PROBLEM SET 01

Day 1

§1. Explain in your own words how the provisional definition implies the $\epsilon - \delta$ definition of limit.

Provisional definition: The function f approaches a limit l near a, if we can make $f(x)$ as close to a as we like to l by requiring that x be sufficiently close to, but unequal to, a.

$\epsilon - \delta$ **definition:** The function f approaches a limit l near a, if for every $\epsilon > 0$ there is some $\delta > 0$ such that, for all x, if $0 < |x - a| < \delta$, then $|f(x) - l| < \epsilon$.

§2. For each of the following functions f and real numbers a,

- Guess the limit $L = \lim\limits_{x \to a} f(x)$.
- Find a δ corresponding to $\epsilon = 0.1$ in the $\epsilon - \delta$ definition of limit.
- For a real number $L' \neq L$ explain why you cannot find any δ that will work.
- Find a δ corresponding to an arbitrary real number ϵ and **prove** that L is indeed the limit.

(1) $f(x) = x + 2, \quad a = 0$
(2) $f(x) = 7x + 2, \quad a = 0$
(3) $f(x) = x^2 - 2, \quad a = 0$
(4) $f(x) = \ln(x), \quad a = 1$

Day 2

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