Problem Session:

1. Rank conjecture

$rk_p(G) \leq n \ \forall p \iff G$ acts freely on a finite CW complex homotopy equivalent to product of n-spheres

2. Telescope Conjecture:

$K(n-1)_*(X) = 0 \Rightarrow K(i)_* X = 0 \quad i < n-1 \qquad X$ finite CW complex, (prime p)

$\Rightarrow \phi: \Sigma^? X \longrightarrow X$ isomorphism in $K(n)_* \varphi$ and 0 in $K(i)$, $i \neq n$

$X \longrightarrow \varphi^{-1} X \qquad \pi_* \varphi^{-1} X = \varphi^{-1} \pi_* X$

ANSS: $\varphi^{-1} E_2(X; MU) \not\Rightarrow \varphi^{-1} \pi_*(X)$

$p < n-1$ hom dim $\leq n^2 \Longrightarrow \pi_*(L_{K(n)} X) \qquad \varphi^{-1} X \longrightarrow L_{K(n)} X$ equivalence?

X ∞-spectra, $K(n)_* X = 0 \iff \exists X_\alpha$ finite such that $X = \varinjlim X_\alpha$ and $K(n)_* X_\alpha = 0$

3. Chromatic Assembly problem:

$L_n X = L_{K(0) \vee \cdots \vee K(n)} X$

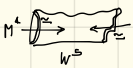
$$\begin{array}{ccc} X & \longrightarrow & \text{holim} \cdots \longrightarrow L_n X \longrightarrow L_{n-1} X \longrightarrow \cdots \longrightarrow L_1 X \longrightarrow L_0 X \\ \uparrow & & \| \\ \text{finite} & & X_{(p)} \end{array}$$

$$\begin{array}{ccccc} L_n X & \longrightarrow & L_{K(n)} X & \Longleftarrow & H^*(G_n, E_* X) \\ \downarrow & & \downarrow & & \\ L_{n-1} X & \longrightarrow & L_{n-1} L_{K(n)} X & & \end{array}$$

Q. Find effective ways of computing $L_{n-1} L_{K(n)} X$. $\quad n=2, \quad L_1 L_{K(2)} X = \text{holim}\ v_1^{-1} L_{K(2)} X \wedge S/p^n$

$L_{n-1} S^0 \longrightarrow L_{n-1} L_{K(n)} S^0$ is split. Complete decomposition for $L_{n-1} L_{K(n)} S^0$.

4. Find invariants of 4-dim topological s-coborbism.



Invariants to distinguish s-cobordism from h-cobordism $\qquad \tau(W) \in$ Whitehead torsion $Wh(\pi_1)$

No new invariants for $\pi_1 = 1$, subexponential growth. Not known for F_2.

5. Spaces of field theories

- Relate dim of FT's to chromatic filtration.

Q. $\quad 2|1$ – EFT $\simeq \Omega^\infty TMF$

6. As manifold invariants how sensitive are TFT's ?

Q. Are there cpt. top n-manifolds M, M' s.t. $\mathrm{conf}_k(M) \cong \mathrm{conf}_k(M')$ $\forall k \geq 0$ but $M \not\cong M'$.

Q. X based space, $\longrightarrow$ $\mathrm{Ran}\, X = \{S_+ \subseteq X, \text{ based finite subset}\}$, $\mathrm{Ran}\, X \longrightarrow \mathbb{N}$, $S_+ \longmapsto |S_+|$ continuous in appropriate top.

How is topology of $\mathrm{Ran}(X)$ dependant on X.

Q. Do there exist $M \not\cong M'$ but $\mathrm{Ran}(M)^* \cong \mathrm{Ran}(M')^*$

Q. $\mathrm{Homeo}_* X \longrightarrow h\mathrm{Aut}_{\mathbb{N}}(\mathrm{Ran}\, X)$

eg: Even for $\mathbb{R}^n_+$

$\mathrm{Top}(n) \longrightarrow h\mathrm{Aut}_{\mathbb{N}}(\mathrm{Ran}\, \mathbb{R}^n_+) \longleftrightarrow h\mathrm{Aut}(\Sigma_n)$

If $R = \mathrm{Ran}(M)$ $\quad \begin{matrix} R \\ \downarrow \\ \mathbb{N} \end{matrix} \rightsquigarrow \begin{matrix} \mathrm{Path}_1^{dec}(R) \\ \downarrow \\ R_1 \end{matrix} \longleftarrow$ fibration with fibers $\mathrm{Ran}(\mathbb{R}^n_+)$

7. $M\mathrm{Spin} \longrightarrow KO$ $\qquad M\mathrm{String} \longrightarrow tmf$ $\rightsquigarrow$ Orientation

$$\begin{matrix} B\mathrm{String} & \\ \downarrow & \\ B\mathrm{Spin} & \longrightarrow K(\mathbb{Z},4) \end{matrix}$$