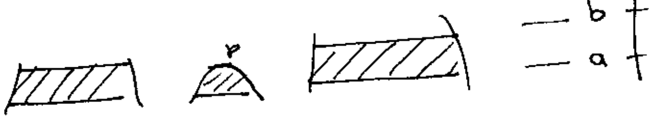
index 2: $M^a \cong M^b \sqcup \mathbb{D}^2$



Prop: $f: M \to \mathbb{R}$ ordered Morse
↳ surface, connected

If f has no index 1 critical point then M is homeomorphic to S^2.
In this case there can be only 2 critical points.

Rm:

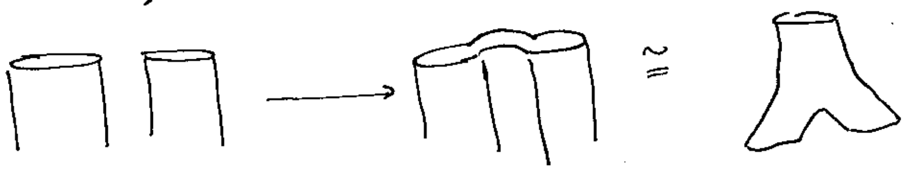
index 1: $M^a \cong M^b \cup_f \mathbb{D}^2$ $\quad f$ attaching map

f can be of two types

1)

orientable

2)

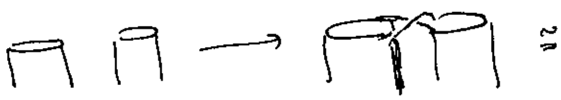


Non-orient~~able~~ed

1)

??

2)



16/01/13

— Pramath

Def^n: Holomorphic map at a pt, isomorphism

Remark. Holomorphic at a pt $\Rightarrow$ Holomorphic in a nbd of the pt

- Holomorphic function := Holomorphic map to $\mathbb{C}$
- $\mathcal{O}(X)$:= Ring of Holomorphic functions, $\mathbb{C}$ algebra
- Every chart is holomorphic in its domain.

Th^m: Let $\psi : X \longrightarrow Y$ non-constant holomorphic map

$x \in X$, $\exists$ charts $\varphi : U \longrightarrow V$ in X, $\varphi' : U' \longrightarrow V'$ in Y

- $\varphi(x) = 0$
- $\varphi' \cdot \psi \cdot \varphi^{-1}(z) = z^k$.

Corollary:

- ψ bijective $\Leftrightarrow$ ψ biholomorphic
- Every non-constant holomorphic map is ~~closed~~ open
- X compact $\Rightarrow$ ψ surjective, Y compact
- A non-constant holomorphic map never achieves its maximum absolute value.
- X compact $\Rightarrow$ $\mathcal{O}(X) \cong \mathbb{C}$.

Lemma: $k \in \mathbb{N}$, $z_0, w_0 \in \mathbb{C} - \{0\}$.

$w_0 = z_0^k$.

Then $\exists$ $U \ni z_0$, $V \ni w_0$ open and biholomorphic $g : V \longrightarrow U$

s.t. $g(w)^k = w$. , $g(w_0) = z_0$.

Proof: Apply implicit function th^m to $p(z,w) = z^k - w$.

Lemma: $x_0 \in U \subseteq \mathbb{C}$ $f(x_0) \neq 0$

$\forall k \in \mathbb{N}$, $\exists$ nbd U of x_0 such that

$\exists$ holomorphic $g : U \longrightarrow \mathbb{C}$ s.t.

$f = g^k$

Proof of Th^m:

Assume charts $(U \ni x, \varphi)$ $(U' \ni \psi(x), \varphi'(x))$

and $\varphi(x) = 0 = \varphi'(\psi(x))$.

$f := \varphi.\psi.\varphi^{-1}$ so $f(0) = 0$

$\exists$ $h : V \longrightarrow \mathbb{C}$ s.t. $f(z) = z^k h(z)$ and $h(0) \neq 0$

WALOG assume $h(V) \cap \{0\} = \emptyset$ so that

h has a k^{th} root i.e. $h = \theta^k$

Then replace φ by $\varphi.(\theta \circ \varphi)$

$z^k h = (z.\theta)^k$