# HOMEWORK 1

## DUE: WEDNESDAY, MAY 30

**1. Radioactive Decay.** For a radioactive object, the amount of radioactive material present, $Q(t)$, satisfies the DE

$$Q' = -rQ$$

where $r$ is a positive constant.

a) Find the order of the DE and determine whether the DE is linear or non-linear.
b) Solve the IVP with initial condition $Q(0) = Q_0$ where $Q_0$ is a constant.
c) The time at which $Q(t) = Q_0/2$ is called the **half-life** of the material. Given that the half-life is equal to $l$, find $r$ in terms of $l$.

**2. Heat diffusion:** For body temperature $T(t)$ the heat diffusion differential equation is given by

$$\dot{T} = -k(T - T_E)$$

where $k$ is a positive constant and $T_E$ is a constant denoting the ambient temperature.

a) Find the order of the DE and determine whether the DE is linear or non-linear.
b) Substitute $T = U + T_E$ in the DE and find the general solution (the final solution should be for $T$ and not $U$).
c) For each of the three conditions,

$$k = 1, T_E = 0$$
$$k = 0, T_E = 1$$
$$k = 1, T_E = 1$$

   1) Draw the direction fields,
   2) Find equilibria,
   3) Solve the IVP for $T(0) = 1$,
   4) Draw the integral curve corresponding to this solution.

**3. Harmonic Oscillator.** The DE for a harmonic oscillator is given by

$$\ddot{x} + kx = 0$$

where $k$ is a positive constant.

a) Find the order of the DE and determine whether the DE is linear or non-linear.
b) The general solution for this DE is of the form

$$x(t) = A\cos(\omega t + \phi)$$

for some constants $A, \omega, \phi$. However not all of these constant parameters are free. Plug this solution back in the DE and determine which constant(s) depends on $k$ and which constants are free.

**4. Forced Harmonic Oscillator.** The DE for a simple forced harmonic oscillator is given by

$$\ddot{x} + kx = \cos(t)$$

where $k$ is a positive constant. The general solution to this problem is not easy to write. Instead we try to find **ONE** solution.

a) Find the order of the DE and determine whether the DE is linear or non-linear.
b) Assuming $k \neq 1$ find *some* constants $A, \omega$ and $\phi$ such that

$$x_1(t) = A\cos(\omega t + \phi)$$

solves the DE.