$K(\mathbb{Z},0)$

$K(\mathbb{Z},0) = \mathbb{Z}$,

$H^* = \mathbb{Z}, 0, 0, \dots$

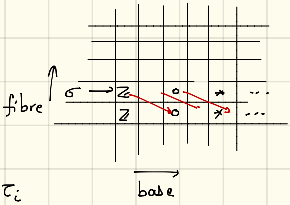
$K(\mathbb{Z},1)$

$K(\mathbb{Z},1) = S^1$

$H^* = \mathbb{Z}[x]/x^2$, $|x|=1$

$K(\mathbb{Z},2)$

$$\begin{array}{ccc} K(\mathbb{Z},1) & \longrightarrow & * \\ & & \downarrow \\ & & K(\mathbb{Z},2) \end{array}$$



$H^1(K(\mathbb{Z},2)) = 0$

$H^{2i}(K(\mathbb{Z},2), \mathbb{Z}) \cong \mathbb{Z}\tau_i$

$d_2 \sigma = \tau_1 \qquad d_2(\sigma\cdot\tau_1) = \tau_2 \ \dots \ d_2(\sigma\cdot\tau_{i-1}) = \tau_i$

$d_2(\sigma\tau_1) = d_2\sigma\cdot\tau_1 - \underbrace{\sigma\cdot d_2\tau_1}_{0}$

$= \tau_1^2$

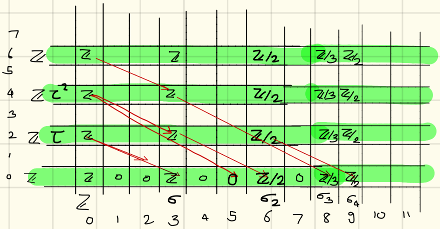
In general we would have $\tau_i = \tau_1^{\,i}$

$H^*(K(\mathbb{Z}/2), n) \cong \mathbb{Z}[x]$, $|x|=2$

Which would have followed straight away by noticing that $K(\mathbb{Z},2) \cong \mathbb{CP}^\infty$

$K(\mathbb{Z},3)$

$$\begin{array}{ccc} K(\mathbb{Z},2) & \longrightarrow & * \\ & & \downarrow \\ & & K(\mathbb{Z},3) \end{array}$$



$H^2 = H^1 = 0$

$H^3 = \mathbb{Z}\sigma$

Already this is getting crazy.

$d_3\tau = \sigma \qquad d_3\tau^2 = 2\tau d_3\tau = 2\tau\sigma$

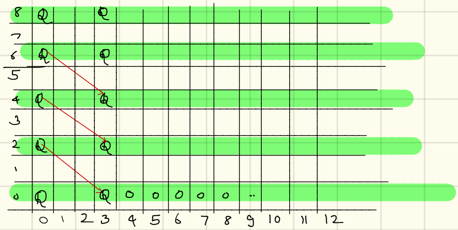
$d_3\tau^n = d_3\tau\cdot\tau^{n-1} + \tau\cdot d_3\tau^{n-1}$

$= \sigma\cdot\tau^{n-1} + \tau\cdot d_3\tau^{n-1}$

$= n\,\sigma\cdot\tau^{n-1}$ by induction

Rational coefficients:

The same relations as before will hold, but now these imply isomorphisms.



So we have

$$H(K(\mathbb{Z},3);\mathbb{Q}) \cong \mathbb{Q}[x]/x^2 \qquad |x|=3$$

The pattern is now clear:

$$H^*(K(\mathbb{Z},2n);\mathbb{Q}) \cong \mathbb{Q}[x] \quad , \ |x| = 2n$$

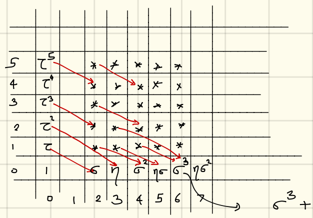
$$H^*(K(\mathbb{Z},2n+1);\mathbb{Q}) \cong \mathbb{Q}[x]/x^2 \ , \quad |x| = 2n+1$$

Can we use the same technique for arbitrary $K(G,n)$'s ?

$K(\mathbb{Z}/2,1) = \mathbb{R}P^\infty$

$K(\mathbb{Z}/2,2)$

$$\begin{array}{ccc} K(\mathbb{Z}/2,1) & \longrightarrow & * \\ & & \downarrow \\ & & K(\mathbb{Z},2) \end{array}$$



$\sigma^3 + \eta^2$

what about η^2 ?

$d_2\tau = \sigma$

$d_2\tau^2 = 0$

$d_2\tau^3 = \sigma\cdot\tau^2 + \tau\sigma\tau + \tau^2\sigma = \tau^2\sigma$

$d_2\sigma\tau = \cancel{d_2\sigma}\,\tau + \sigma\cdot d_2\tau = \sigma^2$

$d_3\tau^2 = \eta$ Note: There is no τ on E_3 page. So cannot do Leibnitz

$d_3\tau^4 = d_3\tau^2\cdot\tau^2 + \tau^2\cdot d_3\tau^2 = 0$

$K(\mathbb{Q},1)$

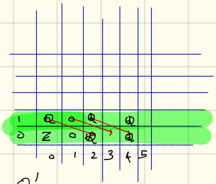
$\mathbb{Q} \to *$
$\downarrow$
$K(\mathbb{Q},1)$

$\mathbb{Q}$ is discrete so this is covering map

$H^1(K(\mathbb{Q},1);\mathbb{Z}) = \mathbb{Q}$, $\quad H^0(\quad) = \mathbb{Z}$

$K(\mathbb{Q},2)$

$K(\mathbb{Q},1) \longrightarrow *$
$\downarrow$
$K(\mathbb{Q},2)$



Except in 0 we have $\mathbb{Q}$'s,

$H^*(\quad;\mathbb{Q}) \cong \mathbb{Q}[x] \qquad |x| = 2.$

$K(\mathbb{Q},3)$

The same reasoning as in $K(\mathbb{Z},n)$ will work

So that we will get:

$$H^*(K(\mathbb{Q},2n);\mathbb{Q}) \cong \mathbb{Q}[x], \quad |x| = 2n$$

$$H^*(K(\mathbb{Q},2n+1);\mathbb{Q}) \cong \mathbb{Q}[x]/x^2, \quad |x| = 2n+1$$

Th^m (Serre):

$H^*(K(\mathbb{Z}/2,n);\mathbb{Z}/2)$ is generated by the Steenrod Squares.