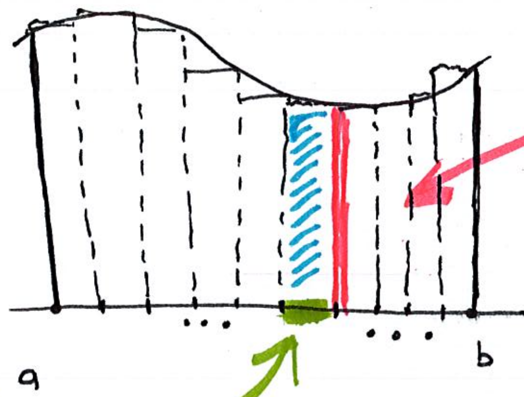
Theorem 4, Section 5.2)

$$\int_a^b f(x)\,dx = \lim_{N\to\infty}\left(\sum_{i=1}^{N} \frac{b-a}{N}\cdot f\left(a+i\cdot\left[\frac{b-a}{N}\right]\right)\right)$$



- divide [redacted] [a,b] into N equal parts

$\Rightarrow$ length of each part is $\frac{b-a}{N}$

- i^{th} endpoint is $a+i\cdot\left(\frac{b-a}{N}\right)$

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eg:

$$\int_2^4 e^x\,dx = \lim_{N\to\infty}\left(\sum_{i=1}^{N} \frac{2}{N}\cdot e^{\left(2+i\cdot\left(\frac{2}{N}\right)\right)}\right)$$

- To go from RHS to LHS :

$$\lim_{N\to\infty}\left(\sum_{i=1}^{N} \frac{2}{N}\cdot e^{\left(2+i\cdot\left(\frac{2}{N}\right)\right)}\right)$$

$\downarrow$

$f(x) = e^x$

$$= \int_2^4 e^x\,dx$$

lower limit : $i=1$ in $2+1\cdot\frac{2}{N}$

$\downarrow \lim_{N\to\infty}$ [redacted]

$2+1.0 = 2$

upper limit : $i=N$ in $2+N\cdot\frac{2}{N}$

$\downarrow \lim_{N\to\infty}$

$2+2=4$