Cartesian fibrations:

Aside: What are Spaces + Spectra as ∞ - cats?

Buth Top, Sp are both Top enriched We can then move from QCat to other $(\infty,1)$ -models.

Idea: $E \xrightarrow{P} B$ isofib. \Rightarrow $E_b \xrightarrow{E} E$ fibers E_b are ∞ -cats $1 \xrightarrow{P} E$

In a cocontesian fibration arrows in B act { covariantly on filers contraviantly

In cocartesian file: can lift 2 cells along p with specified lift on domain

Def " E >>> B iso fib. A natural transform X is E is p-contesion

i) induction: given $X = \begin{bmatrix} c'' \\ b \\ c \end{bmatrix} = \begin{bmatrix} and \\ b \\ c' \end{bmatrix}$ So that b = b

then $\exists \hat{x}: e'' \Rightarrow e$ st $p\hat{x} = x$ "X is universal lift of px with codomain e''

2) conservativity: if $\xi: e \rightarrow e'$ st. $\chi \xi = \chi$ and $\beta \xi = id_{\beta}e'$ then ξ is invertible.

Lemma: $\chi, \chi' : e'' \Rightarrow e'$ are p-cartesian and $p\chi = p\chi'$ then $\exists \varsigma : e'' \stackrel{\sim}{=} e'$ s.t. $p\varsigma = id$.

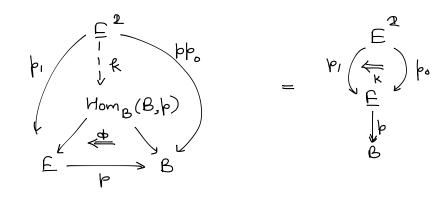
Def: dn usofile £ → B is cartesian of

i) lifting Juven 2-cell $\beta:b \Rightarrow pe$

X_p us p-corresion

Reop · E b B ->> A 9, p cort -> so is eyp

Main "Inhernal char of carbesian fibrations"



 $\mathcal{K} \xrightarrow{\Gamma} \mathcal{I}$ be a cosmological functor \Rightarrow f preserves cartesian fibrations.

Lem:
$$E \xrightarrow{p} B$$
 is a cortesion fib iff the comma cone

Hom_B $(B, p) \xrightarrow{p_i} E$ has a lift = Hom_B $(B, p) \xrightarrow{p_i} E$

such that Xf is rartesian $\forall X \xrightarrow{f} Hom_{B}(B, b)$.

den Consider
$$F \xrightarrow{f} E$$
 p contesian $\Rightarrow q$ -cartesian.
 $q \downarrow f \downarrow p$ $\downarrow p$ \downarrow

Brop A an ∞ -rategory, $A^2 \xrightarrow{p_0} A$ the domain projection functor is a cartesian fiber Moreover, $X \xrightarrow{y \to A} A$ is po-cartesian iff $X \xrightarrow{y \to A} A$ is an iso.

Prop If A has all fullbacks then the codomain force is a contesion fib $A^2 \stackrel{P_1}{\longrightarrow} A$.

Def: K be an ∞ -cosmos \mathcal{F}^{∞} \cdot ob $K = \circ b$ \mathcal{K} \cdot functors $A \longrightarrow B$ are the same \cdot Fun \mathcal{K}^{∞} $(A,B) := \operatorname{Fun}_{\mathcal{K}}(A,B)^{\circ p}$

(A q Cat is a functor: $\triangle^{\circ p} \longrightarrow \operatorname{Set}$ we can precompose with the functor $\triangle^{\circ p} \longrightarrow \triangle^{\circ p}$ sending $(n) \longmapsto (n)$ reversing the order)

Then $h(X^{\circ}) = (hX)^{\circ}$ $A \longrightarrow B \longrightarrow A \longrightarrow B$

Def: $E \xrightarrow{b} B$ is a cocartesian fib iff its p-cartesian fibration in \mathcal{K}^{co}

Def A lei-fibrohon is an isofile that is both cartesian & co-cartesian.

Prof. Let $E \xrightarrow{\beta} B$ be a lir fibration and $X \xrightarrow{b} B \xrightarrow{b} E$ Then \exists an adjunction $E_a \xrightarrow{\bot} E_b$ E_a, E_b dibers over a, b resp.

Special 1 I 2 « E « Luxies def' of adjunction bifibration over 2.