Bockstein:

$$0 \longrightarrow \mathbb{Z} \xrightarrow{p} \mathbb{Z} \longrightarrow \mathbb{Z}/p \longrightarrow 0$$

Let $C_\cdot \in \mathbb{Z}$-mod then we get, $0 \to C_\cdot \xrightarrow{p} C_\cdot \longrightarrow C_\cdot \otimes \mathbb{Z}/p \longrightarrow 0$

The LES,
$$\begin{tikzcd}
H_*(C_\cdot) \arrow[rr, "p"] & & H_*(C_\cdot) \arrow[dl] \\
& H_*(C_\cdot \otimes \mathbb{Z}/p) \arrow[ul] &
\end{tikzcd}$$

The corresponding SS will converge to $H_i(C) \xrightarrow{p} H_i(C) \xrightarrow{p} H_i(C) \xrightarrow{p} \cdots$ direct limit of this

But for this to happen we need stability i.e. $p^r H_i(C) = p^{r+1} H_i(C)$ for sufficiently large r.

i.e. multiplication by p should become an isomorphism.

1) Either p should be invertible
2) There can be p torsion
3) There can be $\mathbb{Z}_{(p)}$ summands, $\mathbb{Q}$ summands
4) Finite generation would be helpful.