Postnikov tower for X:

$$\begin{tikzcd}
 & & K(\pi_{m+1}, m+2) & & & K(\pi_n, n) \\
\cdots \arrow[r] & X_{m+1} \arrow[r] & X_m \arrow[u, "k_m"] \arrow[r] & \cdots \arrow[r] & X_n \arrow[r] \arrow[ur, phantom, "\shortparallel"] & (*) \\
 & & X \arrow[ul, "s_{m+1}"] \arrow[u, "s_m"'] & & X \arrow[u] &
\end{tikzcd}$$

For X $(n-1)$*-connected,*

- $X_n = K(\pi_n(X); n)$
- $\pi_i(X_m) = 0$ for $i > m$
- $\pi_i(X_m) = \pi_i X$ for $i \leq m$
- $X_{m+1} \longrightarrow X_m \xrightarrow{k_m} K(\pi_{m+1}, m+2)$ *fibration*

Let $X = S^n$.

If $\dim K = n$, $[K, S^n] = [K, K(\mathbb{Z}, n)] = H^n(K; \mathbb{Z})$

If $\dim K = n+1$, $[K, S^n] = [K, X_{n+1}]$

$$\begin{tikzcd}
 & X_n = K(\mathbb{Z}, n) \arrow[r, "Sq^2"] & K(\mathbb{Z}/2, n+2) \\
K(\mathbb{Z}/2, n+1) \arrow[r] & X_{n+1} \arrow[u] & \\
K(\mathbb{Z}, n-1) \arrow[u] & &
\end{tikzcd}$$

$$0 \to \operatorname{coker} \_ \to [K, X_{n+1}] \to \ker \_ \to 0$$

$$[K, K(\mathbb{Z}, n-1)] \longrightarrow [K, K(\mathbb{Z}/2, n+1)] \qquad [K, K(\mathbb{Z}, n)] \longrightarrow [K, K(\mathbb{Z}/2, n+2)]$$

$$H^{n-1}(K; \mathbb{Z}) \longrightarrow H^{n+1}(K; \mathbb{Z}/2) \qquad \underline{\underline{H^n(K; \mathbb{Z})}} \longrightarrow H^{n+2}(K; \mathbb{Z}/2) = 0$$

This is very hard to digest.

If $\dim K = n+2$, $[K, S^n] = [K, X_{n+2}]$

$$\begin{tikzcd}
 & X_{n+1} \arrow[r] & K(\mathbb{Z}/2, n+3) \\
K(\mathbb{Z}/2, n+2) \arrow[r] & X_{n+2} \arrow[u] & \\
\Omega X_{n+1} \arrow[u] & &
\end{tikzcd}$$

$$0 \to \operatorname{coker} \_ \to [K, X_{n+2}] \to \ker \_ \to 0$$

$$[K, \Omega X_{n+1}] \to [K, K(\mathbb{Z}/2, n+2)] \qquad \underline{\underline{[K, X_{n+1}]}} \longrightarrow \underset{0}{[K, K(\mathbb{Z}/2, n+3)]}$$

The Spectral sequence:

$$\begin{tikzcd}
D_2 \arrow[rr, "i"] & & D_2 \arrow[dl, "j"] \\
 & E_2 \arrow[ul, "k"] &
\end{tikzcd}$$

$$D_2^{-p,q} = [K, \Omega^p X_{q-p}]$$

$$E_2^{-p,q} = [K, \Omega^p K(\pi_{q-p}, q-p)]$$

Maps:

$$X_{q-p} \longrightarrow X_{q-p-1} \longrightarrow K(\pi_{q-p}, q-p+1)$$

$$[K, \Omega^p X_{q-p}] \xrightarrow[i]{} [K, \Omega^p X_{q-p-1}] \xrightarrow[j]{} [K, \Omega^p K(\pi_{q-p-1}, \mathbb{Z})] \xrightarrow[k]{} [K, \Omega^{p-1} X_{q-p}]$$

$$(0,-1) \qquad (0,0) \qquad (-1,0)$$

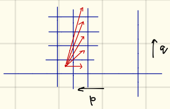
The sequence is cohomological

So filtration is: $\ker([K, X] \to [K, X_i])$

$$d_2 \quad [K, \Omega^p K(\pi_{q-p}, q-p)] \xrightarrow{k} [K, \Omega^{p-1} X_{q-p+1}] \xrightarrow{j} [K, \Omega^{p-1} K(\pi_{q-p+1})]$$

$$H^{q-2p}(K, \pi_{q-p}) \longrightarrow H^{q-2p+2}(K; \pi_{q-p+1})$$

$$E_2^{-p,q} \longrightarrow E_2^{-p+1,q}$$



Brain Fuck. Why grade it like this?

d_r *apparantly has bidegree* $(-1, r-2)$

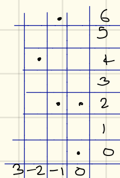
Let's see what we get for $K = S^n$.

$$E_2^{-p,q} = H^{q-2p}(S^n, \pi_{q-p}(X)) = \begin{cases} \pi_p(X) & q = 2p \\ \pi_{p+n}(X) & q = 2p+n \\ 0 & \text{else} \end{cases}$$

Each element can support 1 differential

$$\pi_p(X) \longrightarrow \pi_{p-1+n}(X)$$

$n = 2$



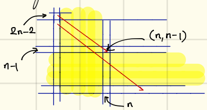
Now on the other hand suppose X is a space whose homotopy groups we know.

$X = \mathbb{Z}\times BU$ $\quad E_2^{-p,q} = H^{q-2p}(K, \pi_{q-p}) = \begin{cases} H^{q-2p}(K,\mathbb{Z}) & \text{if } q-p = \text{even} \\ 0 & \text{else} \end{cases}$

For SS to collapse we need sparse columns (p). But for any p, q can infinite values ∵

Prop: If X is n-connected, then $H^i(X;\mathbb{Z}) \longrightarrow H^{i-1}(\Omega X;\mathbb{Z})$ is isomorphism for

Proof: Look at $\Omega X \to * \to X$. X - n connected $\Rightarrow$ ΩX - $n-1$ connected.



$H^*(\Omega X) \longrightarrow H^{*+1}(X)$ iso for $* < 2n-2$

Further these transgressions are:

$$[X, K(\mathbb{Z}, *+1)] \longleftarrow [\Omega X, \Omega K(\mathbb{Z}, n+1)]$$

And that is by looking at the map between the SS:

$$\begin{tikzcd} \Omega X \arrow[r] \arrow[d] & * \arrow[r] \arrow[d] & X \arrow[d] \\ \Omega K(*+1) \arrow[r] & * \arrow[r] & K(*+1) \end{tikzcd}$$

and noticing that this is true when $X = K(\mathbb{Z}, *+1)$, $f = \mathrm{id}$.

Functional Cohomology Operations:

$f: Y \to X$, $u \in H^n(X;G)$, $f^*u = 0$,

$\theta: H^n(\,;G) \longrightarrow H^q(\,;\pi)$ additive natural transform with a desuspension ${}^1\theta$

$\theta(u) = 0$.

$$\begin{tikzcd} & H^{n-1}(Y;G) \arrow[r] \arrow[d, "{}^1\theta"] & H^n(X,Y;G) \arrow[r] \arrow[d, "\theta"] & H^n(X;G) \arrow[r, "f^*"] \arrow[d, "\theta"] & H^n(Y;G) \\ H^{q-1}(X;\pi) \arrow[r, "f^*"'] & H^{q-1}(Y;\pi) \arrow[r] & H^q(X,Y;\pi) \arrow[r] & H^q(Y;\pi) & \end{tikzcd}$$

Define:

$\theta_f : H^n(X;G) \dashrightarrow H^{q-1}(Y;\pi)/Q$ $\qquad Q = {}^1\theta(H^{n-1}(Y;G)) + f^*(H^{q-1}(X;\pi))$

$$\begin{tikzcd} & \exists\, u' \arrow[r, mapsto] \arrow[d, "\theta"] & u \arrow[r, mapsto, "f^*"] \arrow[d] & 0 \\ u'' \arrow[r, mapsto] & \theta u' \arrow[r, mapsto] & 0 & \end{tikzcd}$$

$\exists$

$u \longmapsto u'' + Q$

Given another tuple with $\begin{tikzcd} Y \arrow[r, "f"] & X \\ Y' \arrow[u, "\eta"] \arrow[r, "f'"] & X' \arrow[u, "\xi"'] \end{tikzcd}$ then we get $\theta_{f'}(\xi^*(u)) = \eta^*(u'' + Q)$

If $f \sim 0$ then $\theta_f(u) = 0$