

CS 601- Advance Algorithm Homework 2

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Solution 1:

```
#include <iostream>
#include <stdlib.h>
using namespace std;
int main()
{
    int i, j, n;
    cout<<" Enter the number of elements in the list- ";
    cin >> n;
    cout<<"Enter elements: ";
    int arr[n], count=0;
    for(i = 0;i < n;i++)
    {
        cin >> arr[i];
    }
    for(i=0;i<n;i++)
    {
        for(j = i+1;j < n;j++)
        {
            if(arr[i] == arr[j])
            {
                count++;
                break;
            }
        }
    }
    cout<<"Total number of Duplicates found- "<<count;
    return 0;
}
```

Analyzing algorithm in worst case:

As there are two for loops, the outer for loop executes n times. Every time the outer loop executes, the inner loop also executes n times. Thus, the complexity is $O(n*n)$. So, in the big O notation, it is $O(n^2)$.

Solution 2:

```
#include<iostream>
#include<bits/stdc++.h>
```

```

using namespace std;
int main()
{
    cout<<"Enter the number of elements in the list";
    int n;
    cin>>n;
    cout<<"Enter elements";
    int* list=new int[n];
    for(int i=0;i<n;i++)
        cin>>list[i];

    // Sort Blackbox used to sort the input

    sort(list,list+n);
    int count=0;
    int i=0, arr[i];
    for(i=0;i<n;i++)
    {
        if(arr[i]==arr[i+1])
        {
            count++;
        }
    }
    cout<<" Total number of Duplicates found -"<<count;
    return 0;
}

```

Time Complexity:

Here I have sorted the list of numbers and scanned the array to find the duplicates. All the elements will have a chance to run continuously. So, in a single scan we can find all the duplicates and also there is one loop which is running n times, so the worst case time complexity in a sorted array will be n (without considering the complexity of sorting). So, we can see that compared to time complexity of the question 1 of unsorted array (n^2) the time complexity in sorted array is improved to (n) .

Solution 3a:

Arranging in ascending order of growth rate:

$$\sqrt{2n} < 2n+100 < n \log n < (n^2)(\log n) < n^{2.5} < n^{10} < 2^n < 10^n < 100^n < n! < n^n$$

Solution 3b:

Proving $f(n)=O(g(n))$

For $f_2(n)$ and $f_9(n)$:

$f_2(n) = \sqrt{2n} = O(n^{0.5})$; and $f_9(n) = 2n+100 = O(n)$;
 $\sqrt{2n} \leq 2n+100$
 So, $f_2(n) < f_9(n)$
 for all values of $n \geq 1$ and $c = 1$
 $(2n)^{0.5} \leq 2n + 100$ // $f_2(n)$ has less degree of 0.5 compared to $f_9(n)$
 Therefore, $f_2(n) = O(f_9(n))$

For $f_9(n)$ and $f_7(n)$:

$f_9(n) = 2n+100 = O(n)$ and $f_7(n) = n \log n = O(n \log n)$
 $2n+100 \leq n \log n$
 For $n \geq 251$ we can see that $2n+100$ is $\text{bigO}(n \log n)$
 So $f_9(n) = O(f_7(n))$ for $c=1$ and $n \geq 251$

For $f_7(n)$ and $f_8(n)$:

$f_7(n) = n \log n$ and $f_8(n) = n^2(\log n)$
 $n \log n \leq (n^2)(\log n)$
 We can see for all values of $n \geq 2$ the above equation satisfies.
 So $f_7(n) = O(f_8(n))$

For $f_8(n)$ and $f_1(n)$:

$f_8(n) = (n^2)(\log n)$ and $f_1(n) = n^{2.5}$
 $f_8(n) < f_1(n)$
 taking log on both side: $2 \log n + \log(\log n) < 2.5 \log n$ ($n \geq 1$)
 $\log(\log n) < 0.5 \log n$ { constant * $\log(n)$ is always greater than $\log(\log n)$ }
 So $f_8(n) = O(f_1(n))$

For $f_1(n)$ and $f_3(n)$:

$f_1(n) = n^{2.5}$ and $f_3(n) = n^{10}$
 2.5 degree is less than 10
 $n \geq 1$, $f_1(n) = O(f_3(n))$

For $f_3(n)$ and $f_6(n)$:

$f_3(n) = n^{10}$ and $f_6(n) = 2^n$
 Taking log on both sides: $10 \log n \leq n \log 2$. ignoring constants 10 and $\log 2$ we get
 $\log n \leq n$ ($n \geq 1$)
 so, n^{10} is $\text{BigO}(2^n)$.

For $f_6(n)$ and $f_4(n)$:

$f_6(n) = 2^n$ and $f_4(n) = 10^n$
 Take log on both sides we get : $n \log 2 \leq n \log 10$
 $\log 10$ is greater than $\log 2$
 2^n is $\text{BigO}(10^n)$
 So $f_6(n) = O(f_4(n))$ $c=1$, for $n \geq 1$

For $f_4(n)$ and $f_5(n)$:

$$f_4(n) = 10^n \text{ and } f_5(n) = 100^n$$

Taking log on both sides: $n \log 10 \leq n \log 100$

$\log 100$ is greater than $\log 10$

10^n is $\text{BigO}(100^n)$

$f_4(n) = O(f_5(n))$ for $c=1, n \geq 1$

For $f_5(n)$ and $f_{11}(n)$:

$$f_5 = 100^n \text{ and } f_{11} = n!$$

$$\log(n!) = n \log n - n$$

$$\log(100^n) = n \log 100,$$

$$\text{divide equations } (n \log n - n) / (n \log 100) = (\log n / \log 100) - (1 / \log 100)$$

$1 / \log 100$ can be ignored

Hence for n is greater than 100

$$(100^n) = O(n!)$$

For $f_{11}(n)$ and $f_{10}(n)$:

$$f_{11}(n) = n! \text{ and } f_{10}(n) = n^n$$

Let $n \geq 2$

$$2! \leq 2^2$$

$$2 \leq 4$$

So $f_{11}(n) = O(f_{10}(n))$ for $n \geq 1$ & $c=1$