Derivations and Square zero extensions

A cdga, M a dg module

Derivation $\delta : A \to M$ $\iff$
$$\begin{tikzcd} A \oplus M \arrow[r] & A \arrow[l, bend right, dashed, "\exists\ a \mapsto (a, \delta(a))"'] \end{tikzcd}$$

More generally,
$$\begin{tikzcd} & A\oplus M \arrow[dl] & \\ A & & B \arrow[ll] \arrow[ul, dashed, "\exists"'] \end{tikzcd}$$
$$U = \operatorname{Spec} A \xrightarrow{f} V = \operatorname{Spec} B$$
$\iff$ derivation $B \xrightarrow{\text{der}} f_* M$

$\Omega^1_B \xrightarrow{B\text{-lin}} f_* M$

$f^*\Omega^1_A \xrightarrow{A\text{-lin}} M$

Given a prestack $X : \mathrm{Aff}^{op} = \mathrm{cdga}_k^{\leq 0} \longrightarrow \mathrm{Spc}$
and a point : $\operatorname{Spec} A = U \xrightarrow{x} X$
and $F \in QCoh(U)^{\leq 0}$, look at extensions

$$\begin{tikzcd} & \operatorname{Spec}(A \oplus F) = U_F \arrow[dr, dashed] & \\ \operatorname{Spec} A = U \arrow[ur] \arrow[rr, "x"] & & X \end{tikzcd}$$

Define : $QCoh_B^{\leq 0} \longrightarrow \mathrm{Spc}$ ☆

$F \longmapsto \mathrm{Map}_{U/}(U_F, X)$

Def

Say X has a <u>cotangent space</u> at x if ☆ is corepresented by $T^*_x(X) \in QCoh(X)^-$

- If $X = \operatorname{Spec} B$ then $T^*_x(X) = f^*\Omega^1_B$