QUIZ 3

Each question is worth 10 points. Please write detailed mathematically correct solutions.

1. Sketch the phase portrait near each equilibrium point and classify the equilibria

$$x' = 10x - 5xy$$

$$y' = 3y + xy - 3y^2$$

For each of the following systems of differential equations

• Find the general solution

• Find $\lim_{t\to\infty}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ for the following initial conditions:

(1)
$$x(0) = 1, y(0) = 1$$

(3)
$$x(0) = 1, y(0) = 0$$

$$(2) \ x(0) = 1, y(0) = -1$$

(4)
$$x(1) = 1, y(1) = 0$$

2.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

3.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

4.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$