Friedrich-Alexander-Universität
Erlangen-Nürnberg

FAU

Decisiontheory

Lecture 1

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Decision Theory

About DT

- A truly fascinating topic!
- Both professionally and privately: How do I make decisions?
- What are good decisions?
- What methods can I employ to find them?
- Blending of philosophical and mathematical aspects

Decision Theory

Organization

- Lecture
  - Mondays 12:15 – 13:45
  - Room H13
- Exercise
  - Exercises (biweekly) immediately after lecture
  - Room H13
- Exam
  - (likely) oral exam
  - Date not yet determined

Decision Theory

Mo	15.04.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	22.04.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	29.04.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	06.05.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	13.05.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
~~Mo~~	~~20.05.2024~~	~~12:00 - 14:00~~			Pfingstmontag
Mo	27.05.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	03.06.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	10.06.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	17.06.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	24.06.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	01.07.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	08.07.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		
Mo	15.07.2024	12:00 - 14:00	12801.01.230 (H13 Johann-Radon-Hörsaal)		

Organization

- Lecture
  - Start: Summary of the last session
  - End: Short quiz
- Exercise
  - We present task
  - You work alone or in small groups
  - We assist
  - Then demonstrate the solution

Organization

- StudOn
  - Course: “Entscheidungstheorie”
  - Exercises, lectures, announcements, etc. via StudOn
  - Please join as soon as possible
- Feedback
  - Your opinion matters
  - Feedback after the end is too late
  - I will check in periodically
  - Please let me know if something is amiss

Decision Theory

Our Deal

- I provide:
  - Materials on StudOn
  - Take your feedback into account (talk to me!)
  - Goal, that you learn something
- I expect from you:
  - Participation
  - Preparation for sessions
  - Support each other (exercises, StudOn)

$\sim$ End of Organization $\backsim$

Decision Theory

What's it about?

A decision refers to...

- Choosing from various alternatives or different options
- By one or more decision-makers
- In connection with immediate or future implementation
- All factors known or not entirely known
- Single-criteria or multi-criteria

Decision Theory

What's it about?

"What are the consequences of a decision?"

Subfields of Decision Theory (DT):

- Normative DT
- Descriptive DT
- Prescriptive DT

Decision Theory

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"What are the consequences of a decision?"

Subfields of Decision Theory (DT):

- Normative DT focuses on the analysis of decisions regarding decision logic and the rationality of the decision. *Axiomatic* approach. "What are the characteristics of a good decision".
- Descriptive DT
- Prescriptive DT

Decision Theory

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"What are the consequences of a decision?"

Subfields of Decision Theory (DT):

- Normative DT focuses on the analysis of decisions regarding decision logic and the rationality of the decision. *Axiomatic* approach. "What are the characteristics of a good decision".
- Descriptive DT *observes and describes* how decisions are made in reality and explains the underlying decision processes.
- Prescriptive DT

Decision Theory

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Subfields of Decision Theory (DT):

- Normative DT focuses on the analysis of decisions regarding decision logic and the rationality of the decision. *Axiomatic* approach. "What are the characteristics of a good decision".
- Descriptive DT *observes and describes* how decisions are made in reality and explains the underlying decision processes.
- Prescriptive DT attempts to *derive strategies* and methods to help people make better decisions. The developed methods should ideally meet the requirements of normative DT. "How do I find the best decision?"

Decision Theory

Topics

- Basics of DT
- Decision under certainty
- Decision under uncertainty
- Decision under risk
- Other decision problems

Literature – I

- Eisenführ, Weber, Langer: Rationales Entscheiden, Springer, Berlin.
- Bamberg, Coenenberg, Krapp: Betriebswirtschaftliche Entscheidungslehre, Vahlen, München.
- Klein, Scholl: Planung und Entscheidung, Vahlen, München.
- French: Decision theory: an introduction to the mathematics of rationality. Halsted Press.
- Amann: Entscheidungstheorie, Springer Spektrum, Wiesbaden.

Literature – II

- Bamberg, Baur, Krapp: Arbeitsbuch zur betriebswirtschaftlichen Entscheidungslehre, Vahlen, München.
- Bitz: Entscheidungstheorie, Vahlen, München.
- Dinkelbach: Entscheidungsmodelle, Berlin New York.
- Kistner: Optimierungsmethoden, Physica, Heidelberg.
- Meyer: Entscheidungstheorie, Gabler, Wiesbaden.
- Saliger: Betriebswirtschaftliche Entscheidungstheorie, Oldenbourg, München / Wien.
- Dinkelbach, Lorscheider: Entscheidungsmodelle und lineare Programmierung, München, Wien.

Introductory Examples

Decision Theory

What makes a decision difficult? Uncertainty!

- A plaintiff wants to pursue a damages claim of 200,000 EUR but doesn't know if they will win in court. The defendant offers a settlement of 100,000 EUR. Should the plaintiff accept?

- Decisions are often individual; finding a general scheme can be challenging
- Consequences of alternatives are partially uncertain
- Exactly one alternative must be chosen

Decision Theory

What makes a decision difficult? Multiple Objectives!

- Personnel decisions: work experience, education, reliability, social skills, leadership qualities...
- Car purchase: price, mileage, color, type, fuel consumption...
- Minimize costs vs. maximize customer satisfaction

- Weighing criteria against each other
- Finding a compromise

Decision Theory

What makes a decision difficult? Many Alternatives!

- A DHL truck can serve n customers in $(n-1)!$ different orders. The cost-effective route needs to be chosen.
  - $5! = 120$
  - $10! = 3,628,800$
  - $15! = 1.3 \cdot 10^{12}$

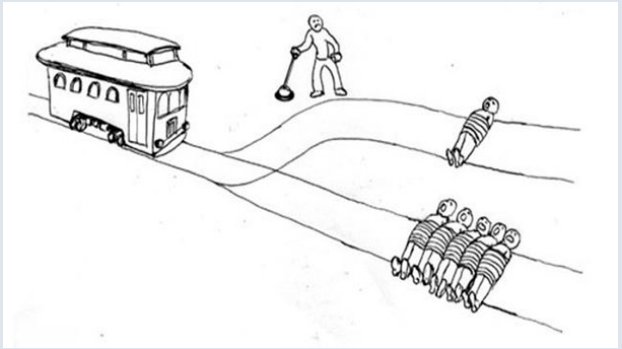
Decision Theory

What makes a decision difficult? Complicated Evaluation!

- Should Germany invest an additional ten billion euros per year to (with certainty) reduce the number of traffic fatalities from 3,300 to 2,500 per year? Where is the limit – 1 billion euros or 100 billion euros?
- How much preparation time (or foregone leisure time) is a grade improvement from 2.0 to 1.3 worth to you?

Complicated Evaluation: Trolley Problem

A trolley is out of control and is about to run over five people. By switching a lever, the trolley can be diverted to another track. Unfortunately, there is another person on that track. Can (by switching the lever) the death of one person be accepted to save the lives of five people?



Source: https://www.br.de/puls/themen/netz/philosophische-memes-memes-kolumne-100.html

Difficult decisions: Examples

- At the roulette table: Red has come up three times in a row; should I bet on red again?
- “Future orientation”: irrelevant factors from the past should not influence future decisions

Sunk Cost Fallacy

- G. W. Bush on the Iraq War, 2005: Many American soldiers have already fallen. “We owe them something. We will finish the task for which they gave their lives.”
- “Sunk Cost Fallacy”: similar to the above. The past cannot be changed; one must choose the best for the future.

Influential Circumstances: Framing

- The way the problem is described can affect the decision
- The type of question can alter my response
- “Do you really want to leave your sick girlfriend alone at home?”
- “Do you want to go to this awesome party?”

Influential Circumstances: Anchoring

- Decisions are influenced by what was seen before
- The number 100 seems large compared to 1, 3, 6, 7
- The number 100 seems small compared to 1024, 3331, 6056, 9732

Difficult Decisions: Examples

Consider the story of the man who met death in Damascus. Death looked surprised, but then recovered his ghastly composure and said, 'I am coming for you tomorrow'. The terrified man that night bought a camel and rode to Aleppo. The next day, death knocked on the door of the room where he was hiding, and said 'I have come for you'.
'But I thought you would be looking for me in Damascus', said the man.
'Not at all', said death 'that is why I was surprised to see you yesterday. I knew that today I was to find you in Aleppo'.

Decision Theory

The Prisoner's Dilemma

You and your accomplice have committed a crime and are arrested. You are in separate cells and must decide: confess or not?

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- You confess
  - Accomplice confesses:
  - Accomplice does not confess:
- You do not confess
  - Accomplice confesses:
  - Accomplice does not confess:

Decision Theory

The Prisoner's Dilemma

You and your accomplice have committed a crime and are arrested. You are in separate cells and must decide: confess or not?

- You confess
  - Accomplice confesses: 4 years in prison for both
  - Accomplice does not confess:
- You do not confess
  - Accomplice confesses:
  - Accomplice does not confess:

Decision Theory

The Prisoner's Dilemma

You and your accomplice have committed a crime and are arrested. You are in separate cells and must decide: confess or not?

- You confess
  - Accomplice confesses: 4 years in prison for both
  - Accomplice does not confess: 1 year in prison for you, 6 years for him
- You do not confess
  - Accomplice confesses: 6 years in prison for you, 1 year for him
  - Accomplice does not confess: 2 years in prison for both

Decision Theory

Decision Theory...

- is interdisciplinary
- is sometimes hard to verify
- can be philosophical

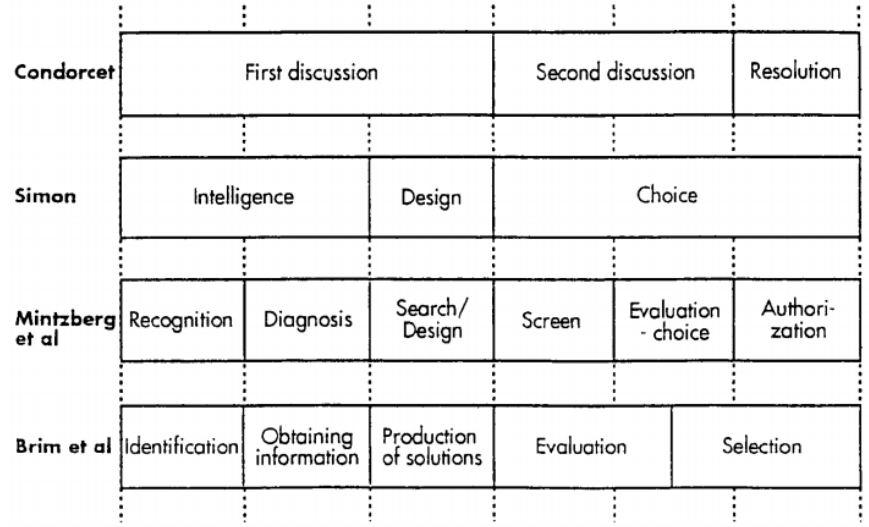

Diagram 2. A comparison of the stages of the decision process according to Condorcet, Simon, Mintzberg et al and Brim et al.

Therefore, we will mainly focus on well defined problems

Decision Theory

How do I evaluate decisions?

Approach 1: Relations.

- $A > B$ means "A is better than B"
- $A \geq B$ means "A is not worse than B"
- $A = B$ means "A is as good as B"
- Transitivity:

$$(A \geq B \text{ and } B \geq C) \quad \Rightarrow \quad A \geq C$$

(same for $>$)

Can you find an example of a non-transitive relation?

Do we only use transitive relations in practice?

Decision Theory

How do I evaluate decisions?

Approach 1: Relations.

- A relation $\geq$ is complete if for all A, B, either $A \geq B$ or $B \geq A$ applies (same for $>$ and =)

Can you find an example of a non-complete relation?

For decision-making, we often assume transitivity + completeness.

Decision Theory

How do I evaluate decisions?

Approach 2: Numerically

- Each decision has a numerical utility (utility function)
- Car purchase: utility of all-wheel drive, lane-keeping assistant, and costs
- Resulting relations are transitive + complete
- Disadvantage: when it's not about money, where do these numbers come from?

Omelette Problem – a typical process

- Five eggs cracked into a bowl
- Sixth egg unopened
- The egg is rotten with a probability of 0.05
- Three options for our omelette:
  1. Crack the egg into the bowl
  2. Crack the egg into a cup first
  3. Discard the egg

Decision Theory

According to the decision-maker's preferences, the results can now be evaluated. They are interested in:

1. the number of (unspoiled) eggs in the omelette,
2. the number of discarded eggs,
3. whether they have to wash the cup.

Decision Theory

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Preliminary Result Matrix:

action a_i \ scenario s_j	s_1 (egg fine) $p_1 = 0.95$	s_1 (egg rotten) $p_1 = 0.05$
a_1: add sixth egg to bowl	e_{11}: 6-egg omelette	e_{12}: no omelette and 5 spoiled eggs
a_2: crack sixth egg into cup	e_{21}: 6-egg omelette and dirty cup	e_{22}: 5-egg omelette and dirty cup
a_3: throw sixth egg away	e_{31}: 5-egg omelette and 1 wasted egg	e_{32}: 5-egg omelette

Decision Theory

We are now evaluating the respective outcomes:

- x_{ij}^1: number of good eggs
- x_{ij}^2: number of wasted eggs
- x_{ij}^3: cup washing

Decision Theory

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a_i \ s_j	s_1 (egg fine) $p_1 = 0.95$	s_1 (egg rotten) $p_1 = 0.05$
a_1: add sixth egg to bowl	$x_{11}^1 = 6$ $x_{11}^2 = 0$ $x_{11}^3 = 0$	$x_{12}^1 = 0$ $x_{12}^2 = 5$ $x_{12}^3 = 0$
a_2: crack sixth egg into cup	$x_{21}^1 = 6$ $x_{21}^2 = 0$ $x_{21}^3 = 1$	$x_{22}^1 = 5$ $x_{22}^2 = 0$ $x_{22}^3 = 1$
a_3: throw sixth egg away	$x_{31}^1 = 5$ $x_{31}^2 = 1$ $x_{31}^3 = 0$	$x_{32}^1 = 5$ $x_{32}^2 = 0$ $x_{32}^3 = 0$

Decision Theory

We are now evaluating the respective outcomes:

- x_{ij}^1: number of good eggs
- x_{ij}^2: number of wasted eggs
- x_{ij}^3: cup washing
- Which decision should be made under rational considerations?

a_i \ s_j	s_1 (egg fine) $p_1 = 0.95$	s_1 (egg rotten) $p_1 = 0.05$
a_1: add sixth egg to bowl	$x_{11}^1 = 6$ $x_{11}^2 = 0$ $x_{11}^3 = 0$	$x_{12}^1 = 0$ $x_{12}^2 = 5$ $x_{12}^3 = 0$
a_2: crack sixth egg into cup	$x_{21}^1 = 6$ $x_{21}^2 = 0$ $x_{21}^3 = 1$	$x_{22}^1 = 5$ $x_{22}^2 = 0$ $x_{22}^3 = 1$
a_3: throw sixth egg away	$x_{31}^1 = 5$ $x_{31}^2 = 1$ $x_{31}^3 = 0$	$x_{32}^1 = 5$ $x_{32}^2 = 0$ $x_{32}^3 = 0$

Decision Theory

Solution

In addition to the information about the results relevant to the decision-maker, information about preferences is also needed.

- One point for each good egg
- Subtract $\frac{1}{10}$ point for each wasted egg
- Subtract $\frac{1}{20}$ point for cup washing

Decision Theory

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- One point for each good egg
- Subtract $\frac{1}{10}$ point for each wasted egg
- Subtract $\frac{1}{20}$ point for cup washing

S_j / a_i	S_1 $p_1 = 0{,}95$	S_2 $p_2 = 0{,}05$
a_1	6	0
	0	5
	0	0
a_2	6	5
	0	0
	1	1
a_3	5	5
	1	0
	0	0

Decision Theory

Solution

In addition to the information about the results relevant to the decision-maker, information about preferences is also needed.

- One point for each good egg
- Subtract $\frac{1}{10}$ point for each wasted egg
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S_j / a_i	S_1 $p_1 = 0{,}95$	S_2 $p_2 = 0{,}05$
a_1	6	0
	0	5
	0	0
a_2	6	5
	0	0
	1	1
a_3	5	5
	1	0
	0	0

$\rightarrow$

S_j / a_i	S_1 $p_1 = 0{,}95$	S_2 $p_2 = 0{,}05$
a_1	6	0
	0	$-\frac{5}{10}$
	0	0
a_2	6	5
	0	0
	$-\frac{1}{20}$	$-\frac{1}{20}$
a_3	5	5
	$-\frac{1}{10}$	0
	0	0

Decision Theory

Afterwards, the utilities are added and weighted by probability:

S_j / a_i	S_1 $p_1 = 0{,}95$	S_2 $p_2 = 0{,}05$
a_1	6 0 0	0 $-\frac{5}{10}$ 0
a_2	6 0 $-\frac{1}{20}$	5 0 $-\frac{1}{20}$
a_3	5 $-\frac{1}{10}$ 0	5 0 0

$\rightarrow$

	u
a_1	5,675
a_2	**5,9**
a_3	4,905

Solution

The optimal alternative is action a_2, breaking the egg into a cup first.

Decision Theory

Terms

- Decision Theory
  - Theory of decision-making
- Theory
  - A self-consistent system of scientific statements (requirements: falsifiability and realism)
- Decision situation
  - Choice situation with multiple alternative courses of action
- Decision
  - Deliberate (model-based) selection among the possible courses of action in a decision situation, taking into account the environmental situation, the alternatives, the consequences of actions, and the level of information

Decision Theory

Purpose and Object of Study of DT

1. Recognize common elements and structures of decisions and decision situations
2. Explain decision behavior (descriptive decision theory)
3. Provide decision recommendations for rational behavior based on decision rules (prescriptive decision theory)

Any questions?

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Thank you and see you next week!