Cohomology of moduli stack of Riemann Surfaces

$$\Sigma$$
 = algebraic curve / \mathbb{C}
 G = complex semisimple Lie group
 $\operatorname{Bun}_{\mathbb{C}}(\Sigma)$ = moduli stack of G -bundles

Goal: Understand H* (Bun E (G))

Q. What is Π_0 Map (Σ, BG) ? Say G = discrete abelian group then this equals $H'(\Sigma, G) \cong H_1(\Sigma, G)$.

Recall:
$$M = \text{oriented } d\text{-manifold}$$
: $H_c^*(M_jA) \cong H_{d-*}(M_jA)$

ey. $M = IR^d$ we get
$$H_c^*(M_jA) \cong \begin{cases} A & *=d \\ 0 & \text{else} \end{cases}$$

$$u \in U(M) = \text{open subsets of } M$$

$$C_{c}^{*}(M,A) \qquad C_{*}(u,A)$$

$$U \longmapsto C_*(U), C_c^*(U)$$

 $U(M) \longrightarrow chain complexes$

Profo: These functors are cosheaves ower M with values in chain complexes

i.e. the following is a homotopy pushout square $C_*(U \cap V) \longrightarrow C_*(V)$

$$C_*(u) \longrightarrow C_*(u \cup V)$$

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Mocolin C_*(u) \cong C_*(M) we have quantities C_*(u) \stackrel{\cong}{\longrightarrow} C_*^{d,*}(u)

which induce is an hordin.

hocolin C_*^*(u) \stackrel{\cong}{\longrightarrow} C_*^*(M)

Which induce is an hocolin.

Non abelian cohomology:

H^{\infty}(M;A) = \text{homotopy classes of maps } M \longrightarrow K(A;m)

Note: If m=1, K(G_2) = BG even for non-cabelian obsceeke group G

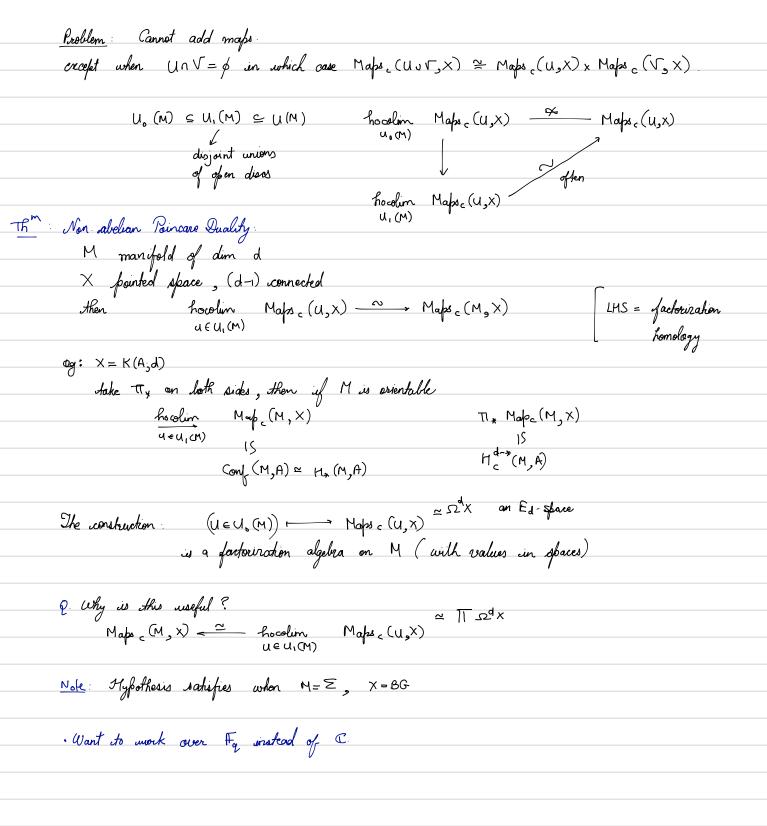
H^{1}(M;G) = [M,BG]

= \text{use classes of } G\text{-bundles on } M

= \text{group homo} \quad T(M) \longrightarrow G / \text{conjugacy}

Oct X be any space H(M;X)^{*} := [M,X]
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$$M = \mathbb{R}^d$$
 Maps $_{\mathcal{C}}(\mathbb{R}^d, \times) = \Omega^d \times$



Ron
$$(M) = \begin{cases} \text{non-empty finite} \end{cases} \leftarrow \text{hopology with basis}$$

$$\begin{cases} \text{Ron } (U_1, \dots, U_n) = \begin{cases} S \leq UU_i \leq M, \\ S \cap U_i \neq \delta \end{cases} \text{ for each } i$$

$$U_i \text{ pairwise disjoint connected rets} \end{cases}$$

F special open sets -> Spaces

 \mathcal{F} (Ran $(u_1, ..., u_n) = Maps_{\epsilon}(u_1 \cup ... \cup u_n, \times)$

The F is a homotopy coshed on Ran (M).

Assume M connected, $\frac{ho(olim)}{u_1, \dots, u_n}$ $F(Ran(u_1, \dots, u_n)) \xrightarrow{\simeq} F(Ran(M))$ IS disjoint open $Maps_c(u_1 u \dots u_{n}, x) \longrightarrow Maps_c(M, x)$

Why (d-1) connected On the dovel of Π_0 this can be shown by triangulating M and winy the simplices. The obstruction for gluing lies precisely in the first of I homotopy groups of X.