

HOMEWORK 3
DUE: MONDAY, JUNE 1

1. For the following autonomous DE's: draw the phase line, the direction fields, sketch several integral curves, and determine the equilibrium points and classify each one as stable, unstable, or semistable.

- a) $y' = -2y^2(y - 1)^2$
- b) $y' = y(y^2 - 1)$
- c) $y' = \sqrt{y}(4 - y^2)$ (assume y is positive)
- d) $y' = y \ln(2/y)$ (assume y is positive)

2. In each of the following problems determine if the DE is exact and if it is solve the IVP

- a) $(2x - y) + (2y - x)y' = 0$, $y(1) = 3$
- b) $(9x^2 + y - 1) - (4y - x)y' = 0$, $y(1) = 0$
- c) $(x \ln y + xy) + (y \ln x + xy)y' = 0$, $y(1) = 1$

3. Find the value of k for which the following DE becomes exact, and then solve it using that value of k

$$(ye^{2xy} + x) + kxe^{2xy}y' = 0$$

4. Show that the given DE's are not exact but become exact when multiplied by the given integrating factor. Then solve the equations

- a) $\left(\frac{\sin y}{y} - 2e^{-x} \sin x\right) + \left(\frac{\cos y + 2e^{-x} \cos x}{y}\right)y' = 0$, $\mu(x, y) = ye^x$
- b) $x^2y^3 + x(1 + y^2)y' = 0$, $\mu(x, y) = x^{-1}y^{-3}$

5. Find the integrating factor and solve the given equations

- a) $1 + (x/y - \sin y)y' = 0$
- b) $y' = e^{2x} + y - 1$

6. Solve the following problems from the book: Chapter 2.3 problems 2, 3, 4, 13