CS738: Advanced Compiler Optimizations

SSAPRE: SSA based Partial Redundancy Elimination

Amey Karkare

karkare@cse.iitk.ac.in

http://www.cse.iitk.ac.in/~karkare/cs738

Department of CSE, IIT Kanpur

PRE without SSA

- Based on well known DF analyses

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  - Availability

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- Identifies paritially redundant computations, make them totally redundant by inserting new compitations

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- Identifies paritially redundant computations, make them totally redundant by inserting new compitations
- Remove totally redundant computations (CSE)

PRE without SSA

- Iterative data flow analysis

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- Operates on control flow graph

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- Operates on control flow graph
- Computes global and local versions of data flow information

SSAPRE

- Information flow along SSA edges

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- No distinction between global and local information

SSAPRE: Challenge

- SSA form defined for variables

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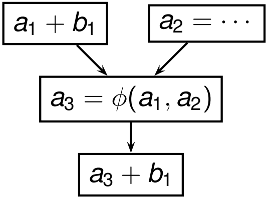

- Here $a_1 + b_1$ is same as $a_3 + b_1$ when control follows the left branch. Lexically different, but computationally identical

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- PRE on SSA form of RCVs (h) to remove redundancies
- Final program will be in SSA form

SSAPRE: Preparations

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  - In the earlier example, $a_3 + b_1$ and $a_1 + b_1$ could be identical

SSAPRE Steps

- Six step algorithm

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  1. Φ-insertion

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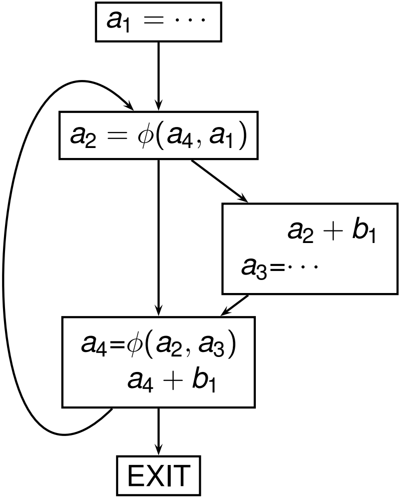
  1. Φ-insertion
  2. Renaming
  3. Down-safety computation
  4. WillBeAvail computation
  5. Finalization
  6. Code Motion

Running Example



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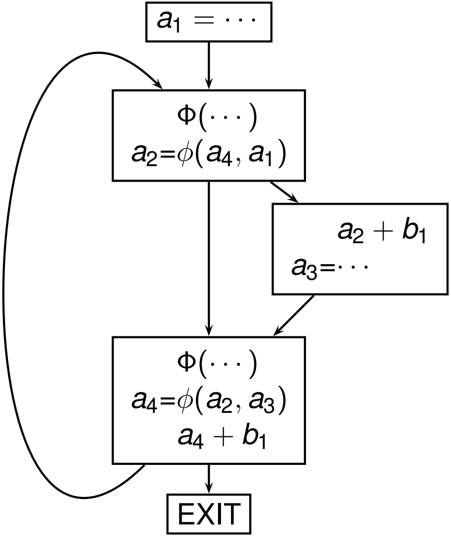
- At each block having a ϕ for some argument of *E*
  - Potential change in the expression's value

Φ-insertion


$a_1 = \cdots$
$\Phi(\cdots)$
$a_2 = \phi(a_4, a_1)$
$a_2 + b_1$
$a_3 = \cdots$
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$a_4 = \phi(a_2, a_3)$
$a_4 + b_1$
EXIT

Rename

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  - Identical SSA instances of h represent identical values of E

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  - Real occurrences (present in original program)
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  - Identical SSA instances of h represent identical values of E
  - A control flow path with two different instances of h has to cross either an assignment to an operand of E or a Φ of h

Rename Algorithm

- Runs with variable renaming

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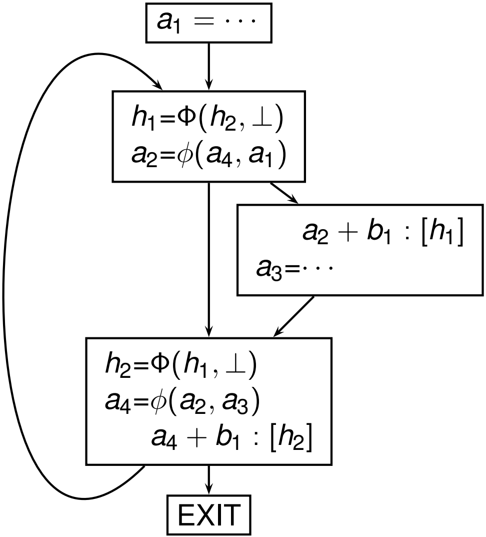
    - if E is operand of Φ, in the corresponding predecessor block
        - for each operand of E, compare the version of operand with the top of the rename stack for operand
        - If all match, h gets same version as the top of E stack
        - If any mismatch, replace E by $\perp$ in the operand push it on E stack (WHY?)

Rename



$a_1 = \cdots$

$h_1 = \Phi(h_2, \perp)$
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$a_2 + b_1 : [h_1]$
$a_3 = \cdots$

$h_2 = \Phi(h_1, \perp)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$

EXIT

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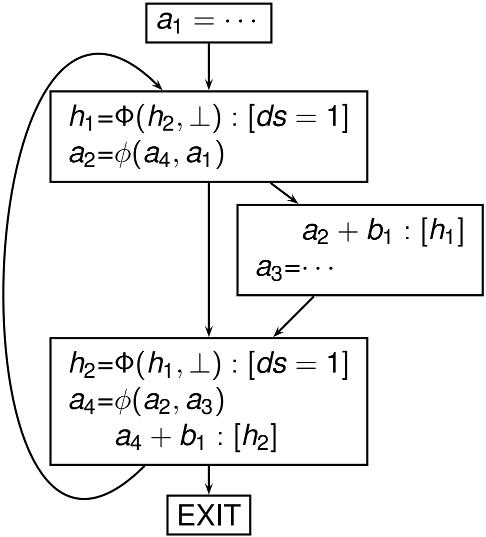
- *HasRealUse*: Real occurrence of an expression

Down-safety (ds = $\cdots$)



WillBeAvail

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- Computation of *two forward* properties:
  - *CanBeAvail*: Φs for which E is either available or anticipable or both
  - *Later*: Φs beyond which insertion can not be postponed without introducing new redundancy

$$\mathit{WillBeAvail} = \mathit{CanBeAvail} \wedge \neg \mathit{Later}$$

CanBeAvail

- Initialized to *true* for all Φs

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  - exclude edges along which *HasRealUse* is *true*

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- Later ⇒ Φs that are CanBeAvail, but do not reach any real occurrence of E

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  - Arg_i is $\perp$; OR
    - (HasRealUse(Arg_i) == *false*), AND

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- Insertion done along i^{th} predecessor of Φ if *Insert* is *true*, i.e.
    - *WillBeAvail*(Φ) == *true*; AND
    - Arg_i is $\perp$; OR
        - (HasRealUse(Arg_i) == *false*), AND
        - Arg_i is defined by Φ' with *WillBeAvail*(Φ') == *false*

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  - If *WillBeAvail* is *true*, it is replaced by SSA temporary with appropriate version (h_x)
  - If *WillBeAvail* is *false*, it is not part of SSA form, and is removed

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    - Real occurrence:
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        - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def

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  - Real occurrence:
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    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]

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  - Real occurrence:
    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)

AvailDef Computation

- Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- During course of traversal, process occurrence x of E
  - Φ occurrence:
    - If WillBeAvail is *false*, ignore.
    - Otherwise AvailDef[x] = this Φ (we must be visiting x for first time) – WHY?
  - Real occurrence:
    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)
    - If WillBeAvail of Φ is false, ignore.

AvailDef Computation

- Initialize: AvailDef[x] = $\perp \forall x$ (all classes of all expressions)
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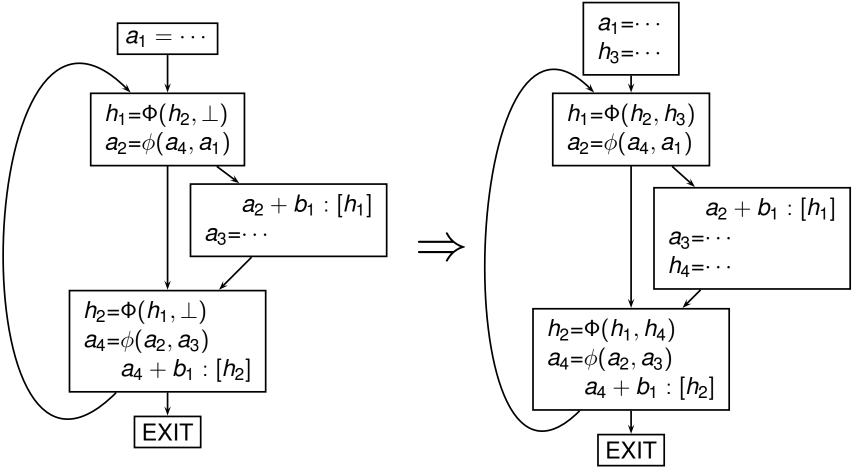
    - Real occurrence:
        - If AvailDef[x] is $\perp$, mark this occurrence as def
        - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
        - Else, mark this occurrence as use of AvailDef[x]
    - Φ operand (processed in predecessor block P)
        - If WillBeAvail of Φ is false, ignore.
        - Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.

AvailDef Computation

- Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- During course of traversal, process occurrence x of E
  - Φ occurrence:
    - If WillBeAvail is *false*, ignore.
    - Otherwise AvailDef[x] = this Φ (we must be visiting x for first time) – WHY?
  - Real occurrence:
    - If AvailDef[x] is $\perp$, mark this occurrence as def
    - Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - Else, mark this occurrence as use of AvailDef[x]
  - Φ operand (processed in predecessor block P)
    - If WillBeAvail of Φ is false, ignore.
    - Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.
    - Else (*Insert* is false), mark this occurrence as use of AvailDef[x]

Finalize



$a_1 = \cdots$

$h_1 = \Phi(h_2, \perp)$
$a_2 = \phi(a_4, a_1)$

$a_2 + b_1 : [h_1]$
$a_3 = \cdots$

$h_2 = \Phi(h_1, \perp)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$

EXIT

$\Rightarrow$

$a_1 = \cdots$
$h_3 = \cdots$

$h_1 = \Phi(h_2, h_3)$
$a_2 = \phi(a_4, a_1)$

$a_2 + b_1 : [h_1]$
$a_3 = \cdots$
$h_4 = \cdots$

$h_2 = \Phi(h_1, h_4)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$

EXIT

Code Motion

- For real *def* occurance of E, compute E in a new version of temporary t

Code Motion

- For real *def* occurance of E, compute E in a new version of temporary t
- For real *use* occurance of E, replace E by current version of t

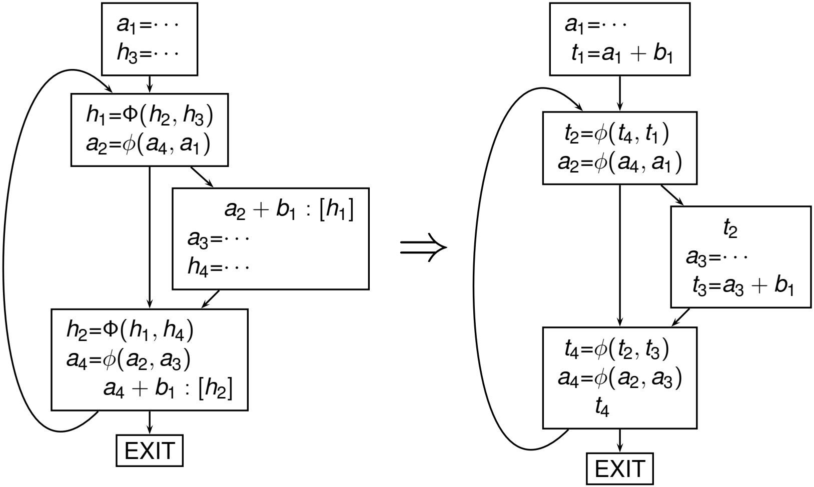
Code Motion

- For real *def* occurance of E, compute E in a new version of temporary t
- For real *use* occurance of E, replace E by current version of t
- For inserted occurrence of E, compute E in a new version of temporary t

Code Motion

- For real *def* occurance of E, compute E in a new version of temporary t
- For real *use* occurance of E, replace E by current version of t
- For inserted occurrence of E, compute E in a new version of temporary t
- For a Φ occurrence, insert appropriate ϕ for t

Code Motion



$a_1 = \cdots$

$h_3 = \cdots$

$h_1 = \Phi(h_2, h_3)$

$a_2 = \phi(a_4, a_1)$

$a_2 + b_1 : [h_1]$

$a_3 = \cdots$

$h_4 = \cdots$

$h_2 = \Phi(h_1, h_4)$

$a_4 = \phi(a_2, a_3)$

$a_4 + b_1 : [h_2]$

EXIT

$\Rightarrow$

$a_1 = \cdots$

$t_1 = a_1 + b_1$

$t_2 = \phi(t_4, t_1)$

$a_2 = \phi(a_4, a_1)$

t_2

$a_3 = \cdots$

$t_3 = a_3 + b_1$

$t_4 = \phi(t_2, t_3)$

$a_4 = \phi(a_2, a_3)$

t_4

EXIT