# CS738: Advanced Compiler Optimizations

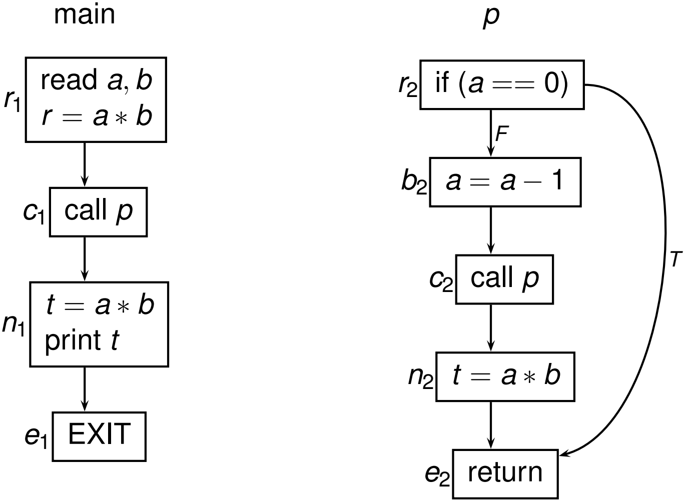
# Interprocedural Data Flow Analysis

Amey Karkare
karkare@cse.iitk.ac.in

http://www.cse.iitk.ac.in/~karkare/cs738
Department of CSE, IIT Kanpur

# Interprocedural Analysis: WHY?

Is $a * b$ available at IN of $n_1$?



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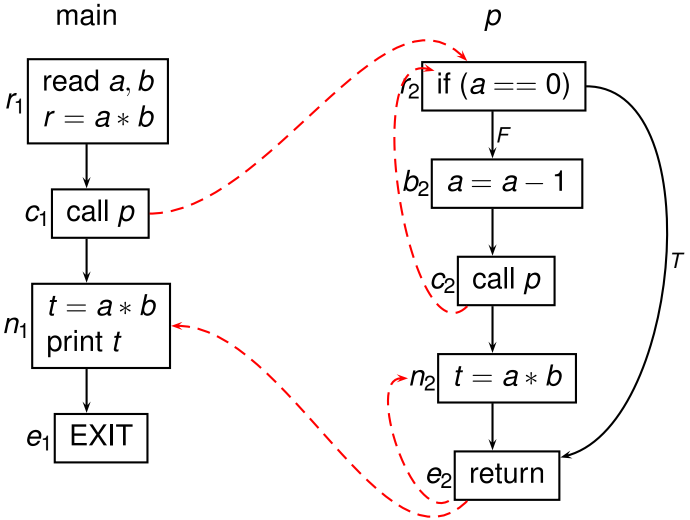
- Function pointers and virtual functions
- Dynamic functions (functional programs)

# Infeasible Paths

How to avoid data flowing along invalid paths?

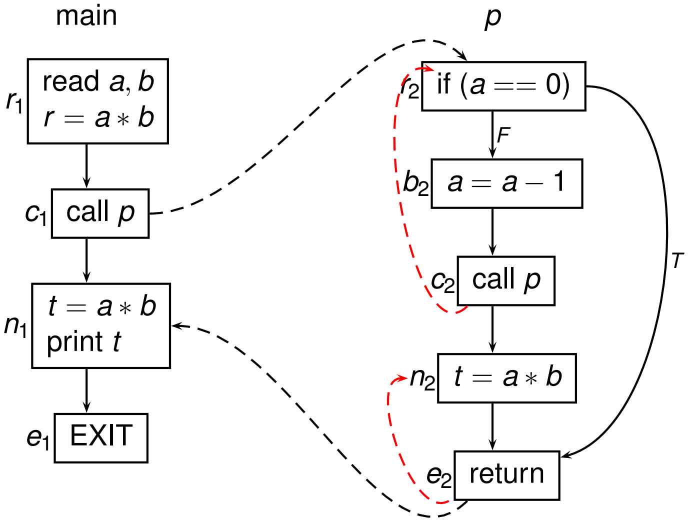
$r_1 \rightarrow c_1 \rightarrow r_2 \rightarrow b_2 \rightarrow c_2 \rightarrow r_2 \rightarrow e_2 \rightarrow n_1$



# Recursion

How to handle Infinite paths?

$\ldots \to r_2 \to c_2 \to r_2 \to c_2 \to r_2 \ldots$



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- No static control flow graph!

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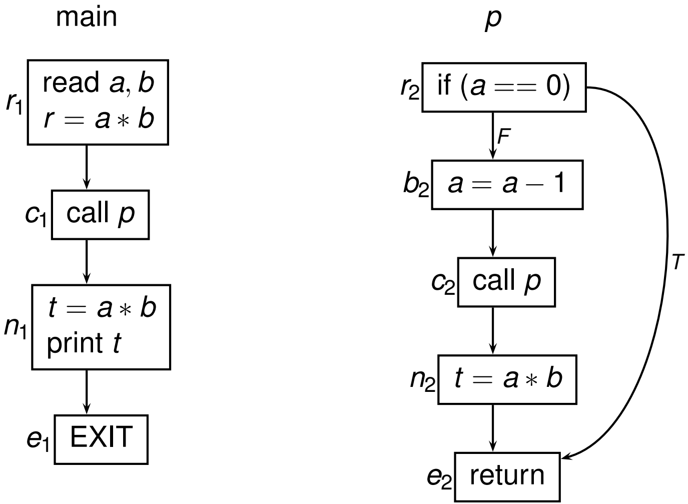
M. Sharir, and A. Pnueli. **Two Approaches to Inter-Procedural Data-Flow Analysis**. In Jones and Muchnik, editors, Program Flow Analysis: Theory and Applications. Prentice-Hall, 1981.

# Notations and Terminology

# Control Flow Graph

One per procedure



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  - $\text{path}_G(m, n)$: Set of all path in graph $G = (N, E)$ leading from $m$ to $n$

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  - recursion stack for formal parameters
- No procedure variables (pointers, virtual functions etc.)

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- $f_{(m,n)} \in F$ represents propagation function for edge $(m, n)$ of control flow graph $G = (N, E)$
  - Change of DF values from the *start* of $m$, through $m$, to the *start* of $n$

# Data Flow Equations

$$
\begin{aligned}
x_r &= \mathit{BoundaryInfo} \\
x_n &= \bigwedge_{(m,n)\in E} f_{(m,n)}(x_m) \qquad n \in N - r
\end{aligned}
$$

- MFP solution, approximation of MOP

$$
y_n = \bigwedge \{ f_p(\mathit{BoundaryInfo}) : p \in \mathrm{path}_G(r, n) \} \quad n \in N
$$

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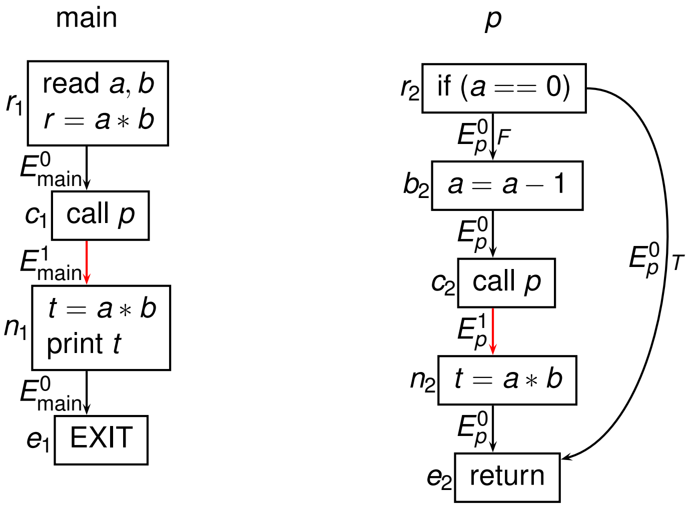
- Computes relationship between DF value at entry node and related data at *any* internal node of procedure
- At call site, DF value propagated directly using the computed relation

# Interprocedural Flow Graph

First Representation:

$$
\begin{array}{rcl}
G & = & \bigcup \{G_p : p \text{ is a procedure in program}\} \\
G_p & = & (N_p, E_p, r_p) \\
N_p & = & \text{set of all basic block of } p \\
r_p & = & \text{root block of } p \\
E_p & = & \text{set of edges of } p \\
 & = & E_p^0 \cup E_p^1 \\
(m,n) \in E_p^0 & \Leftrightarrow & \text{direct control transfer from } m \text{ to } n \\
(m,n) \in E_p^1 & \Leftrightarrow & m \text{ is a call block, and } n \text{ immediately follows } m
\end{array}
$$

# Interprocedural Flow Graph: $1^{st}$ Representation



# Interprocedural Flow Graph

Second representation

$$
\begin{aligned}
G^* &= (N^*, E^*, r_1) \\
r_1 &= \text{root block of main} \\
N^* &= \bigcup_p N_p \\
E^* &= E^0 \cup E^1 \\
E^0 &= \bigcup_p E^0_p \\
(m, n) \in E^1 &\Leftrightarrow (m, n) \text{ is either a } \textit{call} \text{ edge} \\
& \quad\ \text{or a } \textit{return} \text{ edge}
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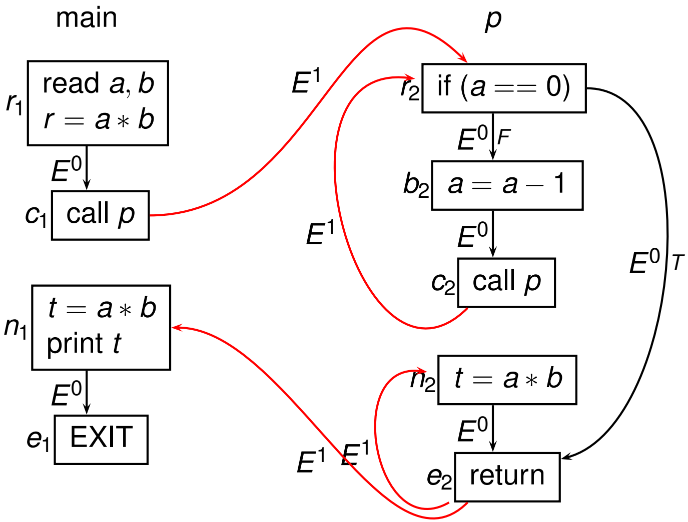
  - if $p = q$ and
  - $(m, n) \in E_s^1$ for some procedure $s$

# Interprocedural Flow Graph: $2^{nd}$ Representation



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- Path $q \in \mathrm{path}_{G^*}(r_1, n)$ is in $\mathrm{IVP}(r_1, n)$
  - iff sequence of all $E^1$ edges in $q$ (denoted $q_1$)is *proper*

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  - $q_1'$ obtained from deleting $q_1[i-1]$ and $q_1[i]$ from $q_1$ is proper

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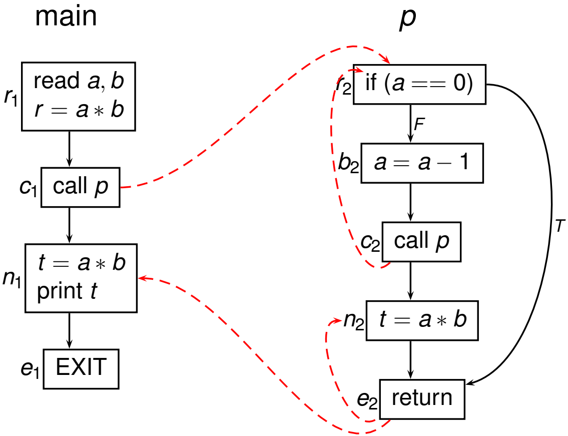
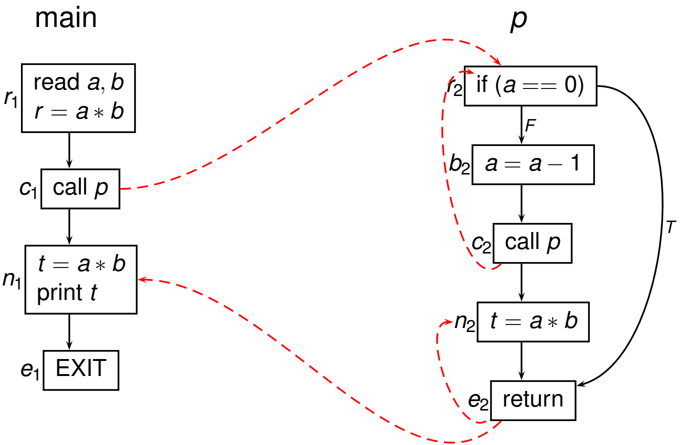
  - Each call edge has corresponding return edge in $q$ restricted to $E^1$

# IVPs



main
p
r1
read a, b
r = a * b
c1
call p
n1
t = a * b
print t
e1
EXIT
r2
if (a == 0)
F
b2
a = a − 1
c2
call p
n2
t = a * b
e2
return
T

# IVPs

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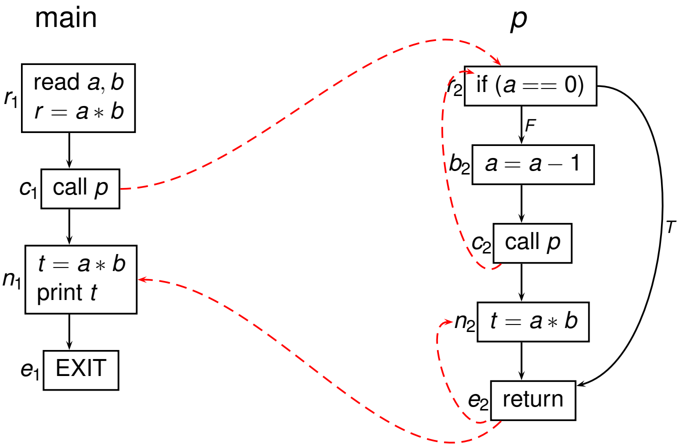


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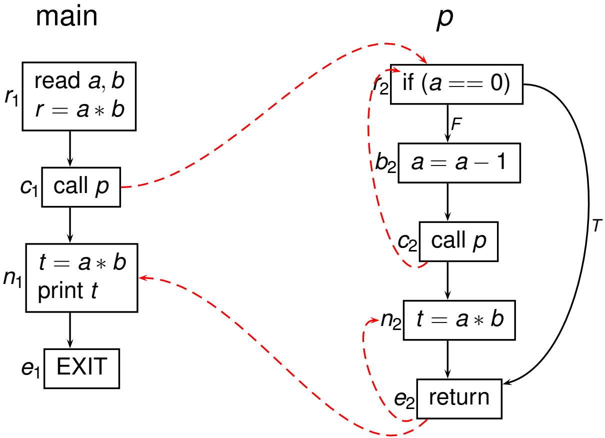


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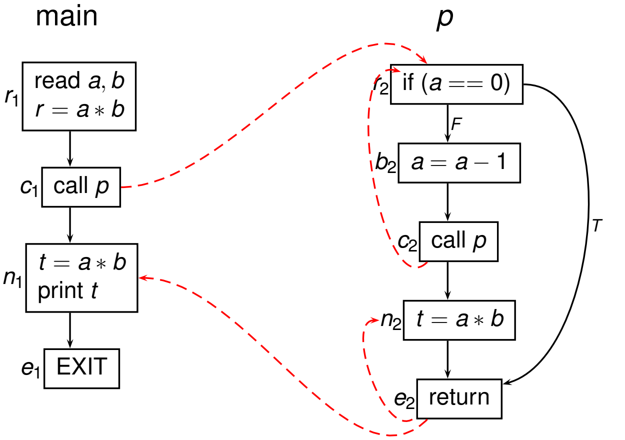


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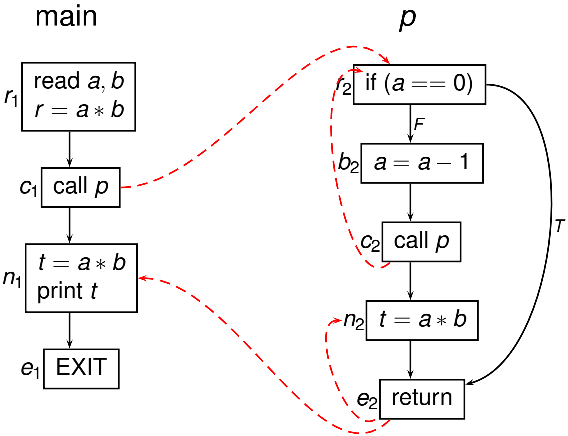
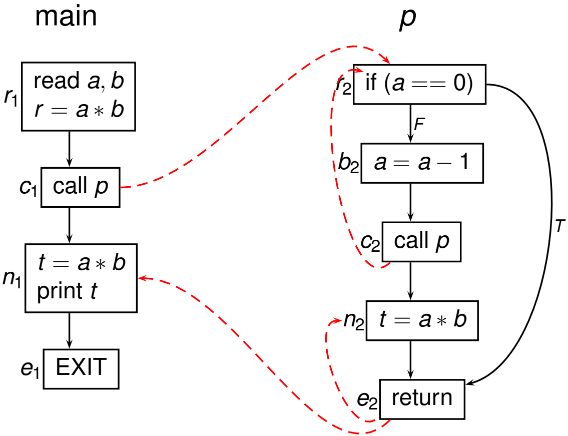



$$r_2 \to c_2 \to r_2 \to e_2 \to n_2$$

# IVPs

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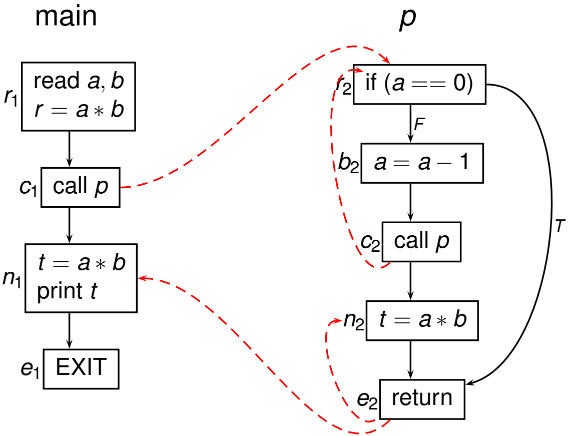


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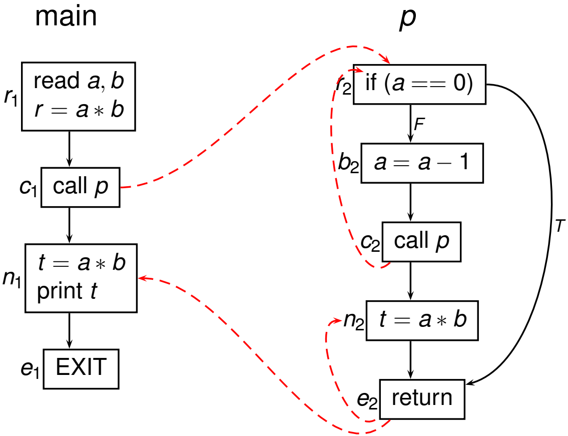


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# Path Decomposition

$$
\begin{array}{rcl}
q & \in & \mathrm{IVP}(r_{\mathrm{main}}, n) \\
& \Leftrightarrow & \\
q & = & q_1 \parallel (c_1, r_{p_2}) \parallel q_2 \parallel \cdots \parallel (c_{j-1}, r_{p_j}) \parallel q_j \\
& & \text{where for each } i < j, q_i \in \mathrm{IVP}_0(r_{p_i}, c_i) \text{ and } q_j \in \mathrm{IVP}_0(r_{p_j}, n)
\end{array}
$$