Decision theory

Exercise 1

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Decision theory – Exercise

Definition: Relation

A relation R on a set M is a subset

$$R \subseteq M \times M = \{(m_1, m_2) : m_1, m_2 \in M\}.$$

For $(m_1, m_2) \in R$, we also write $m_1 R m_2$.

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- $M \times M = \{(m_1, m_1), (m_1, m_2), (m_1, m_3), (m_2, m_1), (m_2, m_2),$
  $(m_2, m_3), (m_3, m_1), (m_3, m_2), (m_3, m_3)\}$

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- There are 2^9 different relations possible

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- There are 2^9 different relations possible
- ”exactly one triangle” $R = \{(m_1, m_2), (m_1, m_3), (m_2, m_1), (m_3, m_1)\}$

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- $M = \{m_1, m_2, m_3\}$ with $m_1 = \square$, $m_2 = \blacktriangle$, $m_3 = \triangle$
- $M \times M = \{(m_1, m_1), (m_1, m_2), (m_1, m_3), (m_2, m_1), (m_2, m_2),$
  $(m_2, m_3), (m_3, m_1), (m_3, m_2), (m_3, m_3)\}$
- There are 2^9 different relations possible
- ”exactly one triangle” $R = \{(m_1, m_2), (m_1, m_3), (m_2, m_1), (m_3, m_1)\}$
- ”no triangle” $R = \{(m_1, m_1)\}$

Decision theory – Exercise

Definition: Transitivity

A relation R on a set M is transitive, when for all $m_1, m_2, m_3 \in M$ with $m_1 R m_2$, $m_2 R m_3$ also $m_1 R m_3$ holds.

Decision theory – Exercise

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- To disapprove a property, we only need a counterexample.

Decision theory – Exercise

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- To disapprove a property, we only need a counterexample.
- To prove a property, we need to find a statement about all elements in $M \times M$.

Decision theory – Exercise

Exercise 1: Transitive or not?

- $M = \mathbb{R}$
  a) $xRy \Leftrightarrow x \leq y + 1$
  b) $xRy \Leftrightarrow x \leq y - 1$
  c) $xRy \Leftrightarrow x^2 \leq y^2$
- $M = \{\text{Students in Erlangen}\}$
  d) aRb, if a and b have attended at least one lecture together
  e) aRb, if the student ID of a is smaller than of b

Decision theory – Exercise

Exercise 2: Transitive or not?

a) $(\mathbb{R}, <)$
b) $(\mathbb{R}, \leq)$
c) $(\mathbb{R}, =)$
d) $(\mathbb{R}, \neq)$
e) (Subset of $\mathbb{R}$, $\subseteq$)
f) $M = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3)\}$
g) $M = \{1, 2, 3, 4, 5\}$, $R = \{(3, 1), (1, 5), (2, 2), (2, 5), (3, 5)\}$

Decision theory – Exercise

Basic Model

A system consisting of:

- $A = \{a_1, \ldots, \}$ a non-empty set of alternatives
- $S = \{s_1, \ldots, \}$ a non-empty set of scenarios,possibly also with probabilities p_i
- E a set of outcomes with $g : A \times S \to E$

Decision Tree

- Decision: □
- Event: ○
- State: ◁

Decision Theory – Exercise

Exercise 3: Lottery

A numbered ball is randomly drawn from a drum containing 100 balls in a lottery. Create a decision matrix and a decision tree.

Lottary A

- 100% Chance of 1 Mio EUR

Lottery B

- 10% Chance of 5 Mio EUR
- 89% Chance of 1 Mio EUR
- 1% Chance of 0 EUR

Decision Theory – Exercise

Exercise 4: Lottery

A numbered ball is randomly drawn from a drum containing 100 balls in a lottery.Create a decision matrix and a decision tree.

Lottery C

- 11% Chance of 1 Mio EUR
- 89% Chance of 0 EUR

Lotterie D

- 10% Chance of 5 Mio EUR
- 90% Chance of 0 EUR

Decision Theory – Exercise

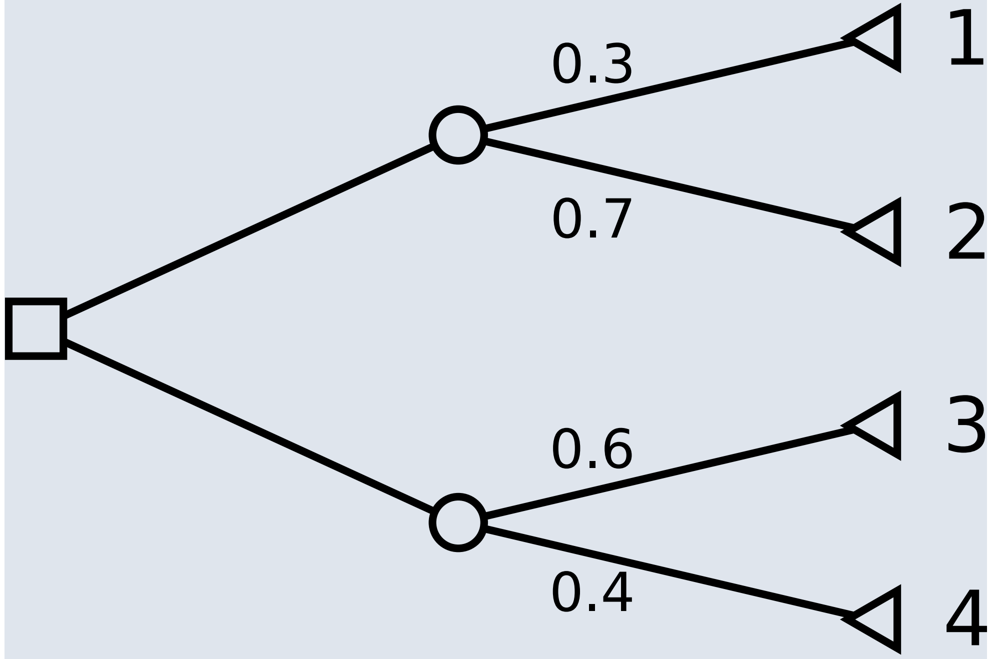
Exercise 5

Convert the following decision matrix into a decision tree:

	s_1	s_2
	0.5	0.5
a_1	5	6
a_2	7	4

Exercise 6

Convert the following decision tree into a decision matrix:



Decision Theory – Exercise

Exercise 7

Convert the following decision tree into a decision matrix