Decision Theory

Lecture 3

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Decision Theory

Recap: What did we do?

- Modelling
- Types of models
- Types of scales
- Basic model: Decision matrix
- Decision trees
- Basics of probability

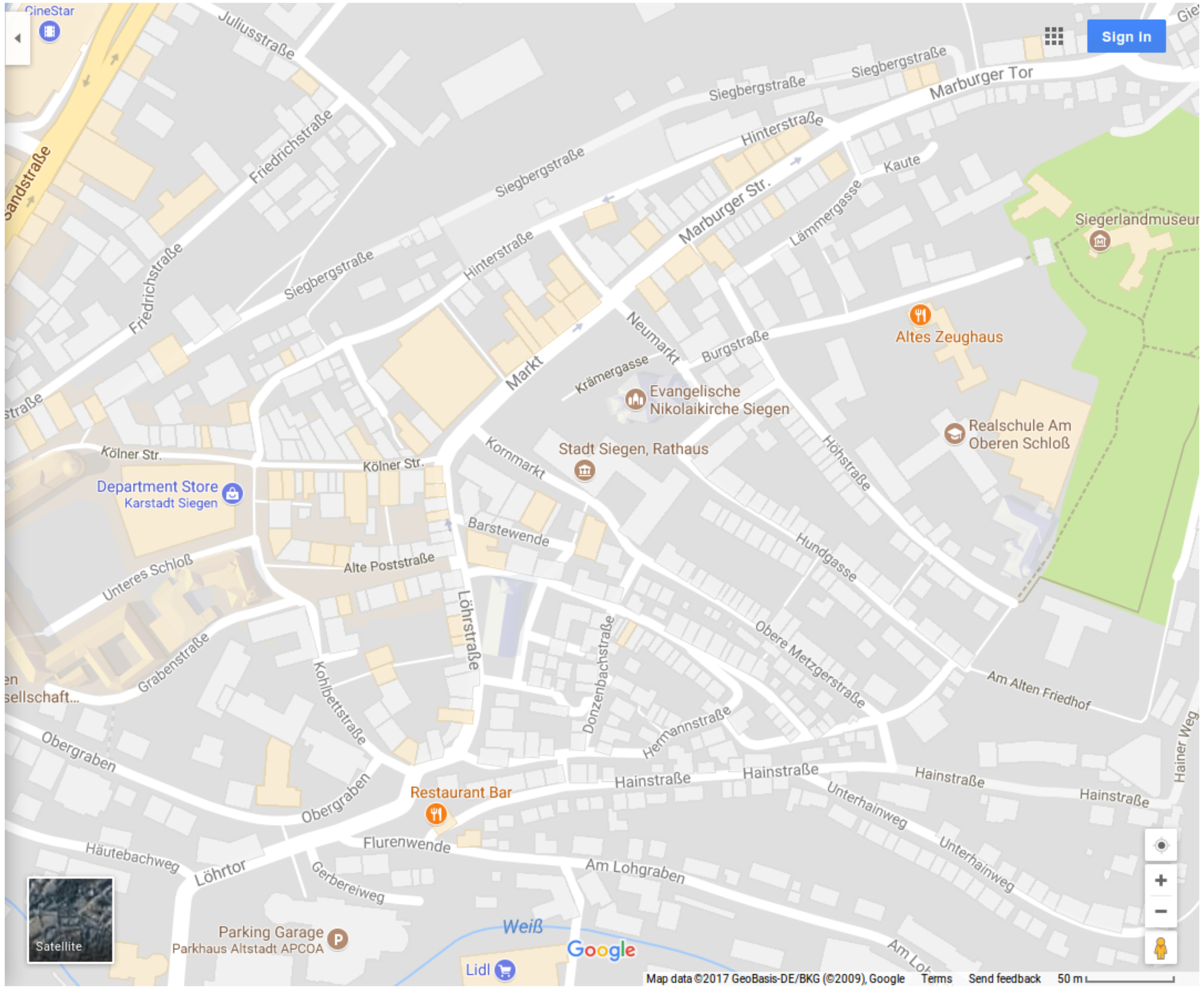
Decision Theory

Modelling

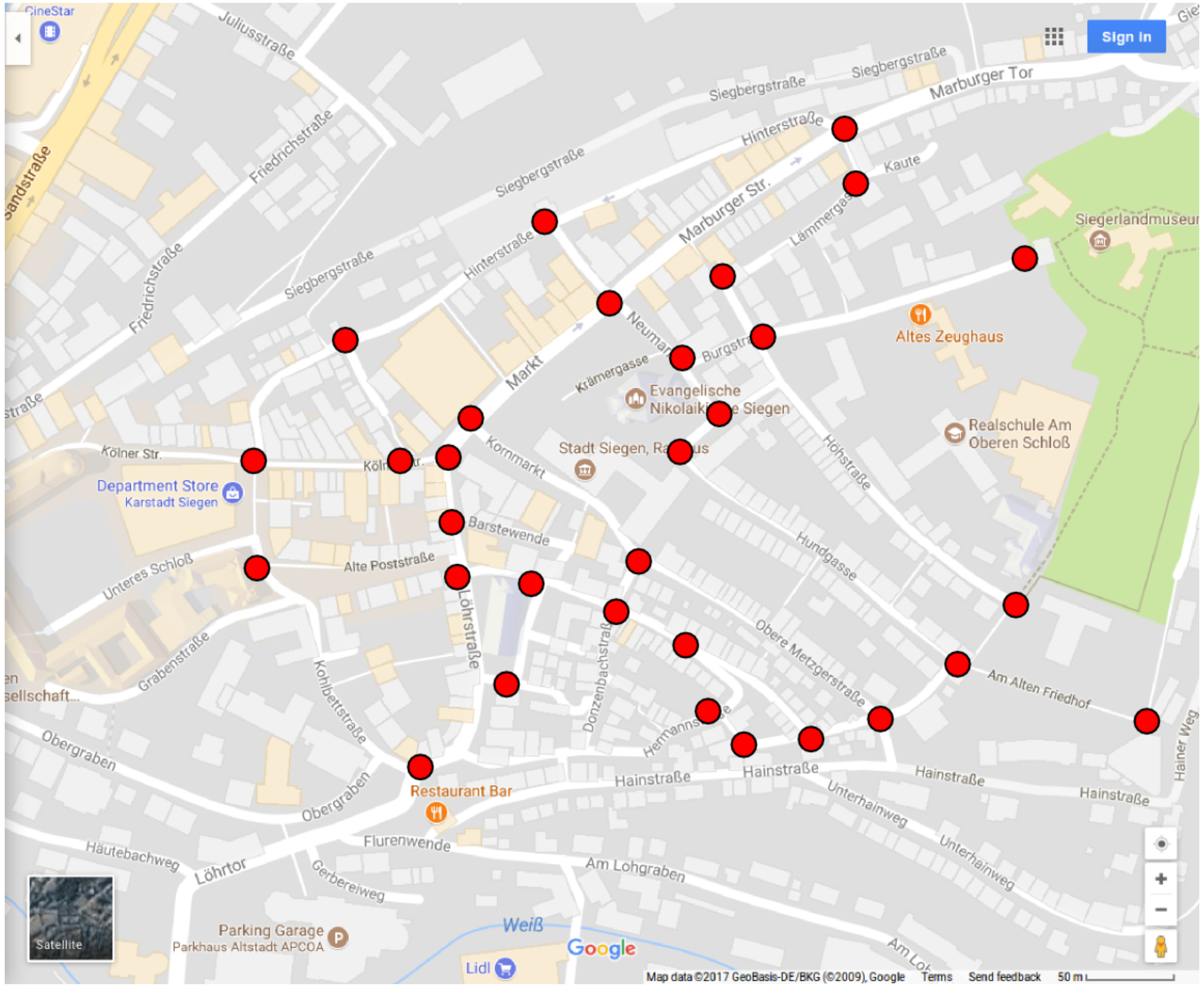
- Represent reality
- Keep only aspects relevant to the purpose
- Description models
- Causality models
- Decision models
- Model cycle

Decision Theory

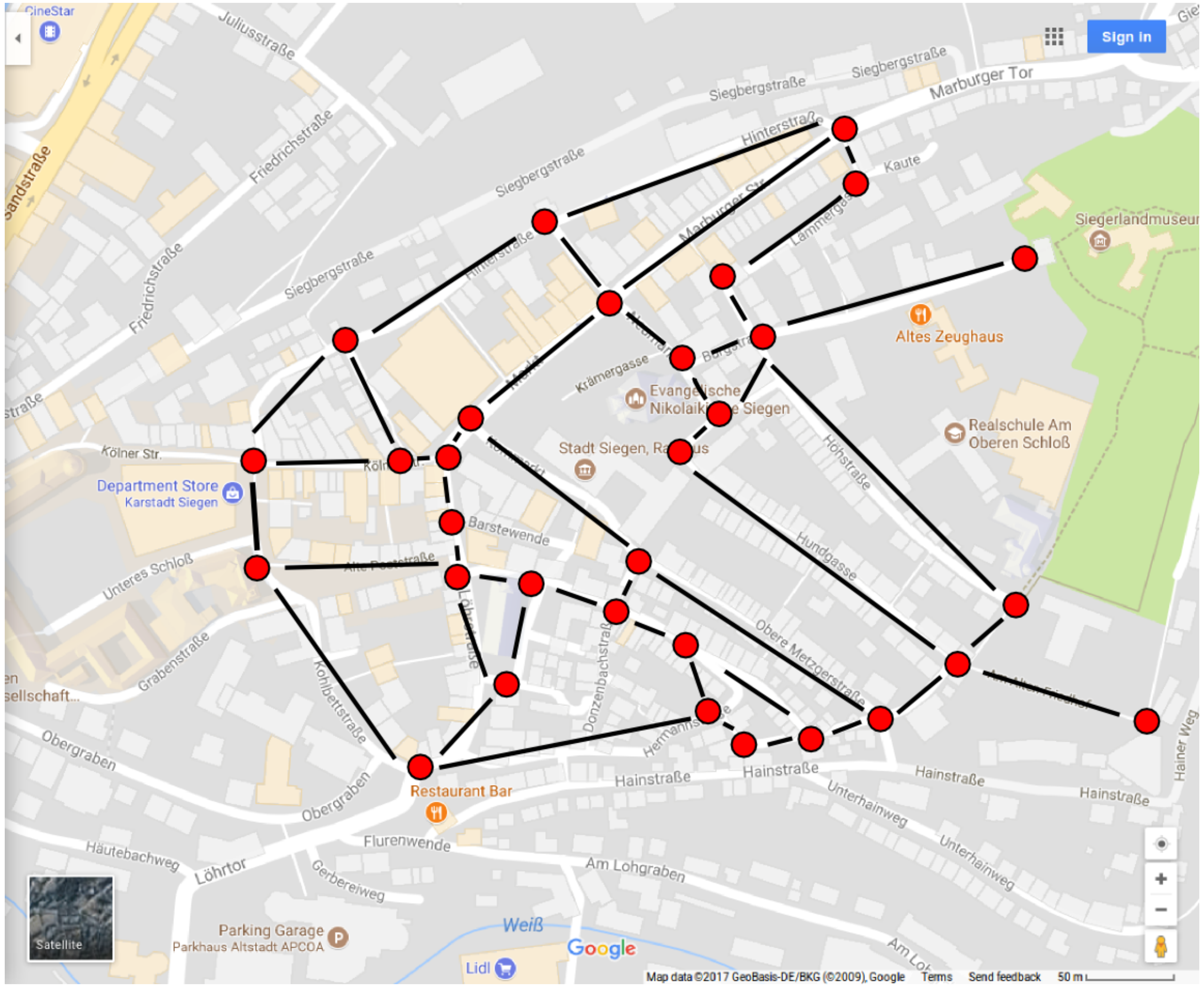
Decision Theory



Decision Theory



Decision Theory



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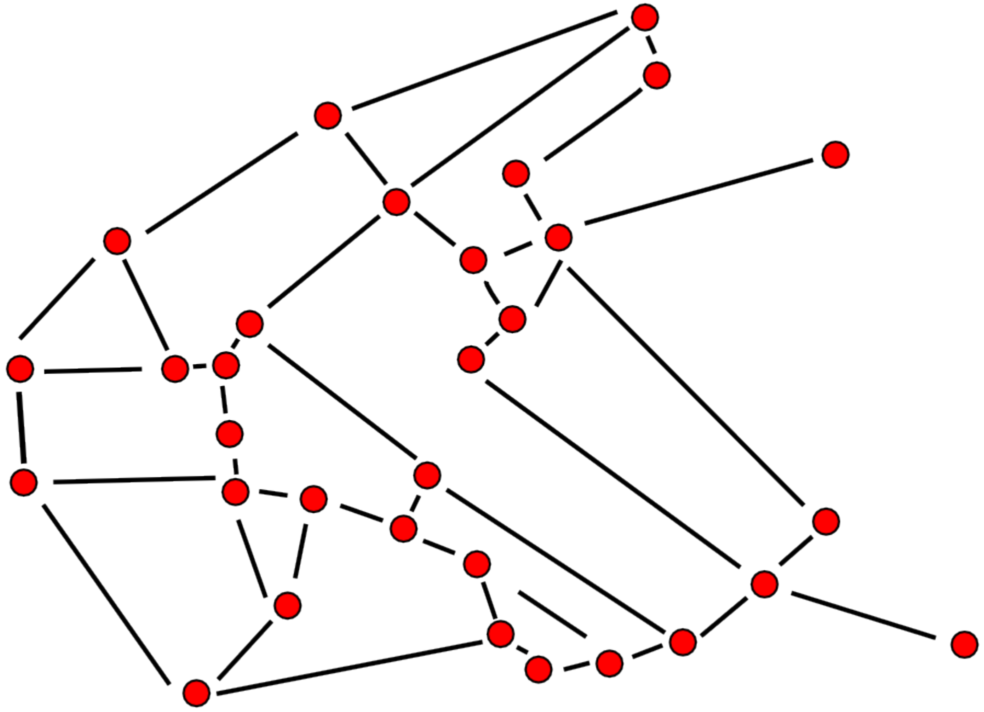
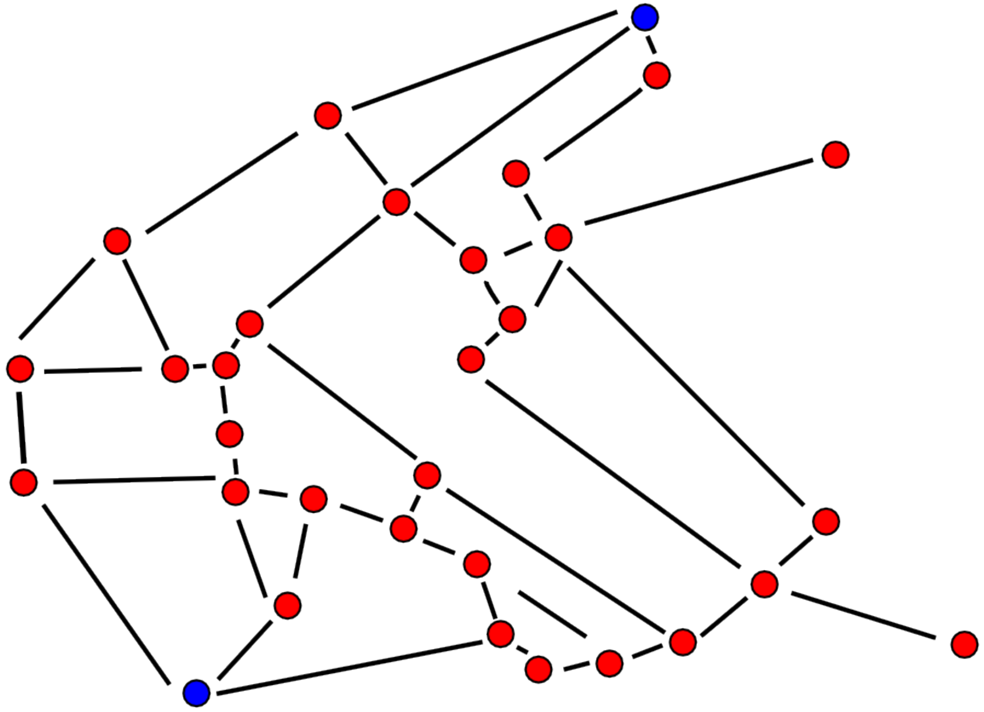
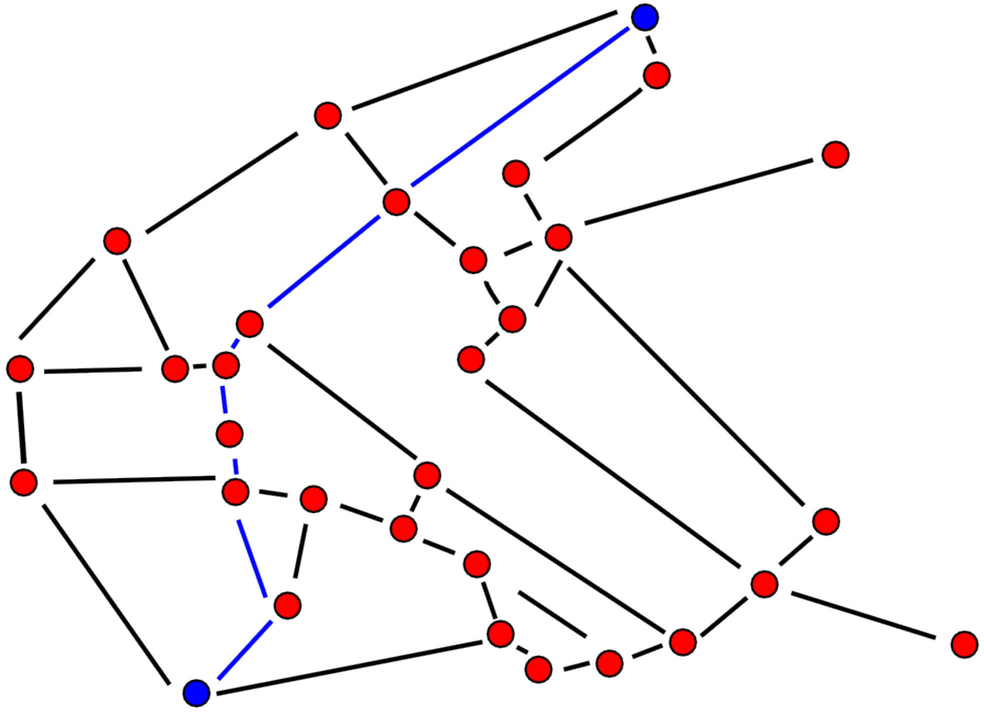


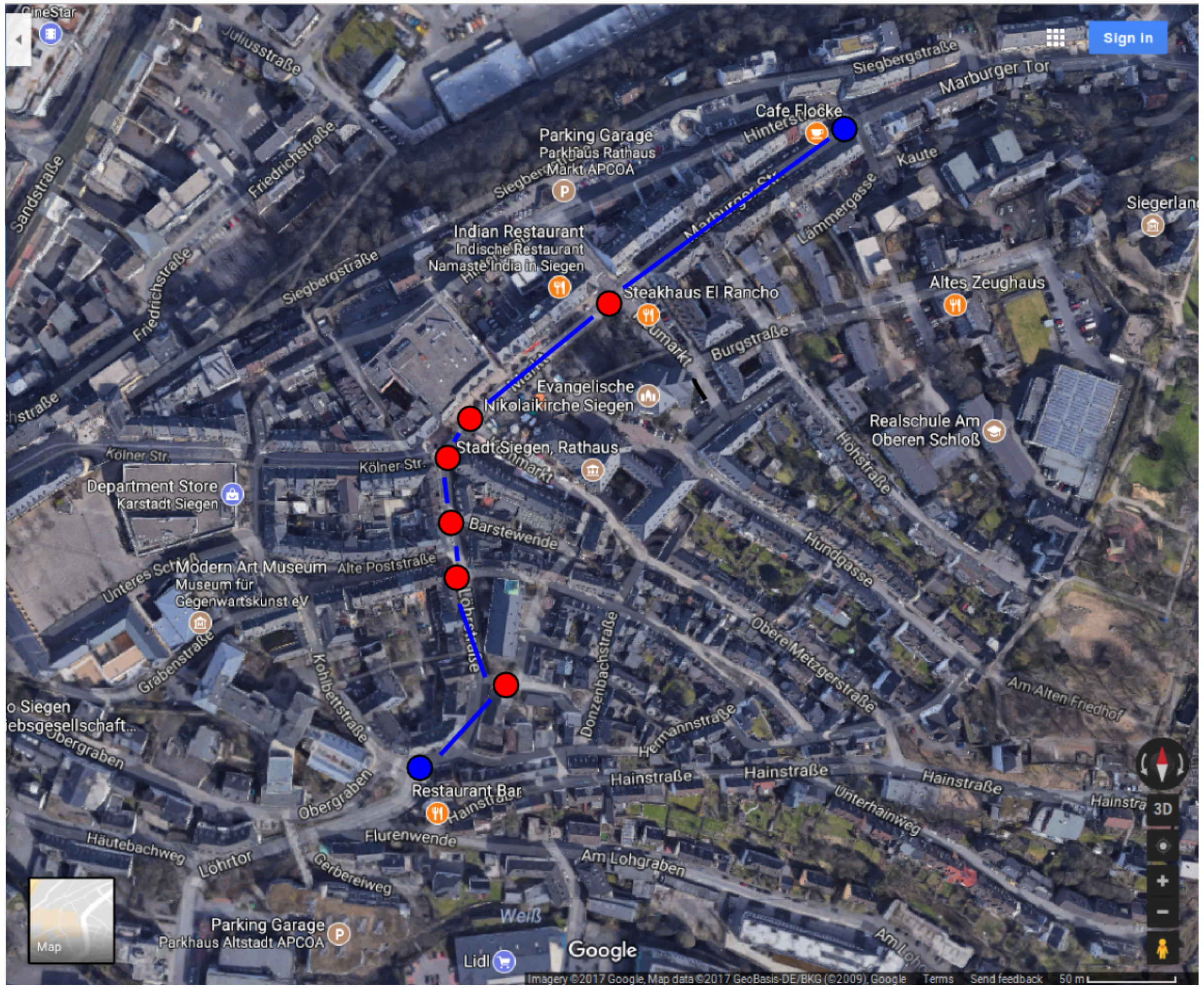
Decision Theory



Decision Theory



Decision Theory



Decision Theory

Scales

- Nominal
- Ordinal
- Interval
- Differential
- Ratio
- Absolute

Decision Theory

Decision Matrix – Basic Version

$$
\begin{array}{c|ccccccc}
 & s_1 & s_2 & \dots & s_j & \dots & s_n \\
\hline
a_1 & e_{11} & e_{12} & \dots & e_{1j} & \dots & e_{1n} \\
a_2 & e_{21} & e_{22} & \dots & e_{2j} & \dots & e_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_i & e_{i1} & e_{i2} & \dots & e_{ij} & \dots & e_{in} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_m & e_{m1} & e_{m2} & \dots & e_{mj} & \dots & e_{mn}
\end{array}
$$

Decision Theory

Decision Matrix – Multiple Criteria

$$
\begin{array}{c|c|c|c}
 & s_1 & \dots & s_n \\
\hline
a_1 & (e_{11}^1, \dots, e_{11}^k) & \dots & (e_{1n}^1, \dots, e_{1n}^k) \\
a_2 & (e_{21}^1, \dots, e_{21}^k) & \dots & (e_{2n}^1, \dots, e_{2n}^k) \\
\vdots & \vdots & \vdots & \vdots \\
a_m & (e_{m1}^1, \dots, e_{m1}^k) & \dots & (e_{mn}^1, \dots, e_{mn}^k)
\end{array}
$$

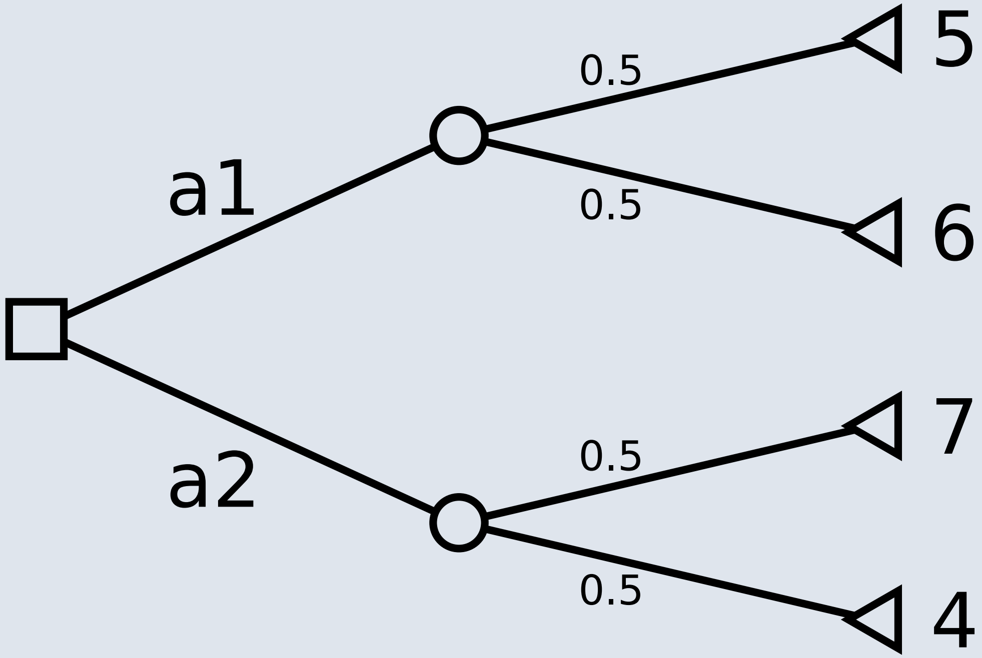
Decision Theory

Decision Matrix – Multiple Criteria and Time Steps

$$
\begin{array}{c|c|c|c}
 & s_1 & \dots & s_n \\
\hline
a_1 & \begin{pmatrix} e_{11}^{11} & \dots & e_{11}^{k1} \\ \vdots & \vdots & \vdots \\ e_{11}^{1t} & \dots & e_{11}^{kt} \end{pmatrix} & \dots & \begin{pmatrix} e_{1n}^{11} & \dots & e_{1n}^{k1} \\ \vdots & \vdots & \vdots \\ e_{1n}^{1t} & \dots & e_{1n}^{kt} \end{pmatrix} \\
\vdots & \vdots & \vdots & \vdots \\
a_m & \begin{pmatrix} e_{m1}^{11} & \dots & e_{m1}^{k1} \\ \vdots & \vdots & \vdots \\ e_{m1}^{1t} & \dots & e_{m1}^{kt} \end{pmatrix} & \dots & \begin{pmatrix} e_{mn}^{11} & \dots & e_{mn}^{k1} \\ \vdots & \vdots & \vdots \\ e_{mn}^{1t} & \dots & e_{mn}^{kt} \end{pmatrix}
\end{array}
$$

Decision Theory

Decision Trees

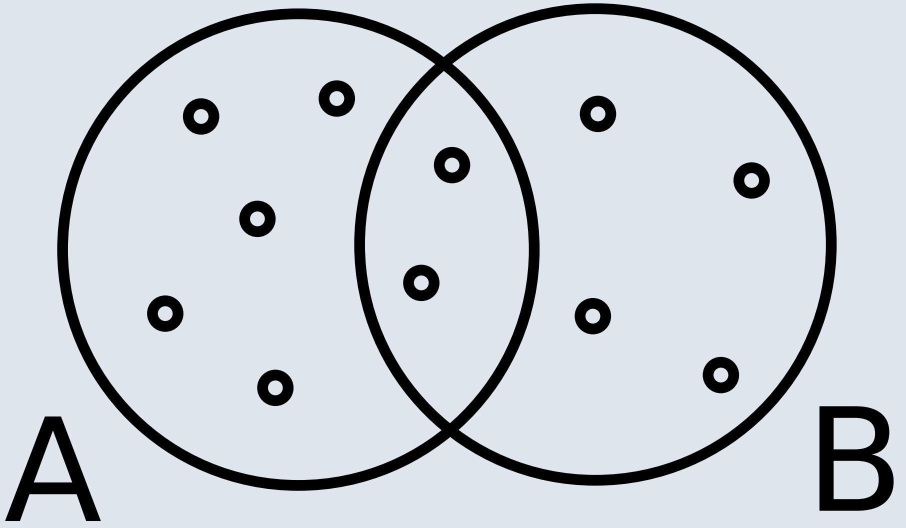


	s_1	s_2
	0.5	0.5
a_1	5	6
a_2	7	4

Decision Theory

Probability Theory

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \emptyset$
- Conditional probability: $P(A|B) = P(A \cap B)/P(B)$
- Independence $\Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$



Decision Theory

Today

- Example decision tree
- Dominance criteria
- Preference functions
- Brief overview of descriptive DT

Decision Theory

Example Decision Tree

- 1981: British ship S.S. Kuniang ran aground in Florida
- New England Electric considered bidding for salvage rights
- Could be repaired for coal transport
- Bid determined using decision tree and simulation

Decision Theory

Example Decision Tree

- 1920 Jones Act: American coastwise trade only with American vessels
- If repair costs are at least three times the salvage value, it qualifies as an American ship
- Repair costs around 15m
- US Coast Guard determines salvage value
- Unknown if less than 5m
- Scrap value certainly less than 5m
- Coast guard will either choose the value of the winning bid or the scrap value

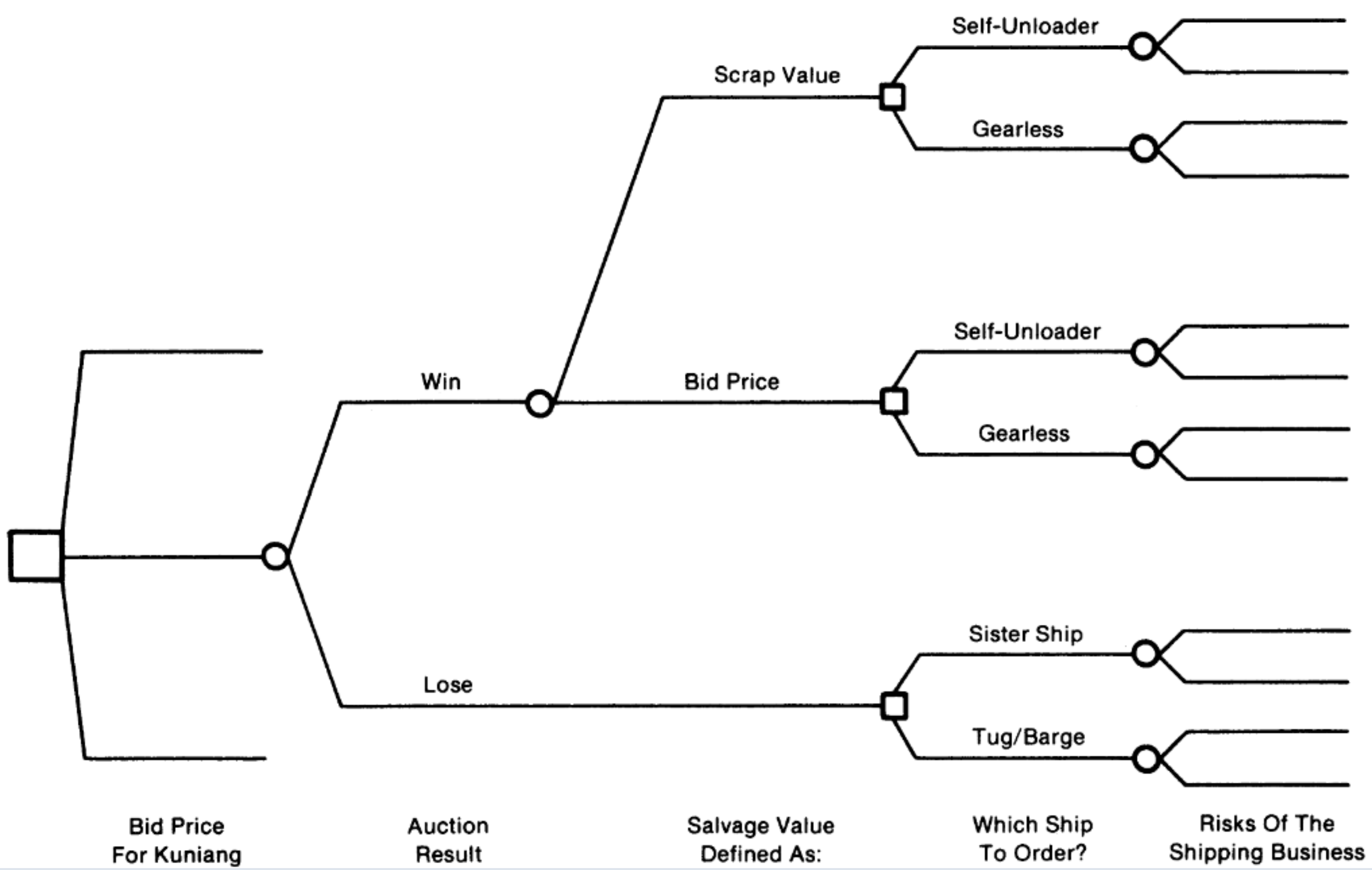
Decision Theory

Example Decision Tree

- Repair costs can be increased:
  - Install additional equipment
  - ”Self-unload” instead of ”gearless”
  - Costs 21m extra
- Alternatively: just buy an American ship
  - Offer from General Dynamics (sister ship)
  - Buy a tug/barge combination

Decision Theory

Example Decision Tree



Decision Theory

Analysis

Bid	Probability of success	Expected value
5	$1/6$	2.85
6	$2/6$	3.15
7	$3/6$	3.05
8	$4/6$	2.40
9	$5/6$	1.30
10	$6/6$	-0.22

Analysis

Bid	Probability of success	Expected value
5	$1/6$	2.85
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- Company bid $6.7m$

Decision Theory

Analysis

Bid	Probability of success	Expected value
5	$1/6$	2.85
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8	$4/6$	2.40
9	$5/6$	1.30
10	$6/6$	-0.22

- Company bid $6.7m$
- Second-highest bid
- Highest bid was $10m$
- Coast Guard set scrap value
- Ship used for grain trade

Decision Theory

Dominance Criteria: Example

Maximize:

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%
a_1	20	30	10	0	20
a_2	0	10	20	10	20
a_3	20	20	-10	0	10
a_4	0	-10	0	0	0

Simplification

- Different types of decision problems can be simplified
- We can find alternatives that can be excluded: dominance or efficiency criteria
- Assumption: no identical alternatives

Decision Theory

Absolute Dominance

- a_1 is definitely preferable to a_2 if the worst outcome is not worse than the best outcome of a_2:

$$\min_{j=1,\ldots,n} e_{1j} \geq \max_{j=1,\ldots,n} e_{2j} \tag{1}$$

- Note: identical alternatives excluded
- Thus, a_1 must be strictly better at least once

Decision Theory

State Dominance

- a_1 is definitely preferable to a_2 if a_1 does not lead to a worse outcome than a_2 in any environmental state:

$$e_{1j} \geq e_{2j} \text{ for all } j = 1, \dots, n \tag{2}$$

- Again: identical alternatives excluded
- Must be strictly better in at least one state

Decision Theory

Relation

- Absolute dominance (1) implies state dominance (2)
- Conversely, state dominance can be satisfied, but not absolute dominance
- Therefore: state dominance is the stronger selection rule, i.e., alternatives can be better restricted

Example (maximization)

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%
a_1	20	30	10	0	20
a_2	0	10	20	10	20
a_3	20	20	-10	0	10
a_4	0	-10	0	0	0

- Which alternatives are dominated?

Decision Theory

Example (maximization)

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20
a_3	20	20	-10	0	10	-10	20
a_4	0	-10	0	0	0	-10	0

- Which alternatives are dominated?

Decision Theory

Example (maximization)

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20
a_3	20	20	-10	0	10	-10	20
a_4	0	-10	0	0	0	-10	0

- Which alternatives are dominated?
- a_4 is dominated by a_1 and a_2 (absolute dominance)

Decision Theory

Example (maximization)

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20
a_3	20	20	-10	0	10	-10	20
a_4	0	-10	0	0	0	-10	0

- Which alternatives are dominated?
- a_4 is dominated by a_1 and a_2 (absolute dominance)
- a_3 is dominated by a_1 (state dominance)

Decision Theory

Example (maximization)

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20
a_3	20	20	-10	0	10	-10	20
a_4	0	-10	0	0	0	-10	0

- Which alternatives are dominated?
- a_4 is dominated by a_1 and a_2 (absolute dominance)
- a_3 is dominated by a_1 (state dominance)
- We only need to consider a_1 and a_2

Decision Theory

Probability Dominance

- a_1 is preferred to a_2 if, for every real number e, the probability of obtaining a result at least e is not smaller for a_1 than for a_2, and for at least one e' is strictly greater:

$$\begin{aligned} P(e_1 \geq e) &\geq P(e_2 \geq e) && \text{for all } e \in \mathbb{R} \\ P(e_1 \geq e') &> P(e_2 \geq e') && \text{for at least one } e' \in \mathbb{R} \end{aligned} \tag{3}$$

- Verifiable based on the result distributions of each alternative

Decision Theory

Example

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20

- Cumulative distribution function for a_1:

e	
$P(e_1 \geq e)$	

Decision Theory

Example

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20

- Cumulative distribution function for a_1:

e	≤ 0	$(0, 10]$	$(10, 20]$	$(20, 30]$	> 30
$P(e_1 \geq e)$	1	0.8	0.5	0.1	0

Decision Theory

Example

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20

- Cumulative distribution function for a_1:

e	≤ 0	$(0, 10]$	$(10, 20]$	$(20, 30]$	> 30
$P(e_1 \geq e)$	1	0.8	0.5	0.1	0

- Cumulative distribution function for a_2:

e	≤ 0	$(0, 10]$	$(10, 20]$	> 20
$P(e_2 \geq e)$	1	0.8	0.5	0

Decision Theory

Example

	s_1 20%	s_2 10%	s_3 30%	s_4 20%	s_5 20%	min	max
a_1	20	30	10	0	20	0	30
a_2	0	10	20	10	20	0	20

- Cumulative distribution function for a_1:

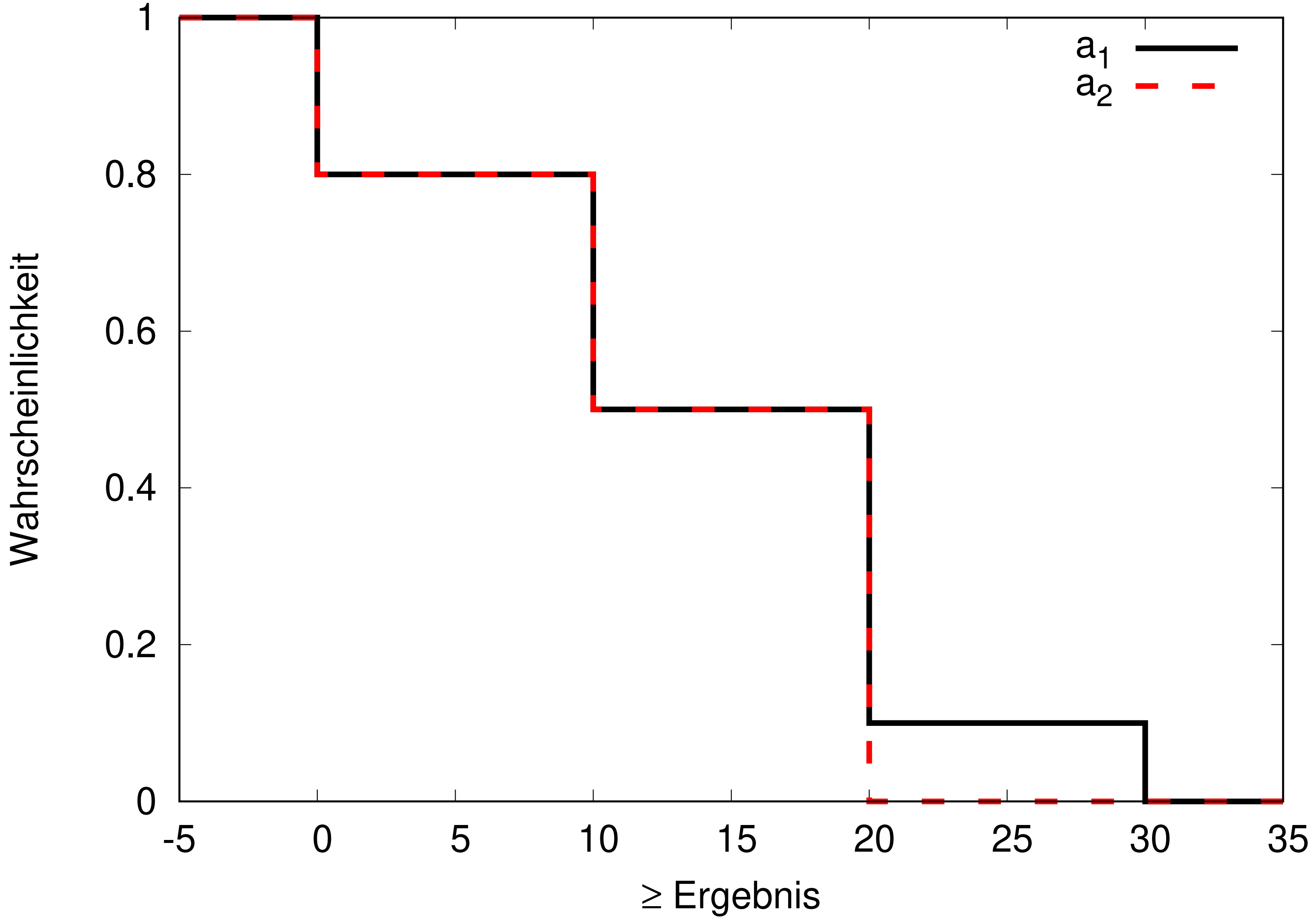
e	≤ 0	$(0, 10]$	$(10, 20]$	$(20, 30]$	> 30
$P(e_1 \geq e)$	1	0.8	0.5	0.1	0

- Cumulative distribution function for a_2:

e	≤ 0	$(0, 10]$	$(10, 20]$	> 20
$P(e_2 \geq e)$	1	0.8	0.5	0

- a_2 is also dominated

Decision Theory



Decision Theory

Connection

- If absolute dominance (1) or state dominance (2) is present, then probability dominance (3) is also satisfied
- However, (3) can be fulfilled without (1) or (2) being satisfied
- Thus, (3) is the strongest criterion

Decision Theory

Another Example (maximization)

- Check the three dominance criteria

	s_1 $p_1 = 0.2$	s_2 $p_2 = 0.3$	s_3 $p_3 = 0.1$	s_4 $p_4 = 0.4$
a_1	0	24	0	36
a_2	20	20	20	0
a_3	6	3	12	23
a_4	0	0	66	0

$$\min_{j=1,\ldots,n} e_{1j} \geq max_{j=1,\ldots,n} e_{2j} \tag{1}$$

$$e_{1j} \geq e_{2j} \qquad \text{for all } j = 1, \ldots, n \tag{2}$$

$$\begin{aligned} P(e_1 \geq e) &\geq P(e_2 \geq e) && \text{for all } e \in \mathbb{R} \\ P(e_1 \geq e') &> P(e_2 \geq e') && \text{for at least one } e' \in \mathbb{R} \end{aligned} \tag{3}$$

Decision Theory

Solution

- (1): no dominance
- (2): no dominance
- (3):

e	≤ 0	$(0, 24]$	$(24, 36]$	> 36
$P(e_1 \geq e)$	1	0.7	0.4	0

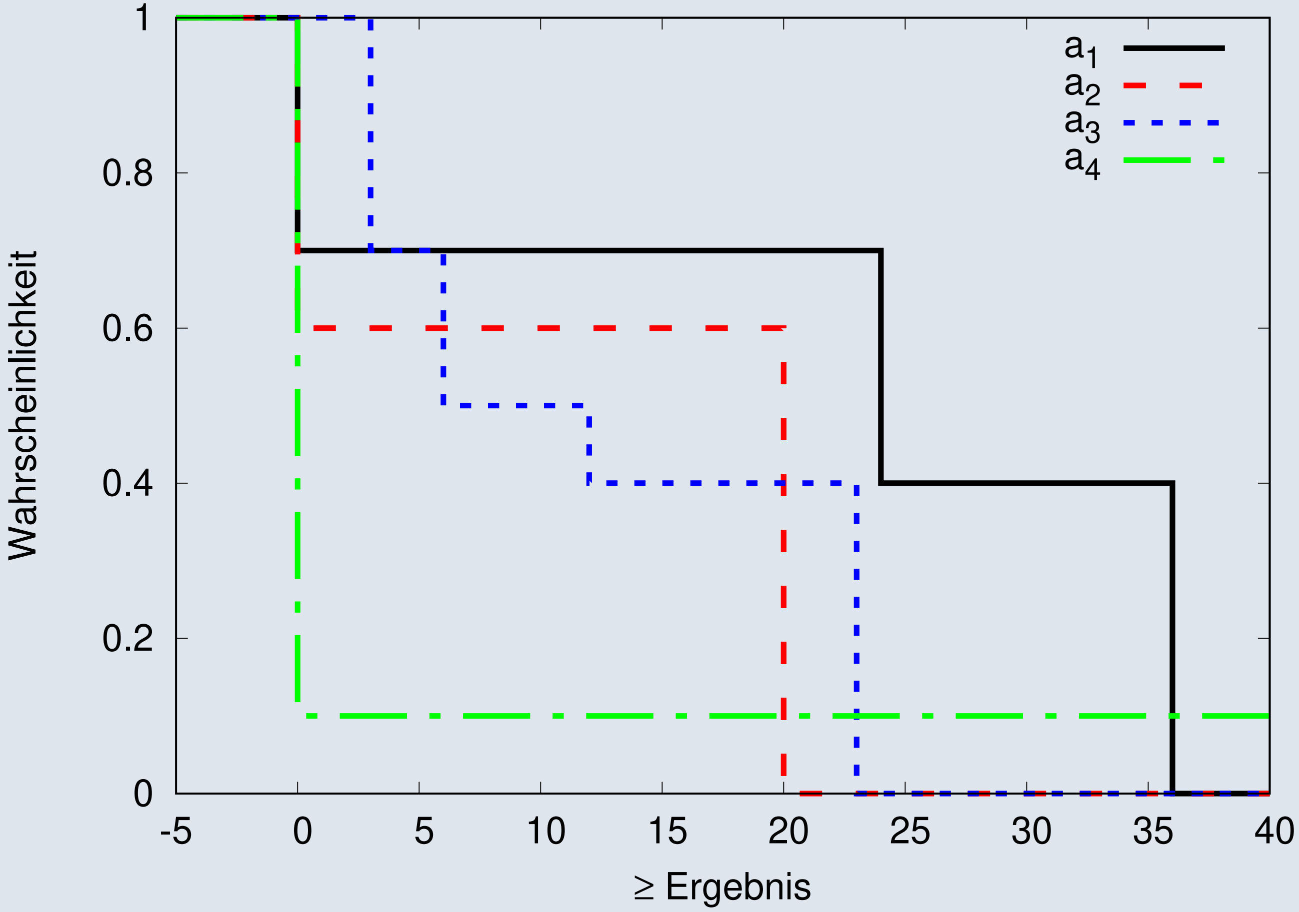
e	≤ 0	$(0, 20]$	> 20
$P(e_2 \geq e)$	1	0.6	0

e	≤ 3	$(3, 6]$	$(6, 12]$	$(12, 23]$	> 23
$P(e_3 \geq e)$	1	0.7	0.5	0.4	0

e	≤ 0	$(0, 66]$	> 66
$P(e_4 \geq e)$	1	0.1	0

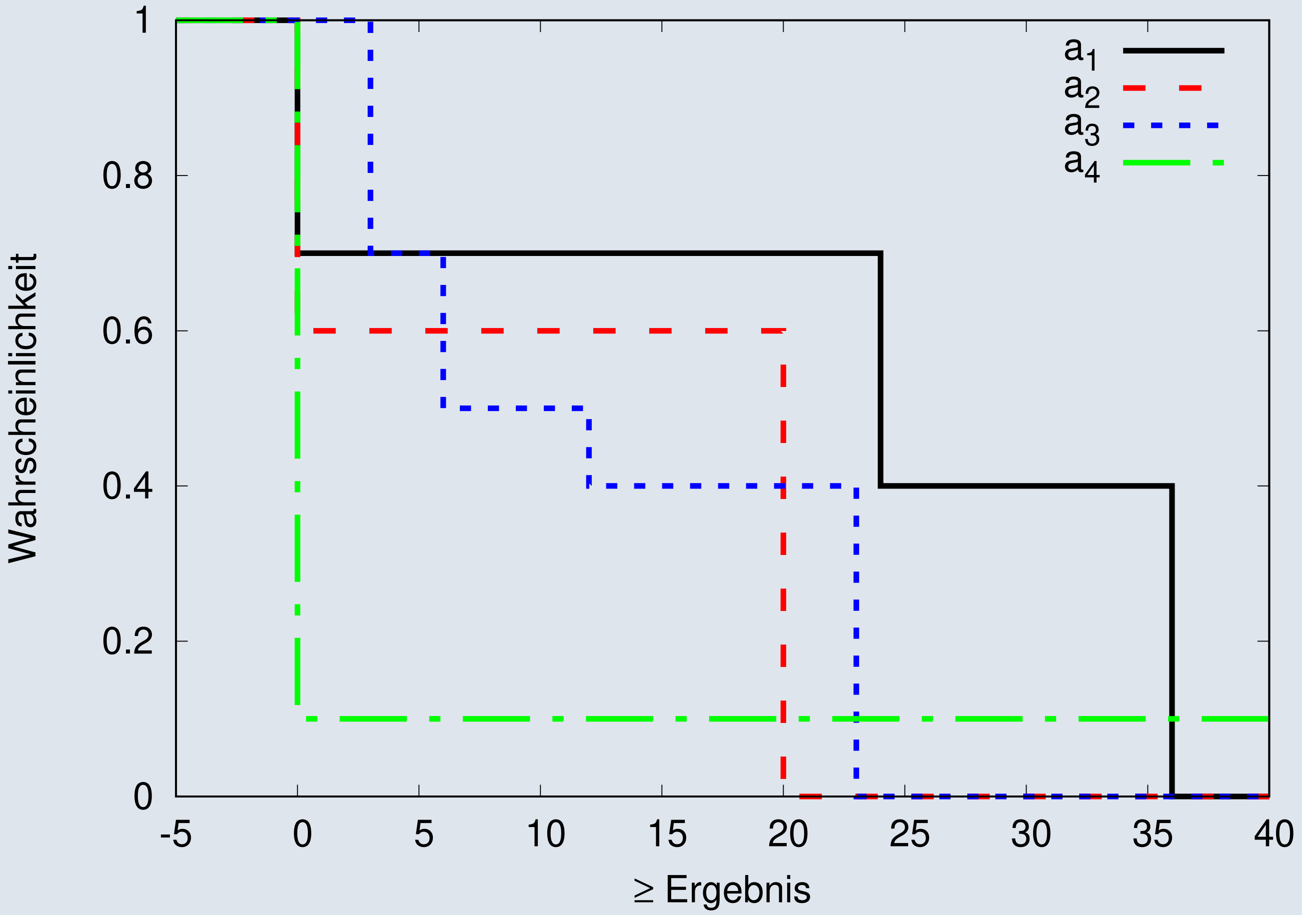
Decision Theory

Solution



Decision Theory

Solution



a_1 dominates a_2

Decision Theory

Last Time

How do I convert results

$$g(a_i, s_j) = \begin{pmatrix} e_{ij}^{11} & \dots & e_{ij}^{k1} \\ \vdots & \vdots & \vdots \\ e_{ij}^{1t} & \dots & e_{ij}^{kt} \end{pmatrix}$$

into a single value $\Phi(a_i)$?

- Height preference relation
- Species preference relation
- Time preference
- Risk or uncertainty preference relation

Decision Theory

Preference Functions

- First, remove dominated alternatives
- If relations $>$, $\sim$, $<$ are known on alternatives, then

$$\begin{aligned} \Phi(a_i) > \Phi(a_j) &\Leftrightarrow a_i > a_j \\ \Phi(a_i) = \Phi(a_j) &\Leftrightarrow a_i \sim a_j \\ \Phi(a_i) < \Phi(a_j) &\Leftrightarrow a_i < a_j \end{aligned}$$

- However, Φ may not be unique

Decision Theory

Preference Functions

- The decision problem is now

$$\max_{i \in [m]} \Phi(a_i)$$

- $[m] := \{1, \ldots, m\}$ is an abbreviated notation
- ”determine the action that gives the highest preference value”

Decision Theory

Examples of Preference Functions

- Let $g(a_i, s_j) = e_{ij} \in \mathbb{R}$
- Choose average

$$\Phi(a_i) = \frac{1}{n} \sum_{j \in [n]} e_{ij}$$

- With probabilities p_j:

$$\Phi(a_i) = \sum_{j \in [n]} p_j e_{ij}$$

Decision Theory

Examples of preference functions

- Let $g(a_i, s_j) = e_{ij} \in \mathbb{R}$
- Find the best result:
$$y_1(a_i) = \max\{e_{ij} : j \in [n]\}$$
- Find the worst result:
$$y_2(a_i) = \min\{e_{ij} : j \in [n]\}$$
- Determine Φ as a function of $y_1(a_i)$ and $y_2(a_i)$
$$\Phi(a_i) = \Phi'(y_1(a_i), y_2(a_i))$$
- How might that look?
- Will learn more examples later

Decision Theory

Types of Optimization Criteria

- Extremization

$$\max_{i\in[m]} \Phi(a_i) \text{ or } \min_{i\in[m]} \Phi(a_i)$$

- Satisficing

$$\Phi(a_i) \geq y^{\text{min}} \text{ or } \Phi(a_i) \leq y^{\text{max}}$$

- Fixation

$$\Phi(a_i) = y^{\text{fix}}$$

Decision Theory

Extremization

- Objective function:

$$\max_{i\in[m]} \Phi(a_i) = -\min_{i\in[m]} -\Phi(a_i)$$

- Minimization and maximization can be transformed into each other

Decision Theory

Satisficing

$$\Phi(a_i) \geq y^{\text{min}} \text{ or } \Phi(a_i) \leq y^{\text{max}}$$

- If all consequences are known, it's better to choose the largest
- Fixation when a specific level is desired
- Satisficing makes sense when consequences are not precisely known
- Equivalent:

$$\max_{i \in [m]} \Phi'(a_i) \text{ with } \Phi'(a_i) = \begin{cases} 1 & \text{, if } \Phi(a_i) \geq y^{\text{min}} \\ 0 & \text{, if } \Phi(a_i) < y^{\text{min}} \end{cases}$$

Decision Theory

Example

	s_1	s_2	s_3
a_1	10	30	30
a_2	25	20	15

- Let's say we don't want to go below the value $y^{\min} = 12$
- For a_1, a minimum of 10 is possible
- For a_2, a minimum of 15 is possible
- Only a_2 satisfies the satisficing condition

Decision Theory

Example

	s_1	s_2	s_3
a_1	10	30	30
a_2	25	20	15

- Formal:

$$\Phi(a_i) = \min\{e_{ij} : j \in [n]\}$$

- Requirement:

$$\Phi(a_i) \geq 12$$

- Equivalent:

$$\max_{i \in [m]} \Phi'(a_i) \text{ with } \Phi'(a_i) = \begin{cases} 1 & \text{, if } \min_{j \in [n]} e_{ij} \geq 12 \\ 0 & \text{, if } \min_{j \in [n]} e_{ij} < 12 \end{cases}$$

Fixation

- Achieve exactly the value y^{fix}
- Equivalent:

$$\max_{i \in [m]} \Phi'(a_i) \text{ with } \Phi'(a_i) = \begin{cases} 1 & \text{, if } \Phi(a_i) = y^{\text{fix}} \\ 0 & \text{, if } \Phi(a_i) \neq y^{\text{fix}} \end{cases}$$

- Alternatively, we can choose the distance to y^{fix} to be as small as possible

Decision Theory

Fixation

- Alternatively, we can choose the distance to y^{fix} to be as small as possible

$$\max_{i \in [m]} -\left|\Phi(a_i) - y^{\text{fix}}\right|$$

- Or:

$$\max_{i \in [m]} -(\Phi(a_i) - y^{\text{fix}})^2$$

Decision Theory

Combinations

- For the combination of best/worst results, we use

$$\Phi(a_i) = \Phi'(y_1(a_i), y_2(a_i))$$

- In general, metrics $y_h(a_i)$, $h = 1, \ldots, H$ can be found, which are then combined into a single criterion
- Amalgamation: assign a weight to each subsidiary target variable
- Combination of extremization, fixation, satisficing possible
- Lexicographic rule:
  - First optimize a target variable y_1
  - Then optimize the next target variable y_2, keeping y_1 constant, etc.
  - Analogous to dictionary search

Decision Theory

Example

	s_1	s_2	s_3	y_1	y_2	y_3
a_1	5	7	0	4	0	7
a_2	1	3	8	4	1	8
a_3	4	2	3	3	2	4
a_4	7	1	4	4	1	7

- First target variable y_1: average profit (maximize)
- Second target variable y_2: smallest profit (maximize)
- Third target variable y_3: largest profit (maximize)

Decision Theory

Example

	s_1	s_2	s_3	y_1	y_2	y_3
a_1	5	7	0	4	0	7
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- First target variable y_1: average profit (maximize)
- Second target variable y_2: smallest profit (maximize)
- Third target variable y_3: largest profit (maximize)
- For y_1, a_1, a_2, a_4 are the best

Decision Theory

Example

	s_1	s_2	s_3	y_1	y_2	y_3
a_1	5	7	0	4	0	7
a_2	1	3	8	4	1	8
a_3	4	2	3	3	2	4
a_4	7	1	4	4	1	7

- First target variable y_1: average profit (maximize)
- Second target variable y_2: smallest profit (maximize)
- Third target variable y_3: largest profit (maximize)
- For y_1, a_1, a_2, a_4 are the best
- Of these, a_2 and a_4 maximize the value y_2

Decision Theory

Example

	s_1	s_2	s_3	y_1	y_2	y_3
a_1	5	7	0	4	0	7
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a_3	4	2	3	3	2	4
a_4	7	1	4	4	1	7

- First target variable y_1: average profit (maximize)
- Second target variable y_2: smallest profit (maximize)
- Third target variable y_3: largest profit (maximize)
- For y_1, a_1, a_2, a_4 are the best
- Of these, a_2 and a_4 maximize the value y_2
- Of these, only a_2 maximizes the value y_3

Lexicographic Rule

- Equivalently, the same effect can be achieved by assigning appropriate weights to the three criteria:

$$\max_{i\in[m]} \Phi(a_i) = 100y_1(a_i) + 10y_2(a_i) + y_3(a_i)$$

- Weights must be chosen sufficiently large!

	s_1	s_2	s_3	y_1	y_2	y_3	Φ
a_1	5	7	0	4	0	7	407
a_2	1	3	8	4	1	8	418
a_3	4	2	3	3	2	4	324
a_4	7	1	4	4	1	7	417

Decision Theory

Procedure

1. For each possible action a_i, determine the distribution of possible outcomes $(e_{i1}, \dots, e_{in})$
2. Determine the metrics $y_h(a_i)$
3. According to the decision rule, some of $y_h(a_i)$ are included in an amalgamation function to be maximized, while others are subject to satisficing or fixation conditions
4. Determine the optimal alternative:
   - Exclude special alternatives that do not satisfy all satisficing and fixation conditions
   - Choose from the remaining alternatives the one that maximizes the amalgamation function

Decision Theory

So Far

- Up to now: basics of decision theory
- Next: decision under certainty
- But first: descriptive DT interlude

Decision Theory

Experiment

- We conduct a short experiment
- Record the last three digits of your phone number

Decision Theory

Experiment

- We conduct a short experiment
- Record the last three digits of your phone number
- Add 400
- Take the result as a year

Decision Theory

Experiment

- We conduct a short experiment
- Record the last three digits of your phone number
- Add 400
- Take the result as a year
- Did Attila the Hun die before or after this year?

Decision Theory

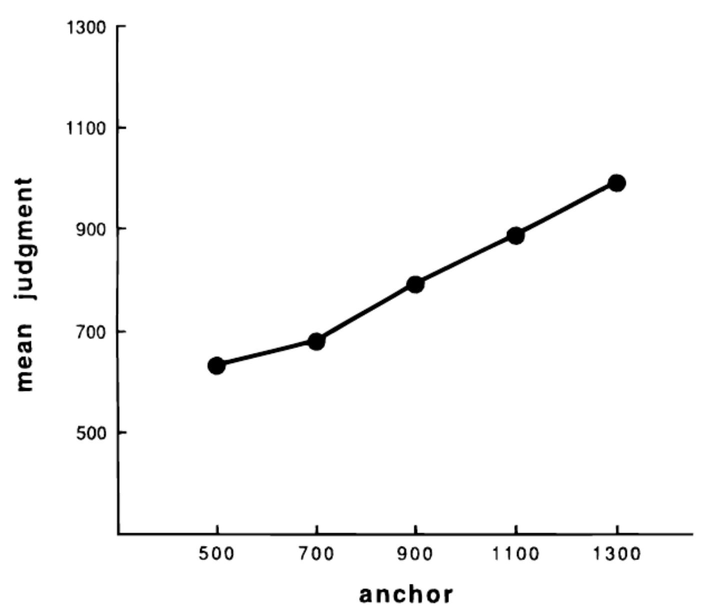
Experiment

- We conduct a short experiment
- Record the last three digits of your phone number
- Add 400
- Take the result as a year
- Did Attila the Hun die before or after this year?
- Estimate the exact year

Decision Theory

Experiment

- The answer is 453
- The phone number (anchor) has nothing to do with the year to be estimated
- Anchoring effect:



Decision Theory

Decision Making in Judges

- Participants:
  - 52 students in the administrative science supplementary study for legal trainees
- Experiment:
  - You are a judge in the following case: Lena M. has stolen from the supermarket for the 12th time. The prosecutor demands X months in prison, and the defendant's lawyer requests 1 month in prison.
  - $X \in \{3, 9\}$ is determined by the participant using a die

Decision Theory

Decision Making in Judges

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Result

	prosecutor's demand	
	3 months	9 months
ruling: avg. (std. deviation)	5.28 (3.21)	7.81 (3.51)
how confident are you? From 1, not confident at all, to 9, absolutely confident.	5.87 (1.86)	

Decision Theory

Anchoring Effect

Observed in:

- Estimation of probabilities
- Self and others' assessments
- Assessment of spouse's preferences
- Prediction of performance
- ...

Decision Theory

Cognitive Bias

Linda is 31, single, straightforward, and very intelligent. In college, she majored in philosophy. As a student, she was involved in anti-discrimination and other social issues and participated in anti-nuclear demonstrations. Which statement is more likely:

Cognitive Bias

Linda is 31, single, straightforward, and very intelligent. In college, she majored in philosophy. As a student, she was involved in anti-discrimination and other social issues and participated in anti-nuclear demonstrations. Which statement is more likely:

1. Linda is a bank teller
2. Linda is a bank teller and actively involved in the women's movement

Decision Theory

Cognitive Bias

- Option 2 is often considered more likely
- From the perspective of probability theory, event 2 is included in 1
- Therefore, 2 cannot be more likely than 1

Decision Theory

Ultimatum Game

Player A receives 10 euros in coins. She must offer an amount (0 to 10 euros) to Player B.

- Player B accepts: both receive their share
- Player B rejects: both lose all the money

Decision Theory

Ultimatum Game

- According to decision theory rules, Player B should always accept (positive gain)
- In practice, this is not the case (rejection at 1 or 2 euros)
- Player A usually offers 4 or 5 euros
- Even in the variant where Player B does not need to agree

Decision Theory

Two Systems

- Literature on the topic: Daniel Kahneman, ”Thinking, Fast and Slow”
- Numerous experiments on the ”two systems” of decision-making
- Here are some of them

Decision Theory

Problem 1

Questionnaire A

Choose between:

a) 25% chance to win \$240 and 75% chance to lose \$760

b) 25% chance to win \$250 and 75% chance to lose \$750

Questionnaire B

Decision (i). Choose between:

a) A sure gain of \$240

b) 25% chance to gain \$1,000 and 75% chance to gain nothing

Decision (ii). Choose between:

c) A sure loss of \$750

d) 75% chance to lose \$1,000 and 25% chance to lose nothing

Decision Theory

Problem 1

Questionnaire A

Choose between:

a) 25% chance to win \$240 and 75% chance to lose \$760 (0%)
b) 25% chance to win \$250 and 75% chance to lose \$750 (100%)

Questionnaire B

Decision (i). Choose between:

a) A sure gain of \$240 (84%)
b) 25% chance to gain \$1,000 and 75% chance to gain nothing(16%)

Decision (ii). Choose between:

c) A sure loss of \$750 (13%)
d) 75% chance to lose \$1,000 and 25% chance to lose nothing (87%)

Decision Theory

Analysis

- Questionnaire A: option B is clearly better than A
- Principle: If an alternative B is better than alternative A in every scenario, then B is preferable to A.
- Questionnaire B:
  - Most common variant is a)+d)
  - In combination: 25% win \$240, 75% lose \$760
  - Rarest variant is b)+c)
  - In combination: 25% win \$250, 75% lose \$750
  - Just like on questionnaire A!

Decision Theory

Problem 2

Questionnaire A

Consider the following two-stage game. In the first stage, there is a 75% chance to end the game without winning anything and a 25% chance to move into the second stage. If you reach the second stage you have the choice between:

a) a sure win of $30

b) 80% chance to win $45

Your choice must be made before the game starts.

Questionnaire B

Which of the following options do you prefer?

a) 25% chance to win $30

b) 20% chance to win $45

Decision Theory

Problem 2

Questionnaire A

Consider the following two-stage game. In the first stage, there is a 75% chance to end the game without winning anything and a 25% chance to move into the second stage. If you reach the second stage you have the choice between:

a) a sure win of $30 (74%)

b) 80% chance to win $45 (26%)

Your choice must be made before the game starts.

Questionnaire B

Which of the following options do you prefer?

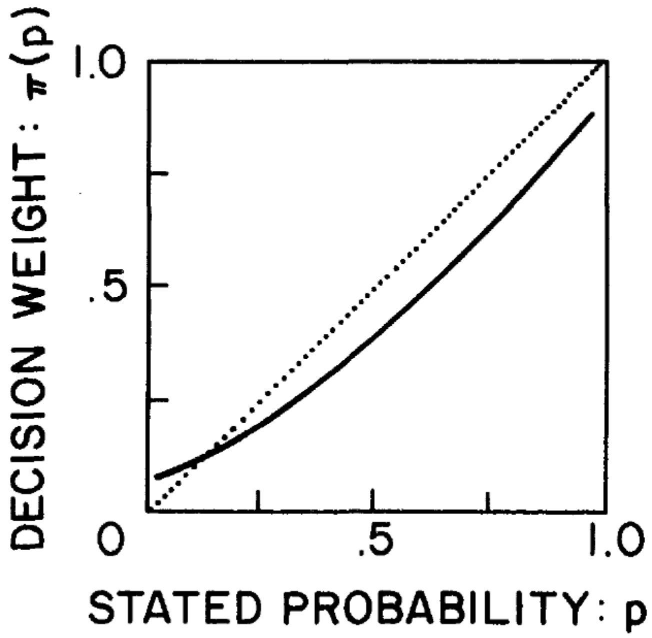
a) 25% chance to win $30 (42%)

b) 20% chance to win $45 (58%)

Decision Theory

Analysis

- Questionnaire A:
  - A gives \$30 with probability $0.25 \cdot 1 = 0.25$
  - B gives \$45 with probability $0.25 \cdot 0.8 = 0.2$
- Same question as on questionnaire B
- Observation: evaluate probabilities nonlinearly



Decision Theory

Problem 3

Questionnaire A

Imagine that you have decided to see a play and paid the admission price of $10 per ticket. As you enter the theater, you discover that you have lost the ticket. The seat was not marked, and the ticket cannot be recovered.
Would you pay $10 for another ticket?

- Yes
- No

Questionnaire B

Imagine that you have decided to see a play where admission is $10 per ticket. As you enter the theater, you discover that you have lost a $10 bill.
Would you still pay $10 for a ticket for the play?

- Yes
- No

Decision Theory

Problem 3

Questionnaire A	**Questionnaire B**
Imagine that you have decided to see a play and paid the admission price of \$10 per ticket. As you enter the theater, you discover that you have lost the ticket. The seat was not marked, and the ticket cannot be recovered. Would you pay \$10 for another ticket? • Yes (46%) • No (54%)	Imagine that you have decided to see a play where admission is \$10 per ticket. As you enter the theater, you discover that you have lost a \$10 bill. Would you still pay \$10 for a ticket for the play? • Yes (88%) • No (12%)

Decision Theory

Analysis

- Both alternatives are financially equal
- Different evaluation: in questionnaire A, costs seem to increase to $20
- Emotions: regret, frustration, satisfaction/contentment
- Similar: sunk cost fallacy

Decision Theory

Problem 4

Questionnaire A	Questionnaire B
Would you accept a gamble that offers a 10% chance to win $95 and a 90% chance to lose $5?	Would you pay $5 to participate in a lottery that offers a 10% chance to win $100 and a 90% chance to win nothing?

Problem 4

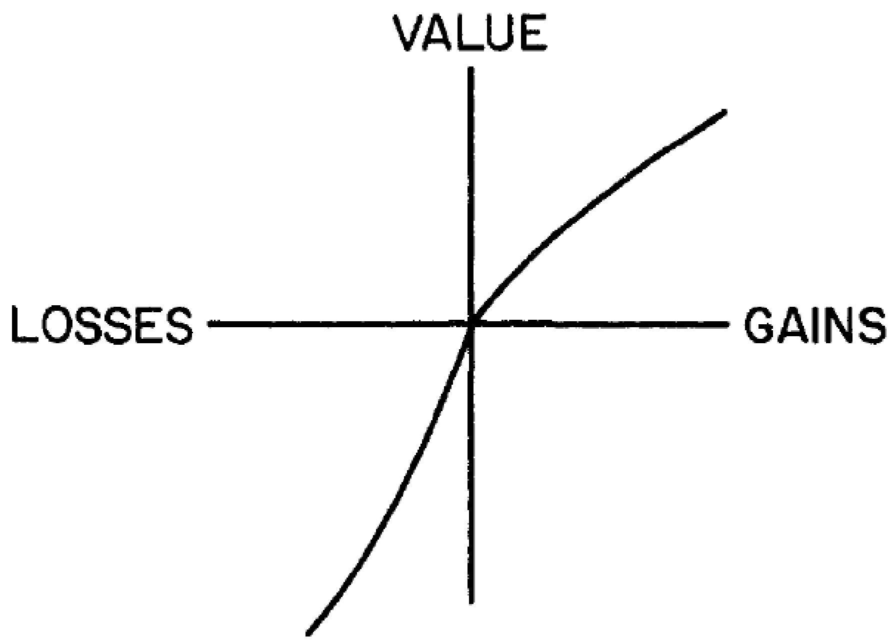
Questionnaire A	Questionnaire B
Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance to lose \$5?	Would you pay \$5 to participate in a lottery that offers a 10% chance to win \$100 and a 90% chance to win nothing?

42% give different answers

Decision Theory

Analysis

- Lottery questionnaire B:
  - 10% win \$100 - \$5 fee = 10% win \$95
  - 90% win \$0 - \$fee = 90% loss \$5
- Both alternatives are equal again
- Gain and loss are not valued equally
- Also, compare: lottery versus insurance



Decision Theory

”Asian Disease Problem”

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed.

- First problem: choose between A and B
  - A 200 people will be saved.
  - B There is a 1/3 probability that 600 people will be saved, and a 2/3 probability that no people will be saved.
- Second problem: choose between C and D
  - C 400 people will die.
  - D There is a 1/3 probability that nobody will die, and a 2/3 probability that 600 people will die.

Decision Theory

Difficult Decisions: Examples

- A is usually preferred over B
- D is usually preferred over C
- But A=C and B=D

Decision Theory

How do we decide?

System 1, ‘intuitive”

- Quick
- Low cognitive effort
- Automatic
- Holistic
- Associative
- Confident

System 2, “analytical”

- Slow
- High cognitive effort
- Controlled
- Analytical
- Rule-based
- Doubtful

Decision Theory

When do we use which system?

- Hypothesis 1: for important decisions, System 2; for the rest, System 1
  - refuted
- Hypothesis 2: system 2 only when triggered in a certain way, e.g., by circumstances or as part of the task
- Through training, System 2 can be used more frequently (expert effect)
- However, the effect has also been observed that experts make worse decisions with more time than with less time

Decision Theory

Heuristics in System 1

- = rule of thumb
- Problem: ”catch the ball”
  - We don’t solve differential equations
  - Heuristic: fixate on the ball, run, keep the angle constant
- Recognition heuristic
  - prefer the known over the unknown
- Tit-for-tat
  - Be friendly at first
  - Then mimic your counterpart’s behavior
- Pearls heuristic
  - Reduce to one dimension (e.g., parties on the left-right continuum)
  - Possibly in a lexicographic fashion

Decision Theory

Quiz: Is an alternative dominated?

Question 1

	s_1	s_2
	0.5	0.5
a_1	3	4
a_2	2	1

Question 2

	s_1	s_2
	0.5	0.5
a_1	3	5
a_2	2	4

Question 3

	s_1	s_2	s_3
	0.5	0.25	0.25
a_1	5	4	3
a_2	4	2	5

Decision Theory

Quiz: Is an alternative dominated?

Question 1

	s_1	s_2
	0.5	0.5
a_1	3	4
a_2	2	1

Question 2

	s_1	s_2
	0.5	0.5
a_1	3	5
a_2	2	4

Question 3

	s_1	s_2	s_3
	0.5	0.25	0.25
a_1	5	4	3
a_2	4	2	5

a_1 dominates a_2
(absolute dominance)

Decision Theory

Quiz: Is an alternative dominated?

Question 1

	s_1	s_2
	0.5	0.5
a_1	3	4
a_2	2	1

Question 2

	s_1	s_2
	0.5	0.5
a_1	3	5
a_2	2	4

Question 3

	s_1	s_2	s_3
	0.5	0.25	0.25
a_1	5	4	3
a_2	4	2	5

a_1 dominates a_2 (absolute dominance)

a_1 dominates a_2 (state dominance)

Decision Theory

Quiz: Is an alternative dominated?

Question 1

	s_1	s_2
	0.5	0.5
a_1	3	4
a_2	2	1

Question 2

	s_1	s_2
	0.5	0.5
a_1	3	5
a_2	2	4

Question 3

	s_1	s_2	s_3
	0.5	0.25	0.25
a_1	5	4	3
a_2	4	2	5

a_1 dominates a_2 (absolute dominance)

a_1 dominates a_2 (state dominance)

a_1 dominates a_2 (probability dominance)

Decision Theory

Quiz: Is an alternative dominated?

Question 1

	s_1	s_2
	0.5	0.5
a_1	3	4
a_2	2	1

Question 2

	s_1	s_2
	0.5	0.5
a_1	3	5
a_2	2	4

Question 3

	s_1	s_2	s_3
	0.5	0.25	0.25
a_1	5	4	3
a_2	4	2	5

a_1 dominates a_2 (absolute dominance)

a_1 dominates a_2 (state dominance)

a_1 dominates a_2 (probability dominance)

Thank you!