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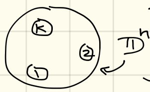
Non-abelian Poincare Duality:

Generalizes:
- Poincare duality
- May's Recognition principle
  A is group like E_n algebra then $A \cong \Omega^n B^n A$
- Goodwillie's Th^m
  $HH_*(C_*(\Omega X)) \cong H_*(LX)$

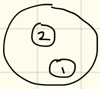
E_n-algebra: Algebra over an E_n operad.

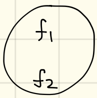
E_n operad $\cong D_n \leftarrow$ n disk operad

$D_n(K) = \Big\{$  $\Big\}$

$$\coprod D_n(K) \underset{\Sigma_K}{\times} A^K \longrightarrow A$$

eg: $f_1, f_2 \in \Omega^n(X)$



$(f_1 f_2) =$ 

$D_n(2) \times A^2 \longrightarrow A$ makes $\pi_0(A)$ a monoid.

Def^n:
A is group like if $\pi_0(A)$ is a group.

Def^n: X space, $F(X)$ free abelian group on X.

Th^m (Dold-Thom): $\pi_*(F(X)) \cong H_*(X)$

if $Y \subseteq X$, $\pi_*(F(X)/F(Y)) \cong H_*(X; Y)$

$\pi_*(\mathrm{Maps}^c(X, K(\mathbb{Z}, n)) \cong H_c^{*-n}(X)$

Orientation System:

$$\mathcal{O}_m = H_n(D_m M, S_m M) \qquad m \in M^n$$

Def^n: Let $B^{TM}\mathbb{Z}$ be the bundle over M with fiber over m

$$F(D_m M)/F(S_m M) \simeq K(\mathbb{Z}, n) \simeq B^n \mathbb{Z}$$

$$\pi_*\left(\Gamma^c\left(\begin{array}{c} B^{TM}\mathbb{Z} \\ \downarrow \\ M \end{array}\right)\right) = H_c^{*-n}(M, \mathcal{O})$$

$$S: F(M) \longrightarrow \Gamma^c\left(\begin{array}{c} B^{TM}\mathbb{Z} \\ \downarrow \\ M \end{array}\right) \qquad \text{scanning map}$$

$$(\xi, m) \longmapsto ? \in F(D_m M)/F(S_m M)$$

This somehow leads to Poincare Duality.

Conf (M, A) configuration space of M with coefficients in A (A: commutative monoid).

Th^m: If A is a graph like monoid

$$\mathrm{Conf}(M, A) \xrightarrow{\simeq} \Gamma^c\left(\begin{array}{c} B^{TM}A \\ \downarrow \\ M \end{array}\right)$$

Cor: Goodwillie's th^m: $\quad M = S^1, \; A = \Omega X$

Salvatore:

Space of ways particles can collide in n-dimensions is a framed E_n-operad.

Properties of $\mathrm{Conf}_n(M, A)$: $\quad A$ - E_n algebra

Functoriality $\quad A \to B$ E_n map then $\mathrm{Conf}_n(M, A) \to \mathrm{Conf}_n(M, B)$

$\quad M \to N \qquad \mathrm{Conf}_n(M, A) \to \mathrm{Conf}_n(N, A)$

$$\mathrm{Emb}(M, N) \times \mathrm{Conf}(M; A) \longrightarrow \mathrm{Conf}(N; A)$$