AS.110.361 - Symmetries & Polynomials

$$
\begin{aligned}
x_1 = &-\frac{b}{3a} \\
&-\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d+\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]} \\
&-\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d-\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]} \\
x_2 = &-\frac{b}{3a} \\
&+\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d+\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]} \\
&+\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d-\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]} \\
x_3 = &-\frac{b}{3a} \\
&+\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d+\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]} \\
&+\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\frac{1}{2}\left[2b^3-9abc+27a^2d-\sqrt{\left(2b^3-9abc+27a^2d\right)^2-4\left(b^2-3ac\right)^3}\right]}
\end{aligned}
$$

Everybody knows the formula for finding the roots of a quadratic polynomial. The much less known formula for finding the roots of a cubic polynomial looks something like the image on the left. The formula for finding the roots of a 4^{th} degree polynomial is even more complicated. And then they stop!

A deep theorem in Galois theory says that there is no general formula for find the roots of a 5^{th} or higher degree polynomial!! And this has to do something with symmetry groups and permutations of roots!!!

In this course we'll try to derive the formula for cubic and 4^{th} degree polynomials using complex numbers and some basic group theory and try to see why NO such formula exists for higher degree polynomials! This will require us to learn the theory of symmetry groups and explore it's connection to polynomials via permutations of roots, in the process we'll catch a glimpse of a much deeper algebraic theory in mathematics called Galois theory.

This course will be taught in the mode of Inquiry Based Learning which means that problem sheets will be handed out in the class instead of a regular lecture and students will learn the subject by actively solving problems instead of passively listening to a lecture.

Prerequisites

Students should be comfortable with Complex numbers, Proof by Induction, and they should be generally interested in mathematical problem solving. Having done a course in Discrete Maths or Algebra will also be helpful.

Text

No text is required for this course. The necessary material (which will be problem sets) will be handed out at the beginning of each class.

Grading

This is a single credit pass-fail course, the grade will be based solely on attendance and class participation.