Decision theory

Exercise 4

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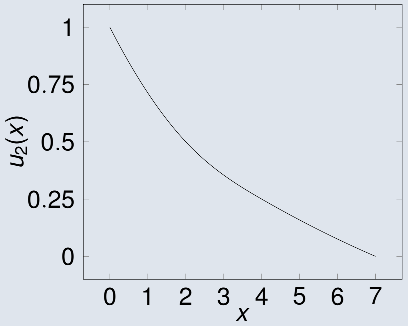
Decision Theory - Exercise

Exercise 1

Tina is looking for a new apartment. Relevant criteria are rent and distance to the university.

- Rent: 200 to 600 euros, linear utility function
- Distance: 0 to 7 km, utility function:

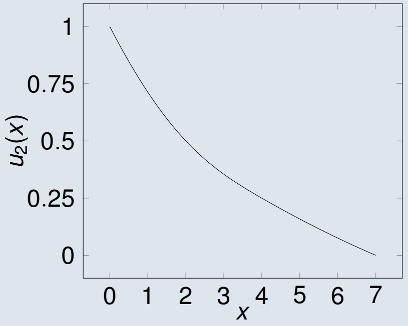


Eisenführ, Weber, Langer, Question 6.2

Decision Theory - Exercise

Data

- Rent: 200 to 600 euros, linear utility function
- Distance: 0 to 7 km, utility function:



Eisenführ, Weber, Langer, Question 6.2

Exercise 1

1. Tina finds an apartment that is 4 km away and costs 300 euros as good as an apartment that is 2 km away but costs 500 euros. What are the weighting factors?
2. How expensive can an apartment directly at the university be so that it is still better than an apartment that is 2 km away and costs 200 euros?

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Solution 1

Let u_1 be utility for rent, u_2 utility for distance. $u_1(x) = -1/400 \cdot (x - 600)$

1. $(300, 4) \sim (500, 2)$

$$\begin{aligned} & \lambda_1 u_1(300) + \lambda_2 u_2(4) = \lambda_1 u_1(500) + \lambda_2 u_2(2) \\ \Rightarrow \quad & 3/4\lambda_1 + 1/4\lambda_2 = 1/4\lambda_1 + 1/2\lambda_2 \\ \Rightarrow \quad & \lambda_1 = 1/3, \lambda_2 = 2/3 \end{aligned}$$

2. $(x, 0) \sim (200, 2)$

$$\begin{aligned} & 1/3u_1(x) + 2/3u_2(0) = 1/3u_1(200) + 2/3u_2(2) \\ \Rightarrow \quad & 1/3u_1(x) + 2/3 = 1/3 + 2/3 \cdot 1/2 \\ \Rightarrow \quad & u_1(x) = 0 \\ \Rightarrow \quad & x = 600 \end{aligned}$$

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Background

Leo wishes to buy a new watch. Relevant criteria are

- low price: 50 to 200 euros
- long warranty: 4 to 16 years
- large water depth: 10 to 50 m

He uses the following utility functions:

- Price x: $u_1(x) = a + b/x$
- Warranty x: $u_2(x) = c + d\sqrt{x}$
- Depth x: $u_3(x) = e + fx$

Exercise 2

1. Determine parameters a to f so that functions are normalized on $[0, 1]$
2. How much does the price of 50 euros have to increase so that the utility of the price is halved?
3. Is the transition from 4 to 9 years of warranty more useful than from 9 to 16 years?
4. Find suitable weights if the following models are equally good:

$$(200, 16, 12) \sim (80, 4, 12)$$
$$(70, 16, 10) \sim (70, 4, 35)$$

Eisenführ, Weber, Langer, Question 6.6

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Solution 2

1. Parameters:

$$u_1(x) = \frac{200}{3x} - \frac{1}{3}$$
$$u_2(x) = \frac{1}{2}\sqrt{x} - 1$$
$$u_3(x) = \frac{1}{40}x - \frac{1}{4}$$

2. Halving utility: 80 euros
3. Warranty: equally good
4. Weights: $\lambda_1 = 10/23$, $\lambda_2 = 5/23$, $\lambda_3 = 8/23$

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Exercise 3

- Model for caffeine addiction
- a person has three life periods: youth, middle age, old age
- decide in each period: drink coffee or not
- if drinking coffee in period t, then addicted in period $t+1$
- Utility values:

	take	not take
not addicted	10	0
addicted	-8	-25

- $\delta = 1$, $\beta = 1/2$
- how do TC, Naive, and Sophisticated decide?

O'Donoghue, Rabin, The Economics of Immediate Gratification (2000)

Decision Theory - Exercise

Solution 3

- TC: only drinks coffee in old age
- Naive: drinks coffee in middle and old age
- Enlightened: always drinks coffee

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Exercise 4

Maximize:

	s_1	s_2	s_3
a_1	8	3	3
a_2	2	9	4
a_3	9	1	2
a_4	9	4	0

Determine the best solution(s) with respect to

- Maximin
- Hurwicz with $\alpha = 1/2$
- Average
- Minimax Regret

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Solution 4

	s_1	s_2	s_3	min	max	Hurwicz	Regret	Average
a_1	8	3	3	3	8	11/2	6	14/3
a_2	2	9	4	2	9	11/2	7	15/3
a_3	9	1	2	1	9	10/2	8	12/3
a_4	9	4	0	0	9	9/2	5	13/3
max	9	9	4					

- Maximin: a_1
- Hurwicz with $\alpha = 1/2$: a_1 and a_2
- Average: a_2
- Minmax Regret: a_4

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Exercise 5: Axioms

Minimax Regret does not satisfy:

- Axiom 5: Independence of irrelevant alternatives
- Axiom 7: Independence of row permutations

Find one example each to demonstrate this.

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Solution 5

- it is minimized
- Axiom 5:

	s_1	s_2	Regret
a_1	3	1	1
a_2	2	3	2

	s_1	s_2	Regret
a_1	3	1	3
a_2	2	3	2
a_3	0	2	1

- Axiom 7:

	s_1	s_2	Regret
a_1	3	1	3
a_2	2	3	2
a_2	3	2	3
a_3	0	2	1

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Exercise 6

- calculate the OWA value for the following solution values:
  - $w = (1/2, 0, 0, 1/2)$, $a = (7, 3, 6, 4)$
  - $w = (1/4, 1/4, 0, 1/2)$, $a = (2, 6, 3, 6)$
  - $w = (1/3, 1/3, 1/3, 0)$, $a = (6, 7, 3, 1)$
  - $w = (0, 0, 1/2, 1/2)$, $a = (6, 3, 5, 8)$

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Solution 6: OWA

- $w = (1/2, 0, 0, 1/2)$, $a = (7, 3, 6, 4)$
  - $OWA(a) = 1/2 \cdot 7 + 1/2 \cdot 3 = 5$
- $w = (1/4, 1/4, 0, 1/2)$, $a = (2, 6, 3, 6)$
  - $OWA(a) = 1/4 \cdot 6 + 1/4 \cdot 6 + 1/2 \cdot 2 = 4$
- $w = (1/3, 1/3, 1/3, 0)$, $a = (6, 7, 3, 1)$
  - $OWA(a) = 1/3 \cdot 7 + 1/3 \cdot 6 + 1/3 \cdot 3 = 16/3$
- $w = (0, 0, 1/2, 1/2)$, $a = (6, 3, 5, 8)$
  - $OWA(a) = 1/2 \cdot 5 + 1/2 \cdot 3 = 4$

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Exercise 7

- Reminder: $WOWA(a_1, \ldots, a_n) = \sum_{i \in [n]} \omega_i a_{\pi(i)}$ with
  - π a permutation such that A is sorted in ascending order
  - $\omega_i = w(\sum_{j \leq i} p_{\pi(j)}) - w(\sum_{j < i} p_{\pi(j)})$
  - $w(x)$ is the interpolating function
- Determine $WOWA(a)$ with $a = (1, 2, 3, 4)$, $p = (0.1, 0.4, 0.3, 0.2)$, $w = (1/2, 1/4, 1/4, 0)$

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Solution 7: WOWA

$$w(x) = \begin{cases} 2x & \text{for } x \in [0, 1/4] \\ 1/4 + x & \text{for } x \in [1/4, 3/4] \\ 1 & \text{for } x \in [3/4, 1] \end{cases}$$

- permuted p-vector: $p' = (0.2, 0.3, 0.4, 0.1)$

$$\begin{aligned} \omega_1 &= w(0.2) - w(0) = 0.4 \\ \omega_2 &= w(0.5) - w(0.2) = 0.75 - 0.4 = 0.35 \\ \omega_3 &= w(0.9) - w(0.5) = 1 - 0.75 = 0.25 \\ \omega_4 &= w(1) - w(0.9) = 1 - 1 = 0 \end{aligned}$$

- $WOWA(A) = 0.4 \cdot 4 + 0.35 \cdot 3 + 0.25 \cdot 2 = 3.15$