CS738: Advanced Compiler Optimizations

Flow Graph Theory

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Agenda

- Speeding up DFA
- Depth of a flow graph
- Natural Loops

Acknowledgement

Rest of the slides based on the material at
`http://infolab.stanford.edu/~ullman/dragon/w06/w06.html`

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Speeding up DFA

- Proper ordering of nodes of a flow graph speeds up the iterative algorithms: **depth-first ordering.**
- “Normal” flow graphs have a surprising property — **reducibility** — that simplifies several matters.
- Outcome: few iterations “normally” needed.

Depth-First Search

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- If not, backtrack to your *parent* (node from which you were visited).

Depth-First Spanning Tree (DFST)

- Root = *Entry*.

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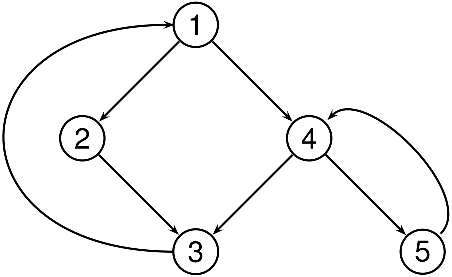
- Tree edges are the edges along which we first visit the node at the head.

DFST Example



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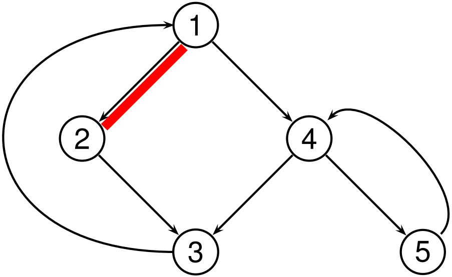

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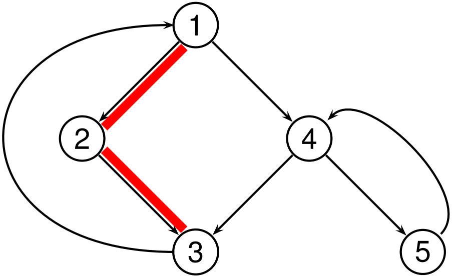

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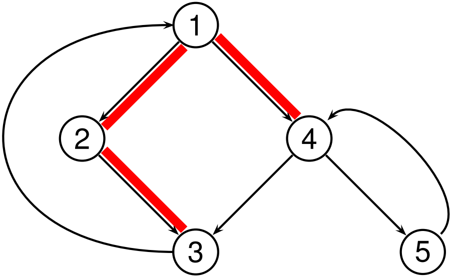

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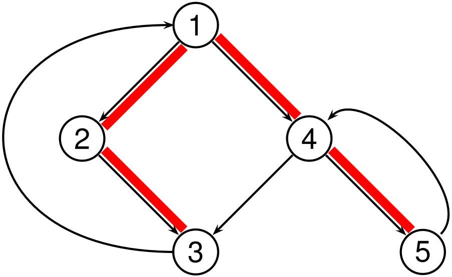

Depth-First Node Order

- The reverse of the order in which a DFS **retreats** from the nodes.

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- Alternatively, reverse of postorder traversal of the tree.

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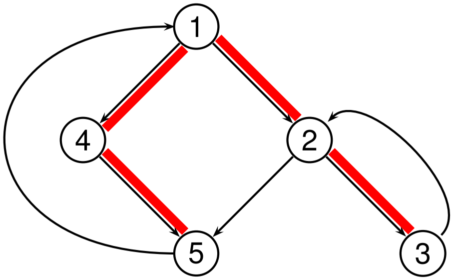

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4. **Cross edges**: between two node, niether of which is an ancestor of the other.

A Little Magic

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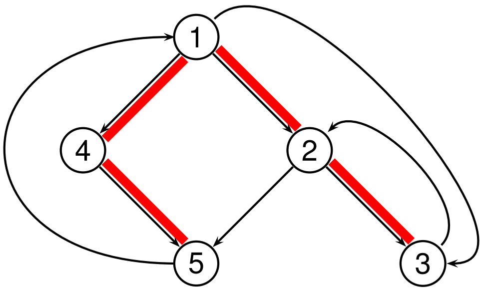
- Most surprising: all cross edges go right to left in the DFST.
  - Assuming we add children of any node from the left.

Example: Non-Tree Edges



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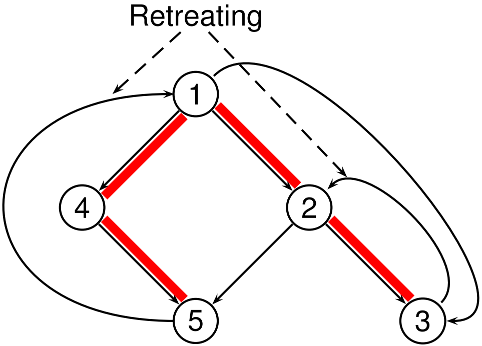

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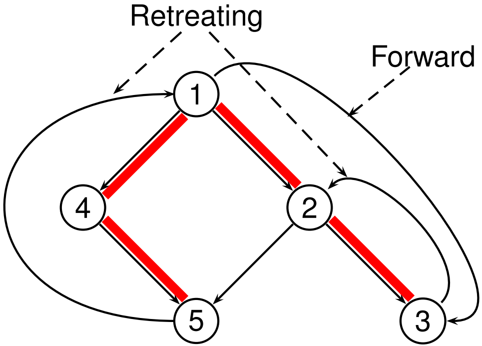

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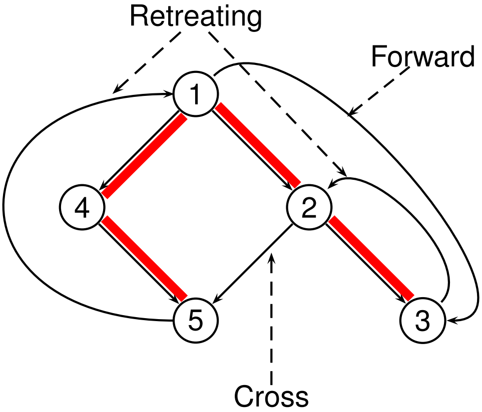

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- “Normal” flow graphs are “**reducible**.”
- “**Dominators**” needed to explain reducibility.
- In reducible flow graphs, loops are well defined, retreating edges are unique (and called “**back**” edges).
- Leads to relationship between DF order and efficient iterative algorithm.

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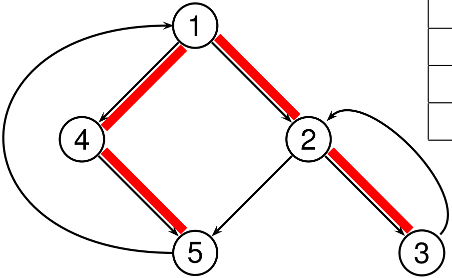

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5	

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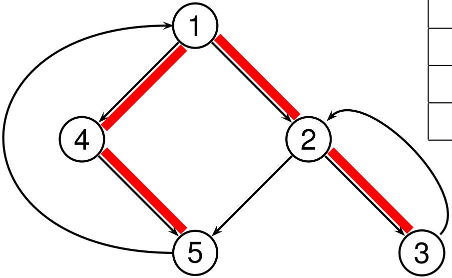

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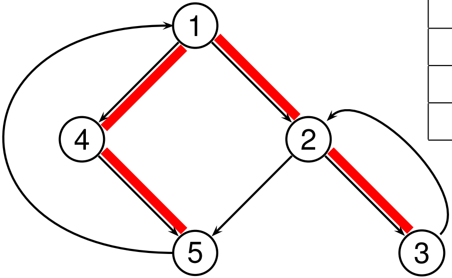

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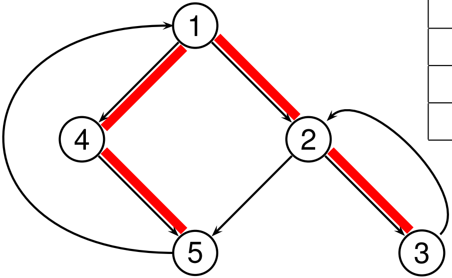

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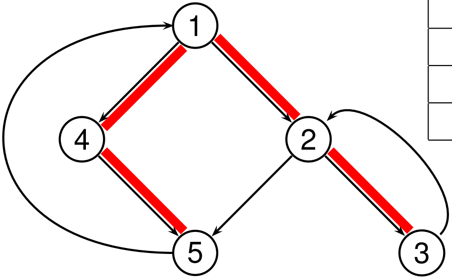

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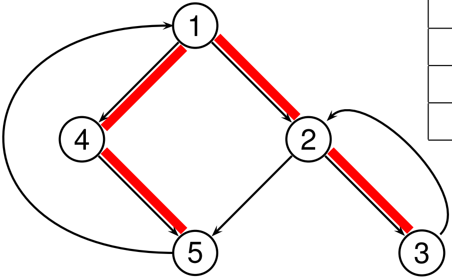

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Common Dominator Cases

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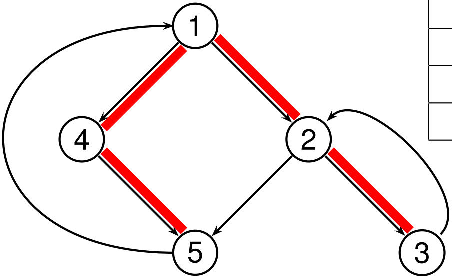
  - Proof? Discuss/Exercise
  - Converse almost always true, but not always.

Example: Back Edges



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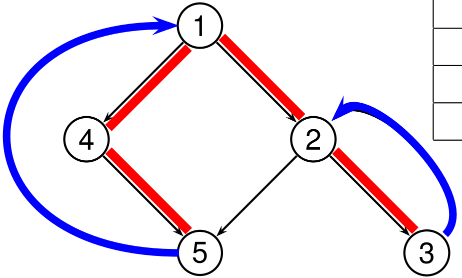

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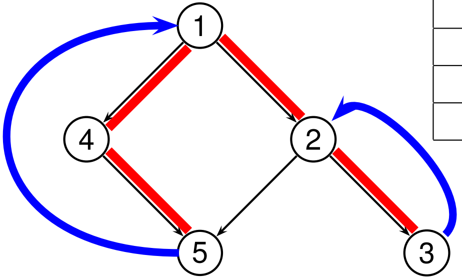
- A flow graph is **reducible** if every retreating edge in any DFST for that flow graph is a back edge.
- **Testing reducibility:** Take any DFST for the flow graph, remove the back edges, and check that the result is acyclic.

Example: Remove Back Edges



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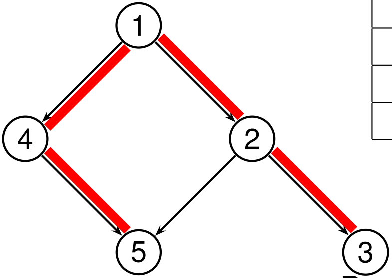

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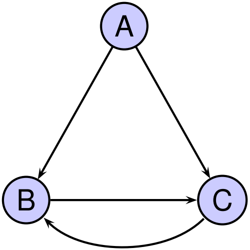
Remaining graph is acyclic.

Why Reducibility?

- **Folk theorem:** All flow graphs in practice are reducible.
- **Fact:** If you use only while-loops, for-loops, repeat-loops, if-then(-else), break, and continue, then your flow graph **is** reducible.

Example: Nonreducible Graph



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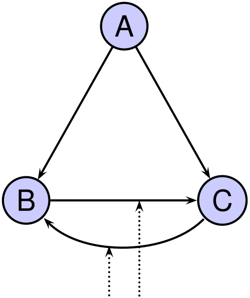

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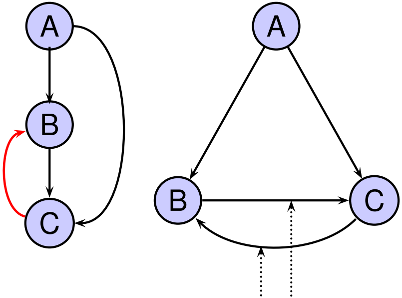

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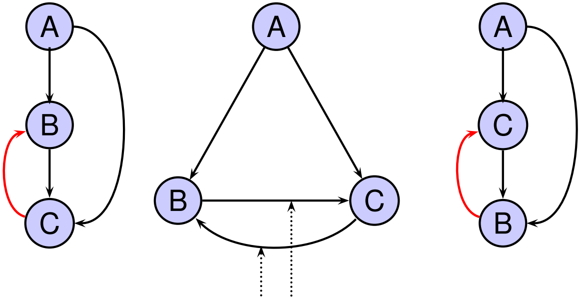

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- Depth of nested loops upper-bounds the number of nested back edges.

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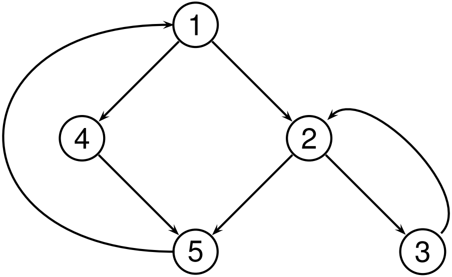
- When *d* arrives along a retreating edge, it is too late to propagate *d* from OUT to IN.

Example: DF Order

Node 2 generates definition d.



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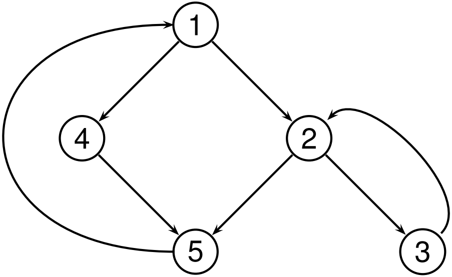

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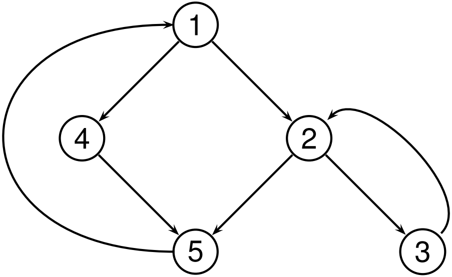
Does d reach node **4**?



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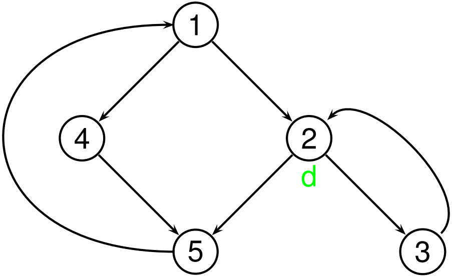

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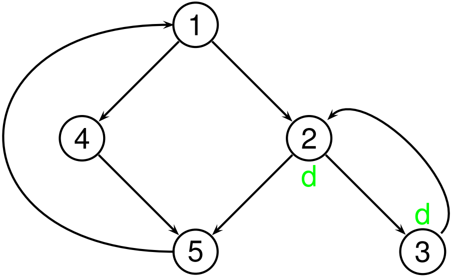

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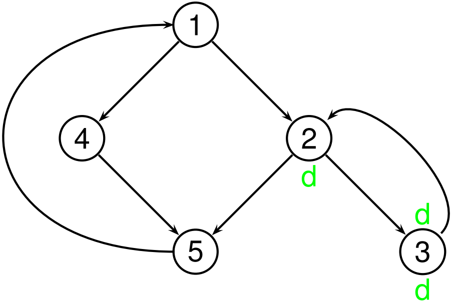

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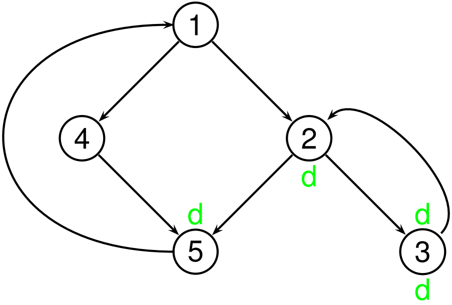

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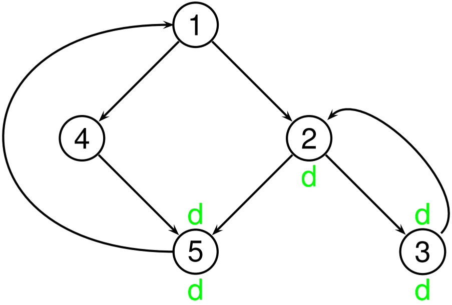

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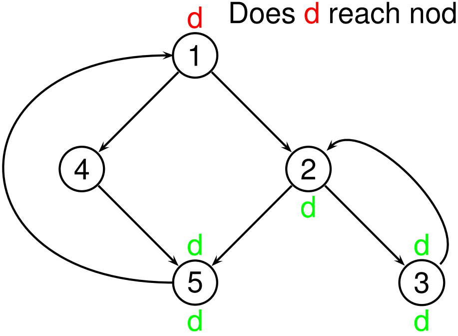

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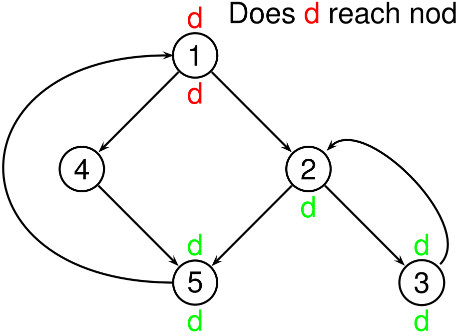

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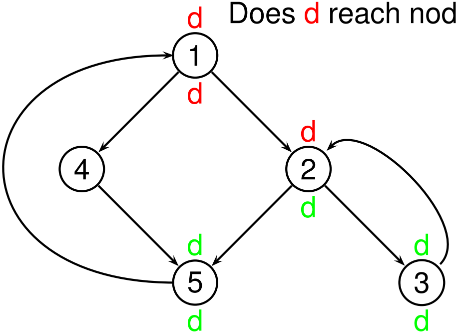

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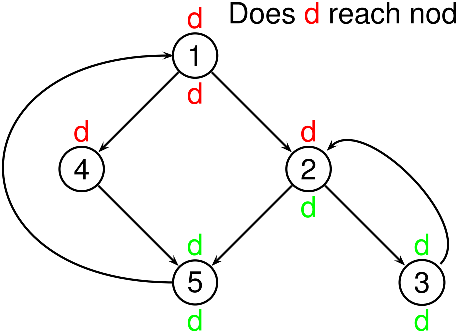

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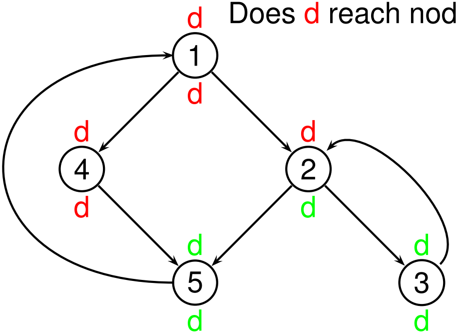

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  - 1 more pass to realize we converged.

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$\xrightarrow[\text{increasing}]{}$

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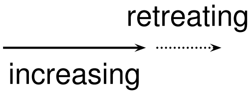

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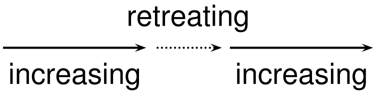

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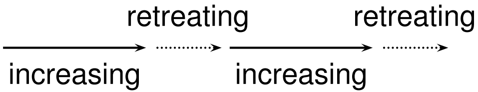

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- AE also works in depth+2 passes.
  - Unavailability propagates along retreat-free node sequences in one pass.
- So does LV if we use reverse of DF order.
  - A use propagates backward along paths that do not use a retreating edge in one pass.

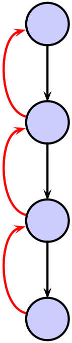
In General . . .

- The depth+2 bound works for any monotone bit-vector framework, as long as information only needs to propagate along acyclic paths.
  - Example: if a definition reaches a point, it does so along an acyclic path.

Why Depth+2 is Good?

- Normal control-flow constructs produce reducible flow graphs with the number of back edges at most the nesting depth of loops.
  - Nesting depth tends to be small.

Example: Nested Loops



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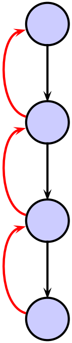

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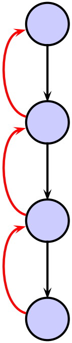
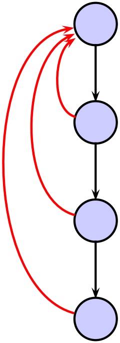
depth = 3.

Example: Nested Loops



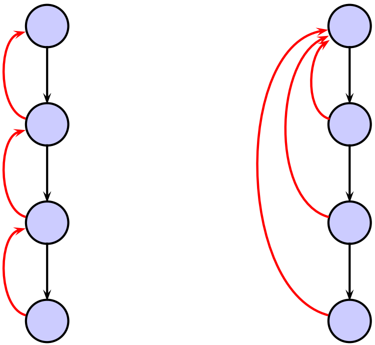


3 nested while loops.

depth = 3.

3 nested do-while loops.

Example: Nested Loops



3 nested while loops.

depth = 3.

3 nested do-while loops.

depth = 1.

Natural Loops

- The **natural loop** of a back edge $a \rightarrow b$ is $\{b\}$ plus the set of nodes that can reach a without going through b.

Natural Loops

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- **Theorem:** two natural loops are either disjoint, identical, or nested.

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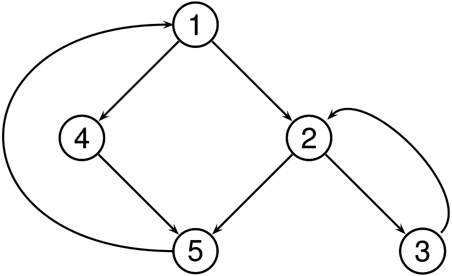
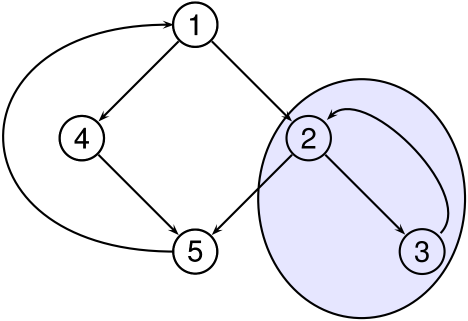
- Proof: Discuss/Exercise

Example: Natural Loops

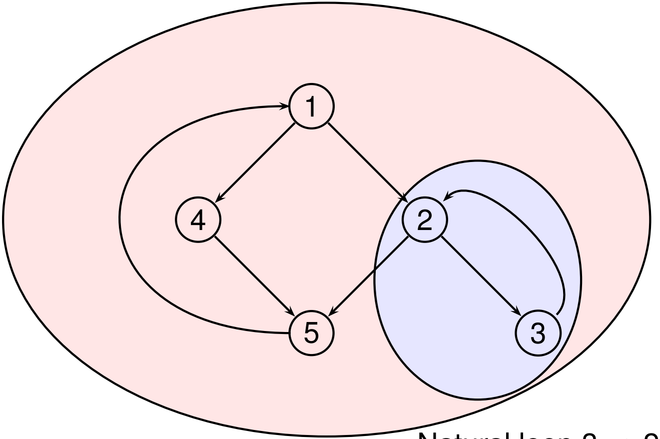


Example: Natural Loops



Natural loop $3 \to 2$

Example: Natural Loops



Natural loop $3 \rightarrow 2$

Natural loop $5 \rightarrow 1$

Reading Assignment

- New Dragon Book (Aho, Lam, Sethi, Ullman)
  - Chapter 9