Quiz 1

Each question is worth 25 points. Please write detailed mathematically correct solutions. Guesses and miracles would not receive any credit.

1. Use the identity $\dfrac{1}{y+y^2} = \dfrac{1}{y} - \dfrac{1}{y+1}$ to solve the initial value problem

$$\frac{dy}{dt} = y + y^2, y(0) = -1/2$$

2. Use the substitution $y = v^{-1/2}$ to find the general solution to

$$\frac{dy}{dt} = y + y^3$$

3. For the following autonomous DE find all the equilibria in the range $-3\pi/2 \leq y \leq 3\pi/2$ and classify them,

$$\frac{dy}{dt} = y \sin y$$

4. Given two differentiable functions $p(x), q(y)$ which of the following two DE's is exact regardless of what p, q are?

$$p(x) + q(y)\frac{dy}{dx} = 0$$

$$q(y) + p(x)\frac{dy}{dx} = 0$$

Solve the one which is exact for the initial condition $y(0) = 0$. (You final answer should be in terms of proper integrals.)