<u>Ch 3</u>

Let $Q \in q$ Cat. • objects of hQ: elements $1 \xrightarrow{q} Q$ • mor of hQ: homotopy classes of 1-simp in Q

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Classical fact:

 $J = small \ category$ $h(Q^3) \neq (hQ)^5$ "homotopy coherent diagrams" \neq "homotopy commutative diagrams"

But there is a map
$$Q^3 \times J \xrightarrow{ev} Q$$

apply $h: qGt \longrightarrow Gt$

 $h(Q^{T} \times T) \cong h(Q^{T}) \times hJ$

 $\cong \mathcal{A}(Q^{\mathsf{T}}) \times \mathsf{T} \longrightarrow \mathcal{A}Q$

 $k(\varrho^3) \longrightarrow (k\varrho)^3$

Jor J= 2= △[1]="0->1"

 $\frac{\text{demma}}{\text{demma}}$ Q any q-lat $-\text{h}(Q^2) \longrightarrow (\text{h}Q)^2 \in \text{Cat}$

The smothering functor is · surjective on objects · full

· conservative (reflects invertibility of objects)

Def A fundor of extict 1-certs is smothering if it has 3 RLP wit to the three morphisms: $\{\phi \hookrightarrow \mathbb{I}, 1 + 1 \hookrightarrow \mathbb{Z}, \mathbb{Z} \hookrightarrow \mathbb{I}\}$

Knot The fibers of a smothering function are connected groupoids

Lemma Juven a fullback of g Cat with p an isofile

$$h(E \times A) \longrightarrow hE \times hA$$

us smothering

A an ∞ -category

A =: ∞ -category of arrows $\exists A^2 = A^{\square} \xrightarrow{\Rightarrow} A^{\square} \cong A \times A$ (p_1, p_2) (dom, cod)

This comes with a canonical 2-cell in hK

simplicial cotensors in X satisfy: Fun $(X, A^2) \cong Fun(X, A)^2$ (p, pi) / (ev., evi)

 $Fun(X,AxA) \cong Fun(X,A)xFun(X,A)$

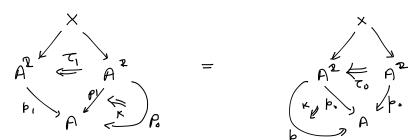
when $X = A^{2}$, im (id_x) is the 2-cell.

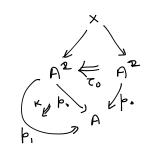
A has a weak UP that supplies 3-operations in h.K:

1-cell induction:

2-cell induction:

Given
$$X \xrightarrow{\alpha} A^{2}$$
 and $\tau_{\alpha}, \tau_{\alpha} \in \mathcal{T}$.





$$X = \frac{1}{b} = \frac{1}{b}$$

$$\Rightarrow t \qquad \Rightarrow \tau_1$$

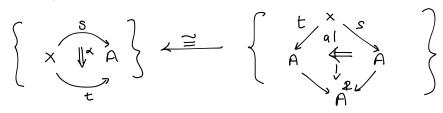
$$\Rightarrow \tau_2 = \tau_1$$

$$\Rightarrow \tau_3 = \tau_3$$

2-cell conservatively:
$$2f \ \tau_o \ \tau_i$$
 are invertible then so is τ_i .

Roof uses the fact that $1 + (x, A^2) \rightarrow (h + (x, A))^2$ is smothering

Rep: Whiskering with the generic 2-cell over A defines a hijection



$$A = \frac{x}{\sqrt{\frac{x}{r}}}$$

Prop The weak UP of A characterizes it up to equiv in JK/AXA.