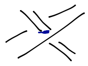
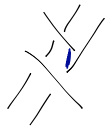
Slice Ribbon Conjecture

Slice knot : $K = \partial D \quad D \subseteq B^4$

Ribbon knot : $K = \partial D \quad D \looparrowright S^3$

self-intersections are ribbon :  not 

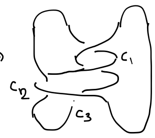
Easy to see ribbon $\Rightarrow$ slice

Conjecture : All slice knots are ribbon.

Why? two knots $K_1 \underset{\text{concordance}}{\sim} K_2 \iff K_1 \# \overline{K_2}$ is slice

This says $K \sim K$. Glue them and join corresponding points which is a ribbon interior.

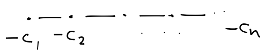
Th^m: If Ribbon Slice conjecture then K_0, K_1 fibered and they support a tight contact structure on ??

Th^m: Ribbon slice conjecture is true for 2-bridge knots. $\rightsquigarrow$ 

$$\cfrac{1}{c_1 - \cfrac{1}{c_2 - \cdots \cfrac{1}{c_n}}}$$

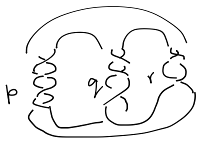
Proof: K slice

$\Sigma =$ double branched cover of (S^3, K)
$\quad \parallel$
∂W^4

Use the continued fraction to construct a 4-manifold  $= P$ with $\partial P = K$

Glue W to P and

$\therefore$ Conclusion ??

Th^m: Also true for 3-stranded Pretzel 

Slice knots have 0 signature.

This forces $p = r$

Donaldson's Theorem : If M^4 is smooth and $H^2(M) \times H^2(M) \longrightarrow H^4(M)$ is $-$ve def. then it is diagonalizable.

Disprove : ? • Try Fibered ribbon surface

Monodromy of Seifert surface extends to the handlebody ?

• $K_1 \# K_2$ is ribbon iff $K_1 = \overline{K_2}$ where $\Delta_{K_1} = \Delta_{K_2} = 0$