Quiz 3

Each question is worth 25 points. Please write detailed mathematically correct solutions.

1. Find eigenvalues and eigenvectors of the following matrices:

a) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ c) $\begin{bmatrix} k & k-1 \\ 0 & 1 \end{bmatrix}$

2. Solve the IVP

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

with initial conditions $x(0) = 1, y(0) = 0$.

3. Find the general solution of

$$x' = y$$
$$y' = -x - 2y$$

4. Describe how the phase portrait of the system

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & k-1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

changes as k varies from -1 to 1.