

n-Groupoids

Category of fibrant objects:

\mathcal{V} a small category.

$\mathcal{W} \subseteq \mathcal{V}$ subcategory of weak equivalences

- contains iso
- 2-out-of-3

Dwyer-Kan : Can construct ^{simplicial} localization of \mathcal{V} respect to \mathcal{W} . (components of this is the derived cat.)

$\mathcal{V}_\bullet \rightarrow \mathcal{V}$ simplicial resolution

$$\text{Ob}(\mathcal{V}_n) = \text{Ob}(\mathcal{V}) \quad n \geq 0$$

\mathcal{V}_n is a free category for $n \geq 0$

$\hookrightarrow n^{\text{th}}$ level = words in \mathcal{V} with parenthesis of depth n .

$$\mathcal{V} \cdot [\mathcal{W}^{-1}] = \mathcal{W}^{-1} \cdot \mathcal{V} \quad \leftarrow \text{Goal: understand this localization}$$

2-out-of-6

$$\mathcal{W} \xrightarrow{f} x \xrightarrow{g} y \xrightarrow{h} z \quad \text{if } gf \text{ and } hg \in \mathcal{W} \text{ then } f, g, h \in \mathcal{W}$$

if 2 out of 6 is satisfied then every weak equiv goes to iso in $\mathcal{V} \cdot [\mathcal{W}^{-1}]$. (Called Saturation property)

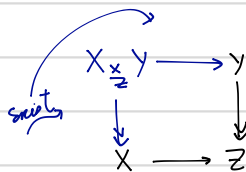
Q. Check this for $\pi_i(-)$.

Th^m

Categories of fibrations

models category of manifolds

- $\mathcal{F} \subseteq \mathcal{V}$ subcategory. All pullbacks in \mathcal{F} exist
- \exists a terminal object.
- all iso $\in \mathcal{F}$



CFO

A category of fibrant objects satisfies following axioms: (has w.e & fibrations)

- 1) Every object is fibrant (i.e. $X \rightarrow c$ is a fibration)
- 2) Pullback of a trivial fibration is a trivial fibration (= w.e. + fib.)
- 3) Every morphism factors into a w.e. followed by a fibration.

- These axioms are weaker than modal category axioms.
- eg: Kan Complexes, manifolds

Vertices of Simplicial Localization for CFO

Generalised morphisms

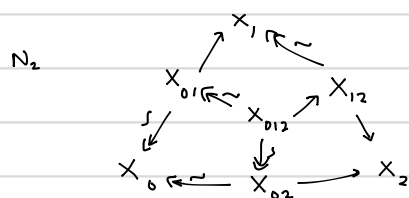
$$\begin{array}{ccc} & P & \\ s \swarrow & & \searrow \text{morphism} \\ X & & Y \end{array} \in \text{Hom}(X, Y)$$

$N.V. [W^{-1}]$ is a quasi-category

N_0 objects of V

N_1 $\begin{array}{ccc} & P & \\ s \swarrow & & \searrow \\ X & & Y \end{array}$

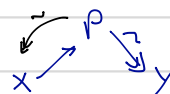
1-simplices



Brouwer's Lemma :

W.E. in a CFO have the following form : section of trivial fib. followed by a fib.

- This lemma allows us to work exclusively with fibrations.



- Fibrations of s.Sets : $X \rightarrow Y$
- for $n \geq 0$, $0 \leq i \leq n$

$$X_n \longrightarrow \text{Hom}(\Delta^n, X) \times_{\text{Hom}(\Lambda_i^n, Y)} Y_n \text{ is surjective}$$

Initial

- Fibration of Kan complexes

- for $n \geq 0$

$$X_n \longrightarrow \text{Hom}(\Delta^n, X) \times_{\text{Hom}(\partial \Delta^n, Y)} Y_n \text{ is surjective}$$

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