Ordinary Differential Equations **Name (Print):** _____
Final
06/28/17
Time Limit: 150 Minutes

- This exam contains 12 pages (including this cover page) and 11 problems.
- Write detailed mathematically correct answers.
- Feel free to use both sides of the paper. If you use extra sheets do not forget to write your name and question number on them.

Problem	Points	Score
1	20	
2	10	
3	20	
4	20	
5	20	
6	20	
7	20	
8	10	
9	20	
10	20	
11	20	
Total:	200	

1. (20 points) Find the general solution of

$$2x + y^2 + (2xy - ye^y)y' = 0$$

2. (10 points) Given that $y_1(t) = 1/t$ is one solution of

$$2t^2y'' + 3ty' - y = 0$$

find the other solution.

3. (20 points) Find one solution of each of the following differential equations:

a)

$$y'' + y = te^t$$

b)

$$y'' + y = \cos t$$

4. (20 points) Find the general solution of

$$x' = y - x, \quad y' = 3y - 4x$$

5. (20 points) Find the general solution of

$$y'' - 2y' + y = \frac{e^t}{1+t^2}$$

6. (20 points)

a) Solve the IVP

$$y'' - 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

b) Find the values of t for which $y(t) = 0$.

c) Draw the graph of the solution $y(t)$ as best as you can.

7. (20 points) Solve the following IVP

$$y'' + y = \begin{cases} 1, & t \le 2\pi \\ 0, & 2\pi < t \le 3\pi \\ -1, & 3\pi < t \end{cases}, \quad y(0) = 1, \quad y'(0) = 0$$

Draw the graph of your solution.

8. (10 points) If $y_1(t) = t\sin(2t)$ is a solution of $my'' + by' + ky = 4\cos(2t)$

   a) Determine m, b, and k.

   b) Find one solution of $my'' + by' + ky = 5\cos(2t)$.

9. (20 points) For the autonomous differential equation

$$y' = y(a^2 - y^2)$$

a) For $a = 1$, draw several integral curves and classify the equilibria.

b) Draw the bifurcation diagram.

10. (20 points) For the system of DE corresponding to the matrix $\begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$

a) Draw the phase portrait for $a = 0$.

b) Find the range of the values of a for which the critical point at $(0, 0)$ will be: (i) a source (unstable) node (ii) a sink (stable) node (iii) a saddle.

11. (20 points) Draw phase portraits near the equilibrium points for the following non-linear system and classify the equilibria.

$$x' = x(1 - x - y)$$
$$y' = y(2 - y - x)$$

Based on your phase portrait, make a guess as to what are the possible initial conditions $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$ for which $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ approaches a stable equilibrium?