HOMEWORK 6
DUE: MONDAY, JUNE 8

1. Find the complex gain of the system given by

$$x'' + 3x' + 2x = 9e^{2it}$$

Find the (real) gain of the system given by

$$x'' + 3x' + 2x = 9\cos 2t$$

Verify the relationship between complex gain and (real) gain.

2. Show that if $a_2, a_1, a_0 > 0$ then the system given by the following DE is stable

$$a_2\ddot{x} + a_1\dot{x} + a_0 x = F(t)$$

(You'll have to argue separately for real distinct, real repeated and complex roots.)

3. For the following forced underdamped harmonic oscillator

$$m\ddot{x} + b\dot{x} + kx = B\cos\omega t$$

a) Find the natural frequency ω_n of the system.
b) Find the gain of the system. What is the gain as $b \to 0$?
c) Find the ω for which this gain is the maximum, this is the *resonant frequency* ω_r.
d) Compute $\omega_n^2 - \omega_r^2$. What is this difference as $b \to 0$?

4. Solve the IVP

$$\ddot{x} + \omega_n x = B\cos\omega t$$
$$x(0) = 0, x'(0) = 0$$

5. Consider the harmonic oscillator

$$m\ddot{x} + b\dot{x} + kx = 0$$

If $f(t)$ is some solution of this DE then the corresponding *energy* is given by

$$E(t) = (m\dot{f}^2 + kf^2)/2$$

Find $E'(t)$ and show that $E(t)$ is a decreasing function of t.

Chapter 4.2. 29, 30

Chapter 4.3. 9, 14

Chapter 4.4. 1, 3