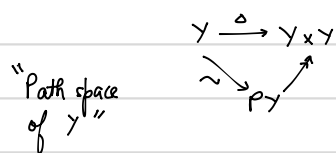
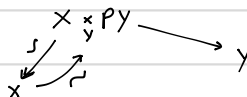


Brown's lemma



project $p_Y \rightarrow Y \times Y \rightarrow Y$ the two endpoints

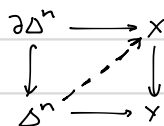
every map $X \rightarrow Y$ factors as



which gives us the Ken Brown's lemma

- Every map factors into section of trivial fibration followed by fibration
- It is a w.e. iff the fibration is a trivial fibration
- If $F: \mathcal{V} \rightarrow \mathcal{C}$ is a functor from a CFO to a category with w.e. then if it takes trivial fib. to w.e., it is a homotopy functor.
- Kan complexes of bounded size form a CFO

$X \rightarrow Y$ trivial fib \Leftrightarrow



Blumberg - Mandell

2-out-of-6 in a CFO \Leftrightarrow 2-out-of-3 + the retract of a w.e. is a w.e.

Lie groupoids

\mathcal{V} category of spaces, has finite limits (small category)

contains a subcategory of covers

- every iso is a cover
- retract of a cover is a cover
- pullbacks of a cover is a cover
- $X \xrightarrow{f} Y \xrightarrow{g} Z$ If f, gf are covers then g is a cover

eg: \mathcal{V} = finite sets

cover = surjective functions

• \mathcal{V} = complex analytic spaces

cover = surjective submersion

• \mathcal{V} = Banach analytic spaces

• $s\mathcal{V}$ category of simplicial spaces

Def: $f: X_i \rightarrow Y_i$ is a fibration of simplicial spaces if $\forall n > 0, 0 \leq i \leq n,$

$$X_n \longrightarrow \text{Hom}(\Lambda_i^n, X) \xrightarrow{\quad \times \quad} \text{Hom}(\Lambda_i^n, Y) \xrightarrow{\quad \gamma_n \quad}$$

for a trivial fibration: replace Λ_i^n by $\partial \Delta^n$.

k dimensional groupoid:

A simplicial space X_\bullet is a k -groupoid if

$$X_n \longrightarrow \text{Hom}(\Lambda_i^n, X) \quad n > 0, 0 \leq i \leq n$$

is a cover, and an iso if $n > k$.

eg: $n=1 \quad X_1 \rightrightarrows X_0$ the two boundary maps are covers.

k -groupoids are $(k+1)$ -cocomplete. $X_n \cong \text{Hom}(sk_{k+1} \Delta^n, X)$

$k=1$

$X_0 =$ space of objects of X

$X_1 =$ space of morphisms

$X_2 \xrightarrow{\cong} \text{Hom}(\Lambda_1^2, X) =$ space of pairs of composable morphisms

$\hookrightarrow \text{Hom}(\Lambda_0^2, X) =$ left inverse

$\hookrightarrow \text{Hom}(\Lambda_2^2, X) =$ right inverse

$\Lambda^* =$ differential graded algebra

$A^i = 0$ if $i \leq -k$