**Name**:

**Roll No**: e.g. 170001 | **Dept.**: e.g. CHE | **Sect.**: e.g. A4

**IIT Kanpur**
**ESC101 Fundam. of Comp.**
**Minor Quiz 4A**
*Date:* February 13, 2018

**Instructions**:

*Total:* **10 marks**

1. Write your name, roll number, department, section on **every side of every sheet** of this quiz paper
2. Write final answers **neatly with a pen** in the given box.
3. Do not give derivations/elaborate steps unless the question specifically asks you to provide these.

**Problem 1 (Fun with Fibonacci: 10 marks).** Two ways to compute Fibonacci numbers were discussed in the class. They are reproduced below. For large $n$, which method runs faster and why? Justify by giving a rough estimate of $T_1(n)$ and $T_2(n)$ in terms of $n$ where $T_1(n)$ is the time taken to compute Ifib($n$) and $T_2(n)$ is the time taken to compute Rfib($n$). **Note:** You do not need to compute the exact constants.

```
1  int Ifib(int n)
2  {
3     int first = 0, second = 1;
4     int next, c;
5     if (n <= 1)
6         return n;
7      for ( c = 1; c < n ; c++ ) {
8         next = first + second;
9         first = second;
10        second = next;
11    }
12     return next;
13 }
```

```
1  int Rfib(int n)
2  {
3     if ( n <= 1 )
4        return n;
5     else
6        return Rfib(n-1) + Rfib(n-2);
7  }
8
9
10
11
12
13
```

Iterative Fibonacci is much faster than Recursive Fibonacci for large inputs. [+3 marks]

## Ifib:

For large inputs, we can ignore the time needed to execute lines 3,4,5 and 12 as the time needed to execute the loop will far exceed it. Suppose the time needed to execute one iteration of the loop is $k$, then

$$T_1(n) = kn \implies T_1(n) \text{ is linear in } n$$

[+3 marks]

## Rfib:

It is easy to observe that, for large inputs,
$T_2(n) = T_2(n-1) + T_2(n-2)$, which is the same as the expression for the Fibonacci numbers.
$T_2(n-1) > T_2(n-2) \implies T_2(n) > 2T_2(n-2) \implies T_2(n) > 2^{n/2}T_2(0)$
Also, $T_2(n-1) > T_2(n-2) \implies T_2(n) < 2T_2(n-1) \implies T_2(n) < 2^nT_2(0)$
$\implies k.2^{n/2} < T_2(n) < k.2^n$, where $k$ is some constant.
Therefore,

$$T_2(n) \text{ is exponential in } n$$

[+4 marks]

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BLANK SPACE: Any answers written here will be left ungraded.
No exceptions.
You may use this space for rough work.