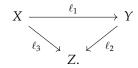
## Homework 05 Galois Correspondence of Covering Space

Algebraic Topology - Winter 2021

Due: March 04, 2021, 11:59 pm

1. Consider the following commuting diagram between path-connected spaces.



- (a) Show that if  $\ell_2$  and  $\ell_3$  are covering maps then so is  $\ell_1$ .
- (b) Find some non-trivial conditions for  $\ell_1$  and  $\ell_2$  under which if  $\ell_1$  and  $\ell_2$  are covering maps then so is  $\ell_3$ . Why does *your* proof not work for arbitrary  $\ell_1$  and  $\ell_2$ .
- 2. Classify connected covers, up to isomorphism of covers, of each of the following spaces:
  - (a)  $\mathbb{RP}^2$ ,
  - (b)  $\mathbb{RP}^2 \times \mathbb{RP}^2$ ,
  - (c) the Mobius strip,
  - (d) (optional)  $\mathbb{RP}^2 \vee \mathbb{RP}^2$ .
- 3. Let *n* be a positive integer greater than 1. Denote by  $\bigvee_n S^1$  the wedge of *n* circles at a point.
  - (a) Construct a covering map  $\ell: X \to \bigvee_2 S^1$  where  $X \simeq \bigvee_n S^1$ .
  - (b) Conclude that there exists an inclusion  $\mathbb{F}_n \hookrightarrow \mathbb{F}_2$  where  $\mathbb{F}_k$  denotes the free group on k generators.<sup>1</sup>
  - (c) Determine generators and index of  $\operatorname{im}(\pi_1(\ell))$  inside  $\mathbb{F}_2$ .
  - (d) (optional) For what positive integers m does there exist an inclusion  $\mathbb{F}_m \hookrightarrow \mathbb{F}_n$ .
  - (e) (optional) Show that every subgroup of a free group is free.

<sup>&</sup>lt;sup>1</sup>See also the ping-pong lemma.

## Suggested exercises for practice from Hatcher

**Pg. 79** 1, 2, 4, 8, 10

**Pg. 80** 11, 12, 13, 14

**Pg. 81** 21, 22