CS738: Advanced Compiler Optimizations

Static Single Assignment (SSA)

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Agenda

- SSA Form
- Constructing SSA form
- Properties and Applications

SSA Form

- Developed by Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck,
  - in 1980s while at IBM.
- *Static Single Assignment* – A variable is assigned only once in program text
  - May be assigned multiple times if program is executed

What is SSA Form?

- An Intermediate Representation
- Sparse representation
  - Definitions sites are directly associated with use sites
- Advantage
  - Directly access points where relevant data flow information is avaliable

SSA IR

- In SSA Form
  - Each variable has exactly one definition
  - ⇒ A use of a variable is reached by exactly one definition
- Control flow like traditional programs
- Some *magic* is needed at *join* nodes

Example

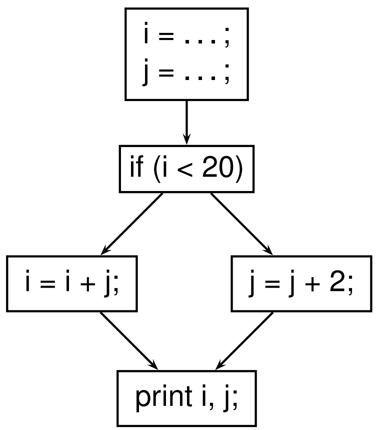
```
i = 0;
...
i = i + 1;
...
j = i * 5;
...
```

SSA ⇒

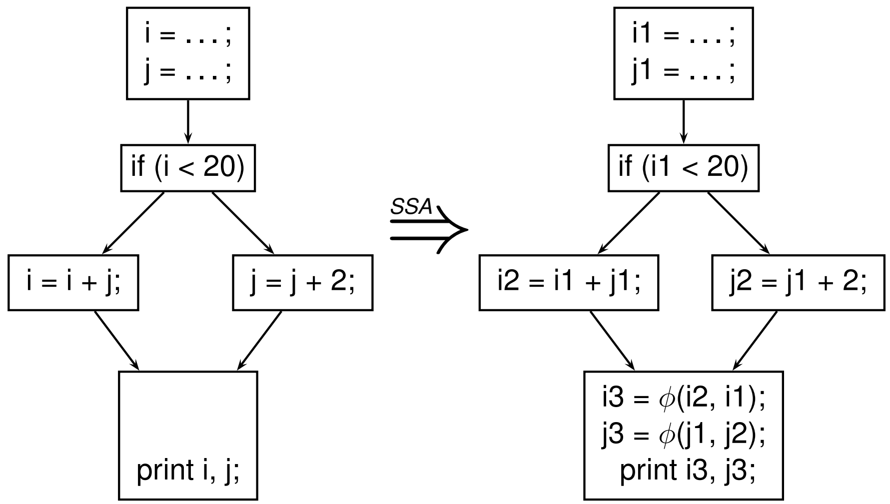
```
i1 = 0;
...
i2 = i1 + 1;
...
j1 = i2 * 5;
...
```

SSA Example

```
i = ...;
j = ...;
if (i < 20)
  i = i + j;
else
  j = j + 2;
print i, j;
```



SSA Example



SSA Example

```
i = ...;
j = ...;
if (i < 20)
  i = i + j;
else
  j = j + 2;


print i, j;
```

$\overset{SSA}{\Longrightarrow}$

```
i1 = ...;
j1 = ...;
if (i1 < 20)
  i2 = i1 + j1;
else
  j2 = j1 + 2;
i3 = φ(i2, i1);
j3 = φ(j1, j2);
print i3, j3;
```

The *magic*: ϕ function

- ϕ is used for selection
  - One out of multiple values at join nodes
- Not every join node needs a ϕ
  - Needed only if multiple definitions reach the node
- Examples?

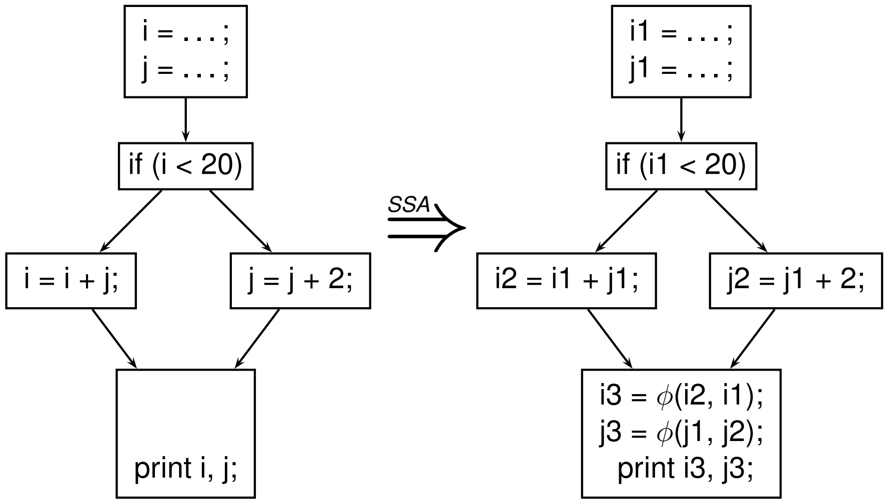
But...What is ϕ?

- What does ϕ operation mean in a machine code?
- ϕ is a conceptual entity
- Statically equivalent to chosing one of the arguments "non-deterministicly"
- No direct translation to machine code
  - typically mimicked using "copy" in predecessors
  - Inefficient
  - Practically, the inefficiency is compensated by dead code elimination and register allocation passes

Properties of ϕ

- Placed only at the entry of a join node
- Multiple ϕ-functions could be placed
  - for multiple variables
  - all such ϕ functions execute concurrently
- n-ary ϕ function at n-way join node
- gets the value of i-th argument if control enters through i-th edge
  - Ordering of ϕ arguments according to the edge ordering is important

SSA Example (revisit)



Construction of SSA Form

Assumptions

- Only scalar variables
  - Structures, pointers, arrays could be handled
  - Refer to publications

Dominators

- Nodes x and y in flow graph
- x dominates y if **every** path from *Entry* to y goes through x
  - $x \operatorname{dom} y$
  - partial order?
- x strictly dominates y if $x \operatorname{dom} y$ and $x \neq y$
  - $x \operatorname{sdom} y$

Computing Dominators

- Equation

$$\mathrm{DOM}(N) = \{n\} \cup \left(\bigcap_{m \in \mathrm{PRED}(n)} \mathrm{DOM}(m) \right)$$

- Initial Conditions:

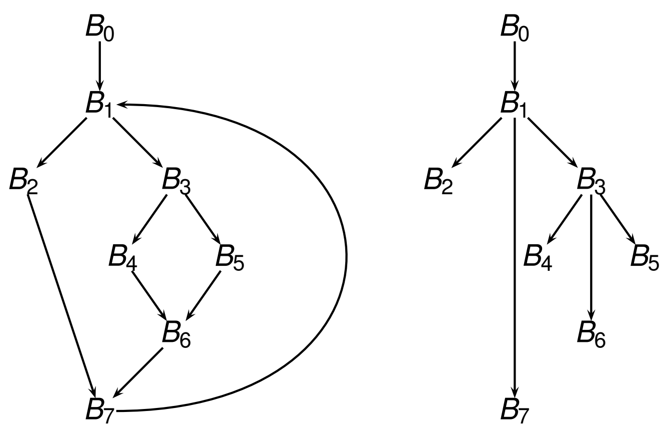
$$\begin{aligned} \mathrm{DOM}(n_{Entry}) &= \{n_{Entry}\} \\ \mathrm{DOM}(n) &= N, \forall n \in N - \{n_{Entry}\} \end{aligned}$$

  where N is the set of all nodes, n_{Entry} is the node corresponding to the *Entry* block.
- Note that efficient methods exist for computing dominators

Immediate Dominators and Dominator Tree

- x is immediate dominator of y if x is the *closest strict dominator* of y
  - unique, if it exists
  - denoted $\mathrm{idom}[y]$
- Dominator Tree
  - A tree showing all immediate dominator relationships

Dominator Tree Example



Dominance Frontier: DF

- Dominance Frontier of x is set of all nodes y s.t.
  - x dominates a predecessor of y AND
  - x does not strictly dominate y
- Denoted $\mathrm{DF}(x)$
- Why do you think $\mathrm{DF}(x)$ is important for any x?
  - Think about the information originated in x.

Computing DF

- PARENT(x) denotes parent of node x in the dominator tree.
- CHILDERN(x) denotes childern of node x in the dominator tree.
- PRED and SUCC from CFG.

$$
\begin{aligned}
\mathrm{DF}(x) &= \mathrm{DF}_{\mathrm{local}}(x) \cup \left(\bigcup_{z \in \mathrm{CHILDERN}(x)} \mathrm{DF}_{\mathrm{up}}(z) \right) \\
\mathrm{DF}_{\mathrm{local}}(x) &= \{ y \in \mathrm{SUCC}(x) \mid \mathrm{idom}[y] \neq x \} \\
\mathrm{DF}_{\mathrm{up}}(z) &= \{ y \in \mathrm{DF}(z) \mid \mathrm{idom}[y] \neq \mathrm{PARENT}(z) \}
\end{aligned}
$$

Iterated Dominance Frontier

- Transitive closure of Dominance frontiers on a set of nodes
- Let S be a set of nodes

$$
\begin{aligned}
\mathrm{DF}(S) &= \bigcup_{x \in S} \mathrm{DF}(x) \\
\mathrm{DF}^1(S) &= \mathrm{DF}(S) \\
\mathrm{DF}^{i+1}(S) &= \mathrm{DF}(S \cup \mathrm{DF}^i(S))
\end{aligned}
$$

- $\mathrm{DF}^+(S)$ is the fixed point of DF^i computation.

Minimal SSA Form Construction

- Compute DF^+ set for each flow graph node
- Place trivial ϕ-functions for each variable in the node
  - trivial ϕ-function at n-ary join: $x = \phi(\overbrace{x, x, \ldots, x}^{n\text{-times}})$
- Rename variables
- Why DF^+? Why not only DF?

Inserting ϕ-functions

```
foreach variable v {
    S = Entry ∪ {B_n | v defined in B_n}
    Compute DF+(S)
    foreach n in DF+(S) {
        insert φ-function for v at the start of B_n
    }
}
```

Renaming Variables (Pseudo Code)

- Rename from the *Entry* node recursively
  - For each variable x, maintain a rename stack of $x \mapsto x_{\text{version}}$ mapping
- For node n
  - For each assignment $(x = \ldots)$ in n
    - If non-ϕ assignment, rename any use of x with the Top mapping of x from the rename stack
    - Push the mapping $x \mapsto x_i$ on the rename stack
    - Replace lhs of the assignment by x_i
    - $i = i + 1$
- For the successors of n
  - Rename ϕ operands through SUCC edge index
- Recursively rename all child nodes in the dominator tree
- For each assignment $(x = \ldots)$ in n
  - Pop $x \mapsto \ldots$ from the rename stack