Calculus II **Name (Print):** _____
Final
7/28/16
Time Limit: 150 Minutes

- This exam contains 12 pages (including this cover page) and 10 problems.
- *Please show all your work! Answers without supporting work will not be given credit. Simplify answers wherever possible.*

Problem	Points	Score
1	20	
2	20	
3	10	
4	20	
5	20	
6	10	
7	10	
8	20	
9	10	
10	10	
Total:	150	

1. (20 points) Evaluate the following integrals

   1. $\int_2^\infty \frac{2x}{(x^2-1)^{1/3}}dx$
   2. $\int_{-\infty}^0 \frac{e^x}{1+e^{2x}}dx$
   3. $\int_1^\infty x^2 \ln x dx$
   4. $\int \frac{1}{x^2-x}dx$

2. (20 points) 1. Solve the differential equation:

$$\frac{dL}{dt} = k(5 - L) \text{ with } L(0) = 2$$

Find k such that $L(4) = 10$.

2. Find the equilibria for the following differential equation and classify them:

$$\frac{dx}{dt} = \left(1 - \frac{x}{2}\right) x\,(x + 1)$$

Find $\lim_{t \to \infty} x(t)$ if $x(0) = 2$.

3. (10 points) Solve

$$\begin{aligned} x - 3y + z &= 4 \\ x - 2y + 3z &= 6 \\ 2x - 6y + 2z &= 8 \end{aligned}$$

4. (20 points) Assume that the population is divided two age classes, 50% of the females of age 0 survive. Females of age 0 have an average of 1.5 offspring and females of age 1 have an average of 2 offspring. At time 0, the population consists of 10 females of age 0, zero females of age 1.

   1. Find the Leslie matrix.
   2. Find the age distribution at time 2.
   3. Find the stable age distribution.

5. (20 points) Find the absolute maxima and minima of x^2+y^2 on the rectangular domain $\{(x, y) : -1 \leq x \leq 1, -1 \leq y \leq 1\}$.

6. (10 points) Find a linear approximation to

$$\vec{f}(x,y) = \begin{bmatrix} \sqrt{2x+y} \\ x - y^2 \end{bmatrix}$$

at $(1,2)$ and find the approximate value of $f(1.05, 2.05)$.

7. (10 points) Consider the system of differential equation:

$$\frac{dx_1}{dt} = x_1 + 2x_2$$
$$\frac{dx_2}{dt} = 3x_1 + 2x_2$$

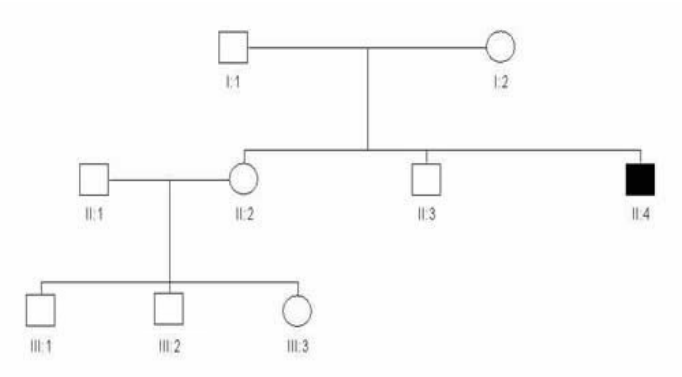
Classify the equilibrium $(0, 0)$ and find the general solution.

8. (20 points) Toss a fail coin 400 times. Find the probability of getting at the most 100 heads
    1. Exactly
    2. Using Poisson approximation
    3. Using normal approximation

9. (10 points) Suppose that the amount of yearly rainfall in a certain area is normally distributed with mean 30 cm and standard deviation 5 cm. What is the probability that,

   1. In a given year the rainfall will exceed 35 cm?
   2. In 5 consecutive years, the rainfall will exceed 35 cm?
   3. In at least 1 out of 5 years, the rainfall will exceed 35 cm?

10. (10 points) For the following Haemophilia pedigree tree find the probability that the females I2, II2 and III3 are carriers.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986