Calculus 1000A - Section 007

This week: § Quick Review

Sections: 1.4 Exponentials

Appendix D Trigo functions

1.5 Logarithms / Inverse

§ ~~R~~ Today: Review, 1.4

§ Precalc - Review:

Q. What is calculus?

Study of functions

- derivatives, rates of change
- approximations of curves
- limits
- integrals
- parametrizations

(series, sequences).

• Properties of functions :

for every x there is y value → ("x, y" values}, variables)

(continuity)

- domain , range
- limits / asymptotes = " end behaviors"
- odd, even
- maxima, minima, critical points, points of inflection
- concavity / convexity
- ~~norm~~ graphs
- normals / tangents

■ = precalc, do not need derivatives, integrals

Q. What is a
~~Q.~~ Function ?

- function outputs a "y-value" for every "x-value"

- input number $\xrightarrow{\text{function}}$ output number

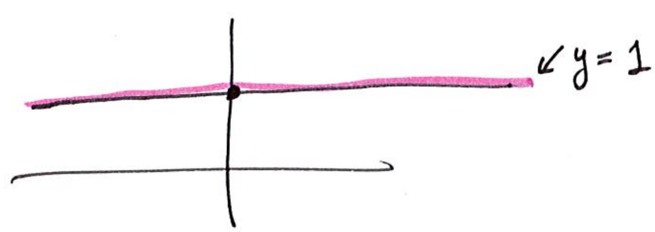
eg:

- $y = mx + b$

$\boxed{f(x) = 2x + 3}$ ← graph of this is a line with slope = 2
y-intercept = 3

$\boxed{f(x) = x}$ slope = 1
y-intercept = 0

- $\boxed{f(x) = 1}$ ← constant function

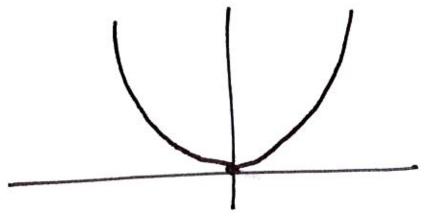


- **Polynomials**

eg: $f(x) = x^2$

$f(x) = x^3 + 10x + 1$

eg: $f(x) = x^2$



$\leftarrow$ parabola

- **domain** of a function is the set of possible x-values ("inputs")

- **range** of a function is the set of y-values ("outputs")

back to $f(x) = x^2$

domain = all real numbers = $\mathbb{R}$

range = non-negative real numbers

$= \{ y \geq 0 \}$

$= [0, \infty)$

~~$[x, y] = \{$~~

- $[a, b] = \{x : a \le x \le b\}$
- $[a, b) = \{x : a \le x < b\}$

$\vdots$ etc.

- Never put ~~$(, \infty]$, or~~ square brackets around ∞.

<u>eg</u>: $f(x) = \dfrac{1}{(x-1)^2}$

domain: $x \neq 1$ (because you cannot divide by 0)

$\circ$ = 

$= (-\infty, 1) \cup (1, \infty)$

$\cup$ ← union

★ range = ??

(range is harder to find than domain)

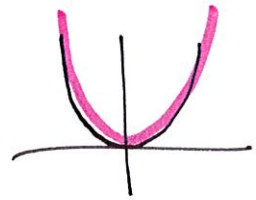
<u>Notice</u>:
- $(x-1)$ is being squared
- $(x-1)^2$ is never negative
- $\dfrac{1}{(x-1)^2}$ can take only positive values

$\Rightarrow$ range $= \{y > 0\}$ ← More precisely $= \{y : y > 0\}$

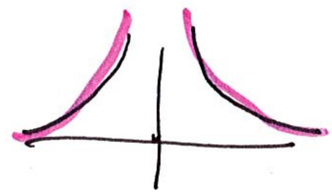
$= (0, \infty)$

- graph:
  - make table values

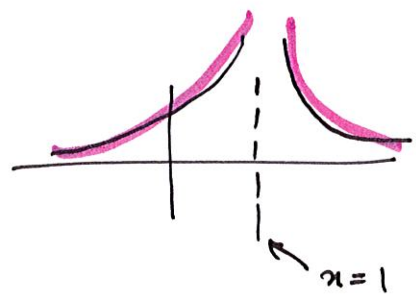
- $x^2 \rightsquigarrow$



$\frac{1}{x^2} \rightsquigarrow$



$\frac{1}{(x-1)^2} \rightsquigarrow$



- changing x to $x-1$ — shifted graph to right by 1

~~Poly, li~~ Polynomials, linear functions

Webwork. $(f \circ g)(x) =$ ~~notat~~ notation for compositions

$$= f(g(x))$$

eg: $f(x) = x^2 - 1$ $\quad$ $g(x) = x + 2$

$$(f \circ g)(x) = f(g(x))$$

$$= (g(x))^2 - 1$$

$(a+b)^2$

$= a^2 + 2 \cdot a \cdot b + b^2$

$$= (x+2)^2 - 1$$

$$= (x^2 + 2 \cdot 2 \cdot x + 2^2) - 1$$

$$= x^2 + 4x + 4 - 1$$

$$= x^2 + 4x + 3$$

* = asterisk

on webwork = x ^ 2 + 4 * x + 3

= x ^ 2 + 4x + 3

§ 1.4 Exponential functions

$f(x) = a^x \leftarrow a^x$ where a is a constant.

[Note: x^n is a polynomial where n is a constant]

$a =$ base of the exponential

eg: $f(x) = 2^x$

- $x =$ positive integer $\qquad 2^x = \underbrace{2 \cdot 2 \cdots \cdot 2}_{x\text{-times}}$

  eg: $2^1 = 2$

  $2^2 = 2 \cdot 2 = 4$

  $2^3 = 2 \cdot 2 \cdot 2 = 8$

  $\vdots$

- $x = 0 \qquad 2^0 = 1$

- $x = \frac{p}{q}$ rational $\qquad 2^{p/q} = \sqrt[q]{2^p}$

  $2^{1/2} = \sqrt[2]{2^1} = \sqrt[2]{2}$

  $2^{2/3} = \sqrt[3]{2^2} = \sqrt[3]{4} \quad \ldots$

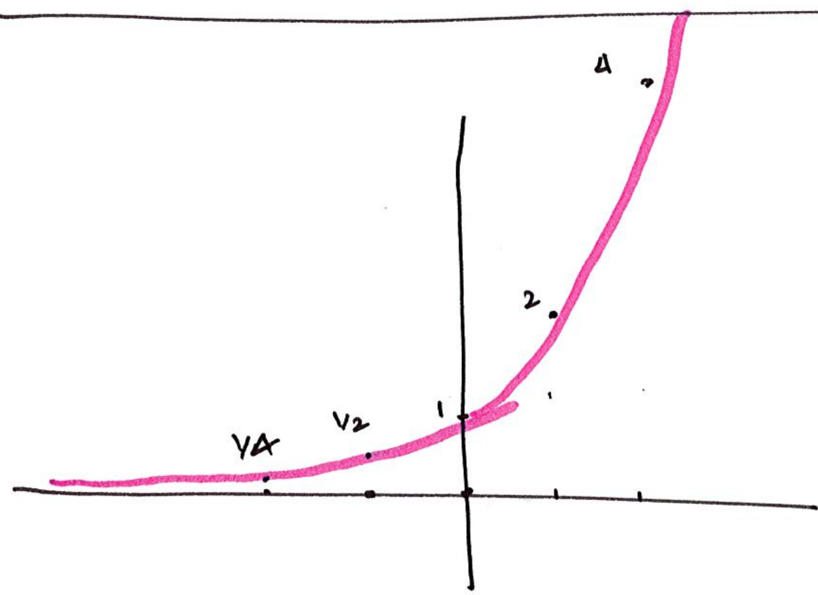
• (x = negative) $\cancel{2^{-x} = \frac{1}{2^x}}$ $\Big|$ $2^{-x} = \frac{1}{2^x}$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$2^{-2/3} = \frac{1}{2^{2/3}} = \frac{1}{\sqrt[3]{2^2}} = \frac{1}{\sqrt[3]{4}}$$

• x = irrational (read from the book)

• 

graph of

$f(x) = 2^x$

table

x	$f(x)$
0	1
1	2
-1	1/2
-2	1/4
2	4

domain : all real numbers

$= \mathbb{R} = (-\infty, \infty)$

range : $\{y : y > 0\}$

$= (0, \infty)$

Identities :

- $$\underbrace{2^x}_{\text{multiplying 2 } x\text{-times}} \cdot \underbrace{2^y}_{\text{mutiply 2 } y\text{-times}} = 2^{x+y}$$

- $$\frac{2^x}{2y} = 2^{x-y}$$

- $$2^{xy} = \left(2^x\right)^y$$

multiplying 2 xy times

$$= \underbrace{2 \cdot 2 \cdot \;\cdots\; 2}_{xy\text{-times}}$$

$$= \underbrace{\underbrace{(2 \cdots 2)}_{x\text{-times}} \cdot \underbrace{(2 \cdots 2)}_{x\text{-times}} \cdots \underbrace{(2 \cdots 2)}_{x\text{-times}}}_{y\text{-times}}$$

$$= \underbrace{2^x \cdot 2^x \cdot \;\cdots\; 2^x}_{y\text{-times}}$$

$$= \left(2^x\right)^y$$

eg. $2^{xyz} = (2^x)^{yz} = ((2^x)^y)^z$

$$2^4 = 2^{2\cdot 2} = (2^2)^2 = 4^2 = 16$$
$$\|$$
$$2\cdot 2\cdot 2\cdot 2$$
$$\|$$
$$16$$

- These are called laws of exponentiation.

- $2^{x+y} = 2^x \cdot 2^y$

  $2^5 = 2^2 \cdot 2^3 = 4\cdot 8 = 32$

- $2^{x-y} = \dfrac{2^x}{2^y}$

- $(2^x)^y = 2^{xy}$

These are true for any base.

- Identify involving multiple bases:

  $(ab)^x = a^x \cdot b^x$ eg: $6^2 = (2\cdot 3)^2 = 2^2\cdot 3^2 = 4\cdot 9 = 36$

eg :- graph of $f(x) = 2^{-x}$ = graph of 2^x but reflected abt y-axis

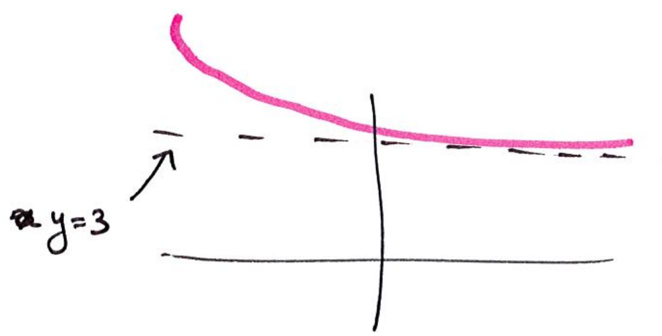
(Sec 1.2)

2^x = "exponential growth"

2^{-x} = "exponential decay"

• graph of $f(x) = 3 + 2^{-x}$

= graph of 2^{-x} shifted up by 3



eg: $\cancel{2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x}$

Solve for x given

$$\boxed{10 \cdot 2^x = 5}$$

$10 \cdot 2^x = 5$

$\Rightarrow \quad 2^x = \frac{\cancel{5}^{\,1}}{\cancel{10}_{\,2}}$ dividing both sides by 10

$\Rightarrow \quad 2^x = \frac{1}{2}$

$\Rightarrow \quad \boxed{x = -1}$ Ans.

(Reason we are studying exponential functions is that they have simple derivatives.)

<u>Later</u>: $\frac{d}{dx}(2^x) = (\ln 2)\, 2^x$

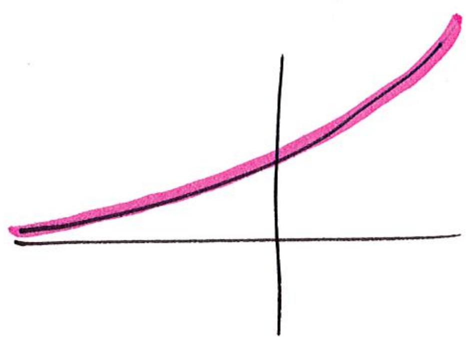
- Natural exponential function

$$f(x) = e^x$$

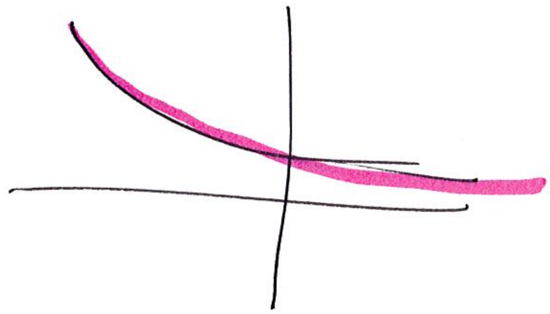
e is a number $\approx 2.71\ldots$ (Euler's number)
$\quad\quad\quad\quad\quad\quad$ || (constant)

e is the unique constant for which

$$\frac{d}{dx}(e^x) = e^x$$



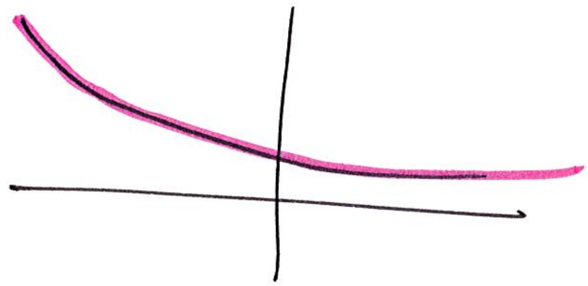
graph of e^x



graph of e^{-x}

$\underline{eg}$:

graph of $\left(\frac{1}{2}\right)^x = 2^{-x}$



graph of $a^x = \begin{cases} \text{exponentially growing} & \text{if } a > 1 \quad \text{eg: } 2^x \\ \text{exponentially decaying} & \text{if } 0 < a < 1 \quad \text{eg: } \left(\frac{1}{2}\right)^x \end{cases}$

~~we never let~~

(negative bases do not always make sense).

Domain of $\sqrt{x}$

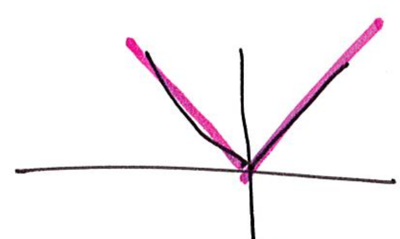
$f(x) = \sqrt{x}$

domain $=$ $[0, \infty)$

range $=$ $[0, \infty)$

absolute ~~ele~~ value $|x|$

$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



domain $= \mathbb{R}$

range $= [0, \infty)$