CS738: Advanced Compiler Optimizations

Data Flow Analysis

Amey Karkare
karkare@cse.iitk.ac.in

http://www.cse.iitk.ac.in/~karkare/cs738
Department of CSE, IIT Kanpur

Agenda

- *Intraprocedural* Data Flow Analysis: Classical Examples

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  - Last lecture: Reaching Definitions

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  - Last lecture: Reaching Definitions
  - Today: Available Expressions

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- *Intraprocedural* Data Flow Analysis: Classical Examples
  - Last lecture: Reaching Definitions
  - Today: Available Expressions
  - Discussion about the similarities/differences

Available Expressions Analysis

- An expression e is available at a point p if

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  - **Every** path from the *Entry* to *p* has at least one evaluation of *e*

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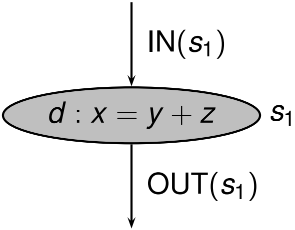
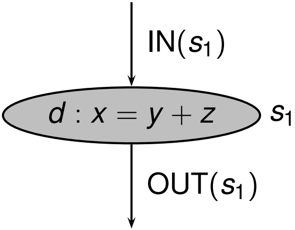
- Expression e is *generated* by its evaluation
- Expression e is *killed* by assignment to its component variables

AvE Analysis of a Structured Program



AvE Analysis of a Structured Program



$$\mathrm{OUT}(s_1) \quad = \quad \mathrm{IN}(s_1) - \mathrm{KILL}(s_1) \cup \mathrm{GEN}(s_1)$$

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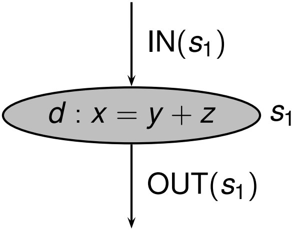
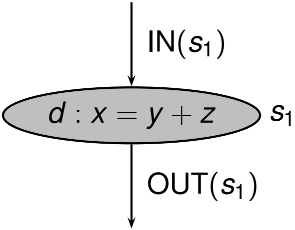


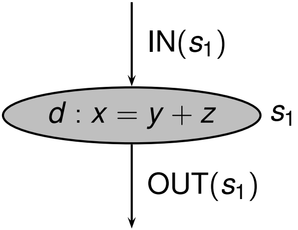
$$
\begin{aligned}
\mathrm{OUT}(s_1) &= \mathrm{IN}(s_1) - \mathrm{KILL}(s_1) \cup \mathrm{GEN}(s_1) \\
\mathrm{GEN}(s_1) &=
\end{aligned}
$$

AvE Analysis of a Structured Program



$$\begin{aligned} \text{OUT}(s_1) &= \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1) \\ \text{GEN}(s_1) &= \{y + z\} \end{aligned}$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\text{OUT}(s_1) &= \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1) \\
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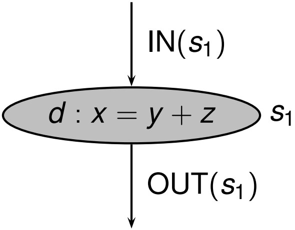
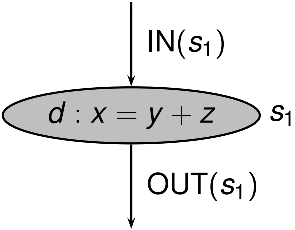


$$
\begin{aligned}
OUT(s_1) &= IN(s_1) - KILL(s_1) \cup GEN(s_1) \\
GEN(s_1) &= \{y + z\} \\
KILL(s_1) &= E_x
\end{aligned}
$$

where E_x: set of all expression having x as a component

AvE Analysis of a Structured Program



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This may not work in general – WHY?

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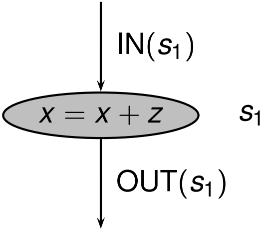
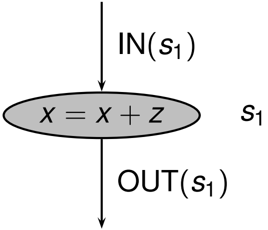


$$\begin{aligned} \text{OUT}(s_1) &= \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1) \\ \text{GEN}(s_1) &= \{x + z\} \\ \text{KILL}(s_1) &= E_x \end{aligned}$$

Incorrectly marks $x + z$ as available after s_1

AvE Analysis of a Structured Program



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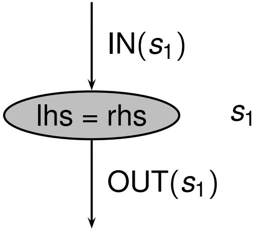
\end{aligned}
$$

Incorrectly marks $x + z$ as available after s_1

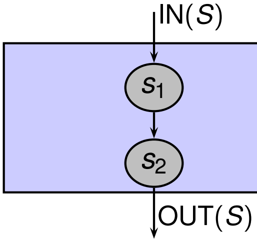
$$
\text{GEN}(s_1) = \emptyset \text{ for this case}
$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\text{OUT}(s_1) &= \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1)\\
\text{GEN}(s_1) &= \{\text{rhs} \mid \text{lhs is not part of rhs}\}\\
\text{KILL}(s_1) &= E_{\text{lhs}}
\end{aligned}
$$

AvE Analysis of a Structured Program



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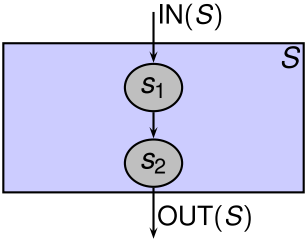
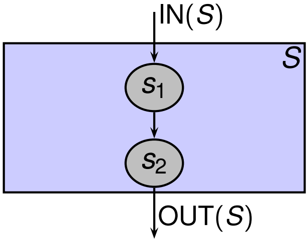


$$\text{GEN}(S) \quad =$$

AvE Analysis of a Structured Program



$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1) - \text{KILL}(s_2) \cup \text{GEN}(s_2)$$

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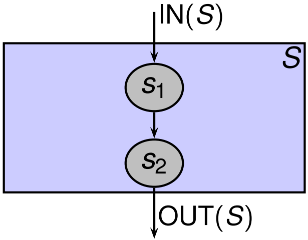

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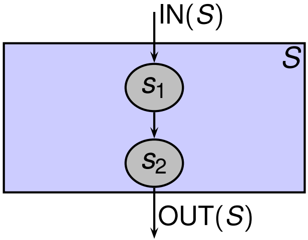

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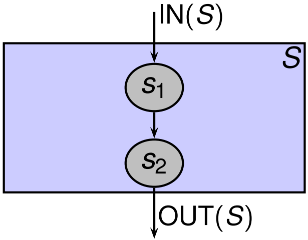

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\mathrm{IN}(s_1) &=
\end{aligned}
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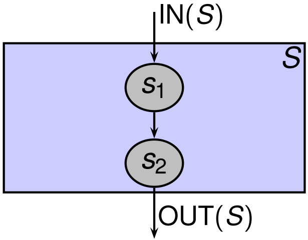

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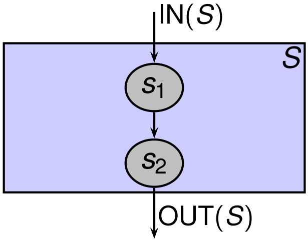

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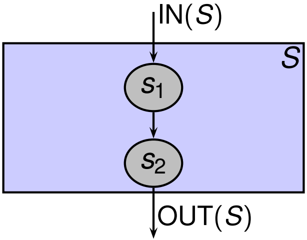

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\end{aligned}
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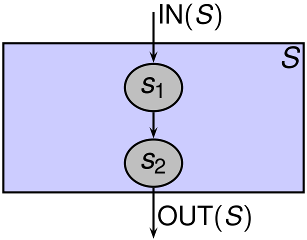

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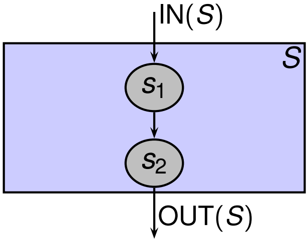

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\mathrm{IN}(s_2) &= \mathrm{OUT}(s_1) \\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_2)
\end{aligned}
$$

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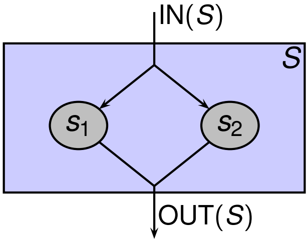

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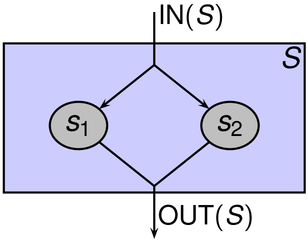
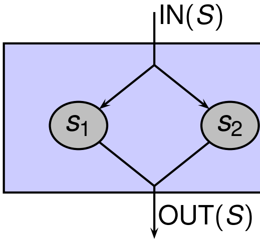


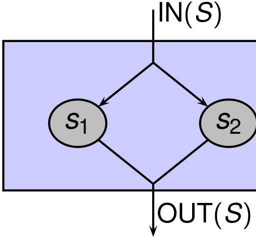
$$\text{GEN}(S) \quad =$$

AvE Analysis of a Structured Program



$$\text{GEN}(S) = \text{GEN}(s_1) \cap \text{GEN}(s_2)$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
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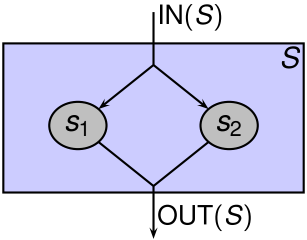
\mathrm{KILL}(S) &=
\end{aligned}
$$

AvE Analysis of a Structured Program



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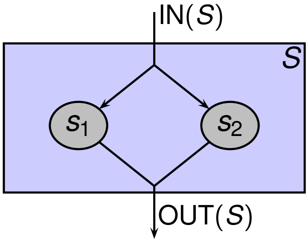

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\end{aligned}
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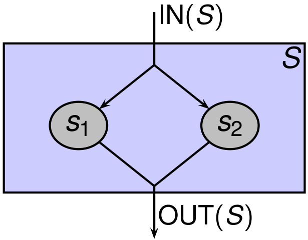

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\mathrm{IN}(s_1) &= \mathrm{IN}(s_2) = \mathrm{IN}(S)
\end{aligned}
$$

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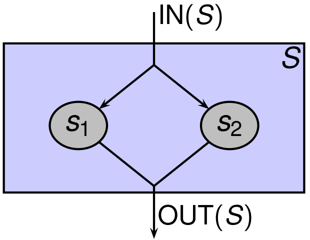

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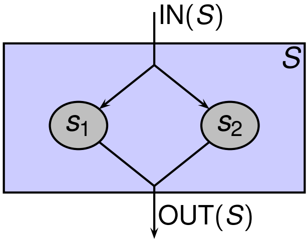
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \cup \mathrm{KILL}(s_2) \\
\mathrm{IN}(s_1) &= \mathrm{IN}(s_2) \quad = \quad \mathrm{IN}(S) \\
\mathrm{OUT}(S) &=
\end{aligned}
$$

AvE Analysis of a Structured Program



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\text{IN}(s_1) &= \text{IN}(s_2) = \text{IN}(S) \\
\text{OUT}(S) &= \text{OUT}(s_1) \cap \text{OUT}(s_2)
\end{aligned}
$$

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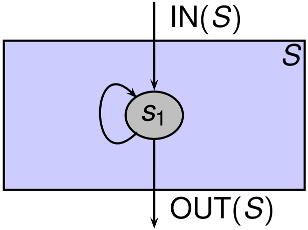

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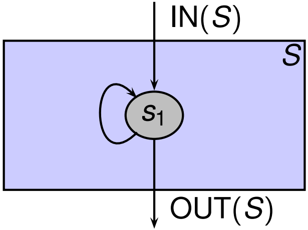

$$\text{GEN}(S) \quad =$$

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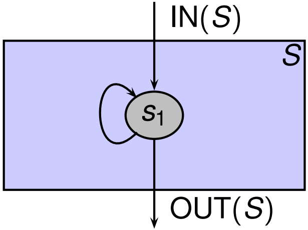
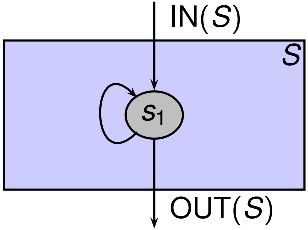


$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1)$$

AvE Analysis of a Structured Program



$$\begin{aligned} \mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\ \mathrm{KILL}(S) &= \end{aligned}$$

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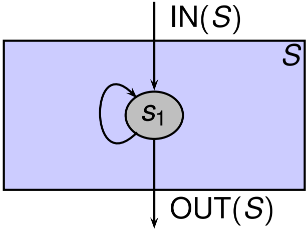
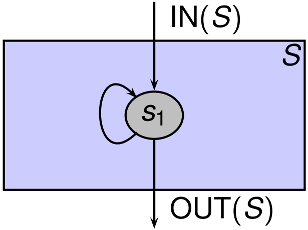


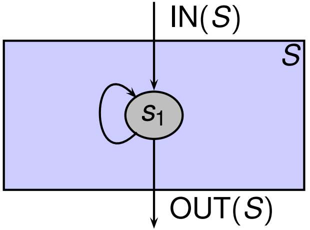
$$\begin{aligned} \mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\ \mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \end{aligned}$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\text{GEN}(S) &= \text{GEN}(s_1) \\
\text{KILL}(S) &= \text{KILL}(s_1) \\
\text{OUT}(S) &=
\end{aligned}
$$

AvE Analysis of a Structured Program



$$
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\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
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\end{aligned}
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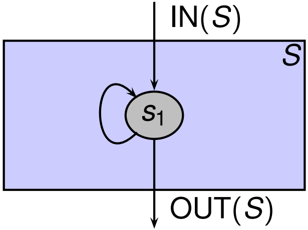

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\text{GEN}(S) &= \text{GEN}(s_1) \\
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\text{IN}(s_1) &=
\end{aligned}
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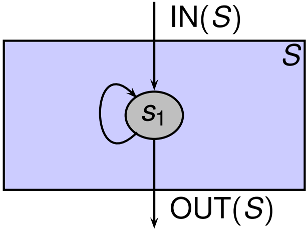
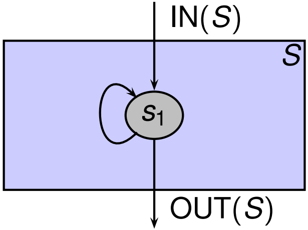


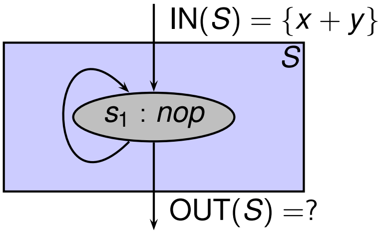
$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_1) \\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cap \mathrm{GEN}(s_1)
\end{aligned}
$$

AvE Analysis of a Structured Program

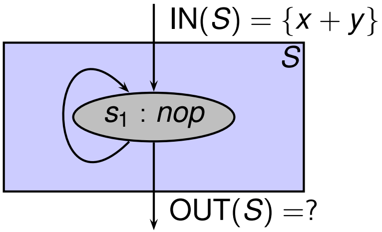


$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_1) \\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cap \mathrm{GEN}(s_1)\ {\color{red}?} \\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cap \mathrm{OUT}(s_1){\color{red}??}
\end{aligned}
$$

AvE Analysis of a Structured Program

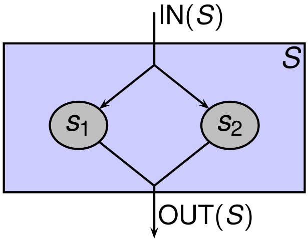


AvE Analysis of a Structured Program



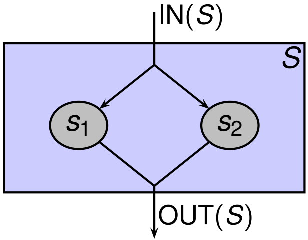
Is x + y available at OUT(S)?

AvE Analysis is Approximate



- Assumption: All paths are feasible.

AvE Analysis is Approximate



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if (true) s1;
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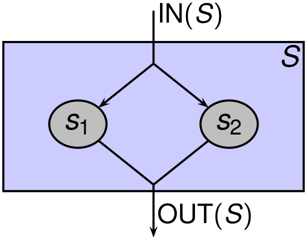

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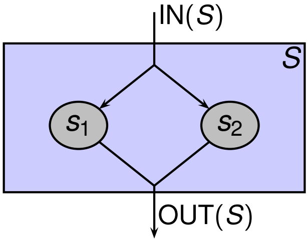

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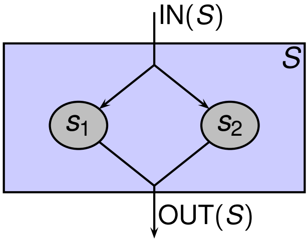
Fact		Computed		Actual
GEN(S)	$=$	GEN(s_1) $\cap$ GEN(s_2)	$\subseteq$	GEN(s_1)

AvE Analysis is Approximate



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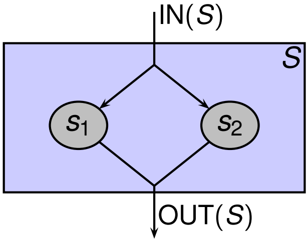
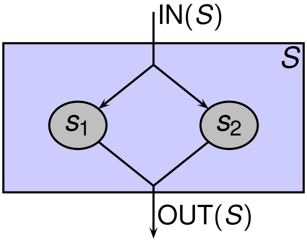
Fact		**Computed**		**Actual**
$\text{GEN}(S)$	$=$	$\text{GEN}(s_1) \cap \text{GEN}(s_2)$	$\subseteq$	$\text{GEN}(s_1)$
$\text{KILL}(S)$	$=$	$\text{KILL}(s_1) \cup \text{KILL}(s_2)$	$\supseteq$	$\text{KILL}(s_1)$

AvE Analysis is Approximate



- Thus,

AvE Analysis is Approximate



- Thus,

  true GEN(S) $\supseteq$ analysis GEN(S)

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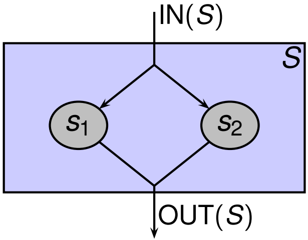

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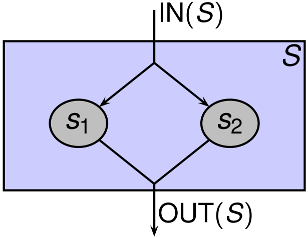

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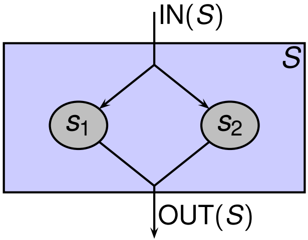

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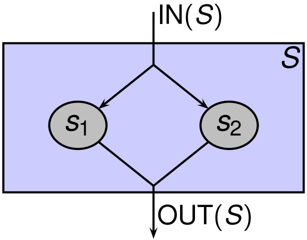

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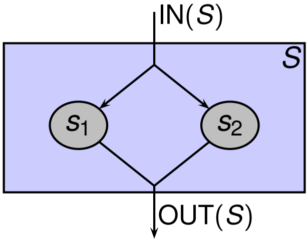

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- Fewer expressions marked available than actually do!
- Later we shall see that this is SAFE approximation
  - prevents optimizations
  - but NO wrong optimization

AvE for Basic Blocks

- Expr e is available at the start of a block if

$$\text{IN}(B) = \bigcap_{P \in \text{PRED}(B)} \text{OUT}(P)$$

AvE for Basic Blocks

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$$\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

- Expr e is available at the end of a block if
  - Either it is generated by the block
  - Or it is available at the start of the block and not killed by the block

$$\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$$

Solving AvE Constraints

- KILL & GEN known for each BB.

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- A program with N BBs has $2N$ equations with $2N$ unknowns.
  - Solution is possible.
  - Iterative approach (on the next slide).

`for each block` B `{`

```
for each block B {
    OUT(B) = U; U = "universal" set of all exprs
```

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for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
}
OUT(Entry) = ∅; // remember reaching defs?
```

for each block B {

    $\text{OUT}(B) = \mathcal{U}$; $\mathcal{U}$ = "universal" set of all exprs

}

$\text{OUT}(\mathit{Entry}) = \emptyset$; // remember reaching defs?

change = true;

while (change) {

    change = false;

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    OUT(B) = U;  U = "universal" set of all exprs
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    for each block B other than Entry {
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while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋂_{P∈PRED(B)} OUT(P);
```

$$\mathrm{OUT}(B) = \mathcal{U}; \quad \mathcal{U} = \text{"universal" set of all exprs}$$

$$\mathrm{OUT}(\mathit{Entry}) = \emptyset$$

$$\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P);$$

for each block B {
    $\mathrm{OUT}(B) = \mathcal{U}$; $\mathcal{U}$ = "universal" set of all exprs
}
$\mathrm{OUT}(\mathit{Entry}) = \emptyset$; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than *Entry* {
        $\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$;
        oldOut = $\mathrm{OUT}(B)$;
        $\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$;

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for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
}
OUT(Entry) = ∅; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋂_{P∈PRED(B)} OUT(P);
        oldOut = OUT(B);
        OUT(B) = IN(B) − KILL(B) ∪ GEN(B);
        if (OUT(B) ≠ oldOut) then {
            change = true;
        }
    }
}
```

Some Issues

- What is $\mathcal{U}$ – the set of *all* expressions?

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- How to compute it efficiently?

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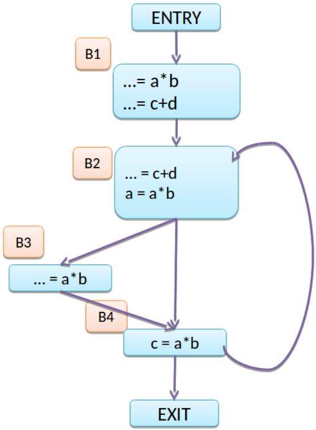
- What is $\mathcal{U}$ – the set of *all* expressions?
- How to compute it efficiently?
- Why *Entry* block is initialized differently?

Available Expressions: Example



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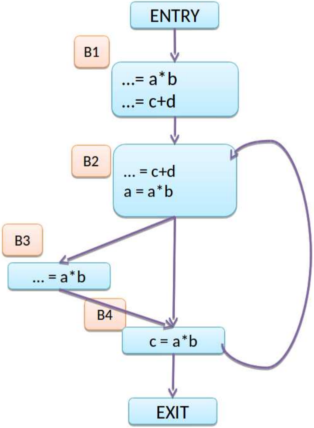

BB	GEN	KILL
B1		
B2		
B3		
B4		

$\mathcal{U}$= {a*b, c+d}

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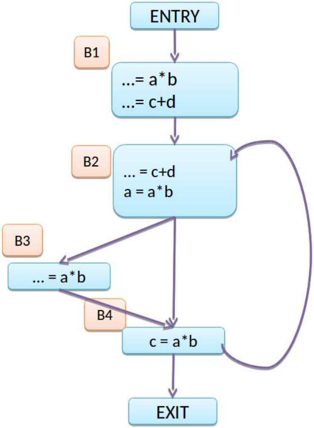

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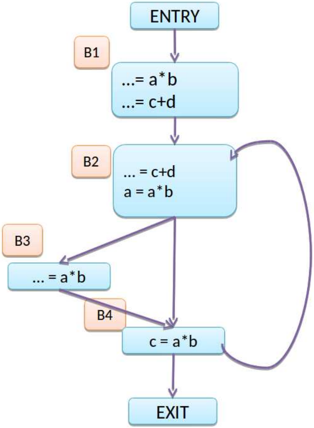

BB	GEN	KILL
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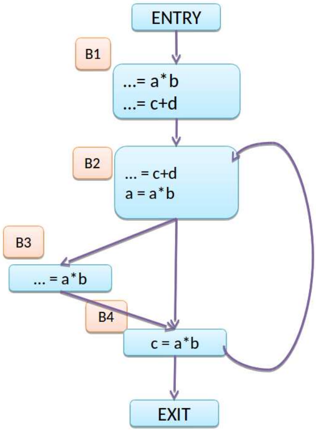

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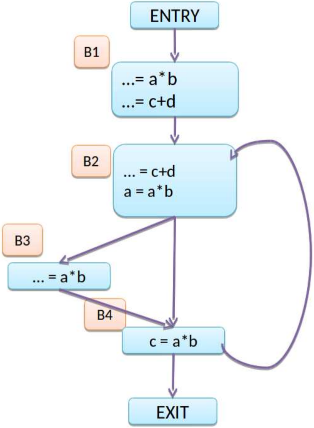

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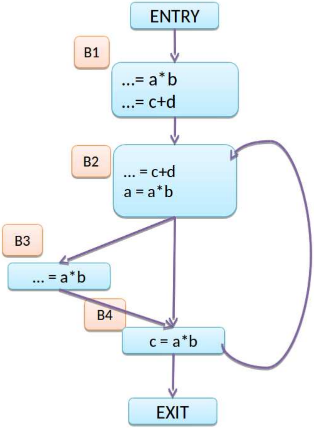

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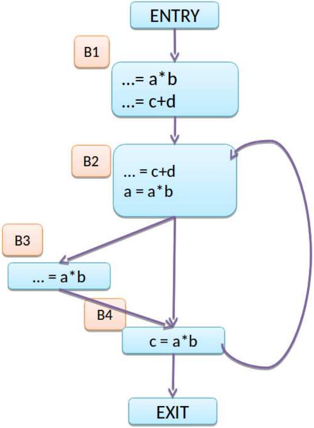

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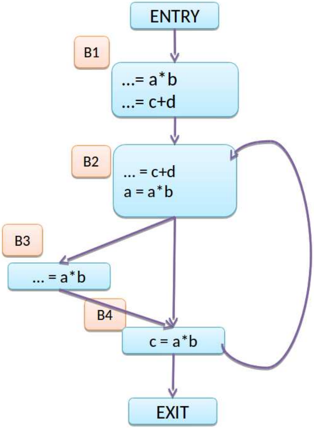

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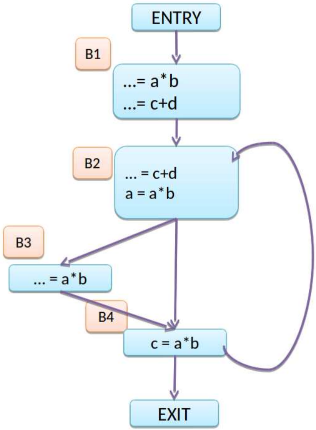

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$\mathcal{U} = \{a*b, c+d\}$

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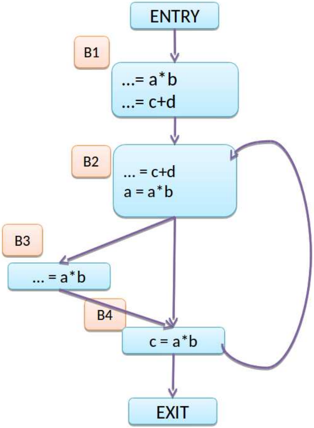
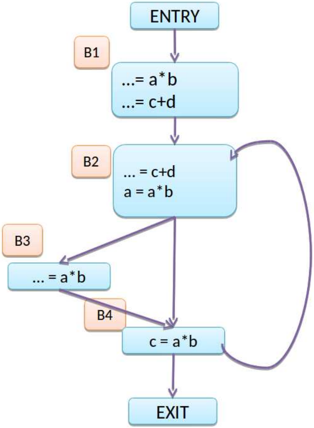


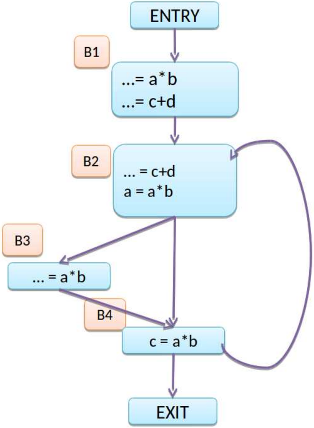
Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$

Available Expressions: Example



Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
1	IN	$\emptyset$	a*b, c+d	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

Available Expressions: Example



Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
1	IN	$\emptyset$	a*b, c+d	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
2	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

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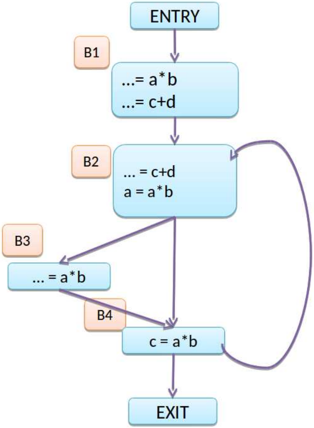

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	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
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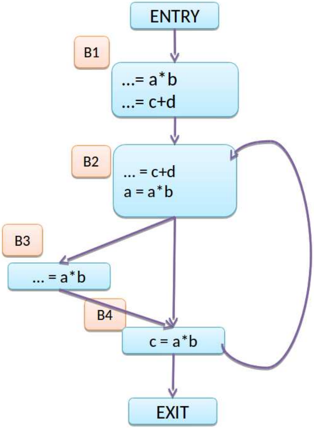
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
2	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
3	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

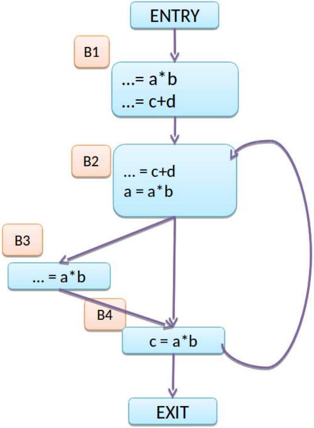
Available Expressions: Bitvectors



a bit for each expression:

a*b	c+d

Available Expressions: Bitvectors



a bit for each expression:

a*b	c+d

Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	11	11	11	11
1	IN	00	11	01	01
	OUT	11	01	11	10
2	IN	00	10	01	01
	OUT	11	01	11	10
3	IN	00	10	01	01
	OUT	11	01	11	10

Available Expressions: Bitvectors

- Set-theoretic definitions:

$$\text{IN}(B) = \bigcap_{P \in \text{PRED}(B)} \text{OUT}(P)$$

$$\text{OUT}(B) = \text{IN}(B) - \text{KILL}(B) \cup \text{GEN}(B)$$

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- Bitvector definitions:

$$\mathrm{IN}(B) = \bigwedge_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

$$\mathrm{OUT}(B) = \mathrm{IN}(B) \wedge \neg \mathrm{KILL}(B) \vee \mathrm{GEN}(B)$$

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- Bitwise $\vee, \wedge, \neg$ operators

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Available Expressions: Application

- Common subexpression elimination in a block B
  - Expression e available at the entry of B
  - e is also computed at a point p in B
  - Components of e are not modified from entry of B to p
- e is "upward exposed" in B
- Expressions generated in B are "downward exposed"

Comparison of RD and AvE

- *All* vs. *Some* path property

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- *All* vs. *Some* path property
- Meet operator: $\cup$ vs. $\cap$

Comparison of RD and AvE

- *All* vs. *Some* path property
- Meet operator: $\cup$ vs. $\cap$
- Initialization of *Entry*: $\emptyset$

Comparison of RD and AvE

- *All* vs. *Some* path property
- Meet operator: $\cup$ vs. $\cap$
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- Safety: "More" RD vs. "Fewer" AvE

AvE: alternate Initialization

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- Otherwise x is dead at p

Live Variables: GEN

- GEN(B): Set of variables whose values may be used in block B prior to any definition
  - Also called "use(B)"
- "upward exposed use" of a variable in B

Live Variables: KILL

- KILL(B): Set of variables defined in block B prior to any use
  - Also called "def(B)"
- "upward exposed definition" of a variable in B

Live Variables: Equations

- Set-theoretic definitions:

$$\text{OUT}(B) = \bigcup_{S \in \text{SUCC}(B)} \text{IN}(S)$$

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- Bitwise $\vee, \wedge, \neg$ operators

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- Also called *Anticipable expression*

QQ

- Expression *e* is very busy at a point *p* if
  - **Every** path from *p* to *Exit* has at least one evaluation of *e* and there is no assignment to any component variable of *e* before the first evaluation of *e* following *p* on these paths.
- Set up the data flow equations for Very Busy Expressions (VBE). You have to give equations for GEN, KILL, IN, and OUT.
- Think of an optimization/transformation that uses VBE analysis. Briefly describe it (2-3 lines only)
- Will your optimization be *safe* if we replace “*Every*” by “*Some*” in the definition of VBE?