

CS738: Advanced Compiler Optimizations

Static Single Assignment (SSA)

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Agenda

- ▶ SSA Form
- ▶ Constructing SSA form
- ▶ Properties and Applications

SSA Form

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- ▶ *Static Single Assignment* – A variable is assigned only once in program text
 - ▶ May be assigned multiple times if program is executed

What is SSA Form?

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 - ▶ Definitions sites are directly associated with use sites
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 - ▶ Directly access points where relevant data flow information is available

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 - ⇒ A use of a variable is reached by exactly one definition
- ▶ Control flow like traditional programs
- ▶ Some *magic* is needed at *join* nodes

Example

```
i = 0;  
...  
i = i + 1;  
...  
j = i * 5;  
...
```

Example

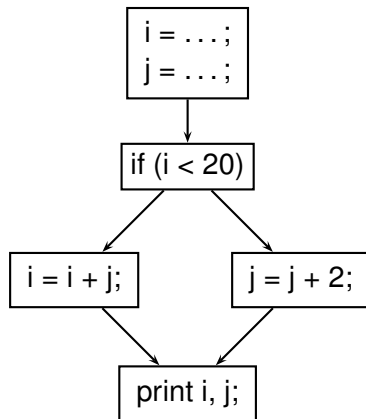
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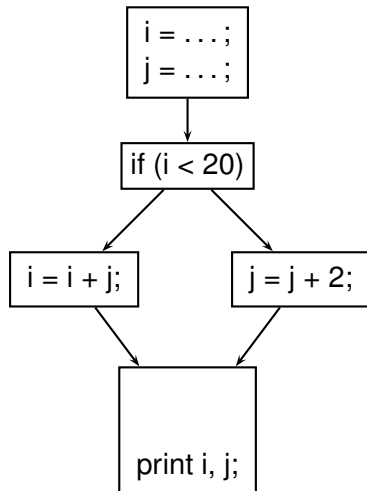
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i1 = 0;  
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j1 = i2 * 5;  
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SSA Example

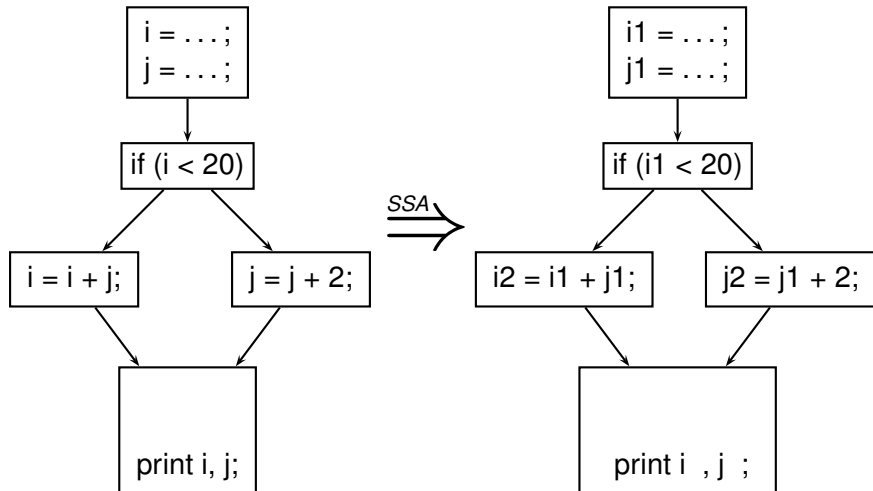
```
i = ...;  
j = ...;  
if (i < 20)  
    i = i + j;  
else  
    j = j + 2;  
print i, j;
```



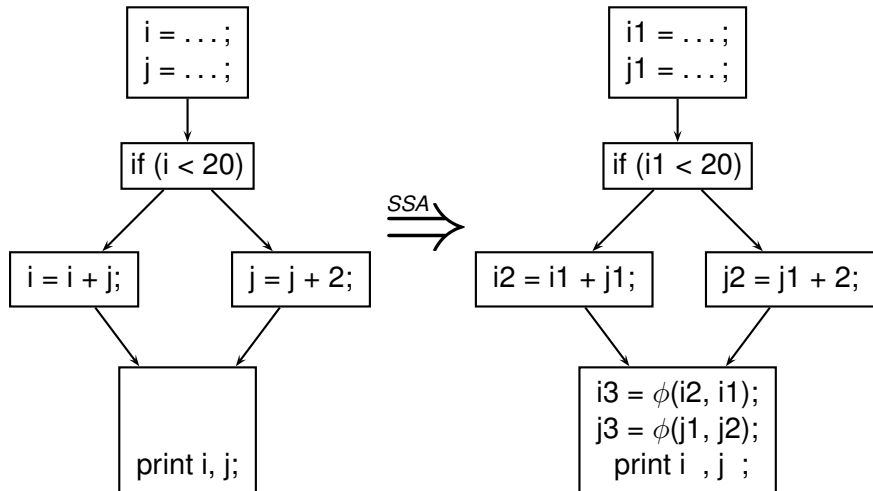
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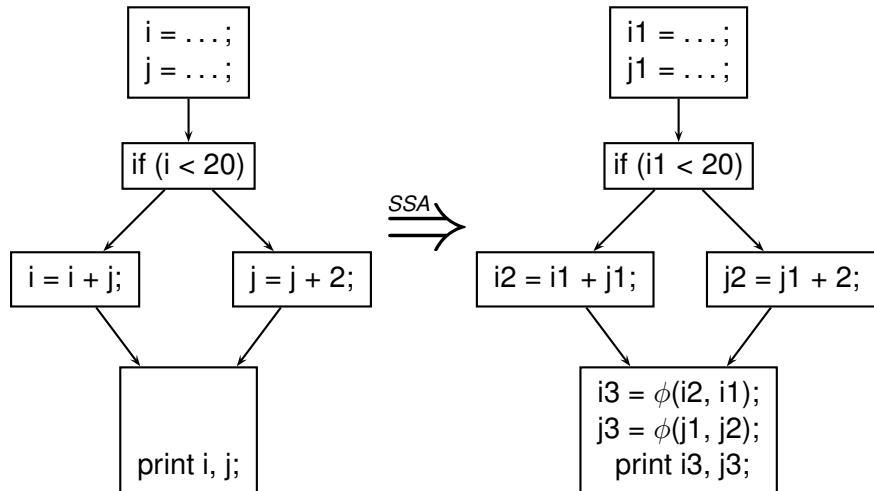
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i1 = ...;  
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if (i1 < 20)  
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    j2 = j1 + 2;  
i3 =  $\phi$ (i2, i1);  
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print i3, j3;
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- ▶ Examples?

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 - ▶ Inefficient
 - ▶ Practically, the inefficiency is compensated by dead code elimination and register allocation passes

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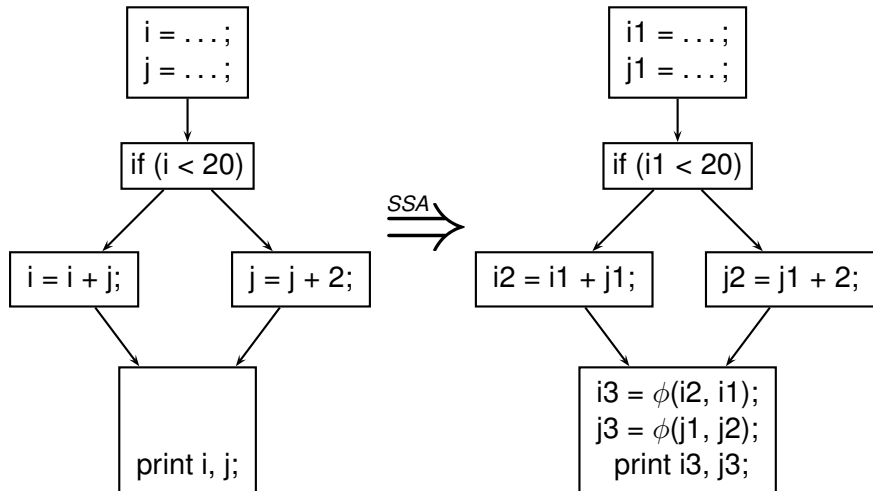
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 - ▶ Ordering of ϕ arguments according to the edge ordering is important

SSA Example (revisit)



Construction of SSA Form

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 - ▶ $x \text{ sdom } y$

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where N is the set of all nodes, n_{Entry} is the node corresponding to the *Entry* block.

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- ▶ Note that efficient methods exist for computing dominators

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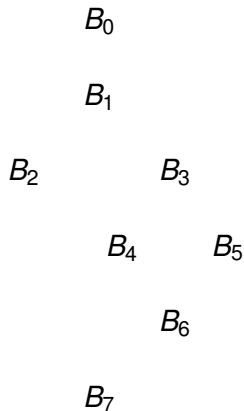
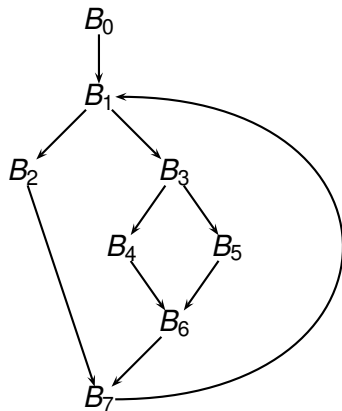
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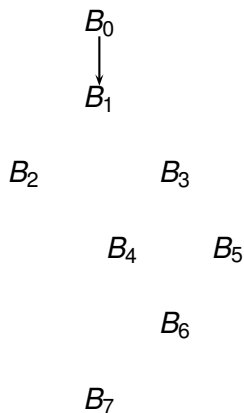
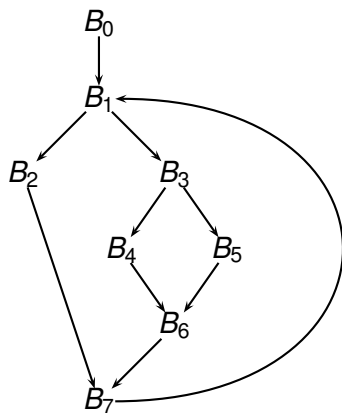
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- ▶ Dominator Tree
 - ▶ A tree showing all immediate dominator relationships

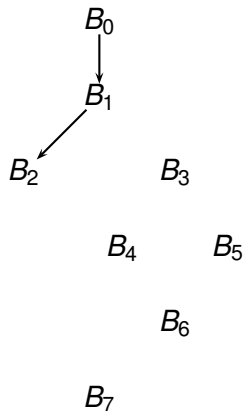
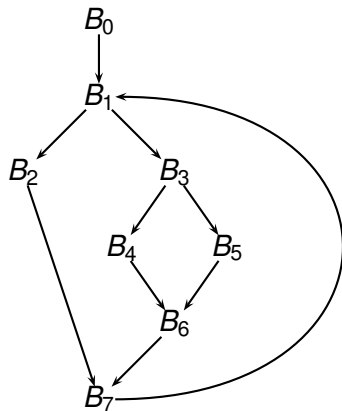
Dominator Tree Example



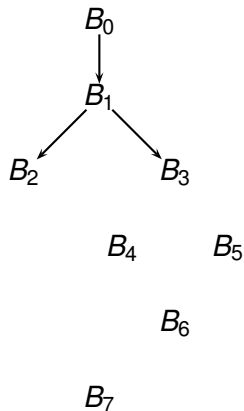
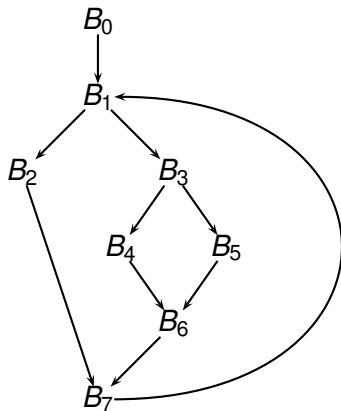
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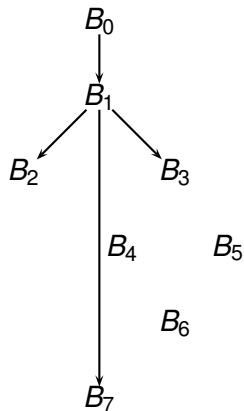
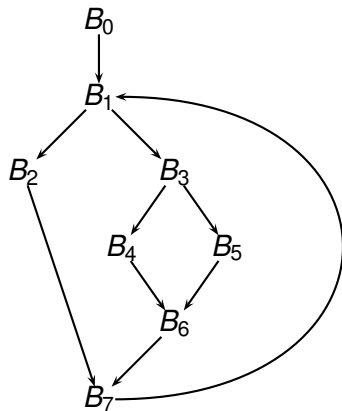
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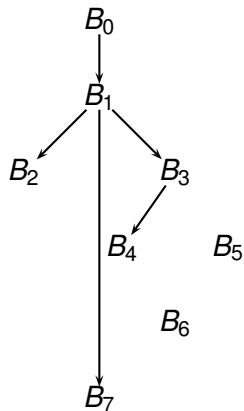
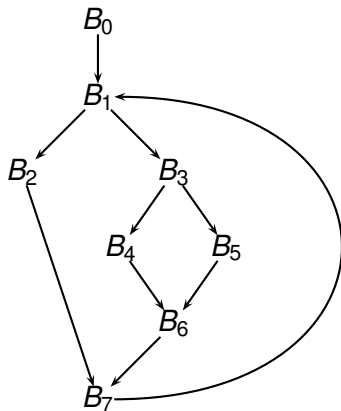
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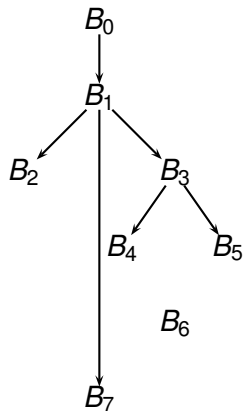
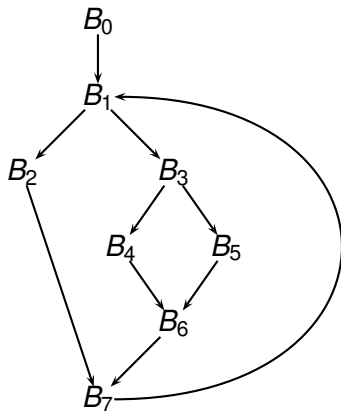
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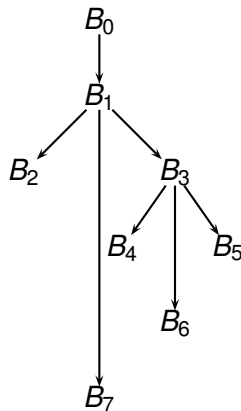
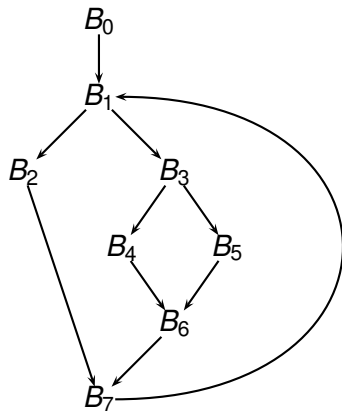
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- ▶ Why do you think $DF(x)$ is important for any x ?
 - ▶ Think about the information originated in x .

Computing DF

- ▶ $\text{PARENT}(x)$ denotes parent of node x in the dominator tree.
- ▶ $\text{CHILDREN}(x)$ denotes children of node x in the dominator tree.
- ▶ PRED and SUCC from CFG.

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Iterated Dominance Frontier

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$$DF^1(S) = DF(S)$$

$$DF^{i+1}(S) = DF(S \cup DF^i(S))$$

- ▶ $DF^+(S)$ is the fixed point of DF^i computation.

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- ▶ **Why DF^+ ? Why not only DF ?**

Inserting ϕ -functions

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        insert  $\phi$ -function for  $v$  at the start of  $B_n$   
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 - ▶ $i = i + 1$
- ▶ For the successors of n
 - ▶ Rename ϕ operands through SUCC edge index

Renaming Variables (Pseudo Code)

- ▶ Rename from the *Entry* node recursively
 - ▶ For each variable x , maintain a rename stack of $x \mapsto x_{\text{version}}$ mapping
- ▶ For node n
 - ▶ For each assignment $(x = \dots)$ in n
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 - ▶ Pop $x \mapsto \dots$ from the rename stack