Friedrich-Alexander-Universität
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FAU

Decision theory

Exercise 4

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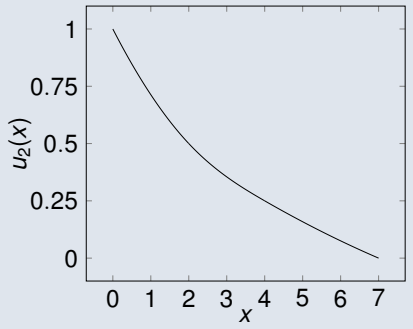
Decision Theory - Exercise

Exercise 1

Tina is looking for a new apartment. Relevant criteria are rent and distance to the university.

- Rent: 200 to 600 euros, linear utility function
- Distance: 0 to 7 km, utility function:

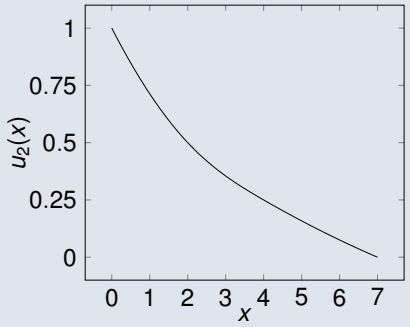


Eisenführ, Weber, Langer, Question 6.2

Decision Theory - Exercise

Data

- Rent: 200 to 600 euros, linear utility function
- Distance: 0 to 7 km, utility function:



Eisenführ, Weber, Langer, Question 6.2

Exercise 1

1. Tina finds an apartment that is 4 km away and costs 300 euros as good as an apartment that is 2 km away but costs 500 euros. What are the weighting factors?
2. How expensive can an apartment directly at the university be so that it is still better than an apartment that is 2 km away and costs 200 euros?

Decision Theory - Exercise

Background

Leo wishes to buy a new watch. Relevant criteria are

- low price: 50 to 200 euros
- long warranty: 4 to 16 years
- large water depth: 10 to 50 m

He uses the following utility functions:

- Price x: $u_1(x) = a + b/x$
- Warranty x: $u_2(x) = c + d\sqrt{x}$
- Depth x: $u_3(x) = e + fx$

Eisenführ, Weber, Langer, Question 6.6

Exercise 2

1. Determine parameters a to f so that functions are normalized on $[0, 1]$
2. How much does the price of 50 euros have to increase so that the utility of the price is halved?
3. Is the transition from 4 to 9 years of warranty more useful than from 9 to 16 years?
4. Find suitable weights if the following models are equally good:

$$(200, 16, 12) \sim (80, 4, 12)$$
$$(70, 16, 10) \sim (70, 4, 35)$$

Decision Theory - Exercise

Exercise 3

- Model for caffeine dependence
- a person has three life periods: youth, middle age, old age
- decide in each period: drink coffee or not
- if drinking coffee in period t, then dependent in period $t+1$
- Utility values:

	take	not take
not dependent	10	0
dependent	-8	-25

- $\delta = 1$, $\beta = 1/2$
- how do TC, Naive, and Sophisticated decide?

O'Donoghue, Rabin, The Economics of Immediate Gratification (2000)

Decision Theory - Exercise

Exercise 4

Maximize:

	s_1	s_2	s_3
a_1	8	3	3
a_2	2	9	4
a_3	9	1	2
a_4	9	4	0

Determine the best solution(s) with respect to

- Maximin
- Hurwicz with $\alpha = 1/2$
- Average
- Minimax Regret

Decision Theory - Exercise

Exercise 5: Axioms

Minimax Regret does not satisfy:

- Axiom 5: Independence of irrelevant alternatives
- Axiom 7: Independence of row permutations

Find one example each to demonstrate this.

Decision Theory - Exercise

Exercise 6

- calculate the OWA value for the following solution values:
  - $w = (1/2, 0, 0, 1/2)$, $a = (7, 3, 6, 4)$
  - $w = (1/4, 1/4, 0, 1/2)$, $a = (2, 6, 3, 6)$
  - $w = (1/3, 1/3, 1/3, 0)$, $a = (6, 7, 3, 1)$
  - $w = (0, 0, 1/2, 1/2)$, $a = (6, 3, 5, 8)$

Decision Theory - Exercise

Exercise 7

- Reminder: $WOWA(a_1, \ldots, a_n) = \sum_{i \in [n]} \omega_i a_{\pi(i)}$ with
  - π a permutation such that A is sorted in ascending order
  - $\omega_i = w(\sum_{j \leq i} p_{\pi(j)}) - w(\sum_{j < i} p_{\pi(j)})$
  - $w(x)$ is the interpolating function
- Determine $WOWA(a)$ with $a = (1, 2, 3, 4)$, $p = (0.1, 0.4, 0.3, 0.2)$, $w = (1/2, 1/4, 1/4, 0)$