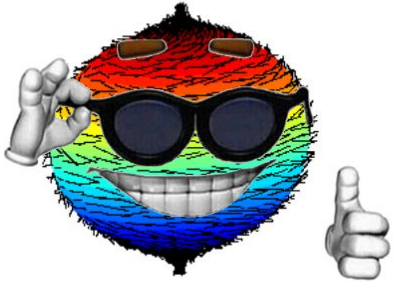
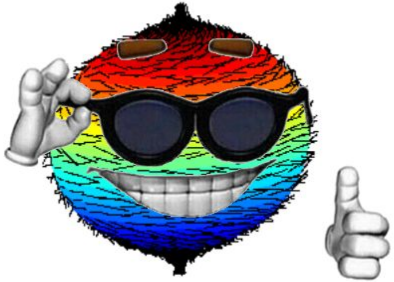
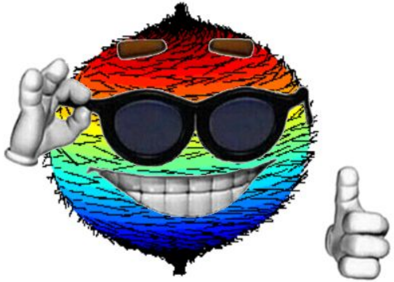
The Hairy Ball Theorem

And Its Applications







Background

First proved by the Dutch mathematician Luitzen Egbertus Jan Brouwer in 1912.

Brouwer’s work extends far beyond working with hairy balls, as he made other significant discoveries in topology such as:

- Simplicial Approximation Theorem
- Invariance of Degree
- Invariance of Dimension
- Brouwer Fixed Point Theory

Also created mathematical philosophy of Intuitionism

What is this Hairy Ball Theorem?

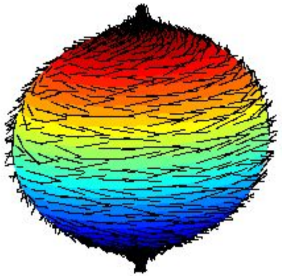
https://www.youtube.com/watch?v=B4UGZEjG02s

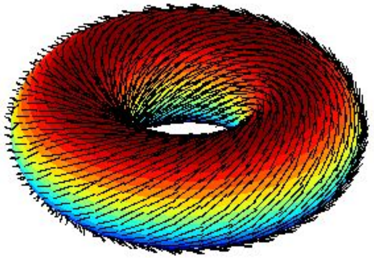
Formal definition:

There is no nonvanishing continuous tangent vector field on even-dimensional n-spheres.

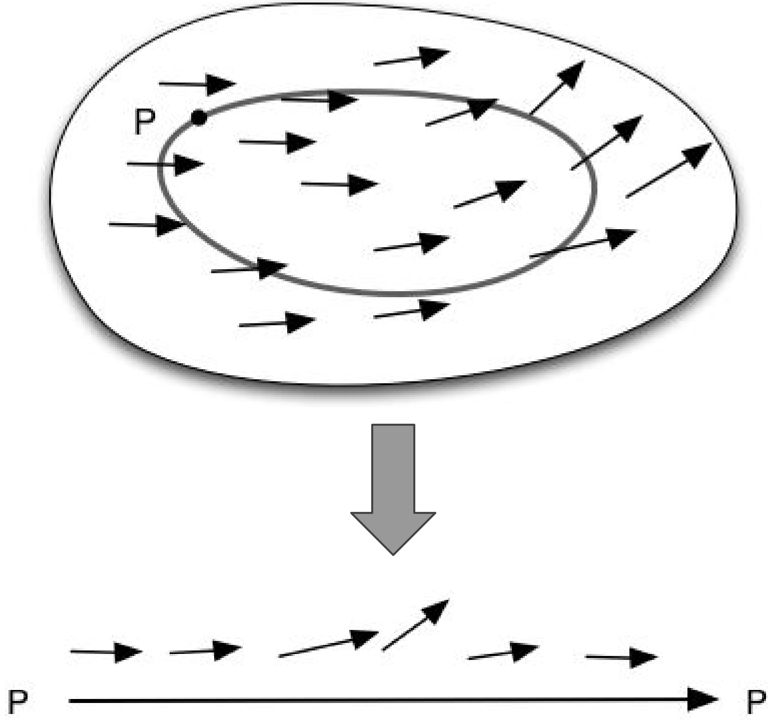
Or

For the ordinary sphere, or 2-sphere, if f is a continuous function that assigns a vector in R_3 to every point p on a sphere such that $f(p)$ is always tangent to the sphere at p, then there is at least one p such that $f(p) = 0$

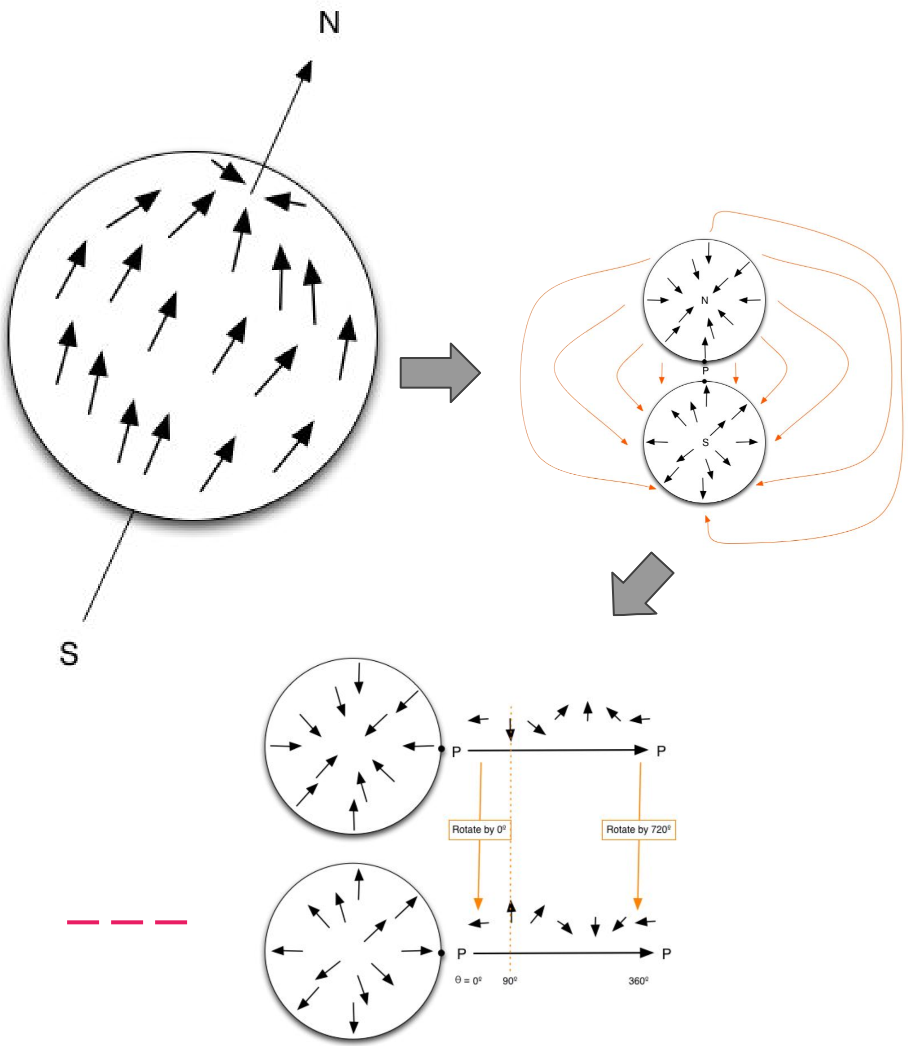




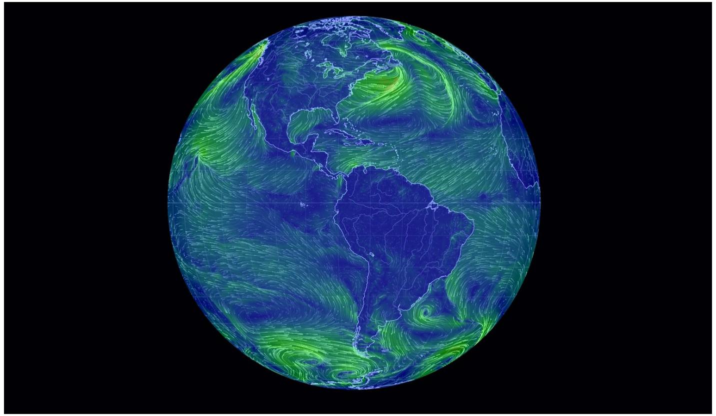
To visualize in 2D:



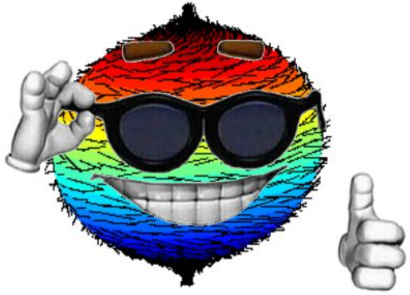
Now in 3D:



Applications:



“You can’t comb the hair on a coconut”



Sources:

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