HOMEWORK 6

DUE: MONDAY, JUNE 12

Note: For the problems where the roots of the characteristic polynomial are purely imaginary, use the $\sin(-)$ and $\cos(-)$ form of the general solution (so *not* the one with amplitude and phase).

1. Use variation of parameters method to find the general solution to the following DEs

a) $y'' + y = \sec t$

b) $y'' + y' - 2y = t$

2. Solve the IVPs

a) $y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 0$

b) $y'' + y' - 2y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 0$

3. Find the general solution to the DE

$$u'' + u = 3\cos \omega t$$

a) For ω some constant *not equal* to 1.

b) For $\omega = 1$.

4. In the absence of damping, the motion of a spring-mass system satisfies the IVP

$$mu'' + ku = 0, \quad u(0) = a, \quad u'(0) = b.$$

The total energy in the system is defined by $E = \dfrac{m(u')^2}{2} + \dfrac{ku^2}{2}$

a) Find the total energy in the system at $t = 0$.

b) Solve the given initial value problem.

c) Using the solution in part (b), determine the total energy in the system at any time t. Your result should confirm the principle of conservation of energy for this system.