Friedrich-Alexander-Universität
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FAU

Decision theory

Exercise 3

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Decision theory – Exercise

DEA – Model 1

$$\min \theta_j$$

$$\text{s.t.} \sum_{i \in I} \lambda_i x_{ik} \leq \theta_j x_{jk} \qquad \forall k \in [N]$$

$$\sum_{i \in I} \lambda_i y_{ik} \geq y_{jk} \qquad \forall k \in [M]$$

$$\theta_j \in \mathbb{R}$$

$$\lambda_i \in \mathbb{R}_+ \qquad \forall i \in I$$

Data

Branch	(1)	(2)	(3)
A	125	50	18
B	44	20	16
C	80	55	17
D	23	12	11

with:

(1) Private transactions

(2) Business transactions

(3) Employees

Exercise 1

Write the DEA LP for DMU A. Which branch is efficient?

Decision Theory - Exercise

AHP: Normalized Columns

How do I compute suitable weights w_1, w_2, w_3 from a comparison matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

such that $r_{ij} \approx w_i / w_j$?

- Using eigenvectors of R
- Through geometric mean
- Via the method of least squares
- Now: normalized columns

Decision Theory - Exercise

AHP: Normalized Columns

1. Normalize R such that the sum of each column equals 1
2. w_i is the mean of each row

The method is simple but lacks a theoretical foundation.

Example

$$R = \begin{pmatrix} 1 & 3 & 1/2 \\ 1/3 & 1 & 2 \\ 2 & 1/2 & 1 \end{pmatrix} \rightarrow \overline{R} = \begin{pmatrix} 3/10 & 6/9 & 1/7 \\ 1/10 & 2/9 & 4/7 \\ 6/10 & 1/9 & 2/7 \end{pmatrix} \rightarrow w \approx \begin{pmatrix} 0.37 \\ 0.30 \\ 0.33 \end{pmatrix}$$

Decision Theory - Exercise

Exercise 2

Model and solve the following problem using AHP:

- For lunch break, you have three options:
  - Going to the cafeteria
  - Pre-cooking for the next day in the evening
  - Going to a snack bar
- Relevant criteria are:
  - Price
  - Taste
  - Healthiness
- Maximize your overall satisfaction
- Use normalized columns

Decision Theory - Exercise

Exercise 3: MAUT

- You want to rent an apartment
- Four possible alternatives
- Three relevant criteria:
  - Price (Euro)
  - Size (m^2)
  - Distance from the city center (m)
- You determine the following metrics:

	Price	Size	Distance
W1	350	35	200
W2	400	50	1500
W3	500	45	300
W4	600	55	500

Decision Theory - Exercise

Exercise 3: MAUT, Continuation

	Price	Size	Distance
W1	350	35	200
W2	400	50	1500
W3	500	45	300
W4	600	55	500

- Determine a utility function for price using the direct rating method
- Determine utility functions for size and distance using the halving method
- Determine weights for the three objective criteria using the trade-off method