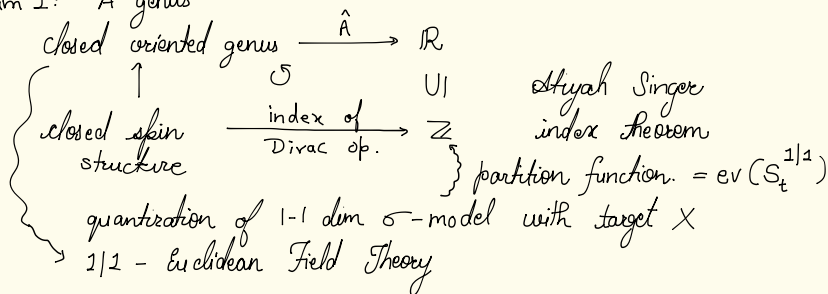


# Twisted field theories, modular forms and Quillen connections

- Invariants in top, analysis, QFT
- Modular
- Computations of the easiest twists
- $\hat{A}$ -genus Atiyah
- $\hat{W}$ -genus Witten

dim 1:  $\hat{A}$  genus



Modularity comes in because  $S'$  is replaced by  $S' \times S'$

Classical  $\Sigma$ -model:  $d$ -dim,

$\Sigma^d$  - space time - Riemannian manifold

$\mathcal{F}(X)$  - Fields  $C^\infty(\Sigma^d, X)$

$A: \mathcal{F}(X) \rightarrow \mathbb{R}$  action map

$\downarrow$  geometric quantization

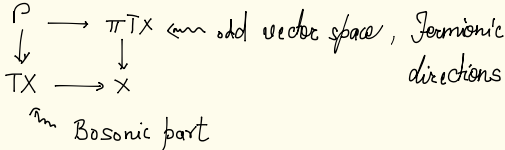
$H = L^2(X)$ , time evolution  $e^{-t\Delta_X}$   
after Wick Rotation

$$\int_{\mathcal{F}} e^{-A(\varphi)}$$

This is Bosonic (?).

If  $\Sigma$  has 1-bosonic, 1-fermionic direction

$\Sigma^{1|1} \rightarrow$  The classical solutions



$\mathcal{P}$  has no Lagrangian  $\hat{=}$   
and hence no polarization

Use the spin structure  $\implies$  polarization on  $P$

$$\mathcal{H} = L^2(\mathbb{S}_x)_{\mathcal{U}_n}$$

$$\begin{array}{ccc} P' & \xrightarrow{\quad} & \pi(TX_g) \oplus (\mathbb{R}^n, -\text{st}) \\ \downarrow & \lrcorner & \downarrow \\ TX & \xrightarrow{\quad} & X \end{array} \quad \begin{array}{c} \uparrow \\ \text{spin} \end{array}$$

$$\text{time evolution} = e^{-t\mathcal{D}_x^2 + \Theta \mathcal{D}_x}$$

Def: A 1-dim Euclidean field theory, is a pair  $(V, \alpha_t: V \rightarrow V)$   $V$ -topological vector space,  $\alpha_t$ -smooth semigroup

$\Leftrightarrow$  symmetric monoidal functor  $(1\text{-E Bord}, \perp) \xrightarrow{F} (TVect, \otimes)$   $\otimes_i$   
 ob: compact 0-manifold eg: pt  $\xrightarrow{\quad} V$  projected tensor product  
 $\mathcal{H}$

not a topological field theory

$$\text{mor: } \begin{array}{c} \text{ } \\ \text{ } \end{array} \xrightarrow{\quad} \begin{array}{c} \text{ } \\ \text{ } \end{array} \quad \alpha_t(\cdot): \mathbb{R} \rightarrow V \otimes V$$

with length

$$E(S_t^1) = \text{tr}(\alpha_t) =: \text{partition function}$$

Now replace  $TVect$  to  $TVect^{\mathbb{Z}_2}$

Lem:  $\{1/1\text{-EFT}\} \cong \begin{cases} \text{super vector space } V \\ \text{super semigroup of operators} \end{cases}$   
 $\alpha_{t,\Theta}: V \rightarrow V$   
 $e^{-t\mathcal{D}^2 + \Theta \mathcal{D}}$ ,  $\mathcal{D}$  odd infinitesimal generator

$$\text{trace}(\alpha_{t,\Theta}) = \text{sdim}(\ker \mathcal{D}) \in \mathbb{Z}$$

$$E(S_{t,\Theta}^1)$$

"super symmetry cancellation"

Q: index of Dirac operator really lives in  $K^n(\text{pt})$ ??

To get twisted 1/1 EFT:  $(1|1 \text{E bord}, \perp) \xrightarrow[\perp]{\quad} (TA\mathbb{B}_g^{\mathbb{Z}_2}, \otimes)$   $\xrightarrow{\quad} \text{2 category}$   
 $\text{0-mor: pts}$ ,  $\text{1-mor: } \curvearrowright$ ,  $\text{2-mor: } \textcircled{\text{11}}$  isometries

Remark:  $\text{TAIg}^{\mathbb{Z}/2}$  is a choice of delooping of  $\text{TVect}$ ,  
 $\text{TVect}$  is a choice of delooping of  $\mathbb{C}$

$$T(\mathbb{R}^{0,1}) \in \text{TAIg}^{\mathbb{Z}/2}$$

$T(\mathbb{C})$  super semigroup of bimodules

$E(\mathbb{R}^{0,1})$  is a  $\mathbb{C}$ - $T(\mathbb{R}^{0,1})$  bimodules

eg:  $T_n(\mathbb{R}^{0,1}) = \mathcal{C}_n = \mathcal{C}_1 \otimes \dots \otimes \mathcal{C}_1 = \text{degree } n\text{-twist}$

$$T_n(\mathbb{C}) = \mathcal{C}_n \mathcal{C}_n \mathcal{C}_n$$

dim 2:

$$\begin{array}{ccc} \text{closed oriented manifold } X^n & \xrightarrow{\hat{w}} & \mathbb{C}[q] \sim \text{comes from } S^1 \text{ action on } L^X \\ \uparrow & & \uparrow \\ \text{spin structure} & \longrightarrow & \mathbb{Z}[q] \end{array}$$

$$\begin{array}{ccc} \uparrow & & U_1 \\ \text{if } P_1 X = \text{torsion} & \longrightarrow & MF_*^{\mathbb{Z}} \\ \text{first Pontryagin class} & & \text{modular form} \end{array}$$

why do we get a modular form?

$$\begin{array}{c} \nearrow \text{partition function} \\ \text{if} \\ \text{ev}(\mathbb{C}/\Gamma) \end{array}$$

$\Gamma$  is a lattice in  $\mathbb{R}^2$

New input  
via factorization  
Algebras

Degree comes in since this is a line bundle on the space of lattices.

eg: degree  $n$  twist comes from delooping  $n$ -th power of Pfaffian line.