Recall:

- derivative $f'(a)$ = slope of tangent line to the graph of $f(x)$ at $x=a$.

Rules of differentiation: f, g differentiable functions.

- $(f+g)' = f' + g'$
- $(f-g)' = f' - g'$
- $(cf)' = c \cdot f'$
  c is a real number.

- $(fg)' = f'g + g'f$
- $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$(e^x)' = e^x$

$(a^x)' = a^x \cdot \ln a$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

Important limits:

① $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$ ② $\lim_{h\to 0} \frac{\sin h}{h} = 1$ ③ $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

Section 3.3

39. Find $\lim_{x\to 0} \frac{\sin(3x)}{5x}$

If we plug in, we get $\frac{0}{0}$.

A: We want to use $\boxed{\lim_{h\to 0} \frac{\sin h}{h} = 1}$

. Multiply numerator & denominator by $\underline{3}$

$$\lim_{x\to 0} \frac{\sin(3x)}{5x} \cdot \frac{3}{3} = \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{3}{5}$$

let $3x = k$,

as $x \to 0$, $k \to 0$

$$= \lim_{k\to 0} \frac{\sin(k)}{k} \cdot \frac{3}{5}$$

$$= 1 \cdot \frac{3}{5}$$

. <u>Example 6</u> : ~~do~~ Find $\lim_{x\to 0} x \cdot \cot x$.

$$= \lim_{x\to 0} x \cdot \frac{\cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{x}{\sin x} \cdot \cos x$$

$$= \lim_{x\to 0} \frac{x}{\sin x} \cdot \lim_{x\to 0} \cos x$$

$$= \lim_{x\to 0} \frac{1}{(\sin x / x)} \cdot \lim_{x\to 0} \cos x$$

$$= \frac{1}{1} \cdot 1$$

$$= 1$$

we're using

① $\lim_{x\to 0} \frac{\sin x}{x} = 1$

② $\cos(0) = 1$

$$a/b = \frac{1}{(b/a)}$$

Chapter :

(02)

45 : Find $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta + \tan\theta}$.

Chapter 2.5

57 ~~56~~) Show that there is a solution to

$$\cos x = x.$$

Proof : Need to use the intermediate value theorem (IVT)

$$\cos x = x \iff \cos x - x = 0$$

Let $\boxed{f(x) = \cos x - x.}$

we need to show that there exists a "c" such that

$$f(c) = 0.$$

Come up with a, b real numbers

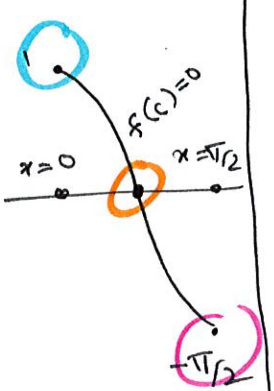
with

$f(a) < 0$ $\qquad$ $f(b) > 0$

$f(x) = \cos x - x$

$\boxed{x = 0, \quad f(0) = \cos 0 - 0 = 1 > 0}$

$\boxed{x = \frac{\pi}{2}, \quad f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} - \frac{\pi}{2} = 0 - \frac{\pi}{2} < 0}$



As $f(x) = \cos x - x$ is continuous, by IVT there is a "c" ~~for which~~ between 0, $\frac{\pi}{2}$ for which

$$f(c) = 0.$$

45) find $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta + \tan\theta}$ (0 3)

$$= \lim_{\theta \to 0} \frac{\sin \theta}{\theta + \sin\theta/\cos\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin \theta / \theta}{(\theta + \sin\theta/\cos\theta)/\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta/\theta}{1 + (\sin\theta/\theta) \cdot \frac{1}{\cos\theta}}$$

$$= \frac{\lim_{\theta \to 0} \sin\theta/\theta}{1 + \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right) \cdot \lim_{\theta \to 0} \frac{1}{\cos\theta}}$$

$$= \frac{1}{1 + 1 \cdot \frac{1}{1}}$$

$$= \frac{1}{2}$$

Chain Rule :

- Derivatives of compositions of functions

- $$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

2-functions f, g

Derivative of the outer function at the inner function

Derivative of the inner function

eg : $(f(g(h(x))))' = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

$\longrightarrow$ Start with derivative of the outermost function and move unward.

---

eg : $(\sin(x^2))' = \sin'(x^2) \cdot (x^2)'$

$= \cos(x^2) \cdot (2x)$

$= 2 \cdot \cos(x^2) \cdot x$

eg: $\left((\sin(x))^2\right)' = 2 \cdot (\sin(x)) \cdot \sin'(x)$

outermost function $= (-)^2$

derivative $= 2 \cdot (-)$

$= 2 \cdot \sin x \cdot \cos x$

$= \sin(2x)$

05

eg

$$\left(e^{(\sin x+\tan x)}\right)' = e^{(\sin x+\tan x)}\cdot(\sin x+\tan x)'$$

outermost function $= e^{x}$

$(e^x)' = e^x$

$$= e^{(\sin x+\tan x)}\cdot\left[(\sin x)' + (\tan x)'\right]$$

$$= e^{\sin x+\tan x}\left[\cos x + \sec^2 x\right]$$

eg

$$\left(\cos\left(e^{(x^2)}\right)\right)' = -\sin\left(e^{(x^2)}\right)\left(e^{(x^2)}\right)'$$

outermost : $\cos(*)$

derivative $= -\sin(*)$

$$= -\sin\left(e^{(x^2)}\right) e^{(x^2)}\cdot(x^2)'$$

$$= -\sin\left(e^{(x^2)}\right)\cdot e^{(x^2)}\cdot 2x$$

eg:

$$\left(\sin\left(\frac{e^x}{1+x^3}\right)\right)' = \cos\left(\frac{e^x}{1+x^3}\right)\cdot\left(\frac{e^x}{1+x^3}\right)'$$

$$= \cos\left(\frac{e^x}{1+x^3}\right)\cdot\left[\frac{(1+x^3)\cdot(e^x)' - e^x\cdot(1+x^3)'}{(1+x^3)^2}\right]$$

$$= \cos\left(\frac{e^x}{1+x^3}\right)\left[\frac{(1+x^3)e^x - e^x\cdot[3x^2]}{(1+x^3)^2}\right]$$

Q. Find slope of tangent at $x=0$ for

$$f(x) = \sqrt{1+\sec x}\ .$$

A. Slope of tangent $= f'(x)$

$$f'(x) = \left(\sqrt{1+\sec x}\right)'$$

$$= \left((1+\sec x)^{1/2}\right)'$$

outermost : $x^{1/2}$

derivative : $\frac{1}{2} \cdot x^{-1/2}$

$$= \frac{1}{2}(1+\sec x)^{-1/2} \cdot (1+\sec x)'$$

$$= \frac{1}{2}(1+\sec x)^{-1/2} \cdot \left(1+\frac{1}{\cos x}\right)'$$

$$= \frac{1}{2}(1+\sec x)^{-1/2} \cdot \left(0 + \frac{\cos x \cdot 1' - 1 \cdot (\cos x)'}{\cos^2 x}\right)$$

$$= \frac{1}{2}(1+\sec x)^{-1/2} \cdot \left(0 + \frac{0 - (-\sin x)}{\cos^2 x}\right)$$

$$= \frac{1}{2} \cdot (1+\sec x)^{-1/2} \left(\frac{\sin x}{\cos^2 x}\right)$$

$$f'(0) = \frac{1}{2} \cdot (1+\sec 0)^{-1/2} \left(\frac{\sin 0}{\cos^2 0}\right)$$

$$= \frac{1}{2} \cdot (1+1)^{-1/2} \cdot \frac{0}{1}\ .$$

$$= 0$$

Q. find $\left(x\cdot\sqrt{1-x^2}\right)'$

A: $\left(x\sqrt{1-x^2}\right)' = x'\left(\sqrt{1-x^2}\right) + x\cdot\left(\sqrt{1-x^2}\right)'$

$$= 1\cdot\sqrt{1-x^2} + x\cdot\underbrace{\left(\sqrt{1-x^2}\right)'}$$

$$\left(\sqrt{1-x^2}\right)' = \left(\left(1-x^2\right)^{1/2}\right)'$$

| outermost function : $x^{1/2}$
| derivative : $\frac{1}{2}x^{-1/2}$

$$= \frac{1}{2}\cdot\left(1-x^2\right)^{-1/2}\cdot\left(1-x^2\right)'$$

$$= \frac{1}{2}\cdot\left(1-x^2\right)^{-1/2}\cdot\left(0-2x\right)$$

$$= -\frac{\cancel{2}}{\cancel{2}}\left(1-x^2\right)^{-1/2}\cdot x$$

Plugging back in

$$\left(x\cdot\sqrt{1-x^2}\right)' = 1\cdot\sqrt{1-x^2} + x\left(-\left(1-x^2\right)^{-1/2}\cdot x\right)$$

$$= \sqrt{1-x^2} - x^2\left(1-x^2\right)^{-1/2}$$