CS738: Advanced Compiler Optimizations

SSAPRE: SSA based Partial Redundancy Elimination

Amey Karkare
karkare@cse.iitk.ac.in

http://www.cse.iitk.ac.in/~karkare/cs738
Department of CSE, IIT Kanpur

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- Remove totally redundant computations (CSE)

PRE without SSA

- Iterative data flow analysis

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- Operates on control flow graph

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- Computes global and local versions of data flow information

SSAPRE

- Information flow along SSA edges

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- No distinction between global and local information

SSAPRE: Challenge

- SSA form defined for variables

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$$\begin{tikzcd}
\boxed{a_1 + b_1} \arrow[dr] & & \boxed{a_2 = \cdots} \arrow[dl] \\
& \boxed{a_3 = \phi(a_1, a_2)} \arrow[d] & \\
& \boxed{a_3 + b_1} &
\end{tikzcd}$$

  - Here $a_1 + b_1$ is same as $a_3 + b_1$ when control follows the left branch. Lexically different, but computationally identical

SSAPRE: Key Idea

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- PRE on SSA form of RCVs (h) to remove redundancies
- Final program will be in SSA form

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  - In the earlier example, $a_3 + b_1$ and $a_1 + b_1$ could be identical

SSAPRE Steps

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  1. Φ-insertion
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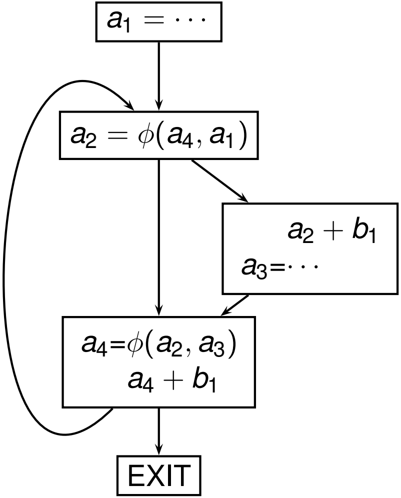
  4. WillBeAvail computation
  5. Finalization
  6. Code Motion

Running Example



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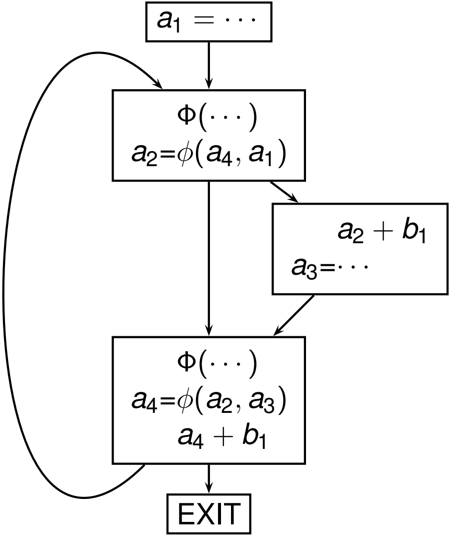
  - Potential change in the expression's value

Φ-insertion



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  - Identical SSA instances of h represent identical values of E
  - A control flow path with two different instances of h has to cross either an assignment to an operand of E or a Φ of h

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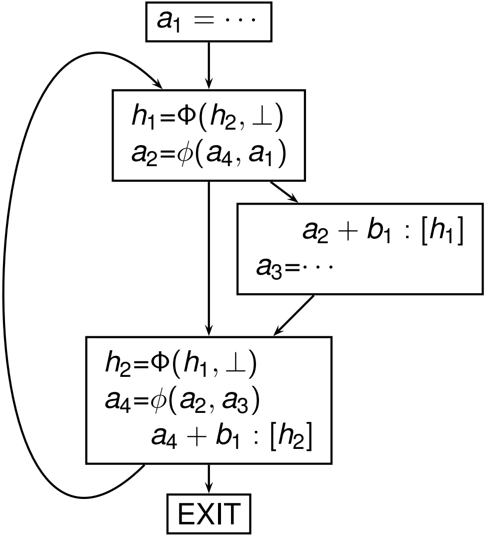
    - for each operand of E, compare the version of operand with the top of the rename stack for operand
    - If all match, h gets same version as the top of E stack
    - If any mismatch, replace E by $\perp$ in the operand push it on E stack (WHY?)

Rename


$a_1 = \cdots$
$h_1 = \Phi(h_2, \perp)$
$a_2 = \phi(a_4, a_1)$
$a_2 + b_1 : [h_1]$
$a_3 = \cdots$
$h_2 = \Phi(h_1, \perp)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$
EXIT

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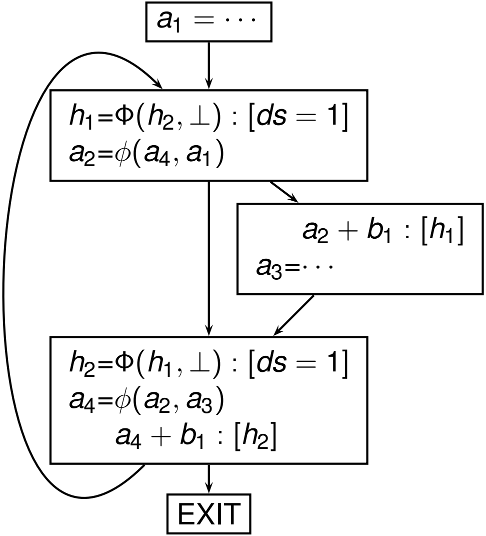
- *HasRealUse*: Real occurrence of an expression

Down-safety (ds = $\cdots$)



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- Computation of *two forward* properties:
  - *CanBeAvail*: Φs for which E is either available or anticipable or both
  - *Later*: Φs beyond which insertion can not be postponed without introducing new redundancy

$$\mathit{WillBeAvail} = \mathit{CanBeAvail} \land \neg \mathit{Later}$$

CanBeAvail

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  - exclude edges along which *HasRealUse* is *true*

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- Later $\Rightarrow$ Φs that are CanBeAvail, but do not reach any real occurrence of E

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- Insertion done along i^{th} predecessor of Φ if *Insert* is *true*, i.e.
  - $WillBeAvail(\Phi)$ == *true*; AND
  - Arg_i is $\perp$; OR
    - (HasRealUse(Arg_i) == *false*), AND
    - Arg_i is defined by Φ' with $WillBeAvail(\Phi')$ == *false*

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  - If *WillBeAvail* is *true*, it is replaced by SSA temporary with appropriate version (h_x)
  - If *WillBeAvail* is *false*, it is not part of SSA form, and is removed

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- Preorder traversal of dominator tree

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    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def

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  - Φ operand (processed in predecessor block P)

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    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)
    - If WillBeAvail of Φ is false, ignore.

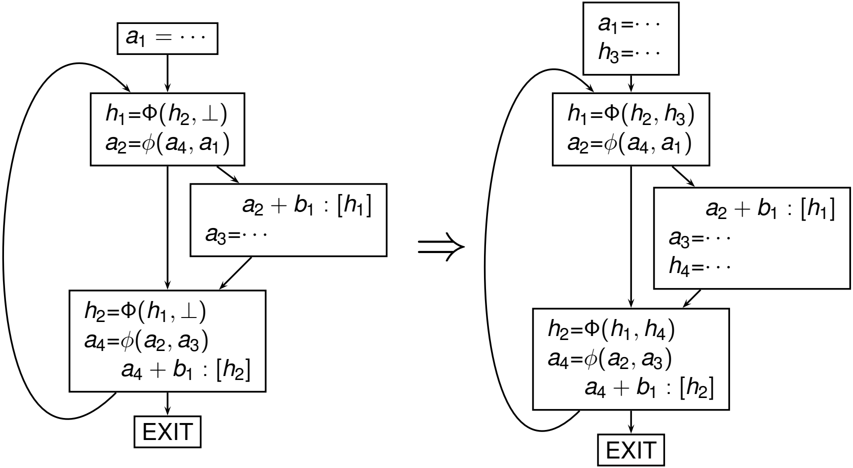
AvailDef Computation

- Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- During course of traversal, process occurrence x of E
  - Φ occurrence:
    - If WillBeAvail is *false*, ignore.
    - Otherwise AvailDef[x] = this Φ (we must be visiting x for first time) – WHY?
  - Real occurrence:
    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)
    - If WillBeAvail of Φ is false, ignore.
    - Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.

AvailDef Computation

- Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- During course of traversal, process occurrence x of E
  - Φ occurrence:
    - If WillBeAvail is *false*, ignore.
    - Otherwise AvailDef[x] = this Φ (we must be visiting x for first time) – WHY?
  - Real occurrence:
    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)
    - If WillBeAvail of Φ is false, ignore.
    - Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.
    - Else (*Insert* is false), mark this occurrence as use of AvailDef[x]

Finalize



$a_1 = \cdots$
$h_1 = \Phi(h_2, \perp)$
$a_2 = \phi(a_4, a_1)$
$a_2 + b_1 : [h_1]$
$a_3 = \cdots$
$h_2 = \Phi(h_1, \perp)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$
EXIT
$\Longrightarrow$
$a_1 = \cdots$
$h_3 = \cdots$
$h_1 = \Phi(h_2, h_3)$
$a_2 = \phi(a_4, a_1)$
$a_2 + b_1 : [h_1]$
$a_3 = \cdots$
$h_4 = \cdots$
$h_2 = \Phi(h_1, h_4)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$
EXIT

Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t

Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t
- For real *use* occurrence of E, replace E by current version of t

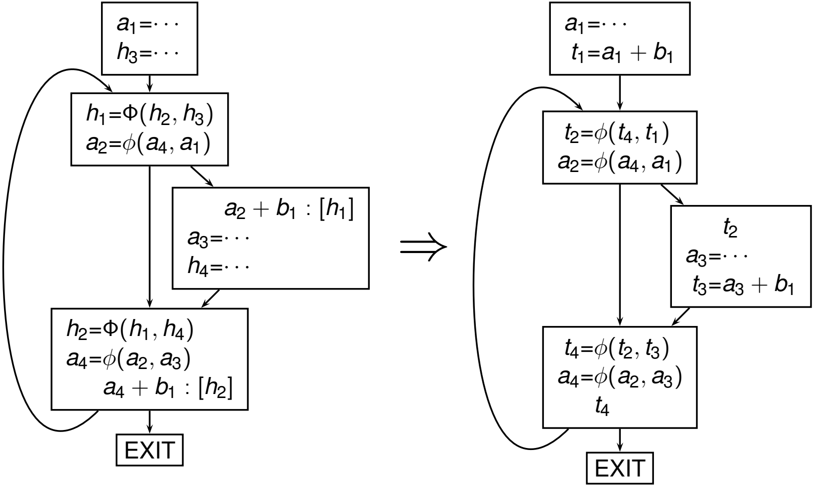
Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t
- For real *use* occurrence of E, replace E by current version of t
- For inserted occurrence of E, compute E in a new version of temporary t

Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t
- For real *use* occurrence of E, replace E by current version of t
- For inserted occurrence of E, compute E in a new version of temporary t
- For a Φ occurrence, insert appropriate ϕ for t

Code Motion



$a_1 = \cdots$
$h_3 = \cdots$
$h_1 = \Phi(h_2, h_3)$
$a_2 = \phi(a_4, a_1)$
$a_2 + b_1 : [h_1]$
$a_3 = \cdots$
$h_4 = \cdots$
$h_2 = \Phi(h_1, h_4)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$
EXIT
$\Rightarrow$
$a_1 = \cdots$
$t_1 = a_1 + b_1$
$t_2 = \phi(t_4, t_1)$
$a_2 = \phi(a_4, a_1)$
t_2
$a_3 = \cdots$
$t_3 = a_3 + b_1$
$t_4 = \phi(t_2, t_3)$
$a_4 = \phi(a_2, a_3)$
t_4
EXIT