(27),I2(28),I2(29),I2(30),I2(31),I2(32),I2(33),I2(34)];

   Aeq = zeros(1,35);

   Beq = [0];

   b=[-O1(i);-O2(i);-O3(i);0;0];

   lb = zeros(1,35);
```

```
    ub = [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
    [x,fval]=linprog(f,A,b,Aeq,Beq,lb,ub);
    z(i)=fval;
end
z
```