Friedrich-Alexander-Universität
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FAU

Decision theory

Exercise 2

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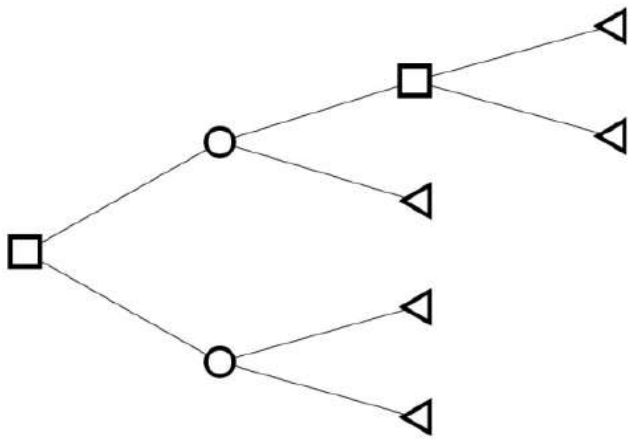
Decision theory – Exercise

Exercise 1

Write the following decision tree as a decision matrix:

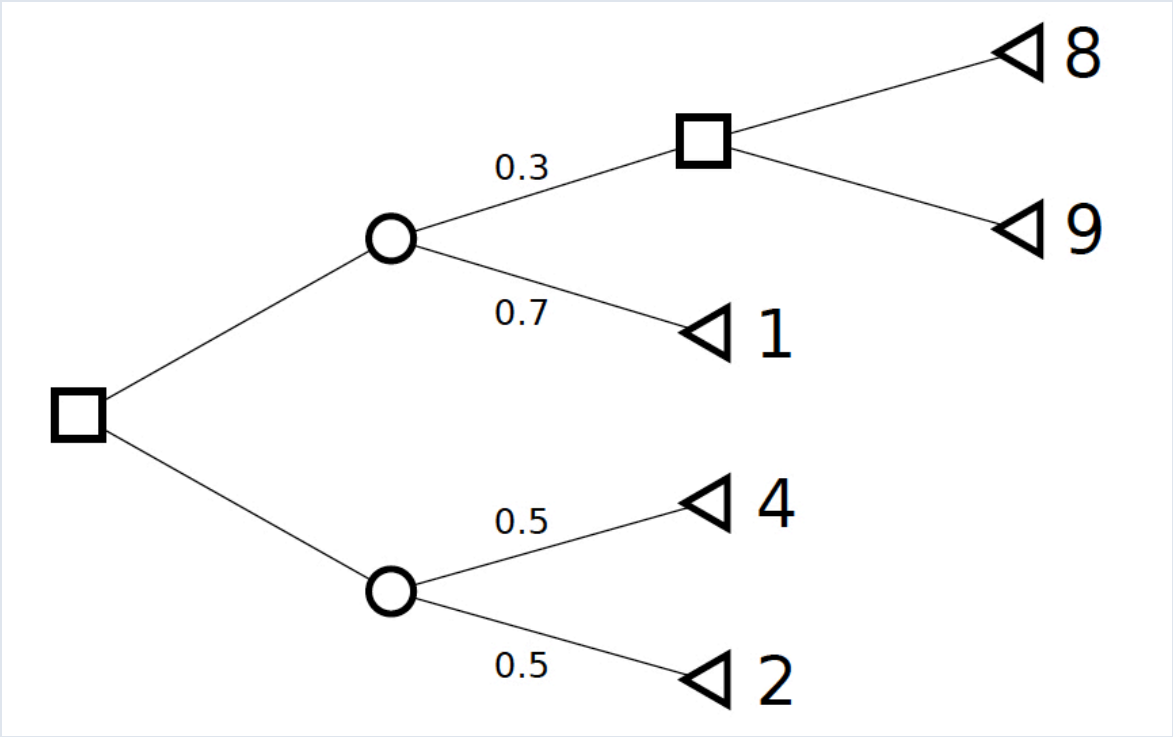
p	0.3	0.2	0.3	0.2
a_1	2	2	4	4
a_2	1	1	8	1
a_3	1	1	9	1

Draw events and probabilities in the following decision tree so that both models are equivalent:



Decision theory – Exercise

Solution 1



Decision theory – Exercise

Exercise 2: Dominance criteria

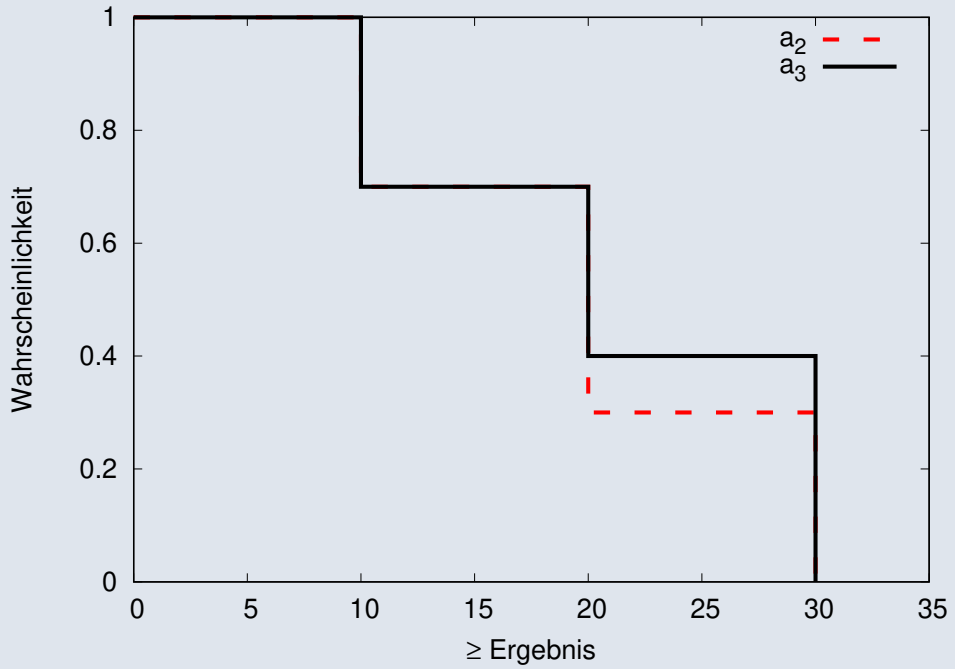
We want to maximize. Check absolute dominance, state dominance, and probability dominance:

	s_1	s_2	s_3	s_4
p	0.3	0.3	0.2	0.2
a_1	5	5	5	5
a_2	30	10	20	20
a_3	10	20	30	30
a_4	15	5	15	20

Decision theory – Exercise

Solution 2

- absolute dominance: a_1 (by a_2, a_3, a_4)
- State dominance: a_4 (by a_2)
- Probability dominance: a_2 (by a_3)



Decision theory – Exercise

Exercise 3: Lexicographic optimization

- given 4 alternatives, and 4 scenarios:

	e^1	e^2	e^3	e^4
a_1	2	5	2	4
a_2	4	3	2	1
a_3	3	3	2	3
a_4	4	2	4	3

- We want to maximize
- Consider
  - y^1: worst-case
  - y^2: average
  - y^3: best-case
- find a lexicographically optimal solution with respect to (y^1, y^2, y^3)

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4			
a_2	4	3	2	1			
a_3	3	3	2	3			
a_4	4	2	4	3			

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2		
a_2	4	3	2	1	1		
a_3	3	3	2	3	2		
a_4	4	2	4	3	2		

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2		
a_2	4	3	2	1	1		
a_3	3	3	2	3	2		
a_4	4	2	4	3	2		

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2	3.25	
a_2	4	3	2	1	1	2.50	
a_3	3	3	2	3	2	2.75	
a_4	4	2	4	3	2	3.25	

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2	3.25	
a_2	4	3	2	1	1	2.50	
a_3	3	3	2	3	2	2.75	
a_4	4	2	4	3	2	3.25	

Decision theory – Exercise

Solution 3

	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2	3.25	5
a_2	4	3	2	1	1	2.50	4
a_3	3	3	2	3	2	2.75	3
a_4	4	2	4	3	2	3.25	4

Decision theory – Exercise

Solution 3

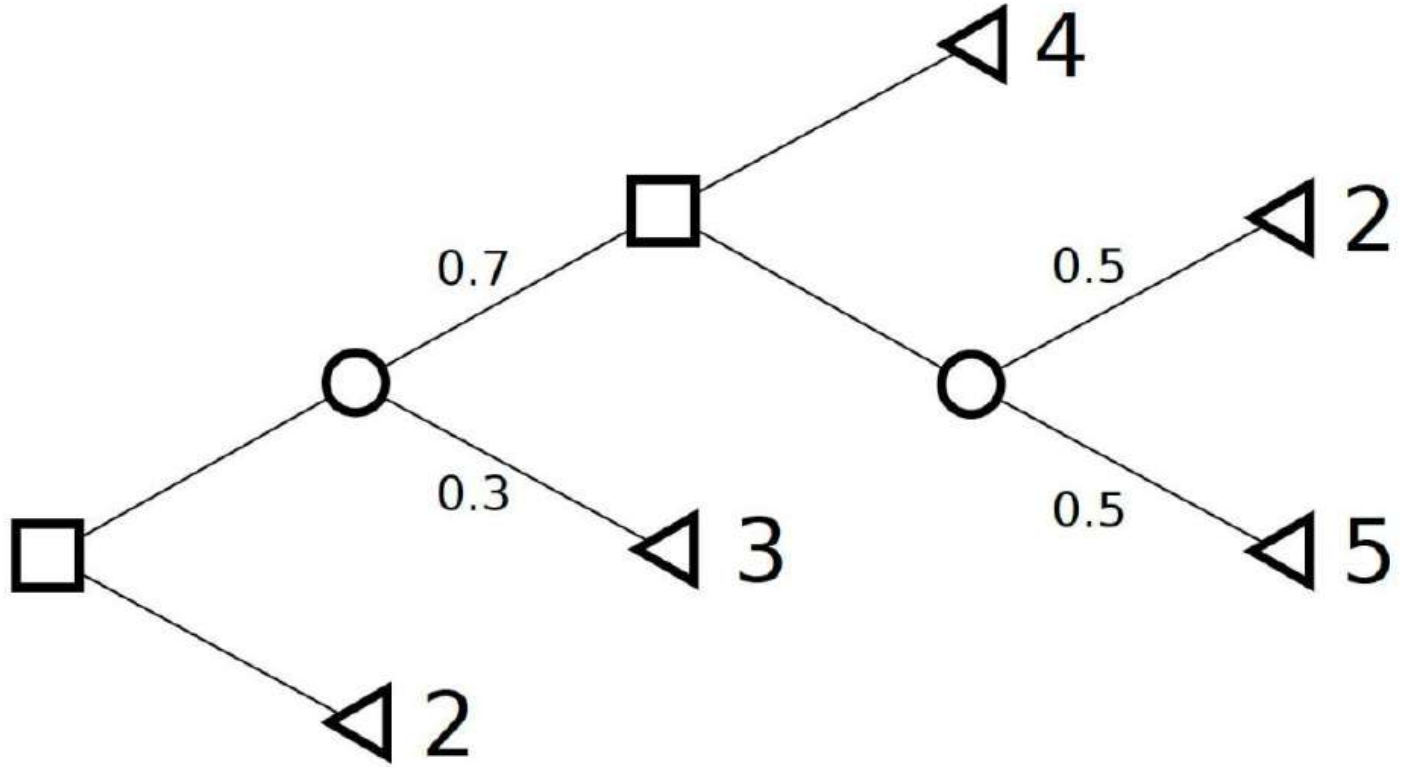
	e^1	e^2	e^3	e^4	y^1	y^2	y^3
a_1	2	5	2	4	2	3.25	5
a_2	4	3	2	1	1	2.50	4
a_3	3	3	2	3	2	2.75	3
a_4	4	2	4	3	2	3.25	4

Alternative a_1 is the only lexicographically optimal solution with respect to (y^1, y^2, y^3)

Decision theory – Exercise

Exercise 4

Write the following decision tree as a decision matrix:



Decision theory – Exercise

Solution 4

	0.3	0.35	0.35
a_1	2	2	2
$a_{2,1}$	3	4	4
$a_{2,2}$	3	2	5

Decision theory – Exercise

Exercise 5: Pareto Efficiency

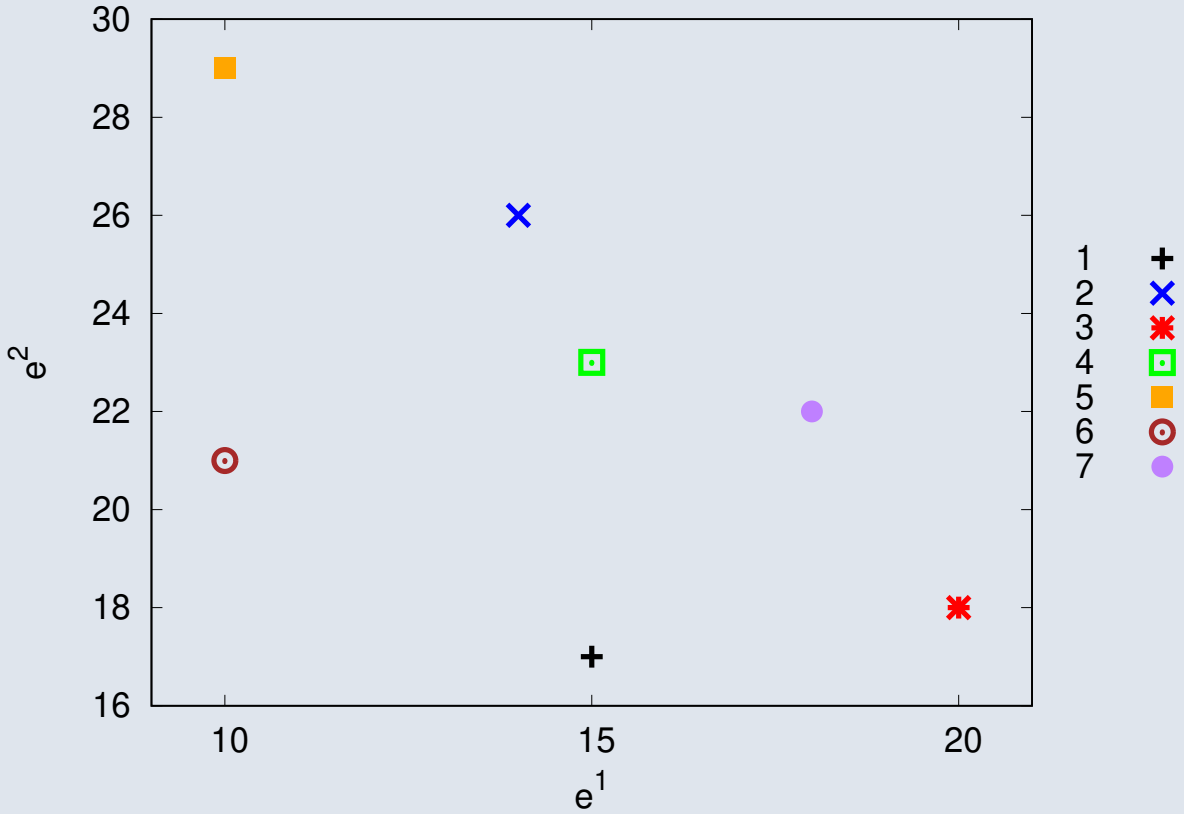
- given 7 alternatives, 2 criteria:

Alternative	1	2	3	4	5	6	7
Criterion 1	15	14	20	15	10	10	18
Criterion 2	17	26	18	23	29	21	22

- sketch alternatives in a coordinate system
- identify efficient solutions
- which are supported, which are not?
- what is a non-graphical method to distinguish between supported and unsupported?

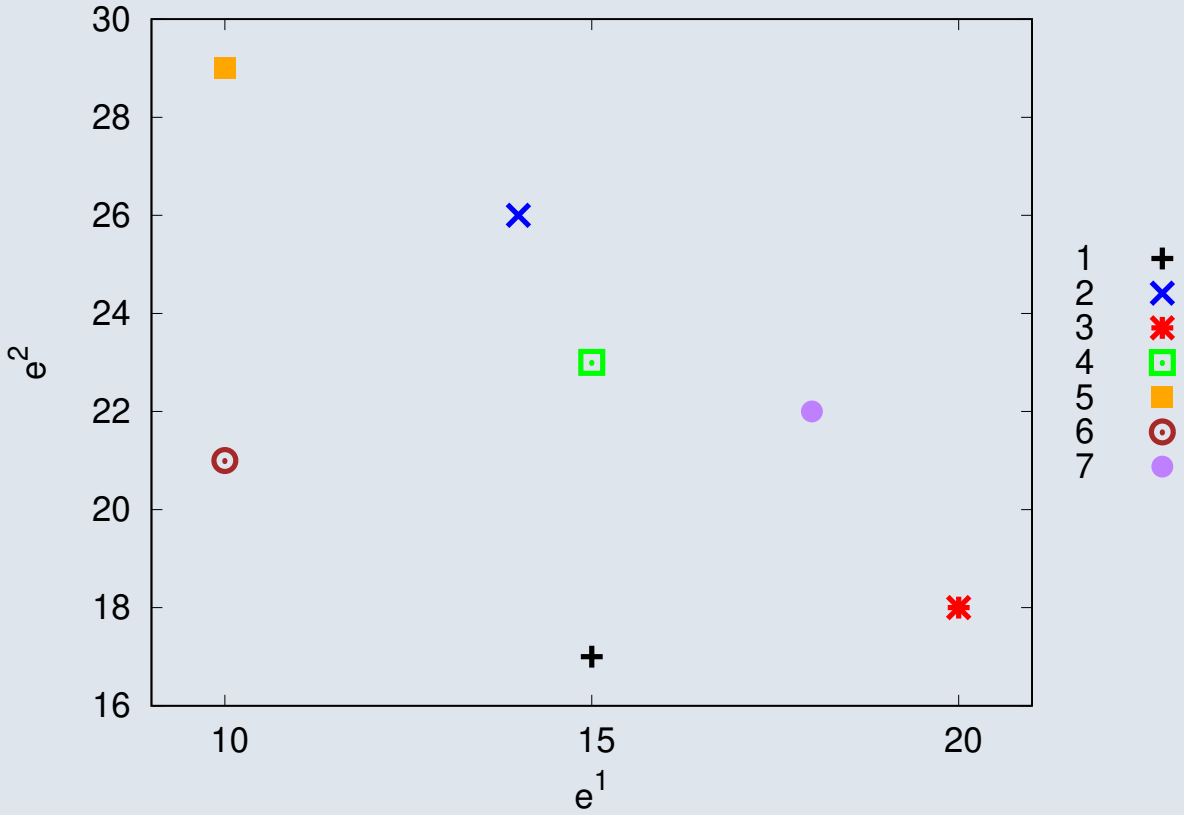
Decision theory – Exercise

Solution 5



Decision theory – Exercise

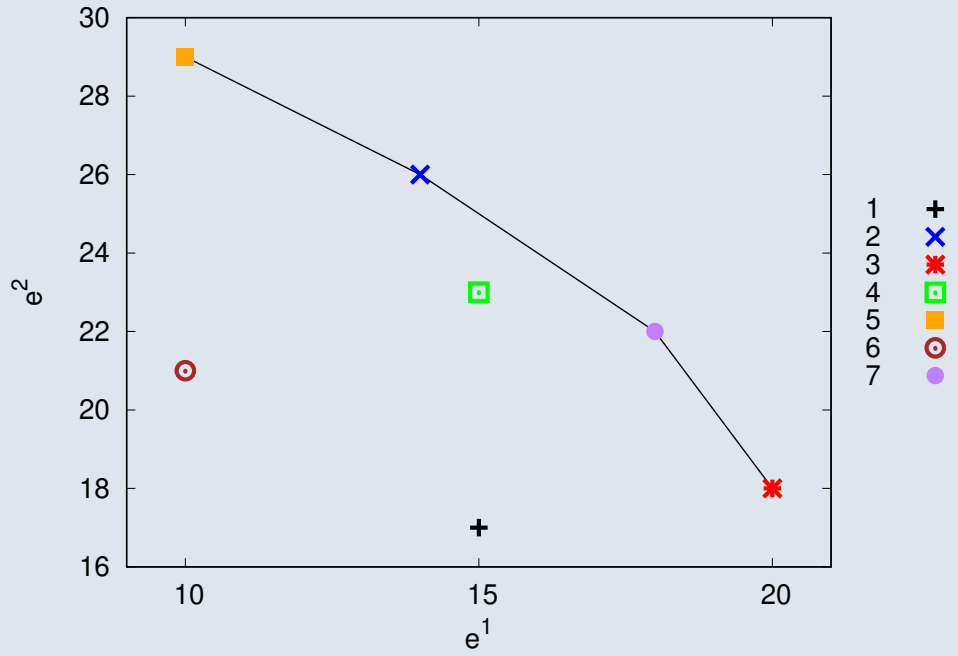
Solution 5



- Pareto efficient: $\{2, 3, 4, 5, 7\}$, among them

Decision theory – Exercise

Solution 5



- Pareto efficient: $\{2, 3, 4, 5, 7\}$, among them
  - supported: $\{2, 3, 5, 7\}$
  - unsupported: $\{4\}$