

Representability of comas :

Th^m : Given functor $C \xrightarrow{g} A \xleftarrow{f} B$ and $C \xrightarrow{r} A \exists$ a bijection :

$$\left\{ \begin{array}{c} B \\ \begin{array}{ccc} & \nearrow r & \\ C & \xrightarrow{g} & A \\ & \searrow f & \end{array} \end{array} \right\} \begin{array}{c} \text{absolute} \\ \text{right} \\ \text{lifting} \end{array} \left\{ \begin{array}{c} \text{Hom}_B(B, r) \simeq_{C \times B} \text{Hom}_A(f, g) \end{array} \right\}$$

Th^m : Given $C \xrightarrow{g} A \xleftarrow{f} B \exists$ an absolute right lifting through f iff $C \xleftarrow{\perp} \text{Hom}_A(f, g)$ admits a right adjoint right inverse in which case $r := p_* \cdot i$ is the representing functor.

Defⁿ : A right adjoint right inverse (rari) for $f: B \rightarrow A$ is a right adjoint i st. the counit $f i \xrightarrow{\varepsilon} \text{id}_A$ is the identity 2-cell ($f i = \text{id}_A$)
By triangle identities : $f \eta = \text{id}_f$, $\eta i = \text{id}_i$.

Def : $A \in \mathcal{K}$. A slice ∞ -cosmos over A
• $\text{ob} = \text{isofib } B \xrightarrow{p} A, C \xrightarrow{q} A$

$$\begin{array}{ccc} \text{mor} = \text{Fun}_A(B \xrightarrow{p} A, C \xrightarrow{q} A) & \longrightarrow & \text{Fun}(B, C) \\ \downarrow & & \downarrow q \\ \mathbb{I} & \xrightarrow{p} & \text{Fun}(B, A) \end{array}$$

• $X \rightarrow A$ defines a map $\mathcal{K}/A \rightarrow \mathcal{K}/X$

• A RARI to $f: B \rightarrow A$ is exactly a terminal element of f over A .

§4. Adjunctions :

- Prop:**
- $B \begin{matrix} \xrightarrow{f} \\ \xleftarrow{u} \end{matrix} A$ are adjoint $f \dashv u$ iff $\text{Hom}_A(f, A) \xrightarrow{\sim}_{A \times B} \text{Hom}_B(B, u)$.
 - Given any $X \xrightarrow{a} A, Y \xrightarrow{b} B$, $\text{Hom}_A(fb, a) \xrightarrow{\sim}_{X \times Y} \text{Hom}_B(b, ua)$.
 - $f \dashv g \Rightarrow 1 \xrightarrow{\varepsilon_a} \text{Hom}_A(f, a)$ is terminal, $1 \xrightarrow{\eta_b} \text{Hom}_B(b, u)$ is initial.

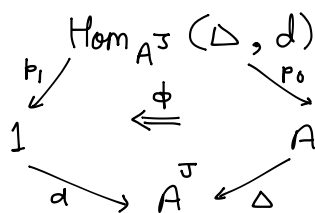
Th^m: Adjoint functor theorem:

$B \xrightarrow{f} A$ admits a right adjoint iff $\text{Hom}_A(f, A) \xrightarrow{p_1} A$ has a terminal element over A . Dually f has a left adjoint iff $\text{Hom}_A(A, f) \xrightarrow{p_0} A$ has an initial element over A .

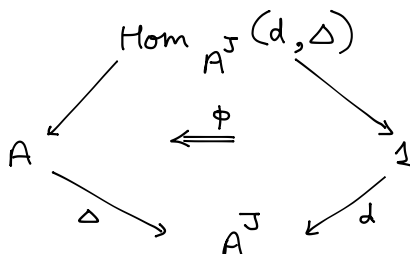
Limits & Colimits :

Def: Given $1 \xrightarrow{d} A^J$, ω -category of

cones over d :



cones under d :



Prop: d has a limit (colimit) iff $\text{Hom}_{A^J}(\Delta, d)$ is right representable. ($\text{Hom}_{A^J}(d, \Delta)$ (left))

• d has a limit iff $\text{Hom}_{A^J}(\Delta, d)$ has a terminal element.