- Implicit differentiation:

**Q.** Given an equation involving $x, y$

Find $y'$.

- $y$ is thought of as a function of $x$ &

(say $y = f(x)$).

- But we do not know $y$ explicitly as a function of $x$.

hence the term "implicit"

in implicit differentiation

- Method: Differentiate both sides but treat $y$ as a function of $x$

eg: Given $\sin(xy) = x^2 + y^2$, find $y'$.

Ans: Differentiate both sides

$$(\sin(xy))' = (x^2 + y^2)'$$

Chain rule

$$= \sin'(xy) \cdot (xy)'$$

Product Rule

$$= \cos(xy) \cdot (x'y + y'x)$$

$$= \cos(xy) \cdot (y + y'x)$$

$$= (x^2)' + (y^2)'$$

Chain Rule

$$= 2x + 2y \cdot y'$$

Note:

$(y^2)' \neq 2y$

Think $y^2 = (f(x))^2$

so that

$(y^2)' = (f(x)^2)'$

$= 2f(x) \cdot f'(x)$

$$\Rightarrow \cos(xy) \cdot (y + y'x) = 2x + 2y \cdot y'$$

From here, we isolate $y'$:

$$\cos(xy)\cdot y + \cos(xy)\cdot y'\cdot x = 2x + 2yy'$$

$$\Rightarrow \quad \cos(xy)\cdot y = -\cos(xy)\cdot y' + 2x + 2yy'$$

$$\Rightarrow \quad \cos(xy)\cdot y - 2x = -\cos(xy)\cdot y' + 2yy'$$

$$= y'(-\cos(xy) + 2y)$$

$$\Rightarrow \quad y' = \frac{\cos(xy)\cdot y - 2x}{-\cos(xy) + 2y}$$

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