CS7646 - Project 1: Martingale

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INTRODUCTION

The goal of this project is to look into the well-known Martingale betting strategy using Monte Carlo simulation in the context of a Roulette game. A Martingale strategy is a type of betting strategy that developed in 18th-century France and gained popularity during that time period. The most basic version of this method was created for a game in which a gambler would win if a coin landed heads up and lose if the coin landed tails up. The method entails doubling the bet after each loss, with the ultimate goal of earning a win that not only recovers all previous losses but also provides a profit equal to the initial stake. For example, if we start with $1, the first bet will be $1. If a player wins the bet, he or she collects the money and the game ends. If a player loses a stake, the next bet is doubled to equal the prior bet. The Martingale gambling simulator was used to carry out the following tests.

1. Using Monte Carlo simulation, investigate the strategy. There is no bankroll limit, but we set a goal of winning $80 before we stop playing.
2. Investigating the strategy With a $256 bankroll. That is, if the player runs out of money, players can not bet more money.

The odds of winning a bet on black is 18/38. Therefore, the experiments use the probability of 18/38. The odds are calculated as following:
Odds of winning on black = (Number of black pockets) / (Total number of pockets)
Odds of winning on black = 18 / 38

QUESTION 1

We can calculate the estimated probability of winning $80 within 1000 sequential bets by counting the number of trials where you reached the target win amount and dividing it by the total number of trials. In experiment 1, the number of trials where the simulator reached the target win amount was 10. The total number of trials is 10. Therefore, the estimated probability of winning $80 within 1000 sequential bets is 100%. When investigating the raw data of 1000 sequential bets, it is observed that in almost all cases winning of $80 is reached before the 200th bet is placed. It should be noted that this is a subset of the available combinations. While it is unlikely that the player will lose a greater number of consecutive bets and never recover enough to reach $80, the chance remains, and hence the probability is 99.999%.

QUESTION 2

If the expected value is calculated using $80 cap, The expected value after 1000 sequential bets would be $80. As seen in the experimental data and explained in question 1, the probability of achieving $80 is infinitesimally close to 100%.

$$E[X] = \sum_{i}^{n} [X_i P_i] \tag{1}$$

$$E[X] = \sum_{i=-\infty}^{80} [X_i P_i] \tag{2}$$

Since $80 is dominating, the equation becomes simplified to be E[X] = 80 * 1.0 = 80.

If the expected value is calculated without using $80 cap, The value of each bet in the martingale strategy is $1 and the probability of winning is 18/38. The expected value for each bet is $1 * 18/38 = $0.4737. After 1000 sequential bets, the expected value of winnings is $0.4737 * 1000 = $473.70.

QUESTION 3

In Experiment 1, the standard deviation oscillated at first before eventually settling to a value of 0. This pattern was characterized by intermittent spikes leading up to the moment of convergence, which represented brief periods of large bets driven by losses. Smaller variations occurred around these significant increases, which helped to stabilize the larger bets. The early volatility in standard deviation was caused by differences in the win and lose sequences between experimental runs. However, as the simulations advanced and all tests achieved the common objective of $80, the standard deviation approached 0. Once the simulation's wins reached the target of $80, both the upper and lower standard deviation bounds converged to the mean line. This convergence happened because the data was continually filled with the number 80 at that point, eliminating any additional deviations and resulting in a standard deviation of 0. It is important to notice that the standard deviation did not settle throughout the simulator's execution, but rather converged to 0 as a result of reaching the $80 goal.

QUESTION 4

In Experiment 2, the probability of winning $80 in a sequence of 1000 consecutive bets is 654, using a more realistic simulation for 1000 episodes. As a result, the likelihood is close to 0.654. Figures 4 and 5 both emphasize this possibility. To lose the game, one must deplete all of their funds, which requires a string of nine consecutive losing bets. The likelihood of such a loss is estimated as $(10/19)^9 = 0.003$.

QUESTION 5

Using the solution to question 4, we can see that 0.654 episodes received $80 and 0.346 (1-0.654) lost $256.

$$(0.654 * 80) + (0.346 * -256) = -36.256. \tag{3}$$

Therefore estimated expected value of winnings after 1000 sequential bets is losing $36.256.

QUESTION 6

The standard deviation lines have indeed stabilized. It should be noticed, however, that neither standard deviation line converged. The loss limit of $256 is responsible for the stabilization. The absence of convergence is primarily owing to the experiment's two conceivable outcomes: earning $80 or losing $256. The significant difference in these two outcomes results in a significant standard deviation.

QUESTION 7

Expected values are a more realistic reflection of a random process's long-term average or central tendency. Outliers or extreme values may occur in a single random event, skewing interpretations. Expected values, on the other hand, are based on outcome probability distributions and take into consideration the likelihood of alternative outcomes. Overall, the concept of expected value tends to provide a more predictable and deterministic perspective.

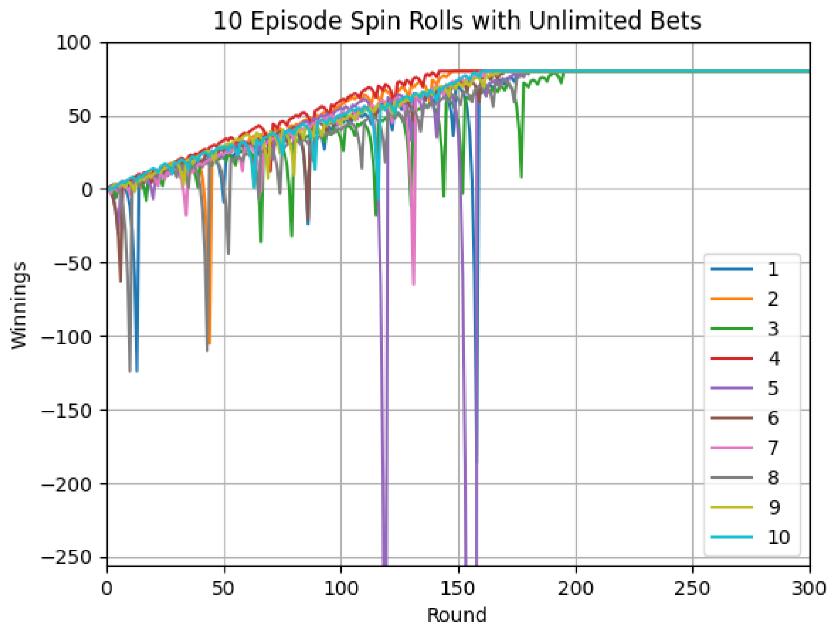

Figure 1—10 Episodes of 1000 Consecutive Spins with Unlimited Bets

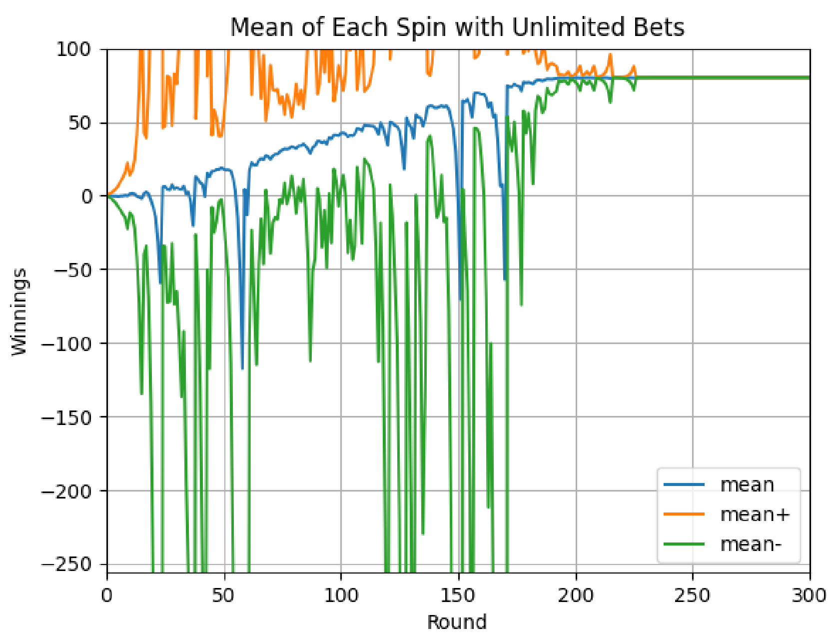

Figure 2—Mean of 1000 Episodes of 1000 Consecutive Spins with Unlimited Bets

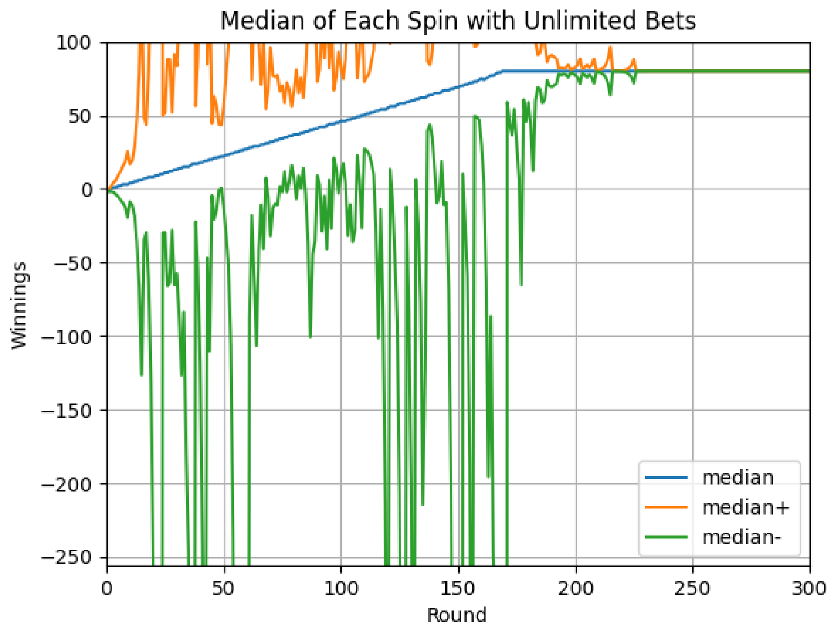

Figure 3—Median of 1000 Episodes of 1000 Consecutive Spins with Unlimited Bets

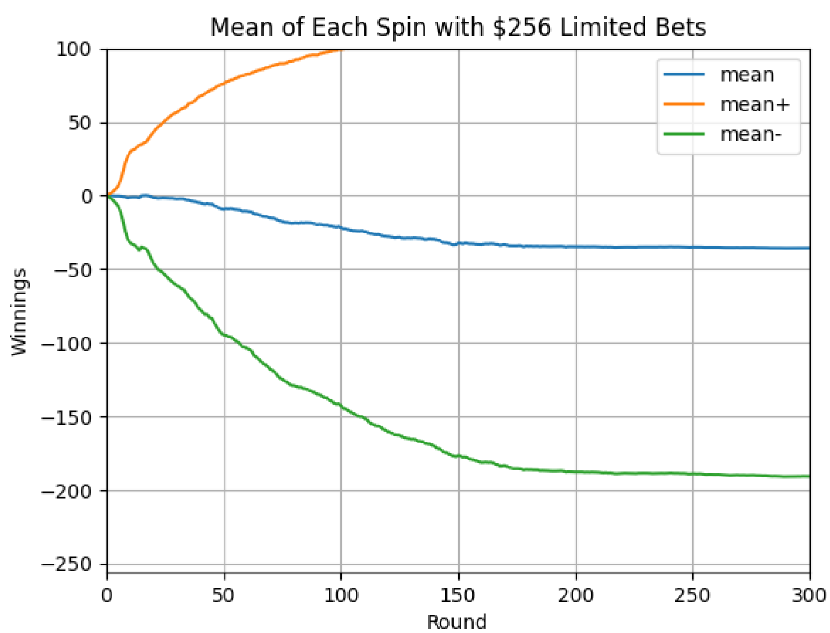

Figure 4—Mean of 1000 Episodes of 1000 Consecutive Spins with $256 Limited Bets

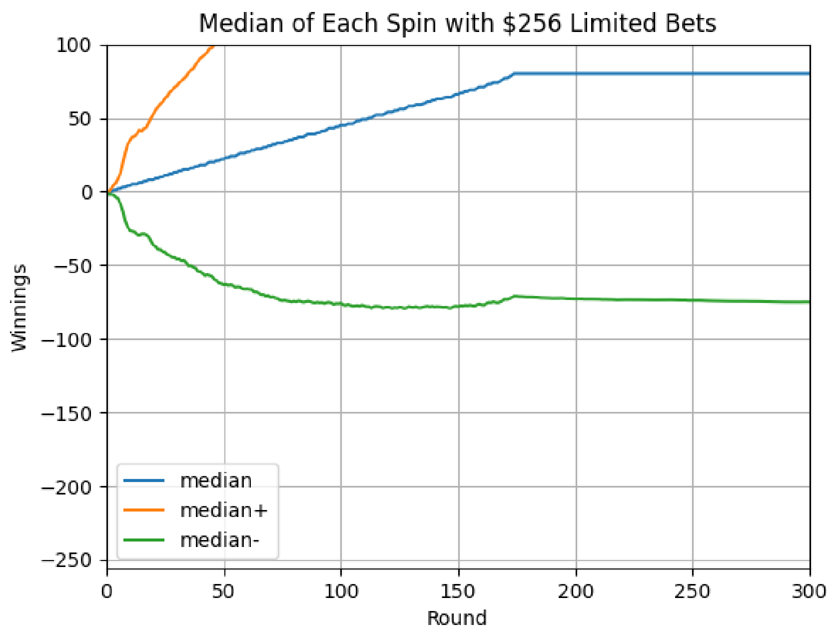

Figure 5—Median of 1000 Episodes of 1000 Consecutive Spins with $256 Limited Bets