Massey Products

- Given a dga A over a field k, Massey products are higher products on $H^*(A)$. These are particularly useful products on $\mathrm{Ext}^*_A(k,k)$.

Massey triple product:

for $[u],[v],[w] \in H^*(A)$ if $[uv] = [vw] = 0$ then

$\langle [u],[v],[w] \rangle = [sw \pm ut]$ where $uv = ds$, $vw = dt$

Check signs.

Geometrically:

$[u],[v],[w] \in H^*(M)$ are submanifolds of M, M_u, M_v, M_w

then

$$uv \rightsquigarrow M_u \cap M_v$$

$$vw \rightsquigarrow M_v \cap M_w$$

$$\left.\begin{array}{l} uv = 0 \Rightarrow M_u \cap M_v = \partial M_s \\ vw = 0 \Rightarrow M_v \cap M_w = \partial M_t \end{array}\right\}$$ for manifolds M_s, M_t

then $\langle u,v,w \rangle \rightsquigarrow M_s \cap M_w + M_u \cap M_t$

- $d\langle u,v,w \rangle = (ds)w \pm u\cdot(dt)$
  $= u\cdot v\cdot w \mp uvw = 0$ Adjust sign accordingly.