Ordinary Differential Equations **Name (Print):** ____________________

Final - Part 1

06/25/15

Time Limit: 80 Minutes

- This exam contains 11 pages (including this cover page) and 10 problems.
- Write detailed mathematically correct answers. **Mysterious or unsupported answers will not receive any credit**.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	
9	25	
10	25	
Total:	250	

1. (25 points) Find an integer n such that x^n is an integrating factor which makes the following DE exact

$$\left(3\frac{y}{x} + \frac{y^2}{x^2}\right) + \left(\frac{y}{x} + 1\right) y' = 0$$

Find the general solution to this DE.

2. (25 points) One of the solutions of the DE

$$(t-1)y'' - ty' + y = 0$$

is e^t. Find a second solution. Show that the two solutions are linearly independent.

3. (25 points) Find the general solution of the system

$$y' = -4x, x' = y$$

Describe, as best as you can, the integral curve for the initial conditions $x(0) = 0, y(0) = 2$.

4. (25 points) Find some k such that the solution of the IVP

$$y'' + 3y' - 4y = 0, \qquad y(0) = k, y'(0) = 1$$

tends to 0 as $t \to \infty$.

5. (25 points) If $y(t)$ is the solution to the IVP

$$y' + y = \begin{cases} e^{-t} & \text{for } t < 3 \\ e^{t} & \text{otherwise} \end{cases}, \qquad y(0) = 0$$

Find $y(2)$ and $y(4)$.

6. (25 points) Find the solution of the following IVP and find it's interval of definition

$$ty' + 2y = \sin t, \qquad y(\pi/2) = 0$$

7. (25 points) Solve the IVP

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \qquad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

8. (25 points) Find a particular solution of the DE

$$\sqrt{1-t^2}y'' + 2\sqrt{1-t^2}y' + \sqrt{1-t^2}y = e^{-t}$$

9. (25 points) Find a particular solution of the DE

$$y''' + y = 1 + 2te^t + t^2$$

10. (25 points) For the following autonomous DE find and classify all the equilbria and draw several integral curves

$$y' = -r\left(1 - \frac{y}{T}\right)^2 \left(1 - \frac{y}{K}\right)^3$$

for constants $r > 0$ and $T > K > 0$.