Bordism groups & Categories

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Motivation

- M- manifold $\quad \chi_M = \sum (-1)^i \beta_i(M) \pmod 2$ $\quad$ Euler Char mod 2

$$= \begin{cases} 0 & \text{if } M \text{ is odd closed} \quad \text{by Poincare Duality} \\ \beta_{n/2}(M) \text{ a Betti number} & \text{closed} \end{cases}$$

If M has a boundary, $\boxed{\chi_{\partial M} = 0}$

Look at $2M = M \underset{\partial M}{\sqcup} M$

use Mayer Vietoris on this to get $\chi_{2M} = 2\chi_M - \chi_{\partial M}$.

Signature: M^{4n} has an orientation

$$B_M : H^{2n}(M;\mathbb{Q}) \otimes H^{2n}(M;\mathbb{Q}) \longrightarrow H^{4n}(M;\mathbb{Q})$$

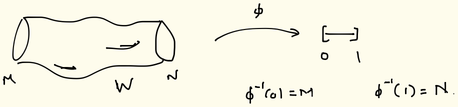
signature of $B_M =: s_M$

again if M^{4n+1} has a boundary, $\boxed{s_{\partial M} = 0}$

Def^n: bordism M to N:

W, $\partial M = M \amalg N^-$

(everything compact).



$\mathcal{N}_n =$ n-manifolds / bordism

Atiyah's definition of TQFT.