CS738: Advanced Compiler Optimizations

Pointer Analysis

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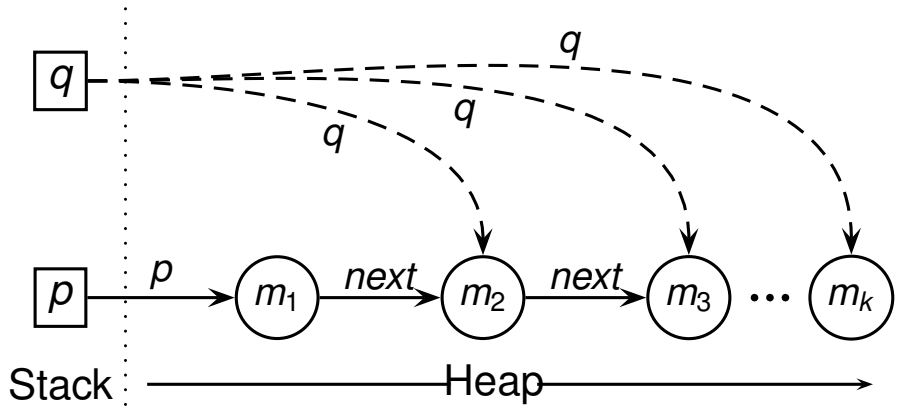
Why Pointer Analysis?

- Static analysis of pointers & references

```
S1.   ...
S2.   q = p;
S3.   do {
S4.       q = q.next;
S5.   } while (...)
S6.   p.data = r1;
S7.   q.data = q.data + r2;
S8.   p.data = r1;
S9.   r3 = p.data + r2;
S10.  ...
```



Superimposition of memory graphs after *do-while* loop

p and *q* are definitely not aliases statement S6 onwards.
Statement S8 **is** redundant.

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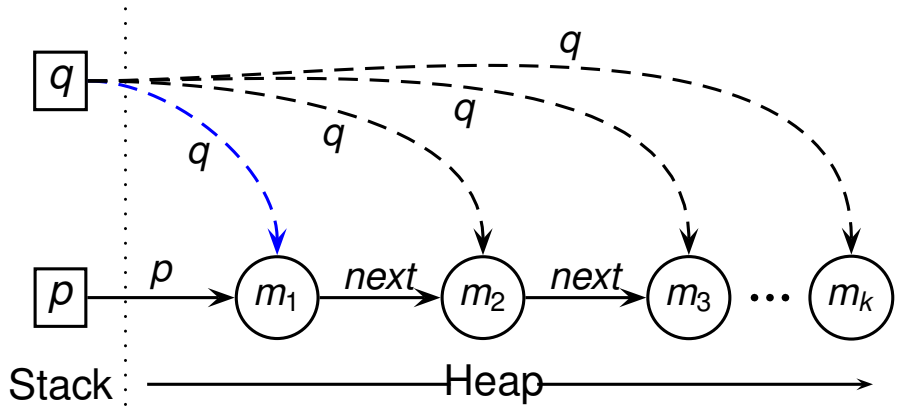

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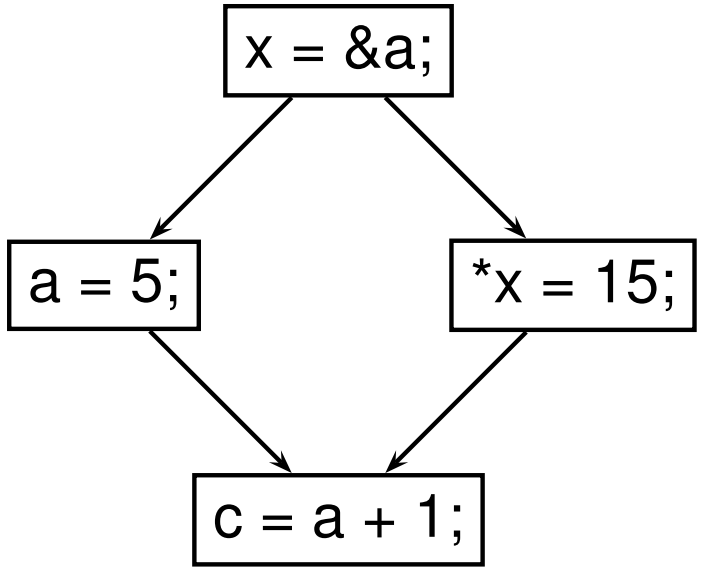

Reaching definitions analysis

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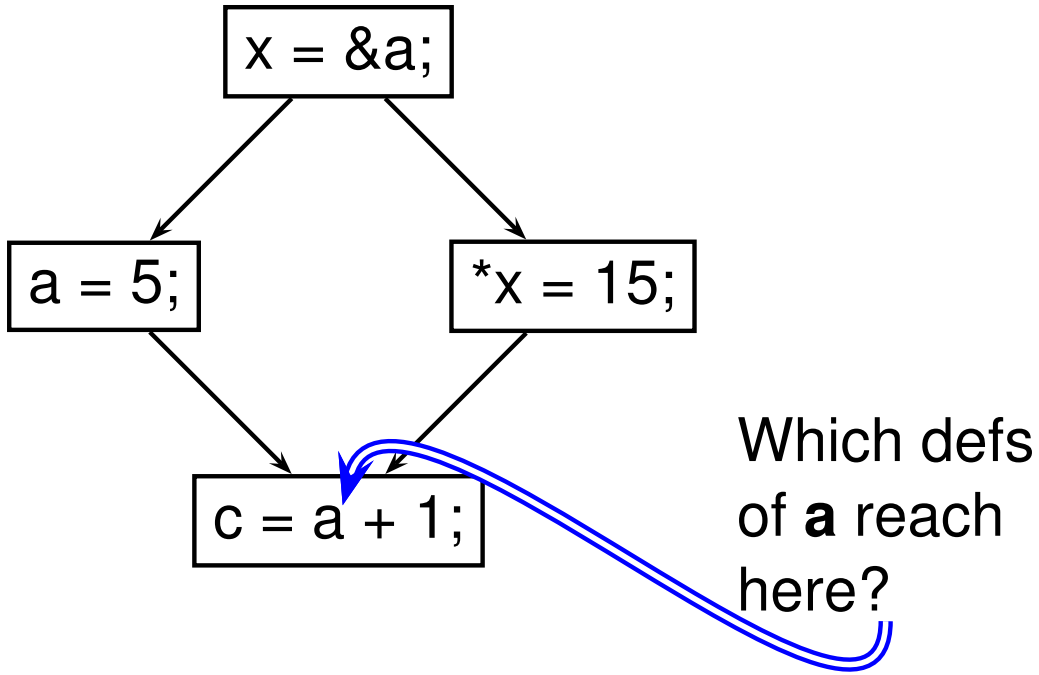

Reaching definitions analysis

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  - A statement can not “override” information computed by another statement
    - *NO* Kill component in the flow function
    - If statement *s* kills some data flow information, there is an alternate path that excludes *s*

Examples of Flow Insensitive Analyses

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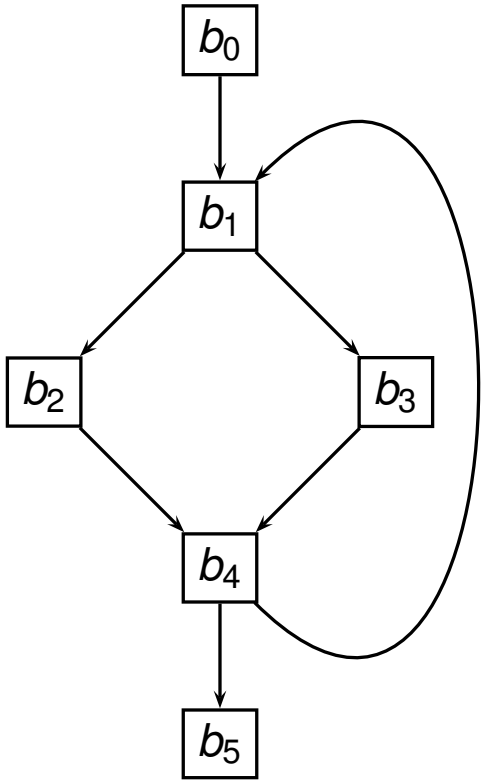
- Side effects analysis
  - Does a procedure modify address / global variable / reference parameter / . . . ?

Realizing Flow Insensitivity



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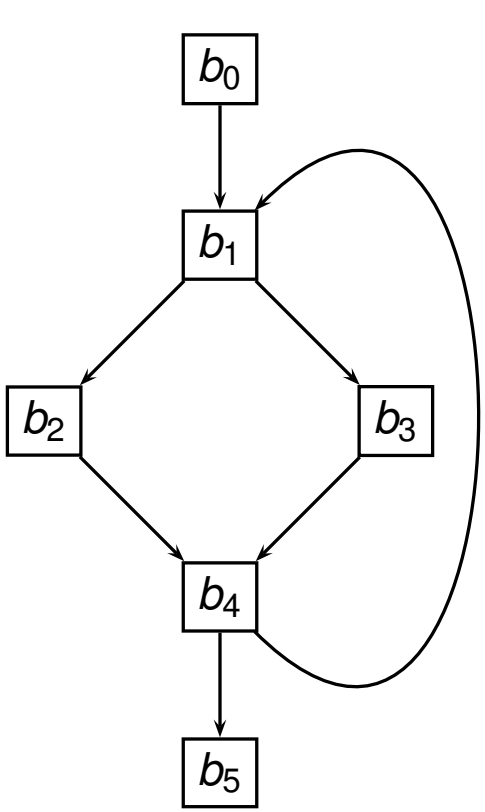

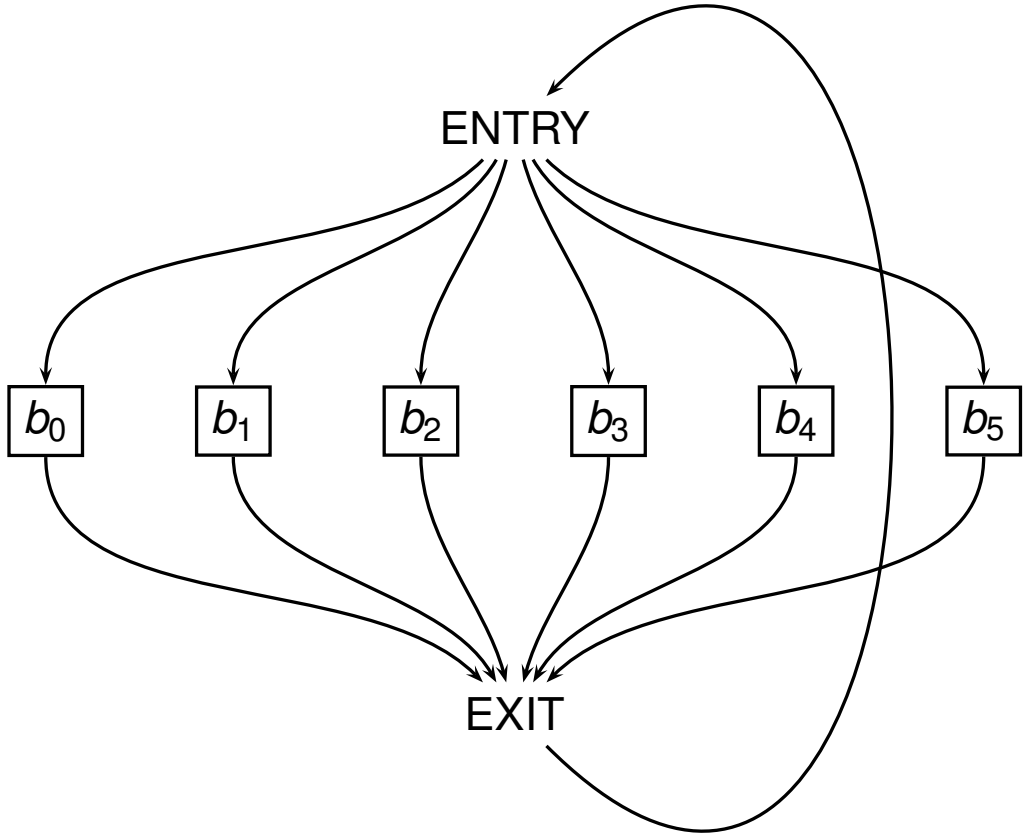

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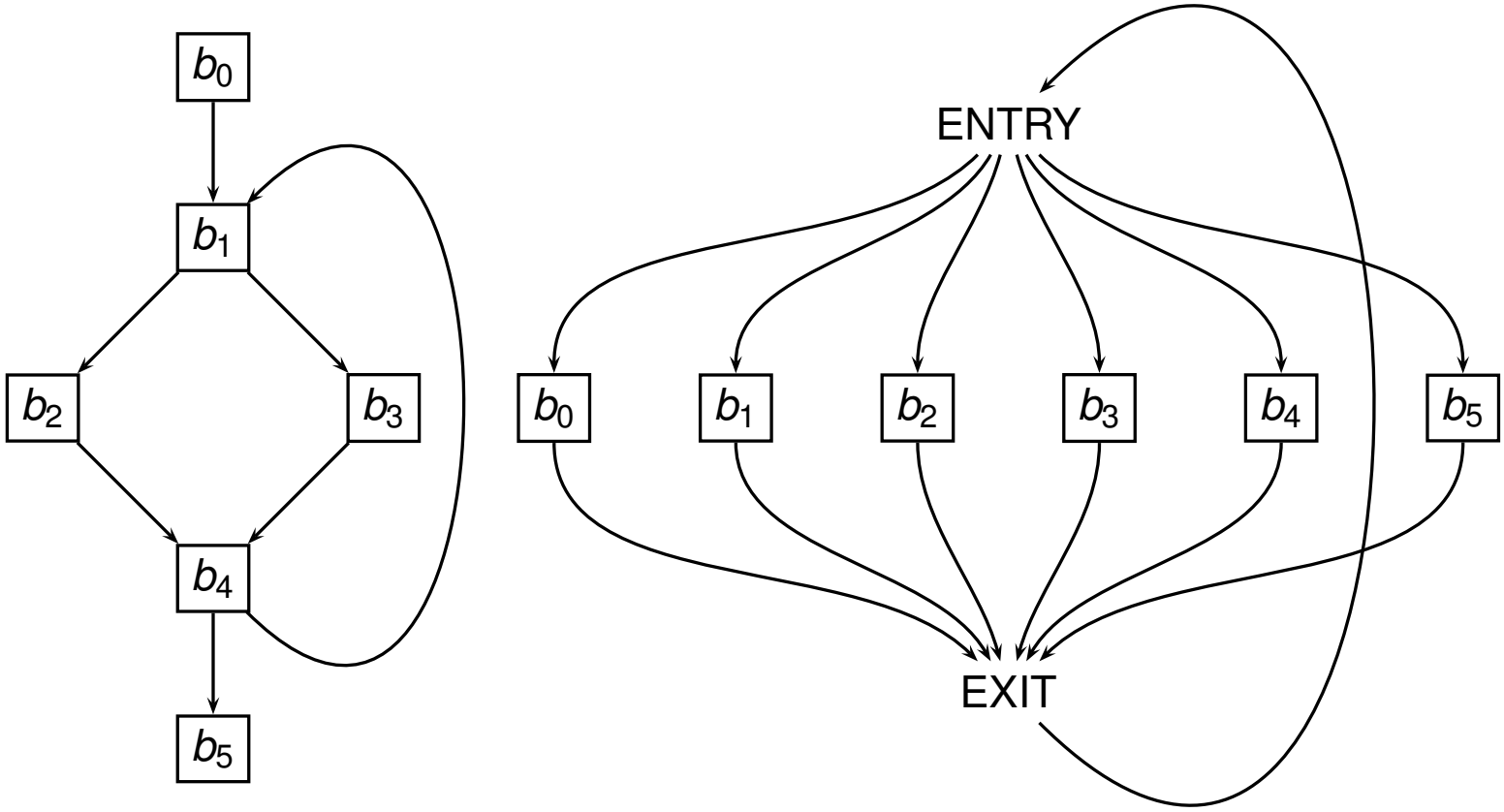

Allows arbitrary compositions of flow functions in any order $\Rightarrow$ *Flow insensitivity*

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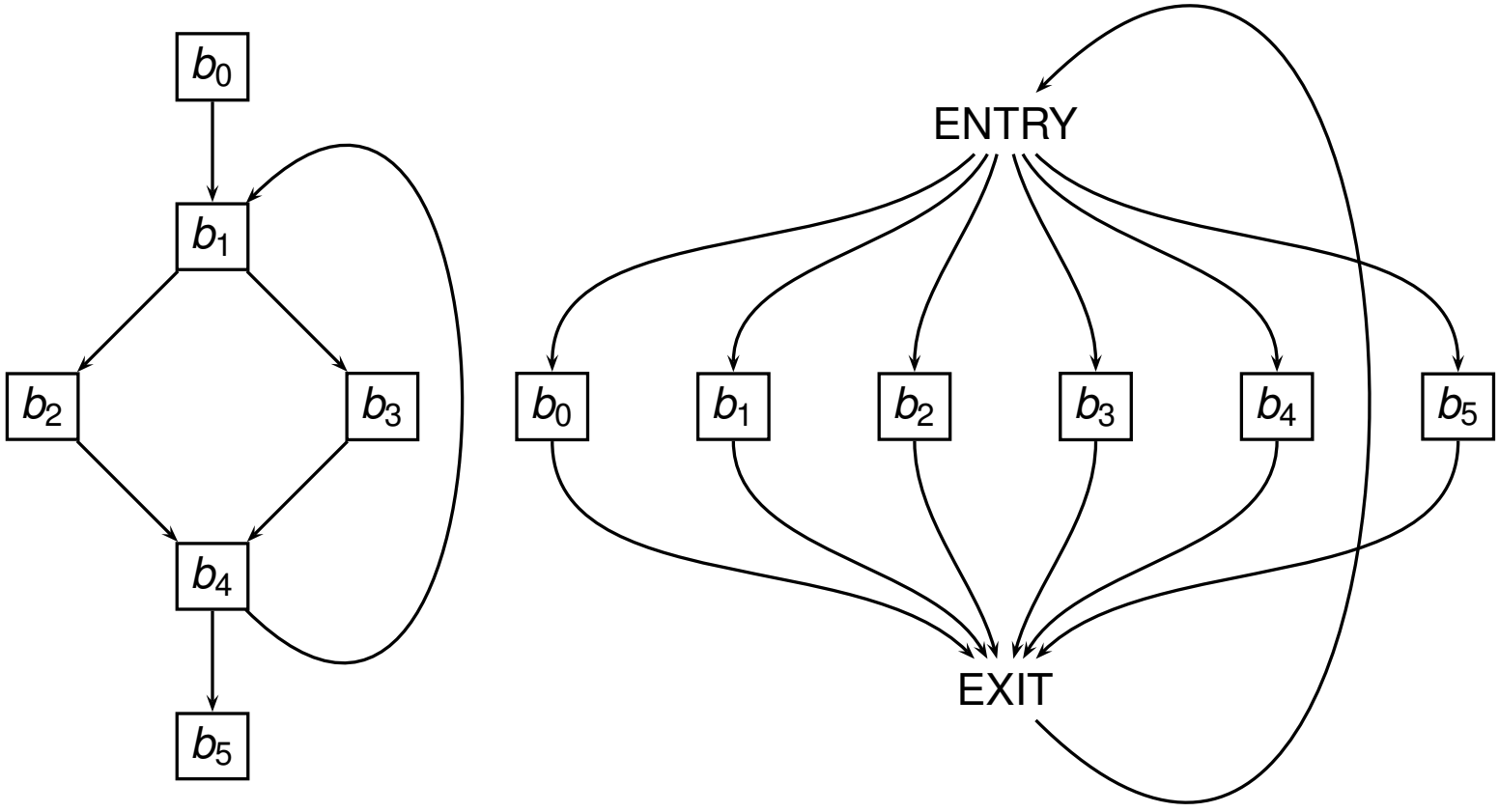

In practice, dependent constraints are collected in a global repository in one pass and solved independently

Alias Analysis vs. Points-to Analysis

	Points-to Analysis	**Alias Analysis**
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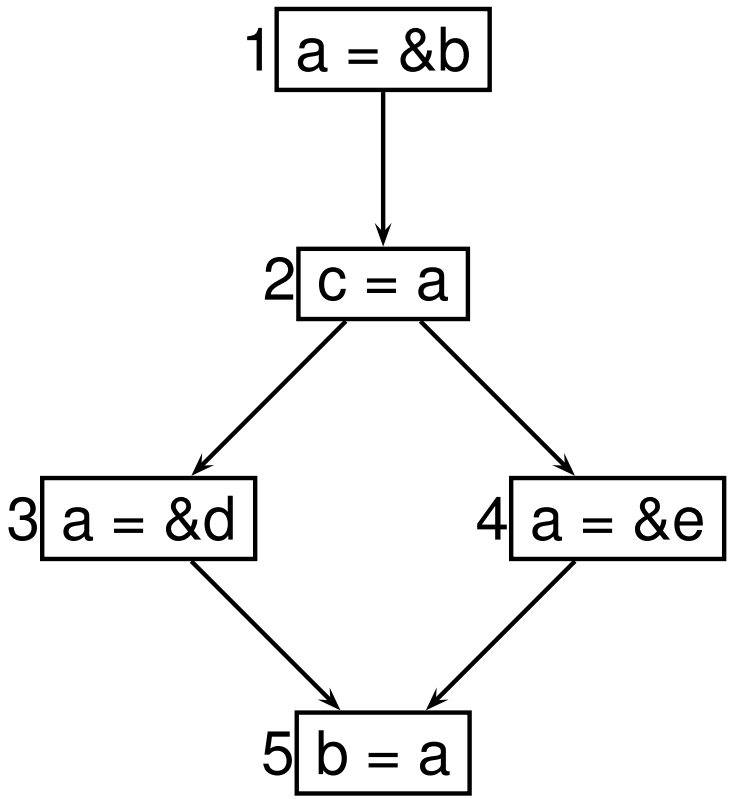
Transitive?	No	Must alias: Yes, May alias: No

Andersen's Flow Insensitive Points-to Analysis

- Subset based analysis

Program	Constraints	Points-to Graph



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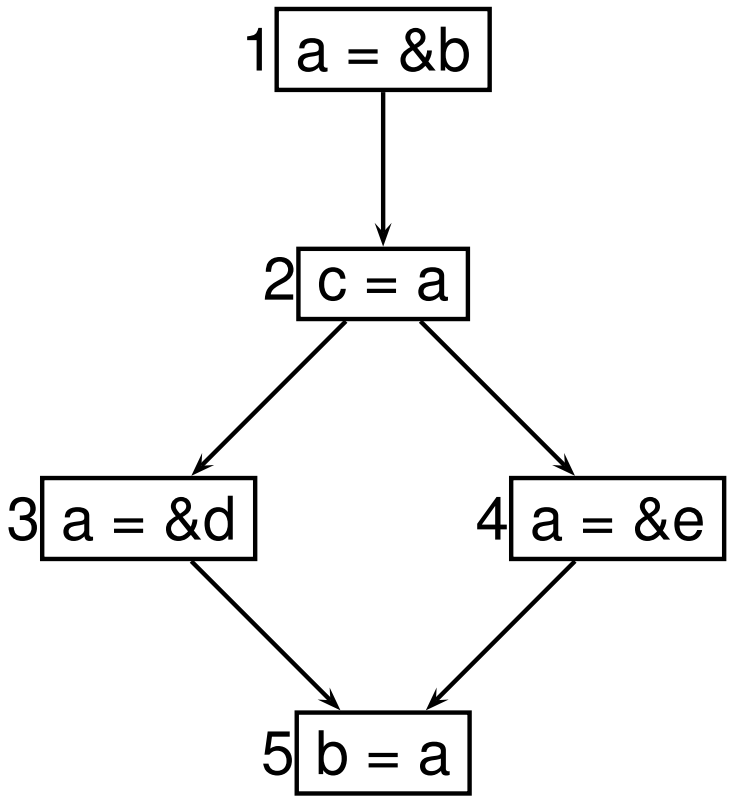

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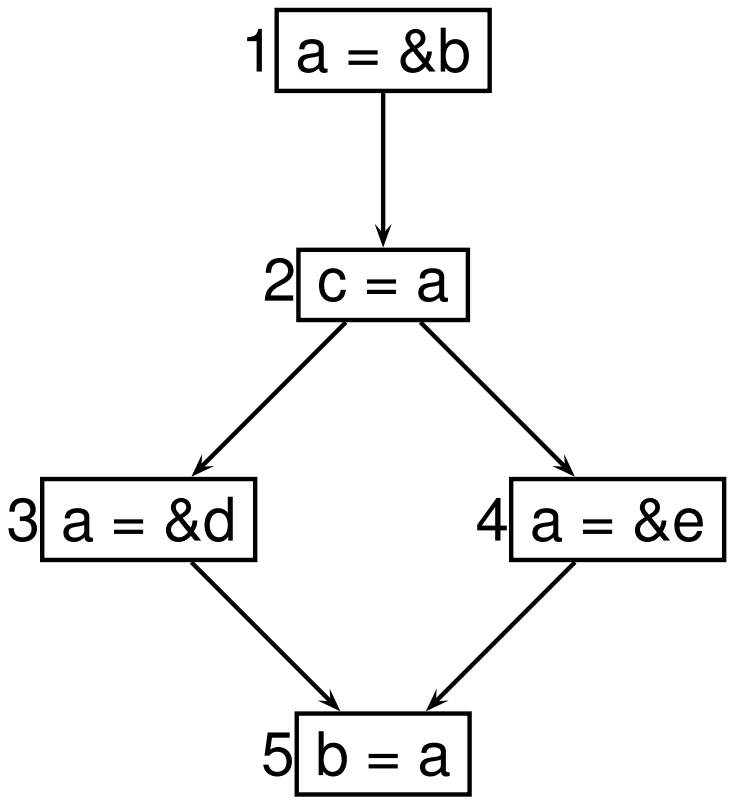

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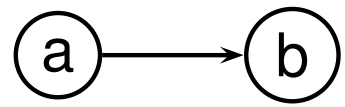

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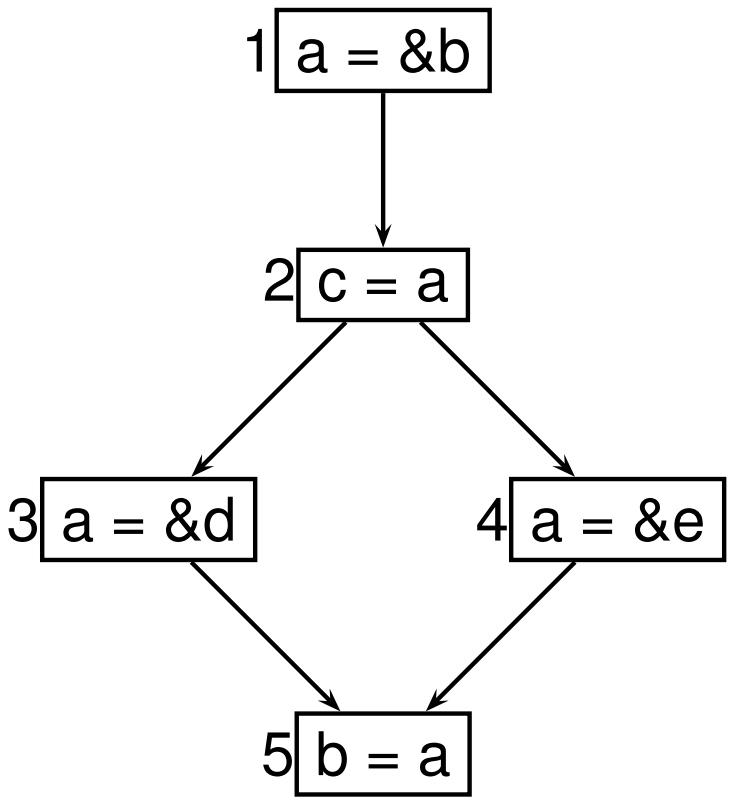

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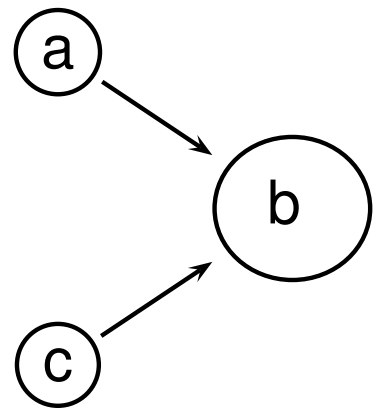

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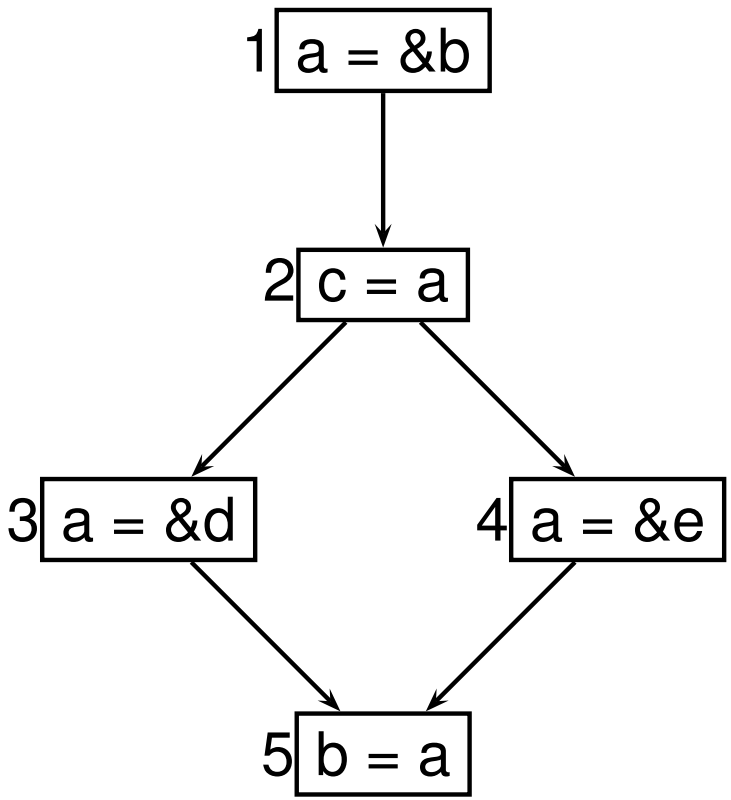

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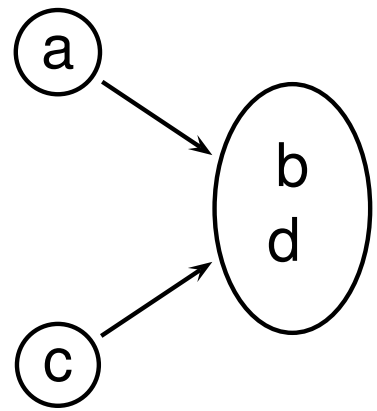

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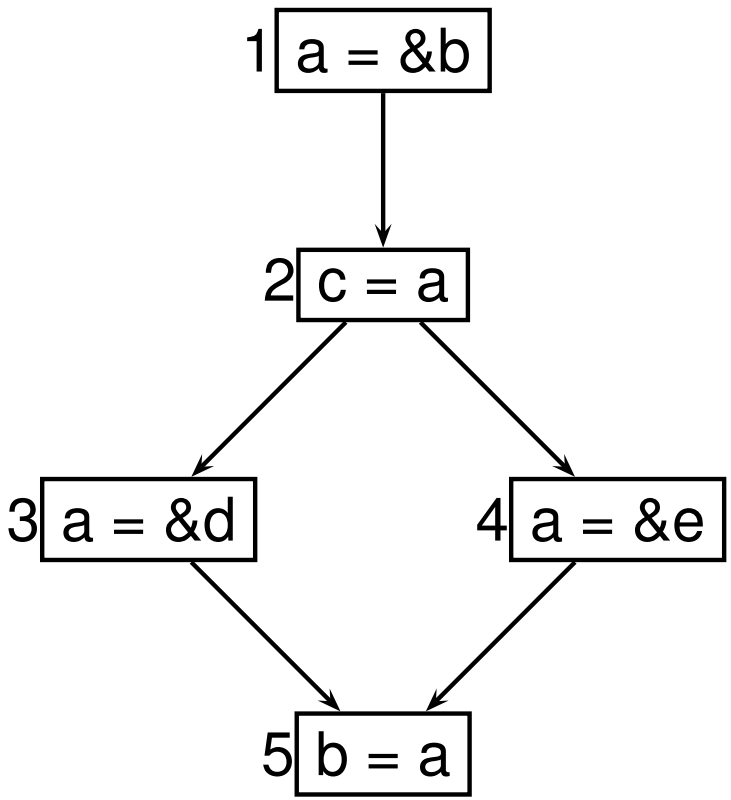

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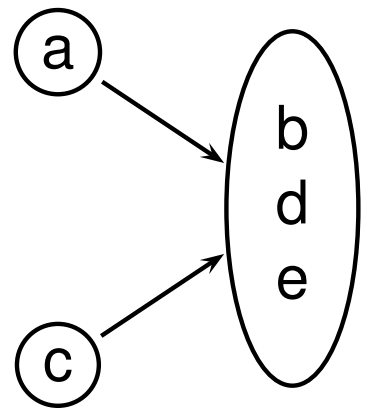

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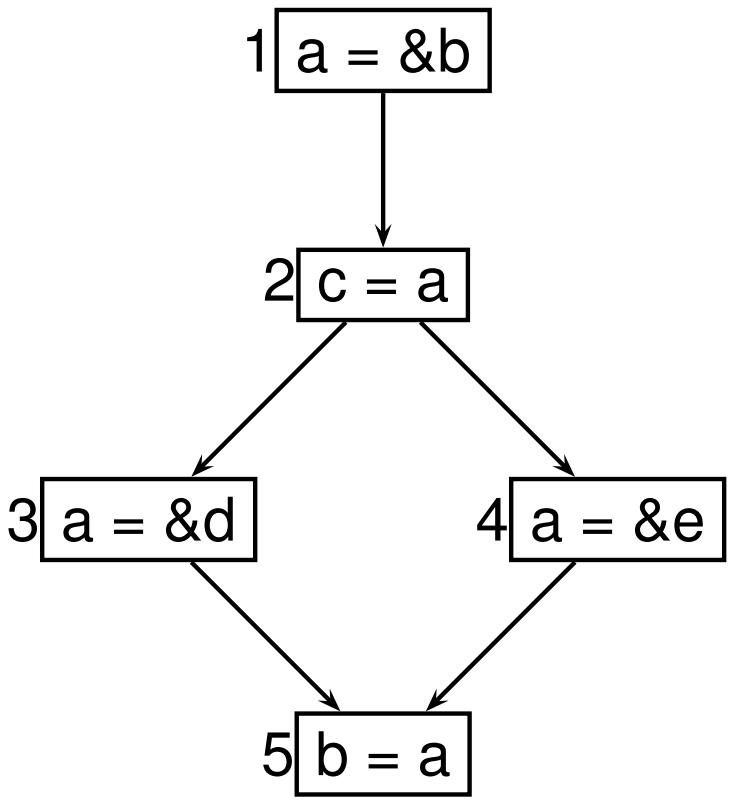

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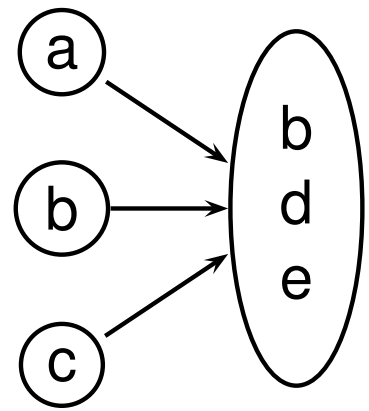

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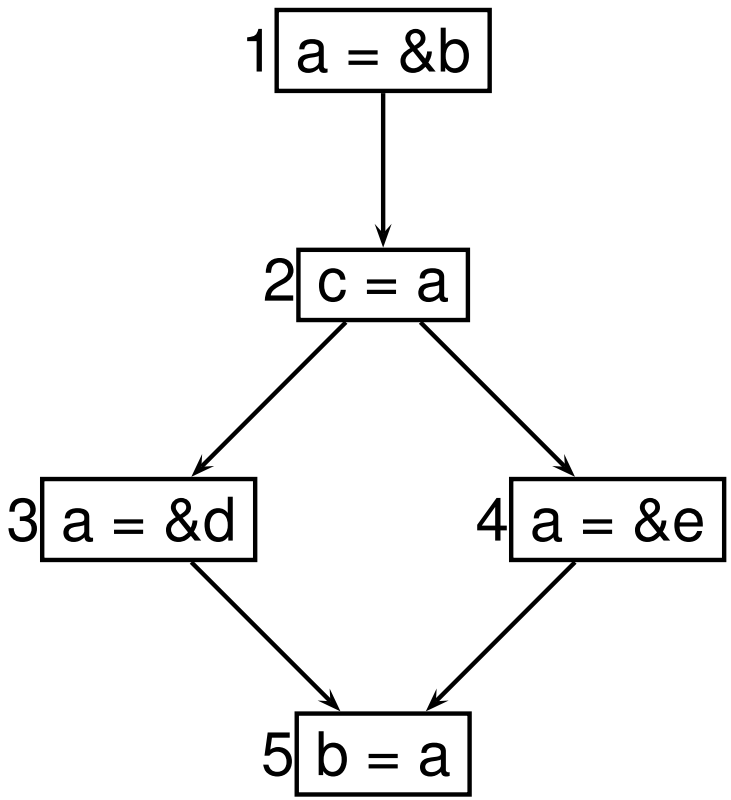

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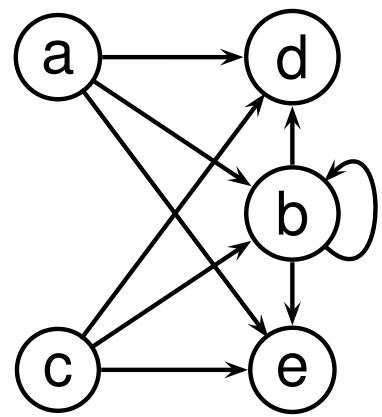

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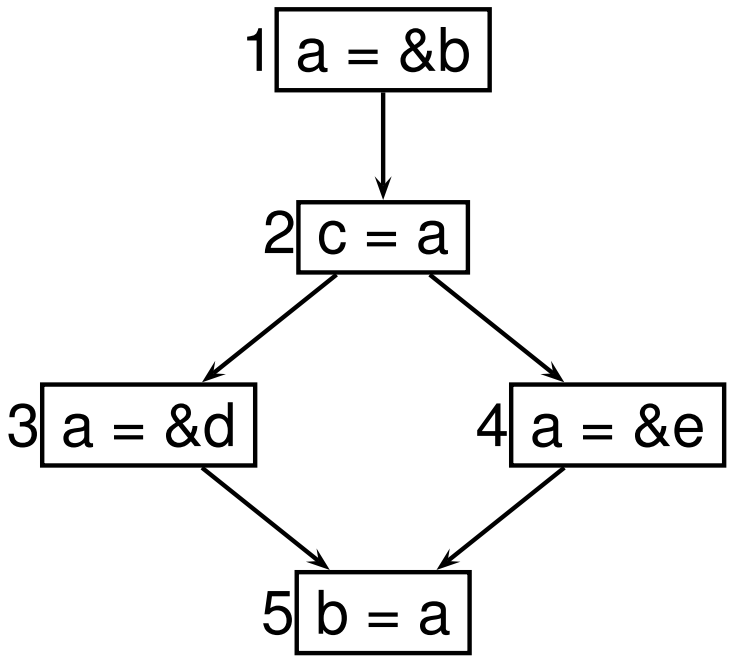

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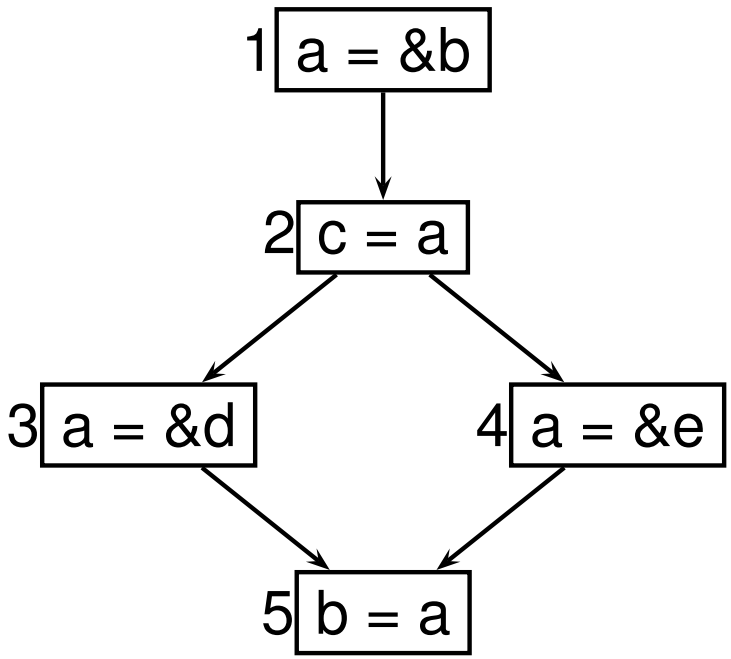

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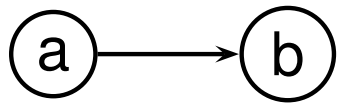

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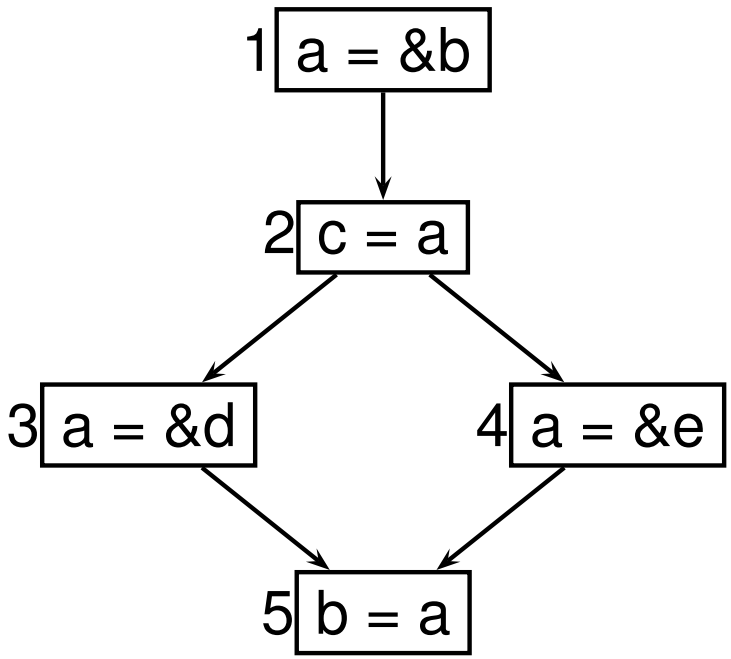

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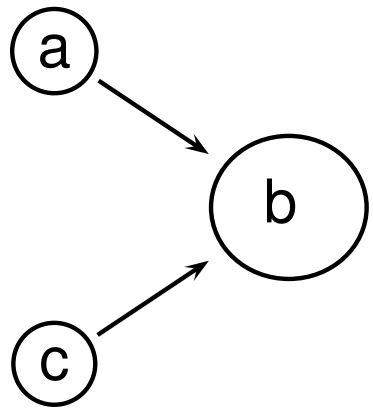

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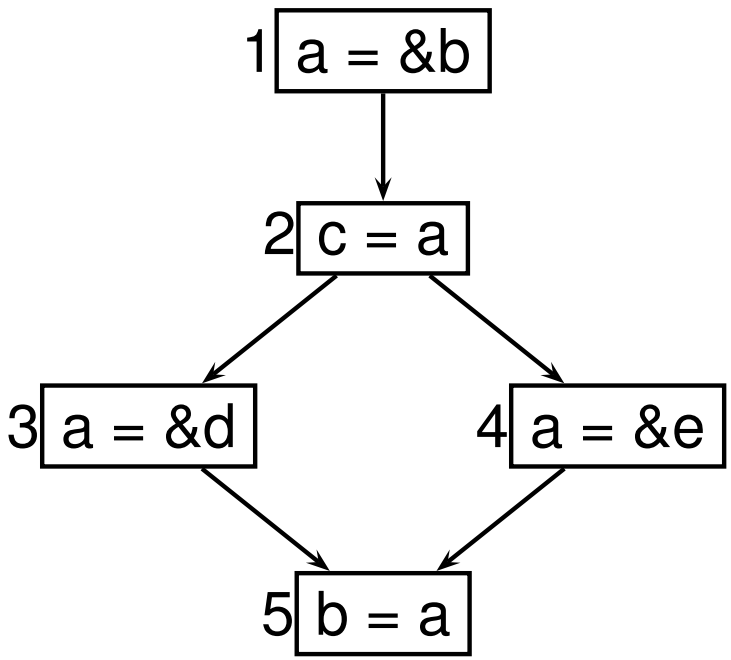

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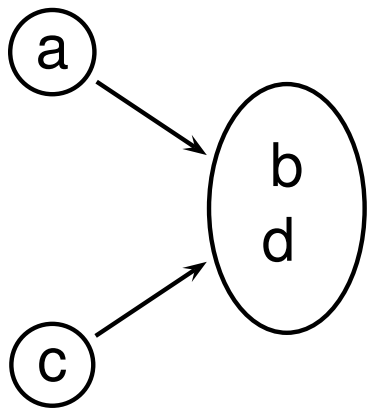
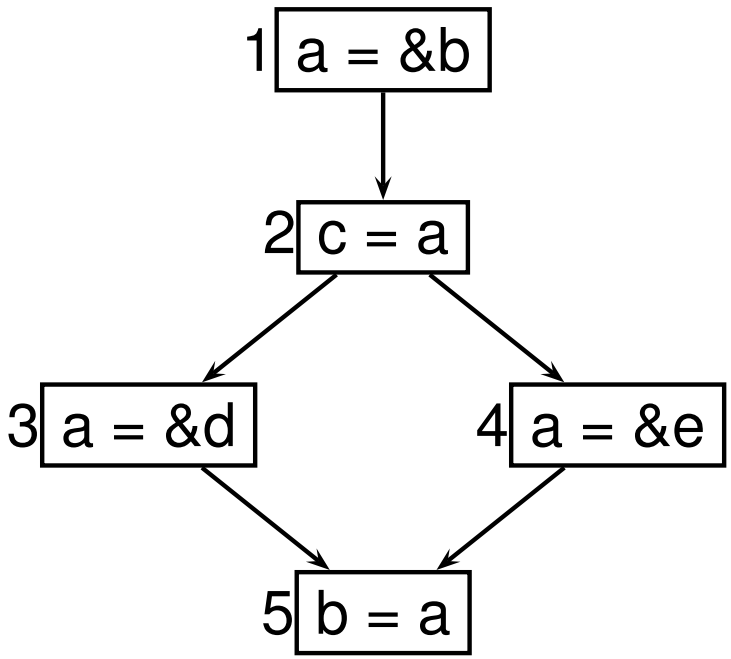
2	MERGE(P_c, P_a)
3	$P_a \supseteq \{d\}$



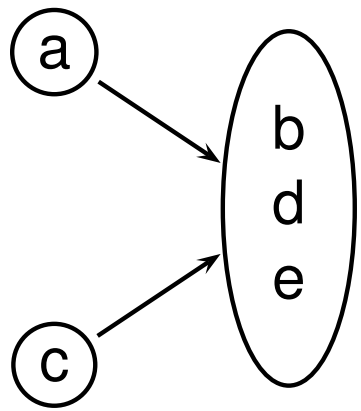
Steensgaard's Flow Insensitive Points-to Analysis

- Equality based analysis: $P_{lhs} \equiv P_{rhs}$
- Only one Points-to successor at any time, merge (potential) multiple successors

Program	Constraints	Points-to Graph



#	Constraint
1	$P_a \supseteq \{b\}$
2	MERGE(P_c, P_a)
3	$P_a \supseteq \{d\}$
4	$P_a \supseteq \{e\}$



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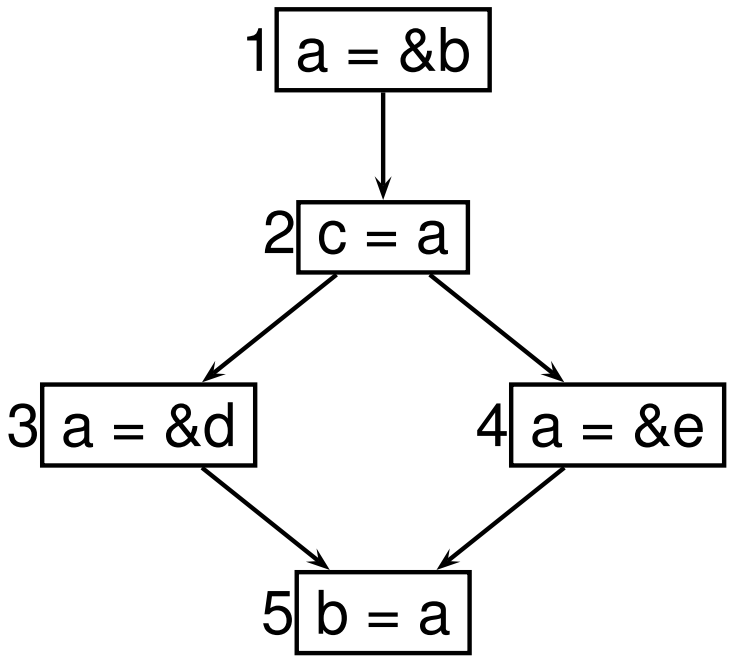
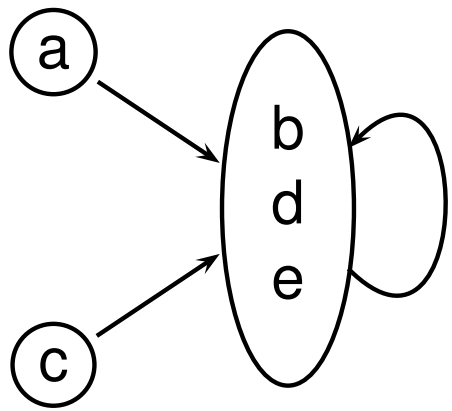
Program	Constraints	Points-to Graph



#	Constraint
1	$P_a \supseteq \{b\}$
2	MERGE(P_c, P_a)
3	$P_a \supseteq \{d\}$
4	$P_a \supseteq \{e\}$
5	MERGE(P_b, P_a)



Steensgaard's Flow Insensitive Points-to Analysis

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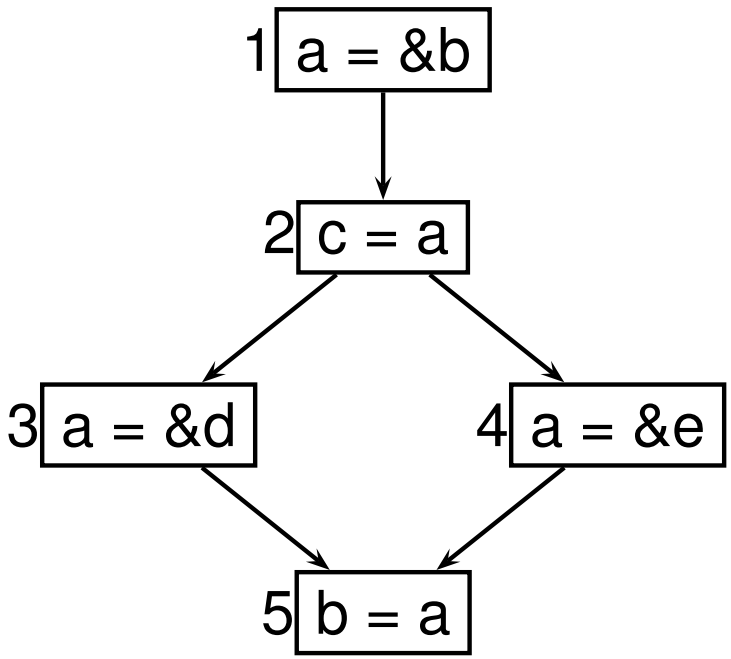
Program	Constraints	Points-to Graph



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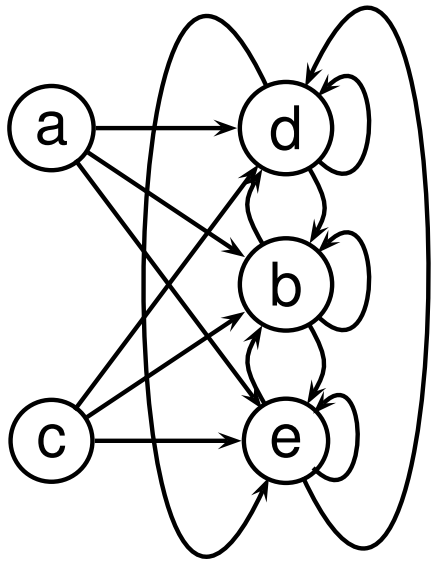
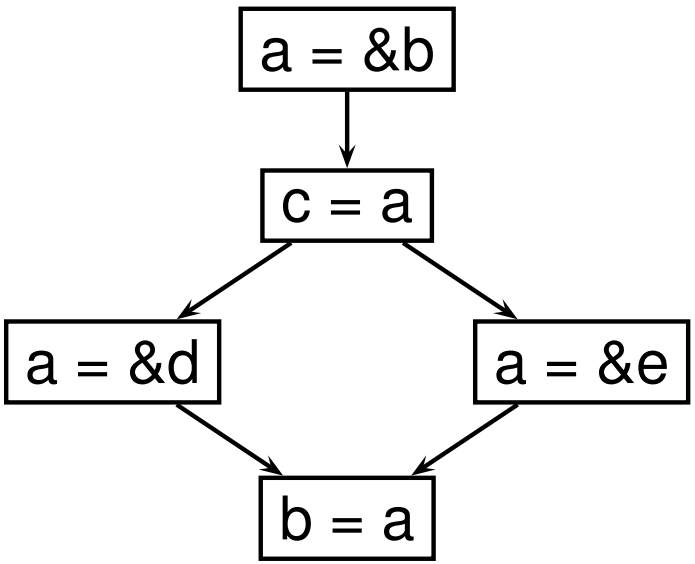
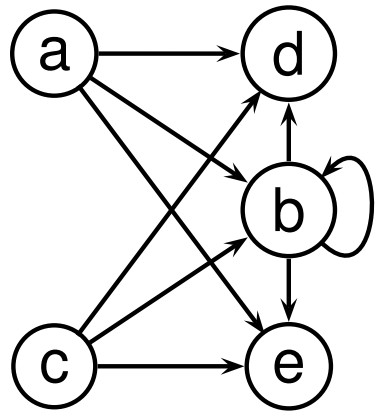
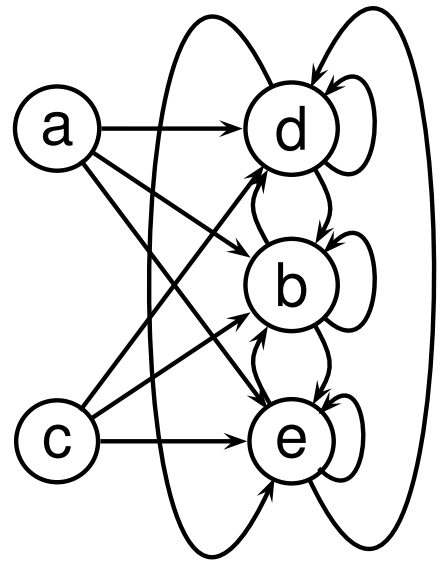


Comparing Anderson's and Steensgaard's Analyses

Program	Subset based Points-to Graph	Equality based Points-to Graph





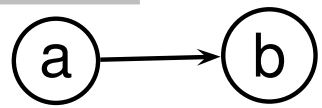


Comparing Anderson's and Steensgaard's Analyses

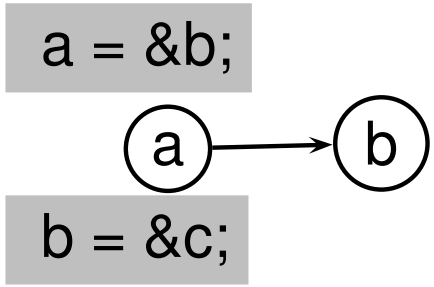
```
a = &b;
```

Comparing Anderson's and Steensgaard's Analyses

```
a = &b;
```



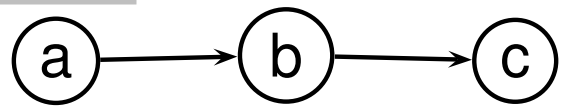
Comparing Anderson's and Steensgaard's Analyses



Comparing Anderson's and Steensgaard's Analyses

```
a = &b;
```

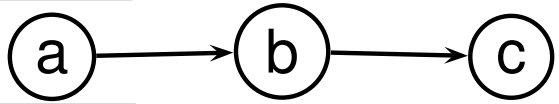

```
b = &c;
```



Comparing Anderson's and Steensgaard's Analyses

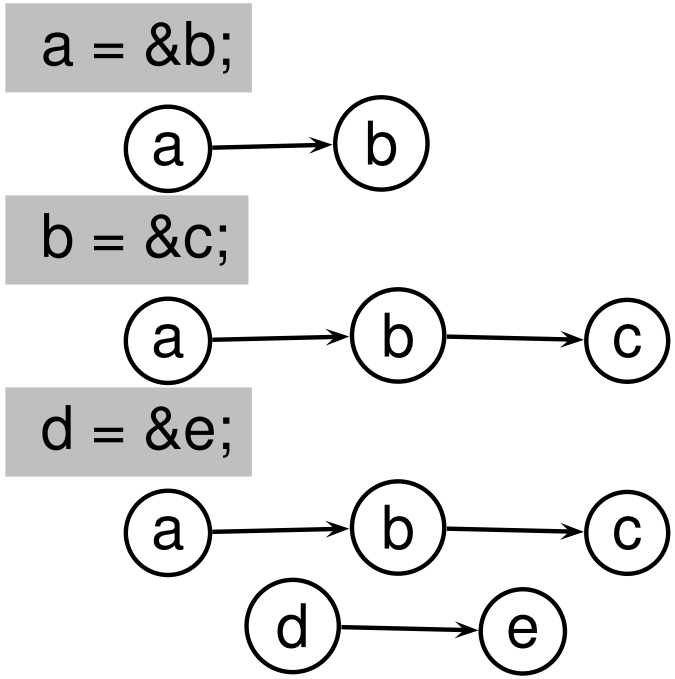
```
a = &b;
```


```
b = &c;
```



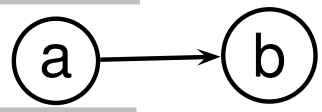

```
d = &e;
```

Comparing Anderson's and Steensgaard's Analyses

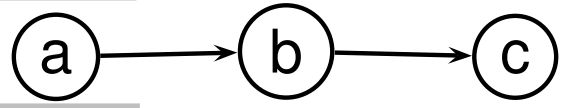


Comparing Anderson's and Steensgaard's Analyses

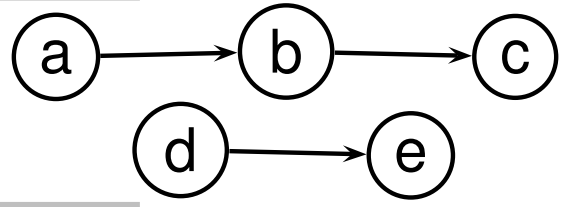
```
a = &b;
```




```
b = &c;
```



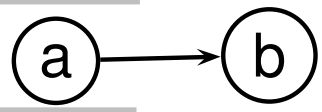

```
d = &e;
```



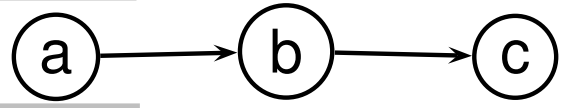

```
a = &d;
```

Comparing Anderson's and Steensgaard's Analyses

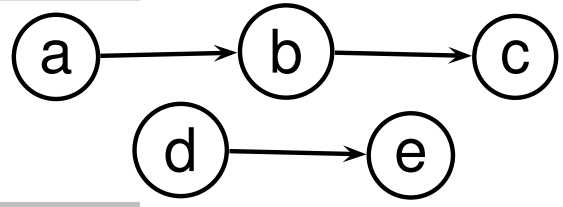
```
a = &b;
```




```
b = &c;
```

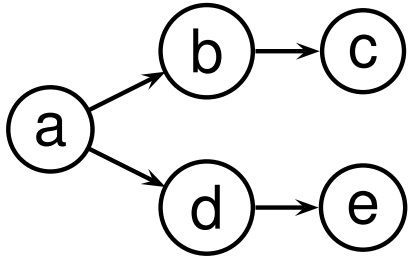



```
d = &e;
```

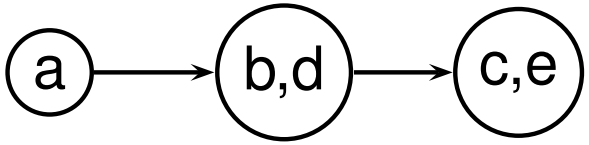



```
a = &d;
```

Subset based



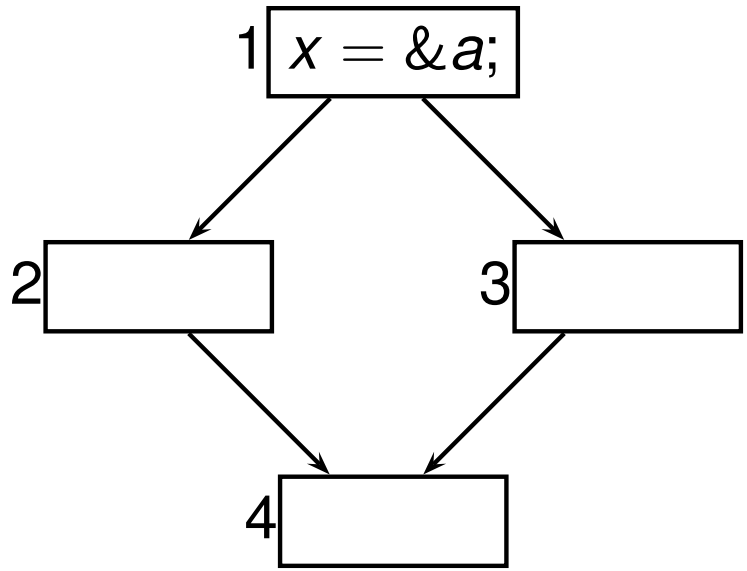
Equality based



Pointer Indirection Constraints

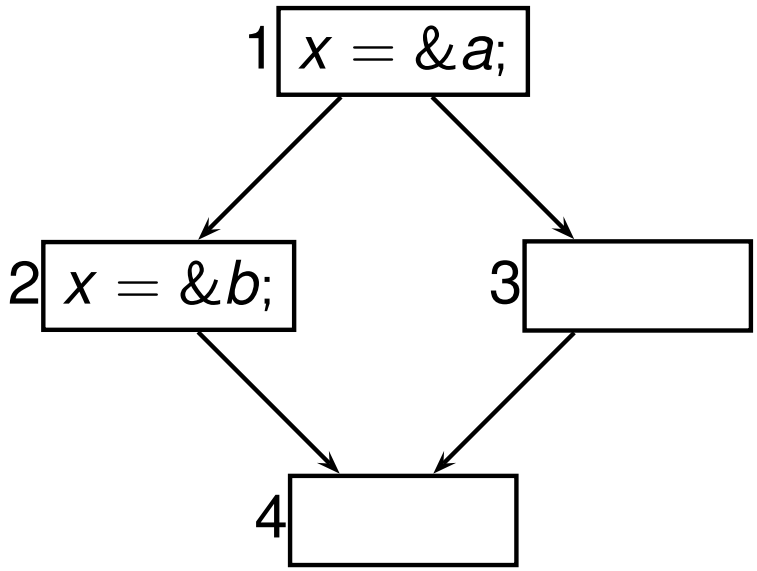
Stmt	Subset based	Equality based
a = *b	$P_a \supseteq P_c, \forall c \in P_b$	MERGE(P_a, P_c), $\forall c \in P_b$
*a = b	$P_c \supseteq P_b, \forall c \in P_a$	MERGE(P_b, P_c), $\forall c \in P_a$

Must Points-to Analysis



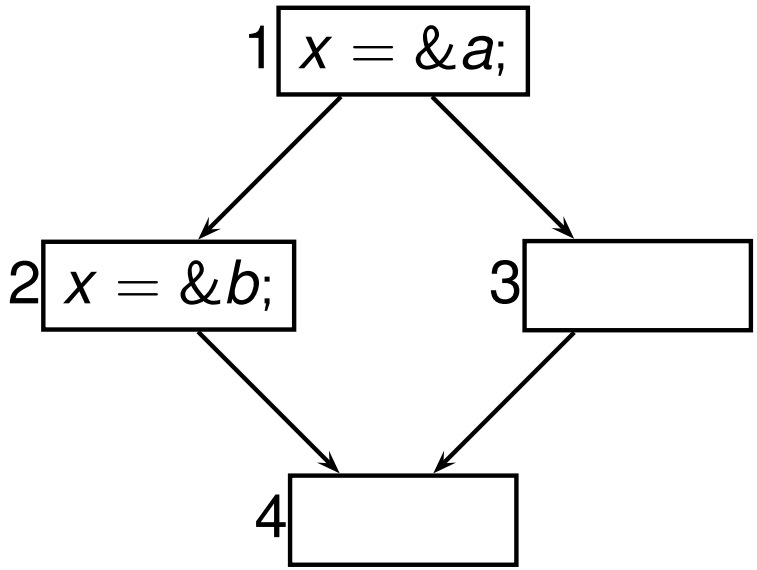
- *x definitely* points-to *a* at various points in the program
- $x \xrightarrow{D} a$

May Points-to Analysis



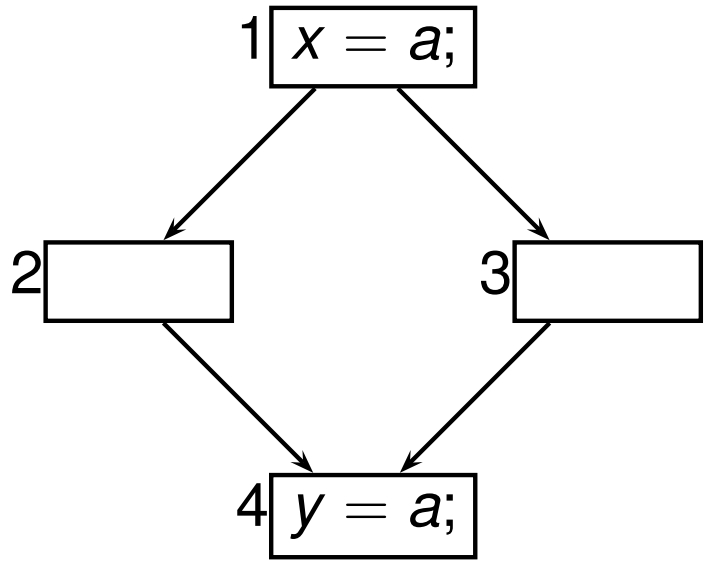
- At OUT of 2, x definitely points-to b
- At OUT of 3, x definitely points-to a
- At IN of 4, x *possibly* points-to a (or b)
  - $x \xrightarrow{P} a$, $x \xrightarrow{P} b$

May Points-to Analysis



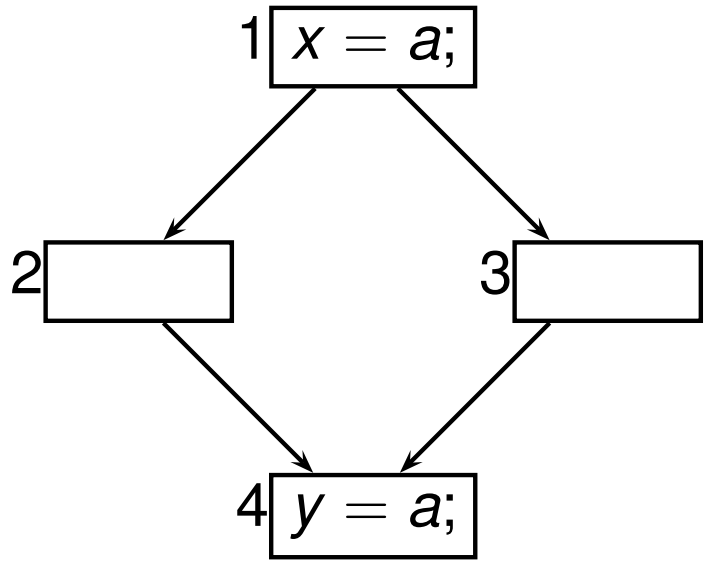
- At OUT of 2, x definitely points-to b
- At OUT of 3, x definitely points-to a
- At IN of 4, x *possibly* points-to a (or b)
  - $x \xrightarrow{P} \{a, b\}$

Must Alias Analysis



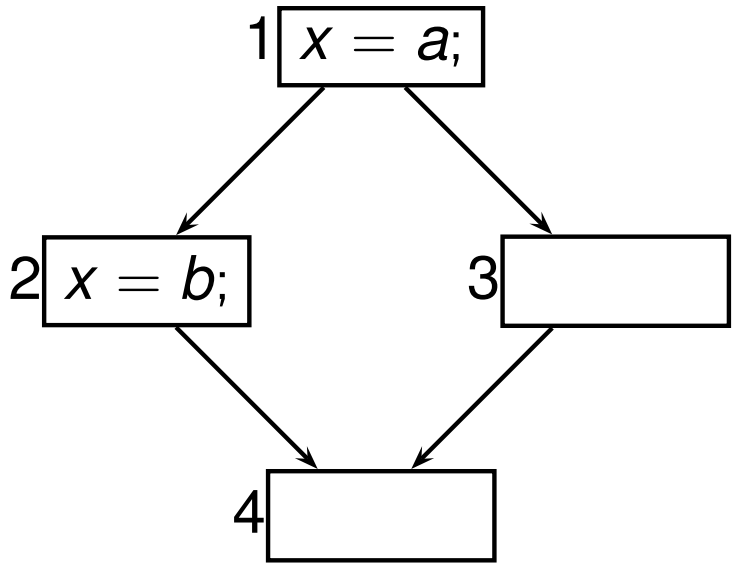
- x and a always refer to same memory location
- $x \overset{D}{\equiv} a$

Must Alias Analysis



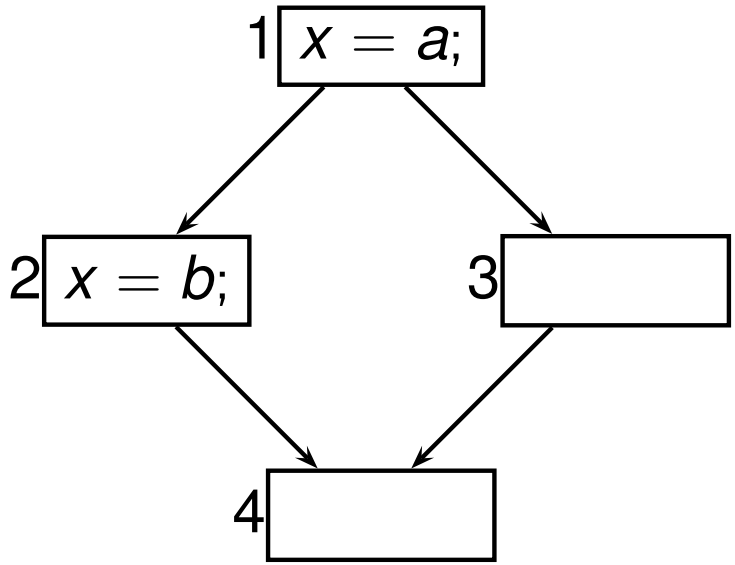
- x and a always refer to same memory location
- $x \overset{D}{\equiv} a$
- x, y and a refer to same location at OUT of 4.
- $x \overset{D}{\equiv} y \overset{D}{\equiv} a$

May Alias Analysis



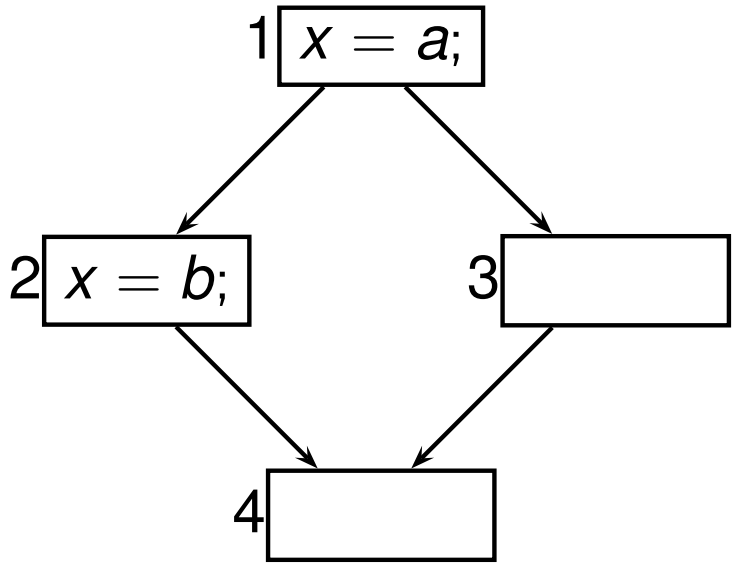
- At OUT of 2, x and b are must aliases
- At OUT of 3, x and a are must aliases
- At IN of 4, x can *possibly* be aliased with either a (or b)
  - $x \overset{P}{\equiv} a$, $x \overset{P}{\equiv} b$

May Alias Analysis



- At OUT of 2, x and b are must aliases
- At OUT of 3, x and a are must aliases
- At IN of 4, x can *possibly* be aliased with either a (or b)
  - $(x, a), (x, b)$

May Alias Analysis



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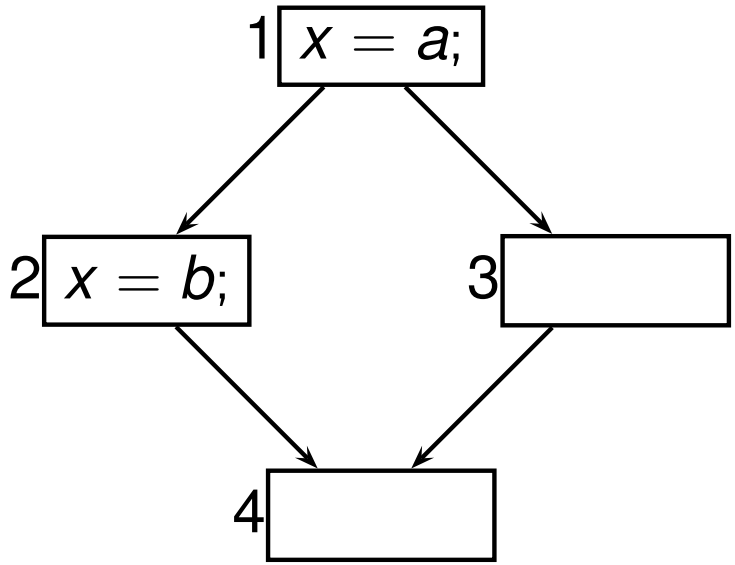
  - $(x, a), (x, b)$

- If we say: (x, a, b), Is it *Precise?*

May Alias Analysis



- At OUT of 2, x and b are must aliases
- At OUT of 3, x and a are must aliases
- At IN of 4, x can *possibly* be aliased with either a (or b)
  - $(x, a), (x, b)$

- If we say: (x, a, b), Is it *Precise? Safe?*

Must Pointer Analysis

- Makes sense only for Flow Sensitive analysis

Must Pointer Analysis

- Makes sense only for Flow Sensitive analysis
- Why?

Must Pointer Analysis

- Makes sense only for Flow Sensitive analysis
- Why?
- Must analysis $\Rightarrow$ Flow sensitive analysis

Must Pointer Analysis

- Makes sense only for Flow Sensitive analysis
- Why?
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- Flow insensitive analysis $\Rightarrow$ May analysis

Must Pointer Analysis

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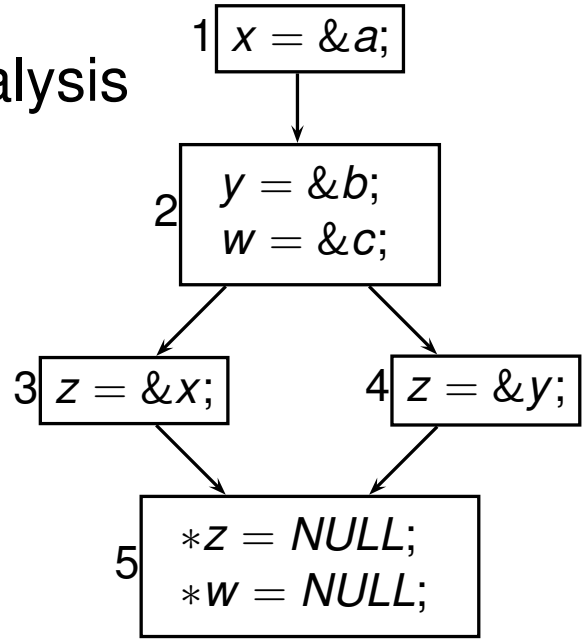
- Why?

Updating Information: When Can We *Kill*?

- Never if flow insensitive analysis

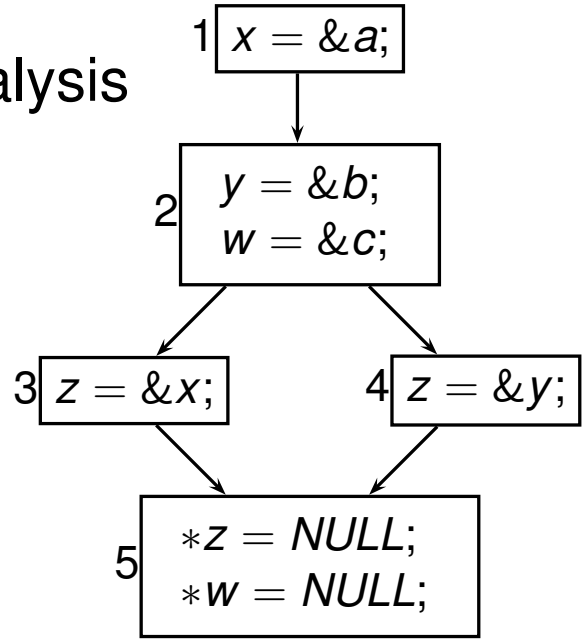
Updating Information: When Can We *Kill*?

- Never if flow insensitive analysis
- For flow sensitive



Updating Information: When Can We *Kill*?

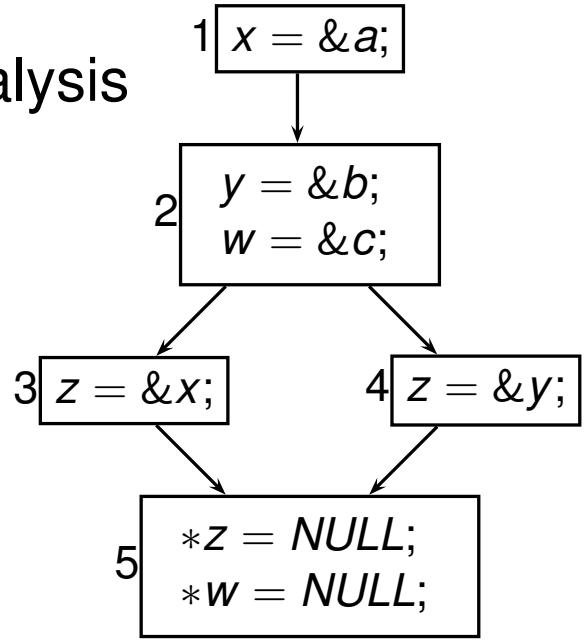
- Never if flow insensitive analysis
- For flow sensitive



- x, y may or may not get modified in 5: *Weak* update

Updating Information: When Can We *Kill*?

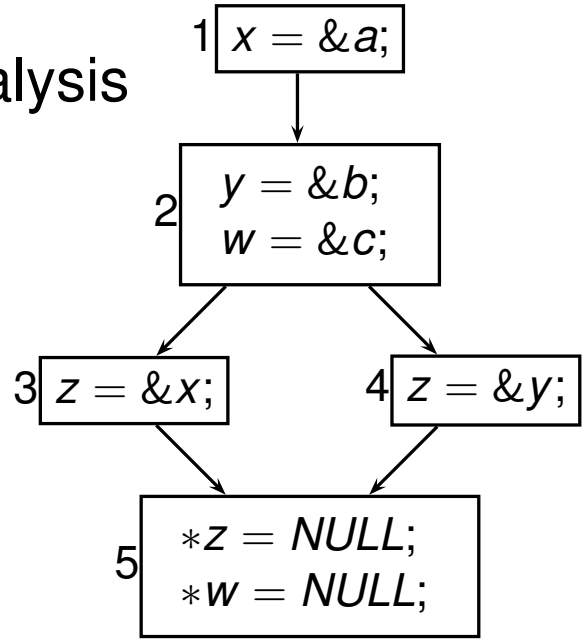
- Never if flow insensitive analysis
- For flow sensitive



- x, y may or may not get modified in 5: *Weak* update
- c definitely gets modified in 5: *Strong update*

Updating Information: When Can We *Kill*?

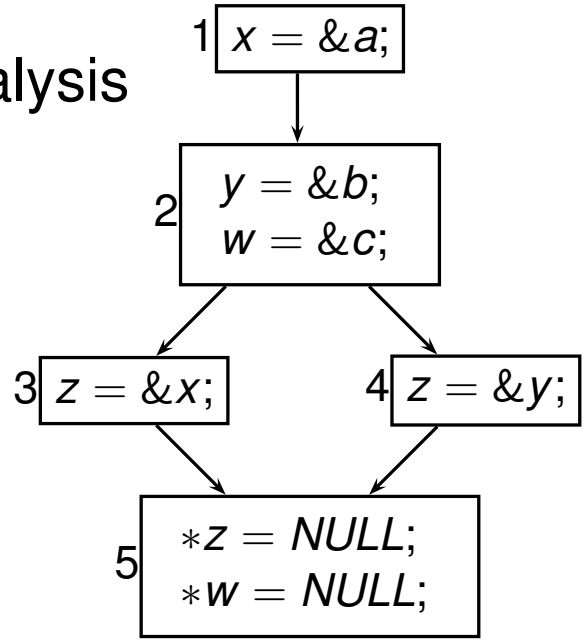
- Never if flow insensitive analysis
- For flow sensitive



- x, y may or may not get modified in 5: *Weak* update
- c definitely gets modified in 5: *Strong update*
- Must information is killed by Strong and Weak updates

Updating Information: When Can We *Kill*?

- Never if flow insensitive analysis
- For flow sensitive



- x, y may or may not get modified in 5: *Weak* update
- c definitely gets modified in 5: *Strong update*
- Must information is killed by Strong and Weak updates
- May information is killed only by Strong updates

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation
  - x = y

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation
  - x = y
  - x = &y

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation
  - x = y
  - x = &y
  - x = *y

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation
  - x = y
  - x = &y
  - x = *y
  - *x = y

Flow Functions for Points-to Analysis

- Basic statements for pointer manipulation
  - x = y
  - x = &y
  - x = *y
  - *x = y
- Other statements can be rewritten in terms of above

Flow Functions for Points-to Analysis

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Weak update!!

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- Must Points-To $\subseteq$ May Points-To

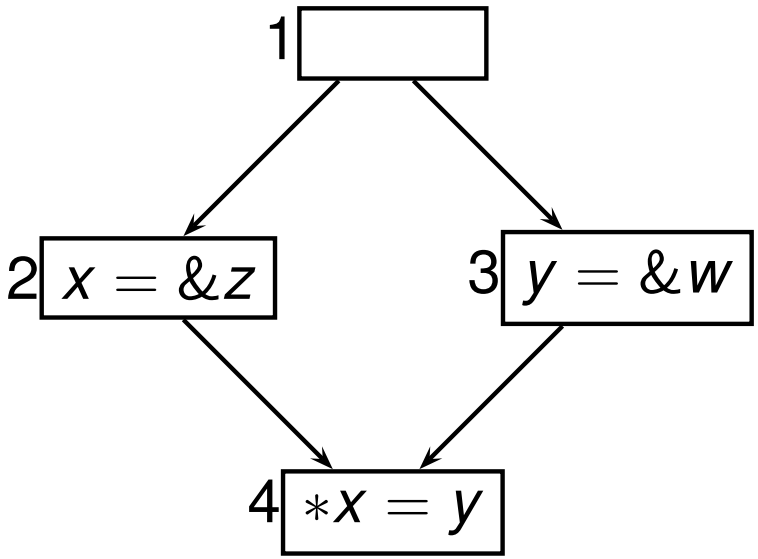
Safe Approximations for May and Must Points-to

- A pointer variable

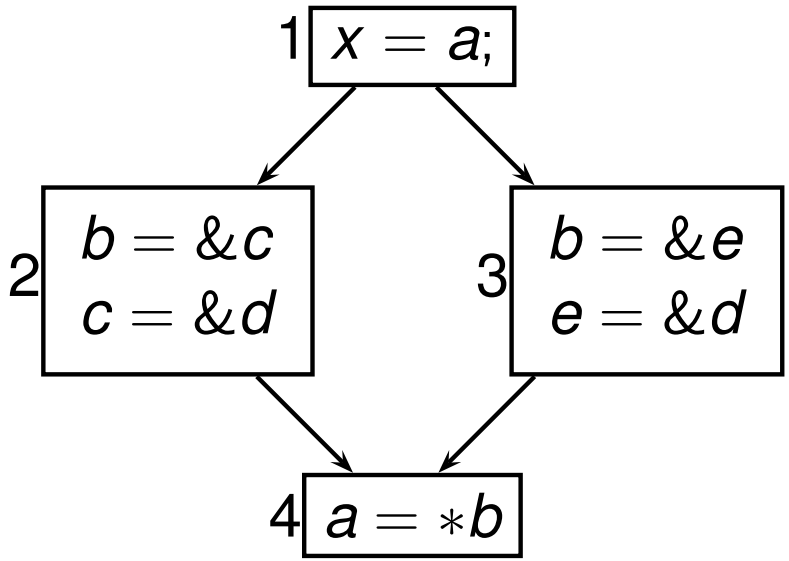
	May	**Must**
Points-to	points to every possible location	points to nothing
Alias	aliased to every other pointer variable	only to itself

Non-Distributivity of Points-to Analysis

May Information



Must Information



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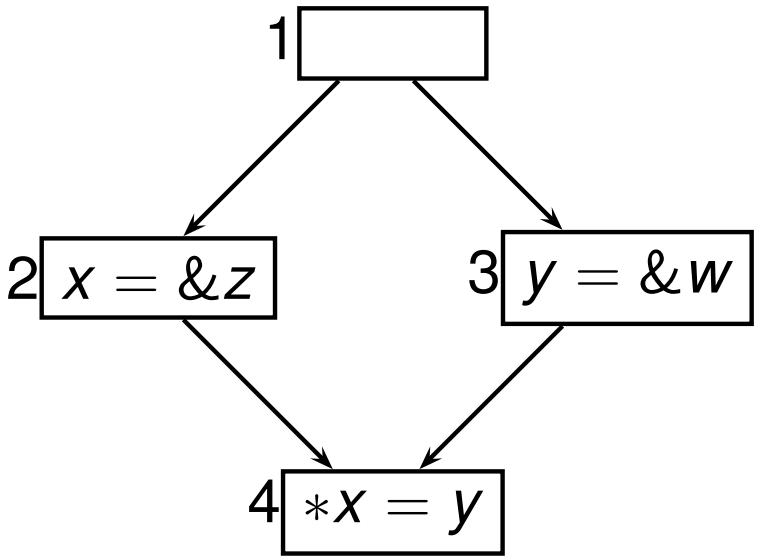

$z \rightarrow w$ is spurious

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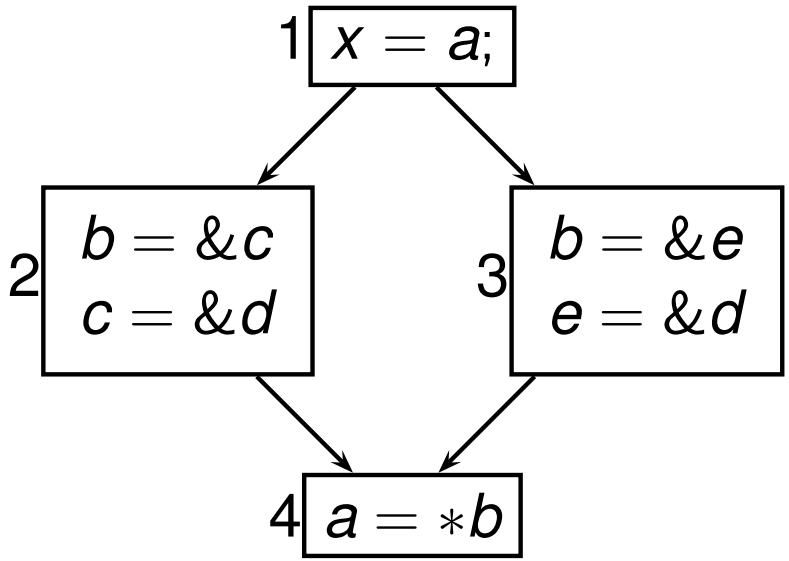

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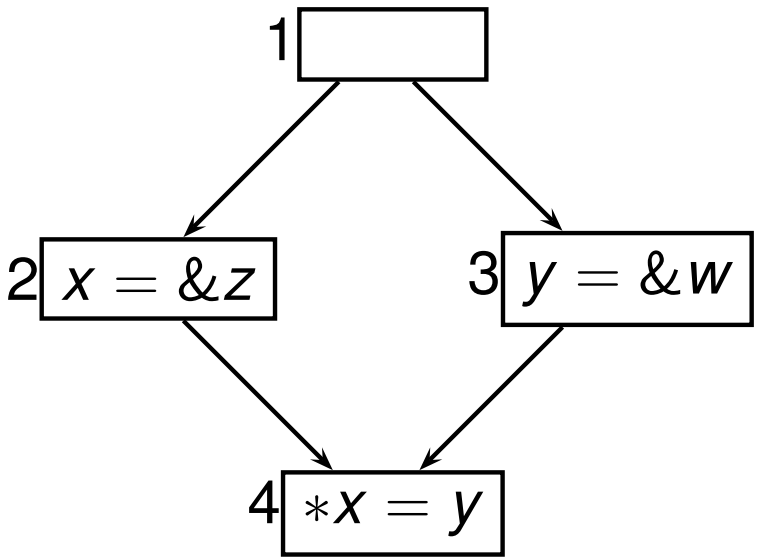

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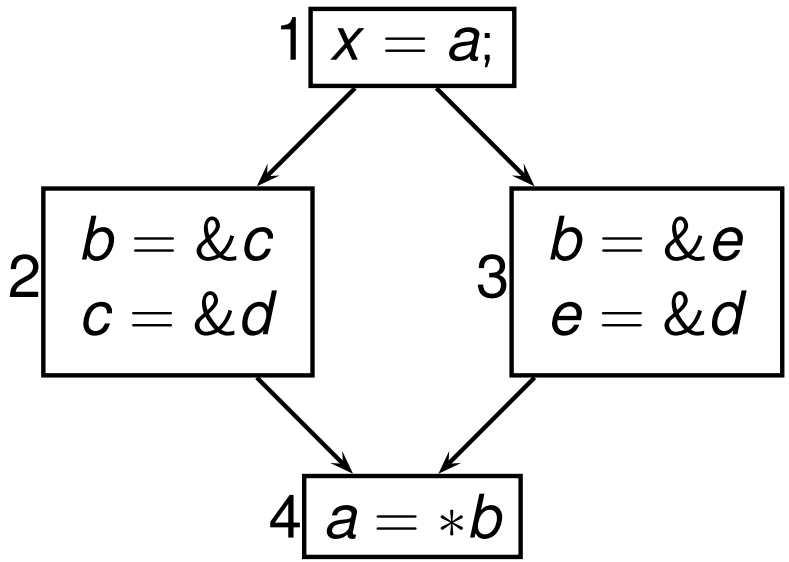

$a \rightarrow d$ is missing