— Robin Koytcheff

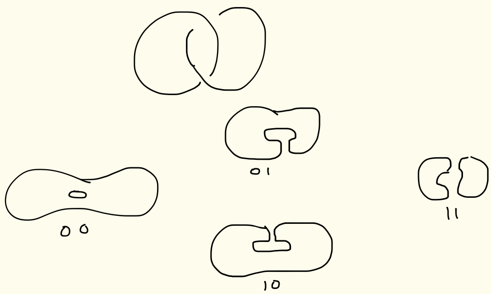
Knot/Link $\rightsquigarrow$ cube of circles in $\mathbb{R}^2$ $\rightsquigarrow$ chain complex of graded vector spaces $\rightsquigarrow$ homology is Link invariant

Kh generalizes Jones polynomial,
graded Euler Char χ of Kh $= \hat{J}$

$$\dim_q V = \sum_i q^i \dim Gr_i(V)$$

generalisation: $L \rightsquigarrow$ chain complex of objects in a Cobordism Category $\overset{\rightsquigarrow}{\scriptstyle TQFT}$ Kh

Can also form a category of links. Kh is then a functor between certain cats.



n crossing link

$\rightsquigarrow$

2^n size cube of smoothings S_α, $\alpha \in \{0,1\}^n$

$k :=$ # circles in S_α

$r := |\alpha| :=$ no. of 1's in α

$n_\pm :=$ no. of $\pm$ crossings

Jones polynomial:

$$\hat{J} = \sum_\alpha m_\alpha \cdot (-1)^{n_-} \cdot q^{n_+ - 2n_-}$$

where $m_\alpha = (-1)^r q^r (q^{-1} + q)^k$

Kh: $\alpha \rightsquigarrow V_\alpha = V^{\otimes k}\{r\}$; $V = \underset{\substack{\uparrow \\ \text{degree } 1}}{\mathbb{R}v_+} \oplus \underset{\substack{\uparrow \\ \text{degree } -1}}{\mathbb{R}v_-}$

$$C_r := \bigoplus_{|\alpha|=r} V_\alpha$$

For Hopf link: $0 \to V^{\otimes 2} \to V \oplus V \to V^{\otimes 2} \to 0$

Differentials: draw cobordisms between pictures and apply $\underbrace{\text{2-dim TQFT}}$.

$v_+ \longleftrightarrow 1$ $\quad H^*(\mathbb{CP}^1)$

$v_- \longleftrightarrow x$

Thm: Homology of $C_*(L)$ depends only on the isotopy class of L.

Defn: Preadditive category $\mathcal{C}$, Matrices over $\mathcal{C}$: $Mat(\mathcal{C})$

$Cob^3(\phi)$: obj = disjoint collection of simple closed curves in $\mathbb{D}^2$

mor = cobordism embedded in $\mathbb{R}^3$ upto isotopy.

$Cob^3(B)$: B finite set of points on $\partial \mathbb{D}^2$

ob:  embedded 1-manifolds in $\mathbb{D}^2$ with boundary B

Category of chain Complexes in a category: $Kom(\mathcal{C})$

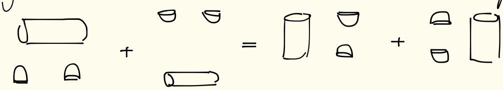
Cube $[T] \in Kom(Mat(Cob^3))$

To get invariant we need to mod out by chain homotopies and by 'local' relations.

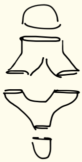
Local Relations l: on morphisms in Cob^3

S: If a cobordism has S^2 component, set it to 0.

T: disjoint union with $S^1 \times S^1$ is same as multiplying by 2.

4Tube: 

• S  $\varepsilon \circ \eta = 0$ in Frob Algebra $H^*(\mathbb{CP}^1)$

• T  $\varepsilon \circ m \circ \Delta \circ \eta = 2$

Th^m: $Kh(T) \in Kom_{/h}(Mat(Cob^3_{/l}))$ is an isotopy invariant.

Th: Kh is a functor
$$Cob^4_{/i} \longrightarrow Kom_{/h}(Mat(Cob^3_{/l}))/\pm 1$$

Dan Freed

F – 3d Chern Simons theory $\quad X^3$ – 3 manifold

$K^1 \subset X^3$ knot

$F(X) \in \mathbb{C} \qquad F(X, K) \in \mathbb{C}$

Cut Tubular neighborhood of K $\nu_K \cong S^1 \times \mathbb{D}^1$

$X_K := X \setminus \nu_K$ compact 3-manifold

$X_K : \underset{S^1 \times S^1}{\partial \nu_K \,\wr\wr} \longrightarrow \phi^2$