Decision Theory

Lecture 10

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Decision Theory

Recap: What did we do?

- Decision under risk
- μ-principle
- μ-σ-principle
- Functions based on other measures

Decision Theory

Decisions Under Risks

$$
\begin{array}{c|ccc}
 & p_1 & \dots & p_n \\
 & s_1 & \dots & s_n \\
\hline
a_1 & e_{11} & \dots & e_{1n} \\
a_2 & e_{21} & \dots & e_{2n} \\
\vdots & \vdots & \vdots & \vdots \\
a_m & e_{m1} & \dots & e_{mn}
\end{array}
$$

- Know probability p_j for each scenario s_j

Decision Theory

μ-Prinziple

- Decide only based on the expected value

$$\mu_i = \sum_{j \in [n]} p_j e_{ij}$$

- Pro:
  - Law of large numbers
  - If decision is repeated, this is the best we can do
- Cons:
  - Decision once, law of large numbers does not hold
  - Never play lottery or take insurance
  - Petersburg paradox

Decision Theory

Petersburg Paradox

- How good is this game?

	s_1	s_2	s_3	$\dots$	s_n	$\dots$
$p(s_i)$	$1/2$	$1/4$	$1/8$	$\dots$	2^{-n}	$\dots$
profit	2	4	8	$\dots$	2^n	$\dots$

- Expected profit:

$$\mu = 2 \cdot \frac{1}{2} + 4\frac{1}{4} + 8 \cdot \frac{1}{8} + \dots = \infty$$

- Clearly not how we decide

Decision Theory

Certainty Equivalent

- Value CE such that
$$\Phi(CE) = \Phi(a_i)$$
- For how much are you willing to sell your ticket?
- In the Petersburg paradox: infinite CE

Decision Theory

μ-σ-Principle

- Include both expected value μ and standard deviation σ (or variance σ^2)

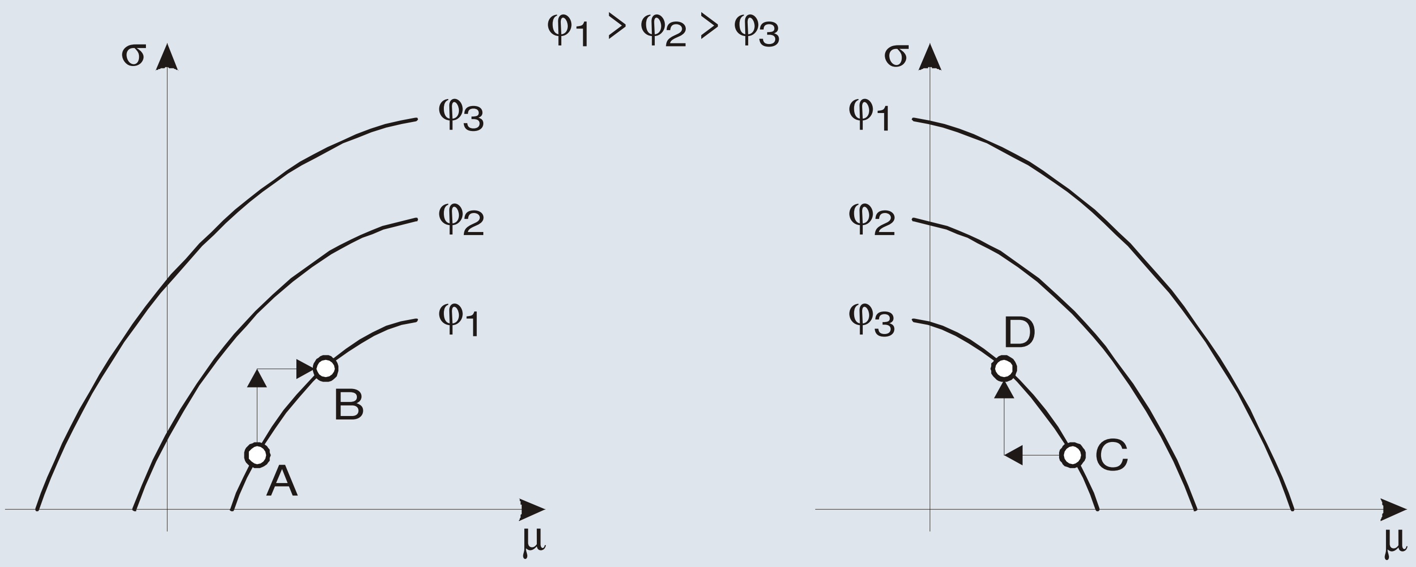
$$\sigma = \sqrt{\sum_{j=1}^{n} p_j (x_j - \mu)^2}$$

$$\sigma^2 = \sum_{j=1}^{n} p_j (x_j - \mu)^2$$

- $\partial \Phi(\mu, \sigma) / \partial \sigma > 0$: risk-seeking
- $\partial \Phi(\mu, \sigma) / \partial \sigma = 0$: risk-neutral
- $\partial \Phi(\mu, \sigma) / \partial \sigma < 0$: risk-averse

Decision Theory

Level Sets



- Draw line through points which give same value
- Map of a landscape: points on the same height
- On the left: risk-averse
- On the right: risk-seeking

Decision Theory

μ-σ Preference Functions

$$\Phi(\mu, \sigma) = \mu - \sigma^2$$

- Looks reasonable: compromise between expected profit and risk
- But can choose dominated solutions!

	s_1	s_2	s_3	s_4	μ	σ^2
a_1	20	20	20	20	20	0
a_2	20	20	20	30	22.5	18.75

Decision Theory

Functions Based on other Measures

- μ and σ are used to represent the distribution of results
- Many more measures exist in statistics
- Depending on what is important for the decision at hand, one might rather want to use one of these
- Examples:
  - Worst case
  - Quantiles
  - Loss probability
  - Loss expectation
  - Range of variation

Today

- CVaR
- Application example: portfolio planning
- Bernoulli principle

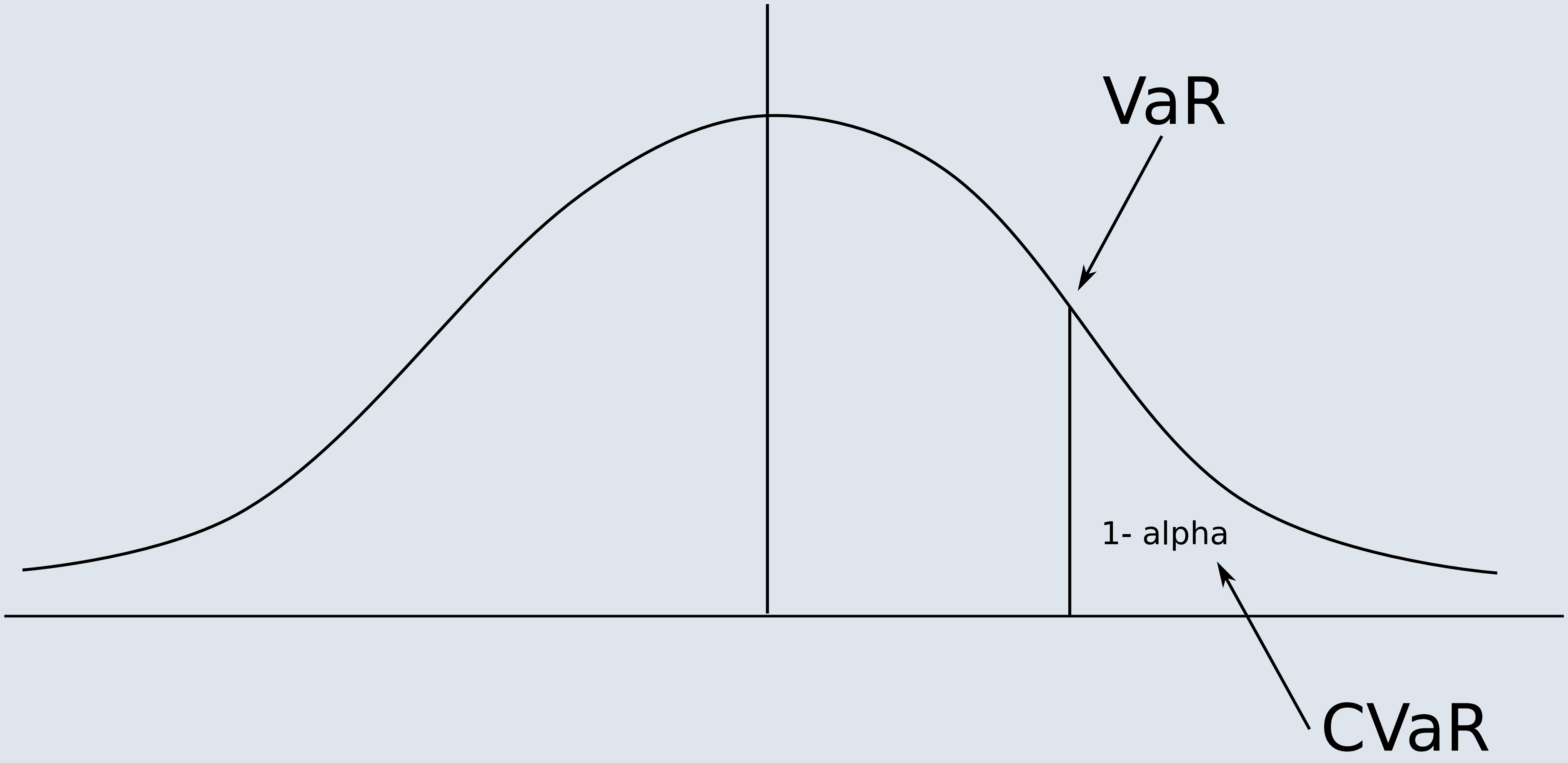
Decision Theory

Conditional Value at Risk

- Extensively studied (also for portfolio): conditional value at risk (CVaR)
- also referred to as “expected shortfall”
- estimates risk of an investment; focus on less profitable outcomes (conservative measure)
- Value at risk VaR_α (Minimization):
  - What is the value x such that the probability of being less than or equal to x is α? (Quantile)
- Conditional value at risk $CVaR_\alpha$ (Minimization):
  - What is the expected value of all outcomes greater than VaR_α?

Decision Theory

CVaR



Decision Theory

CVaR

- What is the expected value of the $1 - \alpha$ worst outcomes?
- Minimize:

p	0.3	0.4	0.3
a_1	4	6	8

- CVaR for $\alpha = 0$:
  - Expected value of all outcomes

Decision Theory

CVaR

- What is the expected value of the $1-\alpha$ worst outcomes?
- Minimize:

p	0.3	0.4	0.3
a_1	4	6	8

- CVaR for $\alpha = 0$:
  - Expected value of all outcomes
- CVaR for $\alpha = 0.2$:
  - 30% outcome 8
  - 40% outcome 6
  - 10% outcome 4
  - CVaR is 6.5

Decision Theory

CVaR

- What is the expected value of the $1 - \alpha$ worst outcomes?
- Minimize:

p	0.3	0.4	0.3
a_1	4	6	8

- CVaR for $\alpha = 0$:
  - Expected value of all outcomes
- CVaR for $\alpha = 0.5$:
  - 30% outcome 8
  - 20% outcome 6
  - CVaR is 7.2

Decision Theory

CVaR

- What is the expected value of the $1 - \alpha$ worst outcomes?
- Minimize:

p	0.3	0.4	0.3
a_1	4	6	8

- CVaR for $\alpha = 0$:
  - Expected value of all outcomes
- CVaR for $\alpha = 0.8$:
  - 20% outcome 8
  - CVaR is 8
  - For α close to 1, CVaR is equal to worst case

Decision Theory

CVaR

- What happens if everything is equally probable?

p	0.25	0.25	0.25	0.25
a_1	1	2	3	4

- CVaR for $\alpha = 1/4$ is the average of the 3 worst outcomes
- CVaR for $\alpha = 2/4$ is the average of the 2 worst outcomes
- CVaR for $\alpha = 3/4$ is the average of the 1 worst outcome

Decision Theory

Connection to OWA

- Reminder: OWA with weights $(1/2, 1/2, 0, 0)$ gives

$$\frac{1}{2}(\text{largest outcome}) + \frac{1}{2}(\text{second-largest outcome})$$

- Therefore: for n scenarios (all equal probability), the following is equivalent
  - OWA with $w = (1/k, 1/k, \ldots, 1/k, 0, 0, \ldots, 0)$ (the first k terms are $1/k$)
  - CVaR with $\alpha = (n-k)/n$

Decision Theory

Example

	s_1	s_2	s_3	s_4	s_5
p	0.4	0.3	0.1	0.15	0.05
a_1	10	15	18	20	23

$$CVaR_{0.1} =$$

$$CVaR_{0.5} =$$

$$CVaR_{0.9} =$$

Decision Theory

Example

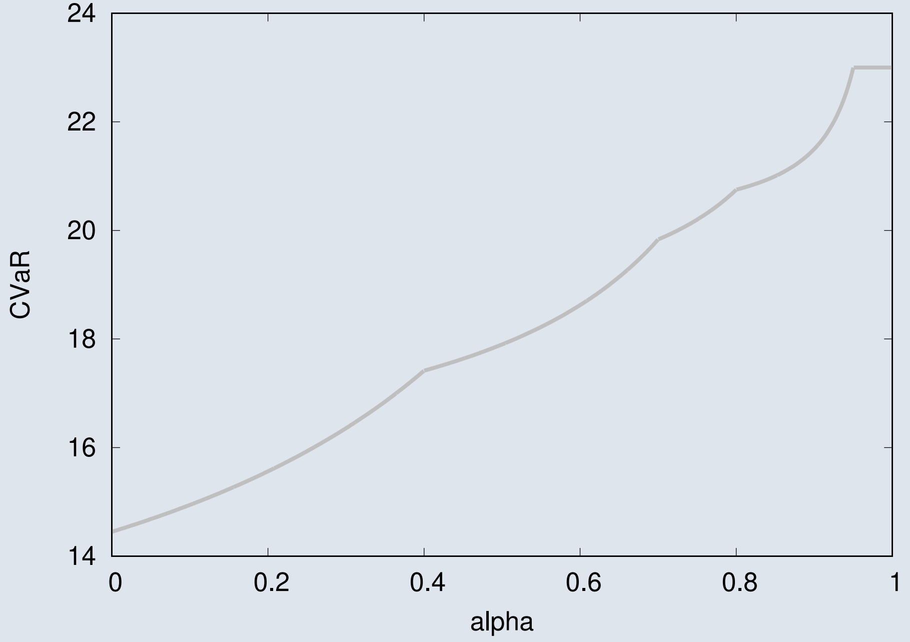
	s_1	s_2	s_3	s_4	s_5
p	0.4	0.3	0.1	0.15	0.05
a_1	10	15	18	20	23

$$
\begin{aligned}
CVaR_{0.1} &= (0.05 \cdot 23 + 0.15 \cdot 20 + 0.1 \cdot 18 + 0.3 \cdot 15 + 0.3 \cdot 10)/0.9 \\
&= 14.9 \\
CVaR_{0.5} &= (0.05 \cdot 23 + 0.15 \cdot 20 + 0.1 \cdot 18 + 0.2 \cdot 15)/0.5 \\
&= 17.9 \\
CVaR_{0.9} &= (0.05 \cdot 23 + 0.05 \cdot 20)/0.1 \\
&= 21.5
\end{aligned}
$$

Decision Theory

Example

	s_1	s_2	s_3	s_4	s_5
p	0.4	0.3	0.1	0.15	0.05
a_1	10	15	18	20	23



Decision Theory

Application of $\mu - \sigma$: Portfolio Theory

- Aim to find the optimal composition of our stock portfolio
- Portfolio return e_j, $j = 1, \dots, n$ possible environmental states

$$
\begin{aligned}
\mu &= \sum_{j \in [n]} e_j p_j \\
\sigma^2 &= \sum_{j \in [n]} (e_j - \mu)^2 \cdot p_j \\
&= \sum_{j \in [n]} e_j^2 p_j - 2\mu \sum_{j \in [n]} e_j p_j + \mu^2 \sum_{j \in [n]} p_j \\
&= \sum_{j \in [n]} e_j^2 p_j - \mu^2
\end{aligned}
$$

Decision Theory

Portfolio Example

- Let's assume there are two stocks
- Return in scenario j is e_{1j}, e_{2j}
- We hold shares x_1 and x_2
- $e_j = e_{1j}x_1 + e_{2j}x_2$
- $\mu = \sum_{j \in [n]} p_j e_j = \mu_1 x_1 + \mu_2 x_2$

Decision Theory

Portfolio Example

- Investor wants to invest 100,000 EUR for exactly one year
- Achieve maximum return
- Applies μ-σ principle
- Decides only between stock 1 and 2
- Beginning of year 0
- Determines the annual returns of the last 5 years

Decision Theory

Portfolio Example

	Year				
	-5	-4	-3	-2	-1
Stock	s_1	s_2	s_3	s_4	s_5
1	15	5	-5	25	10
2	-10	50	10	30	20

- The investor considers all five development possibilities equally likely, $p_j = 0.2$
- $\mu_1 = 10$, $\sigma_1 = 10$
- $\mu_2 = 20$, $\sigma_2 = 20$

Decision Theory

Portfolio Example

- The investor perceives the risk of a_2 as too high
- He considers two alternatives:
  - a_1: Invest entirely in Stock 1
  - a_2: Invest 90,000 EUR in stock 1
    and 10,000 EUR in stock 2

Decision Theory

Portfolio Example

- a_1 evidently has $\mu_{a_1} = 10$, $\sigma_{a_1} = 10$
- For a_2:

	s_1	s_2	s_3	s_4	s_5
a_2	12.5	9.5	-3.5	25.5	11.0

- $\mu_{a_2} = 11$ and $\sigma_{a_2} = 9.2$
- The mixture is better in terms of expected value and standard deviation

Decision Theory

Portfolio Example

- The expected value $e_j = e_{1j}x_1 + e_{2j}x_2$ is linear
- Therefore, the new expected value is a combination of the old ones,

$$\mu_1 = 10, \ \mu_2 = 20, \ \mu_{a_2} = 0.9 \cdot 10 + 0.1 \cdot 20 = 11$$

- This does not apply to standard deviation:

$$\sigma_1 = 10, \ \sigma_2 = 20, \ \sigma_{a_2} = 9.2$$

- We use a riskier stock to reduce risk!

Decision Theory

Correlation

- Why is this so?
- Need correlation ρ
- Determines how two random variables respond similarly to changes in the environment
  - $\rho = 1$: Same
  - $\rho = -1$: Opposite
  - $\rho = 0$: Independent
- Formula:

$$\rho_{12} = \frac{cov_{12}}{\sigma_1 \cdot \sigma_2}$$

with cov_{12} as the covariance between variables 1 and 2:

$$cov_{12} = \sum_{j \in [n]} p_j (e_{1j} - \mu_1) \cdot (e_{2j} - \mu_2)$$

Decision Theory

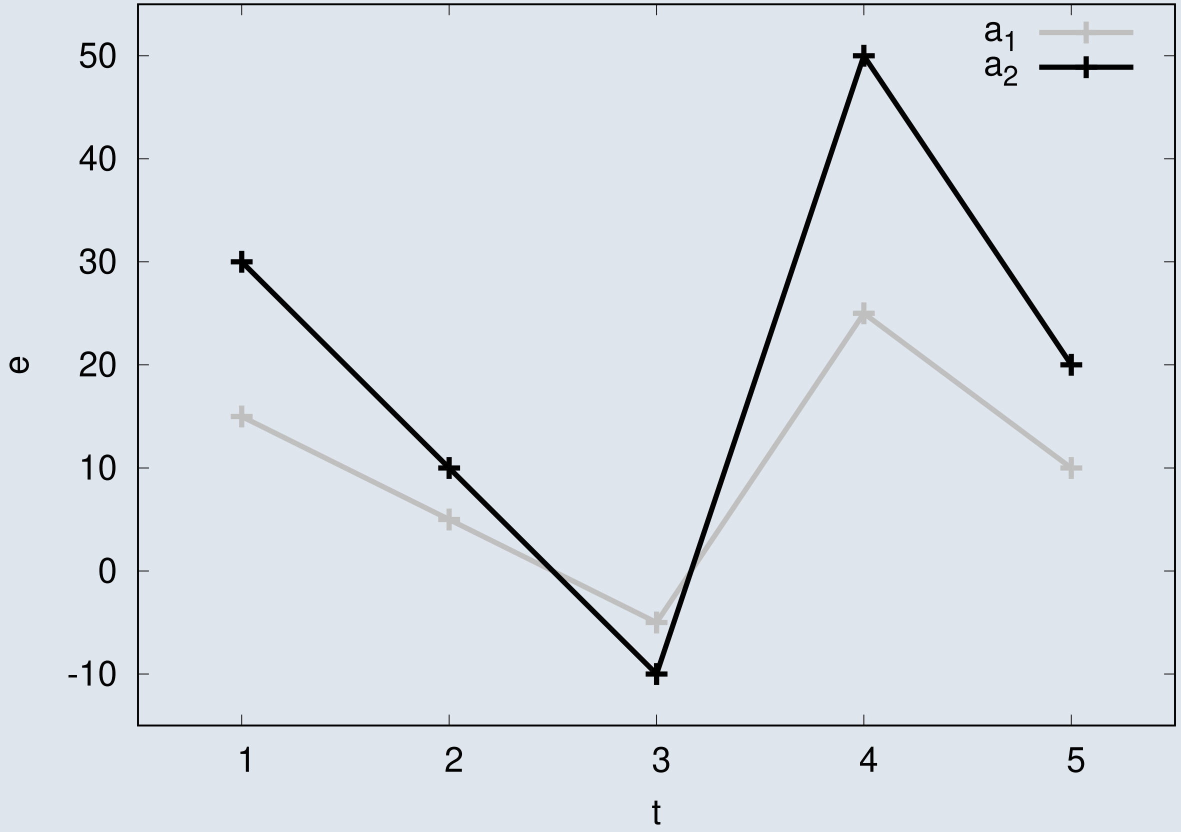
Example

	s_1	s_2	s_3	s_4	s_5
1	15	5	-5	25	10
2	30	10	-10	50	20

- $\mu_1 = 10$, $\sigma_1 = 10$
- $\mu_2 = 20$, $\sigma_2 = 20$
- $cov_{12} = \frac{1}{5}(50 + 50 + 450 + 450 + 0) = 200$
- $\rho_{12} = cov_{12}/\sigma_1\sigma_2 = 200/(10 \cdot 20) = 1$
- Stock 1 and 2 are 100% positively correlated

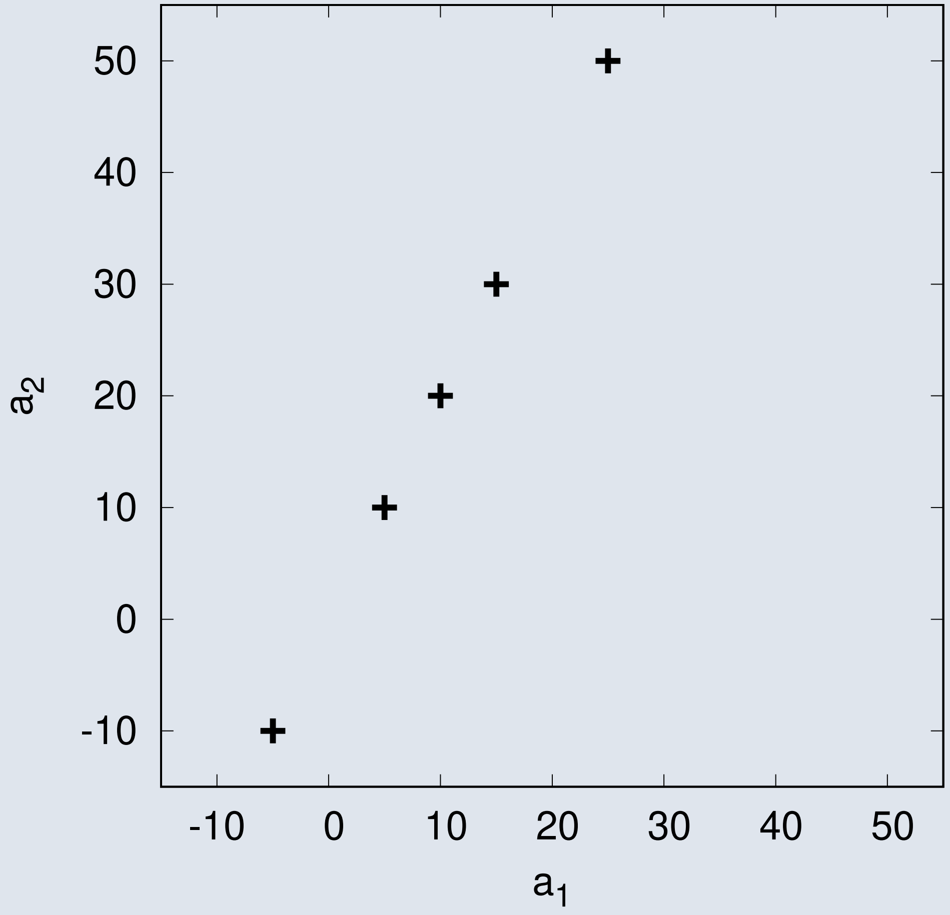
Decision Theory

Example



Decision Theory

Example



Decision Theory

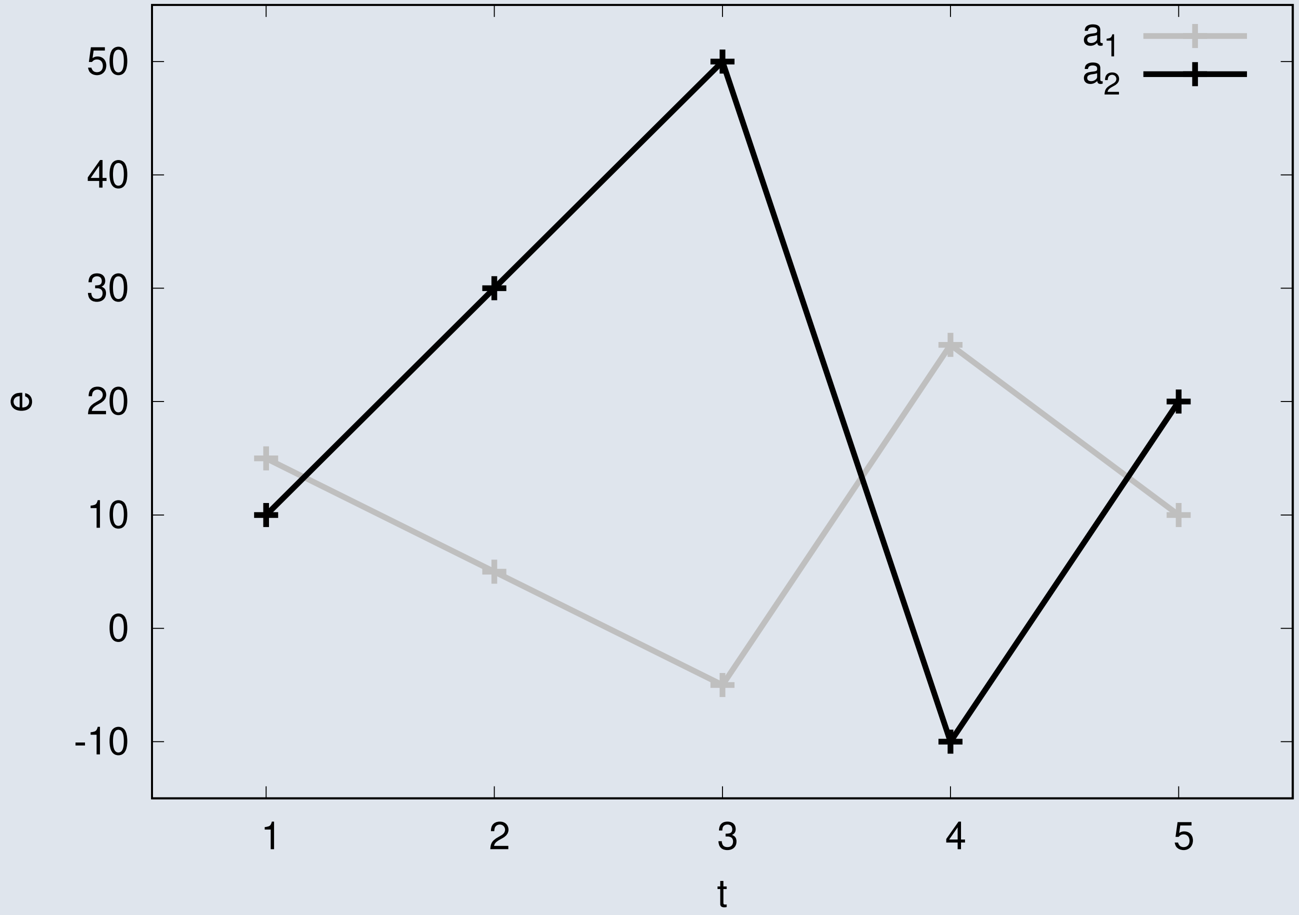
Example

	s_1	s_2	s_3	s_4	s_5
1	15	5	-5	25	10
2	10	30	50	-10	20

- $\mu_1 = 10$, $\sigma_1 = 10$
- $\mu_2 = 20$, $\sigma_2 = 20$
- $cov_{12} = -200$
- $\rho_{12} = cov_{12}/\sigma_1\sigma_2 = -200/(10 \cdot 20) = -1$
- Stock 1 and 2 are 100% negatively correlated

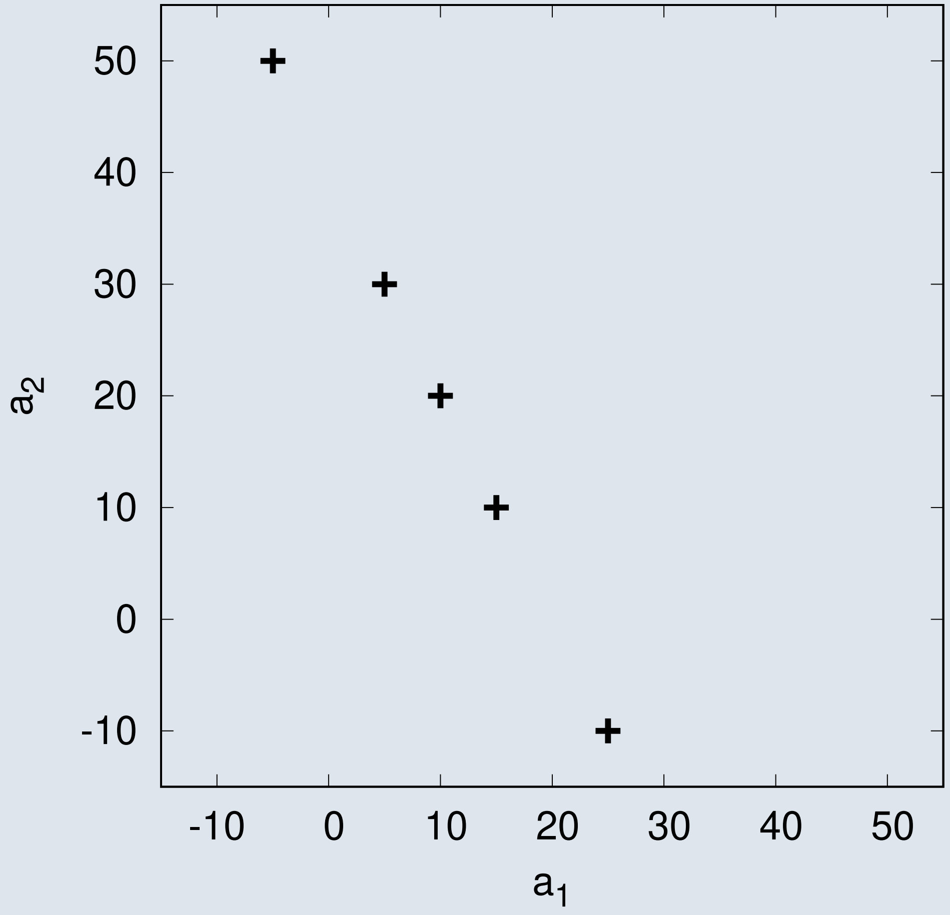
Decision Theory

Example



Decision Theory

Example



Decision Theory

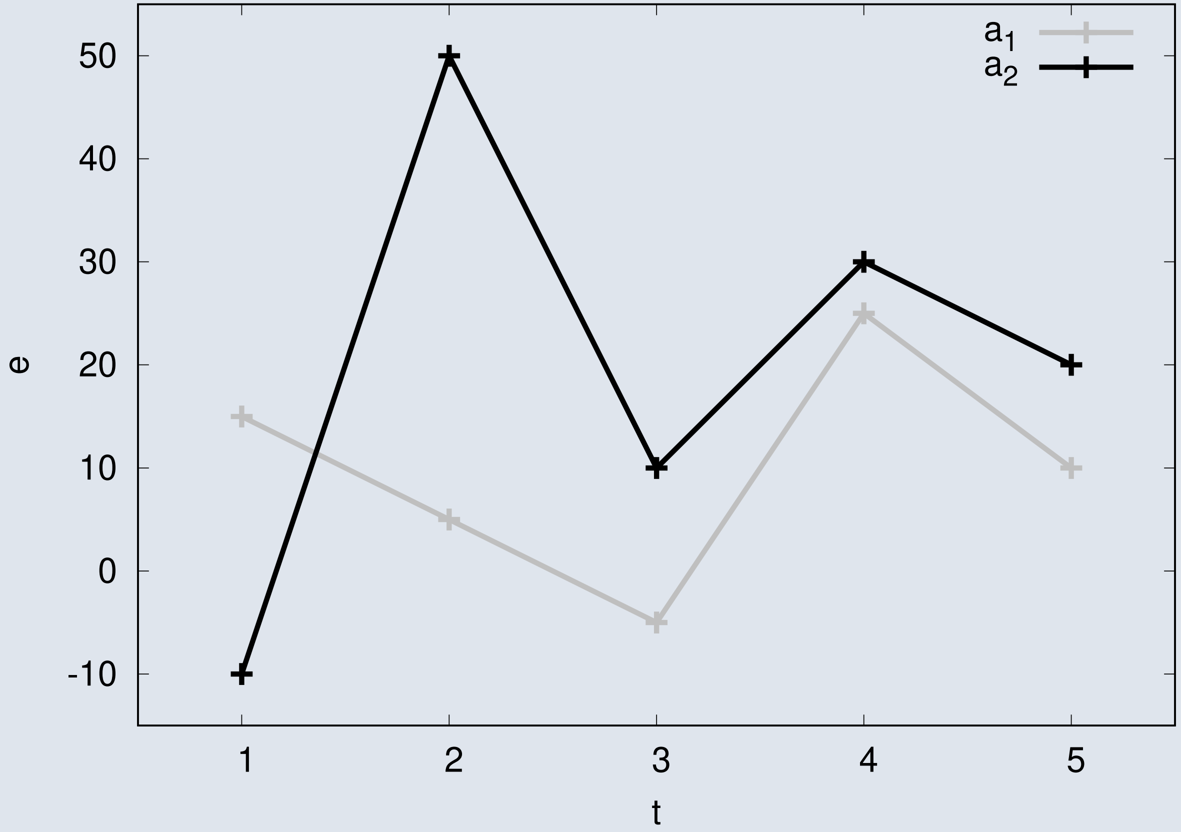
Example

	s_1	s_2	s_3	s_4	s_5
1	15	5	-5	25	10
2	-10	50	10	30	20

- $\mu_1 = 10$, $\sigma_1 = 10$
- $\mu_2 = 20$, $\sigma_2 = 20$
- $cov_{12} = 0$
- $\rho_{12} = cov_{12}/\sigma_1\sigma_2 = 0/(10 \cdot 20) = 0$
- Stocks 1 and 2 are not correlated

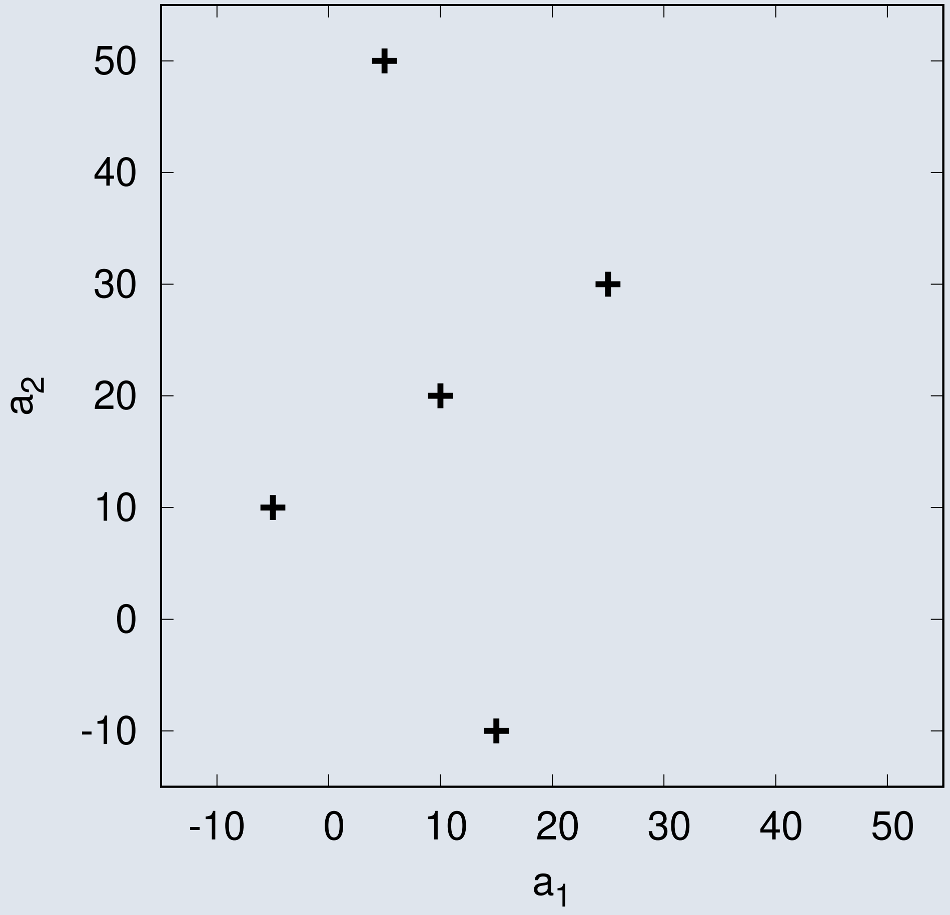
Decision Theory

Example



Decision Theory

Example



Correlation

- Depending on the proportions x_1 and x_2, the variance of the portfolio is given by:

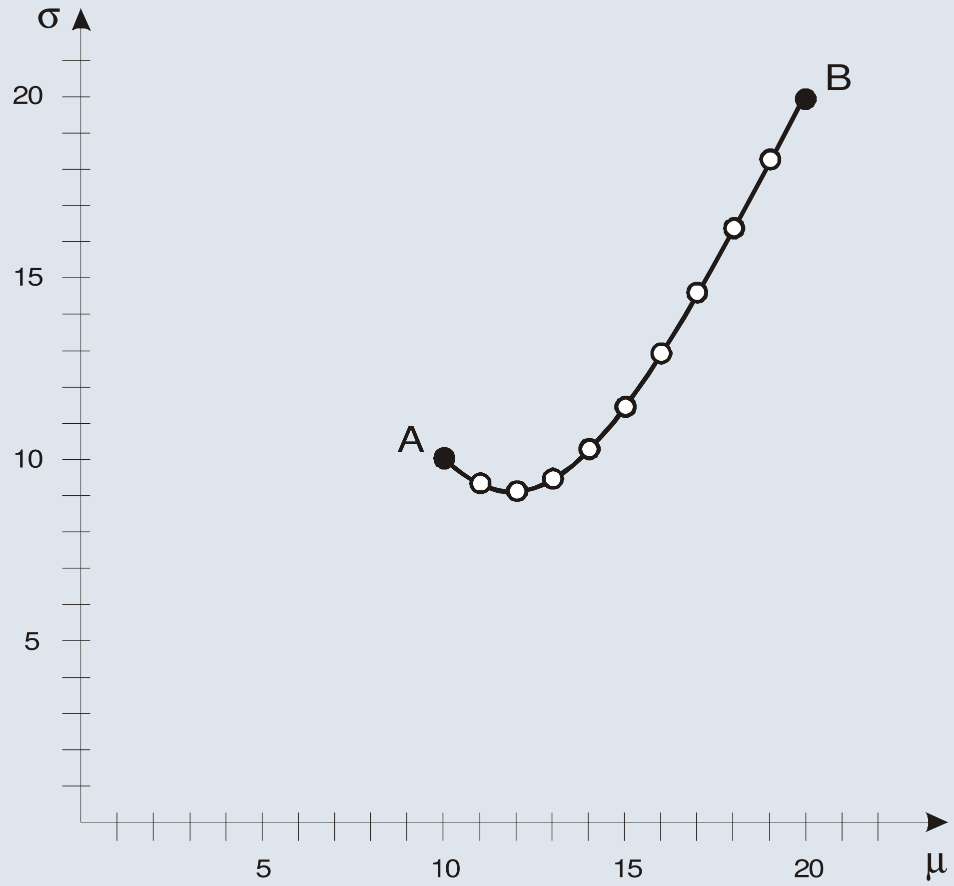
$$\sigma^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12}$$

- In special cases:
  - $\rho = 1 \rightarrow \sigma^2 = (x_1\sigma_1 + x_2\sigma_2)^2$
    - ▶ Standard deviation is linear in x_1, x_2 (similar to expected value)
  - $\rho = -1 \rightarrow \sigma^2 = (x_1\sigma_1 - x_2\sigma_2)^2$
    - ▶ We can reduce variance by combining appropriately
    - ▶ Also known as "Hedging"
  - $\rho = 0 \rightarrow \sigma^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2$
    - ▶ Reduction in variance is also possible here

Decision Theory

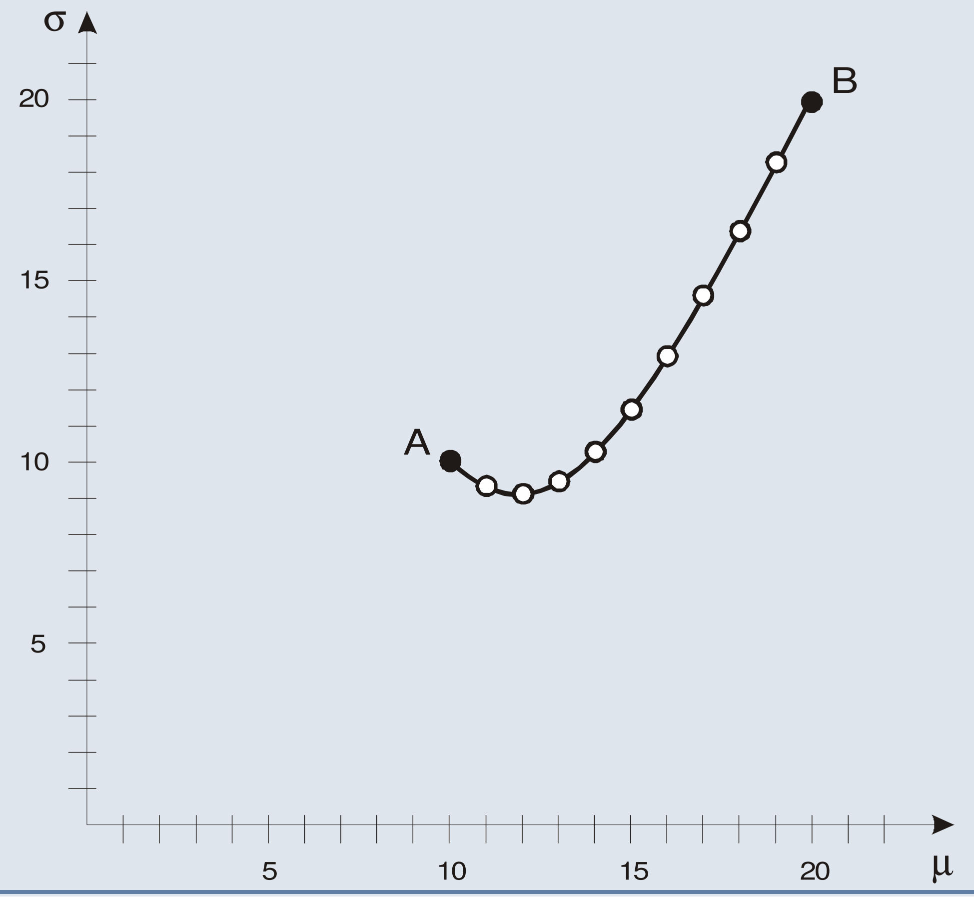
Correlation

- We only need to set x_1, and x_2 is then defined as $1 - x_1$
- Consider all combinations for $x_1 = 1$ (A) to $x_1 = 0$ (B)
- Plot μ and σ in a diagram for each combination (portfolio line)



Decision Theory

Correlation



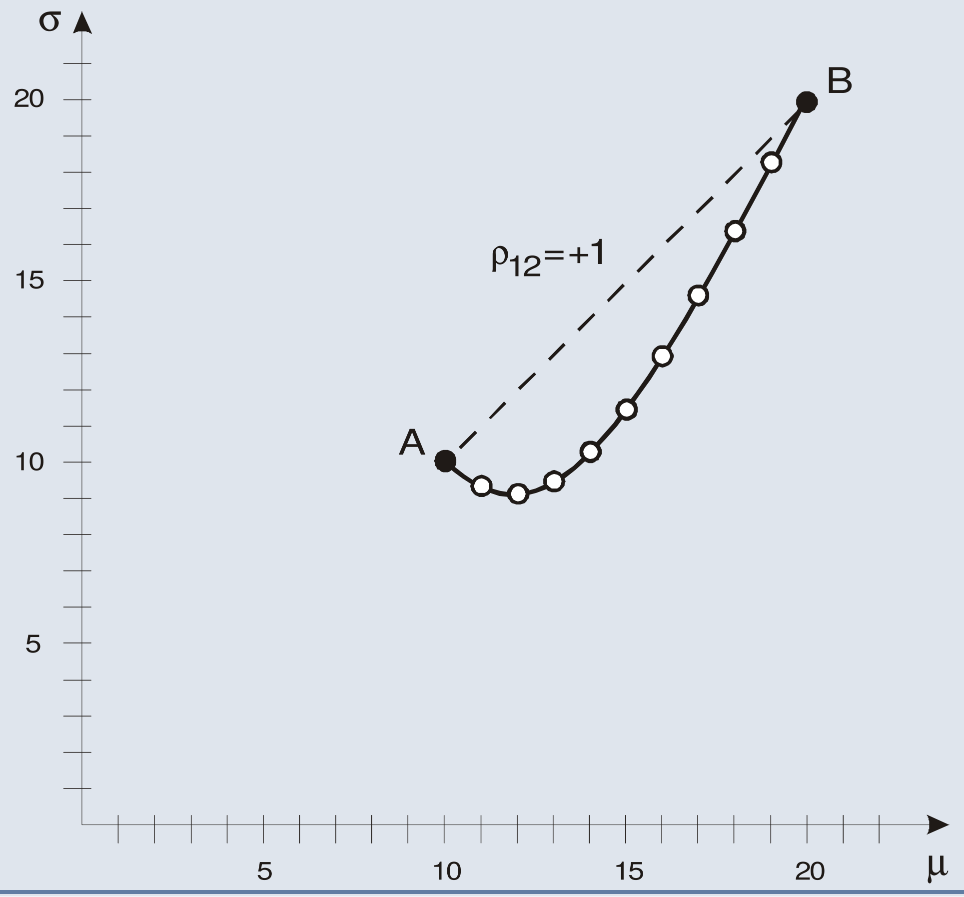
- μ-σ diagram:
- Suppose we have $\rho = 1$

$$\begin{aligned}\sigma &= x_1\sigma_1 + x_2\sigma_2 \\ &= \sigma_2 + (\sigma_1 - \sigma_2)x_1\end{aligned}$$

- We can only increase expected value if we increase standard deviation equally
- Linear relationship

Decision Theory

Correlation



- μ-σ diagram:
- Suppose we have $\rho = 1$

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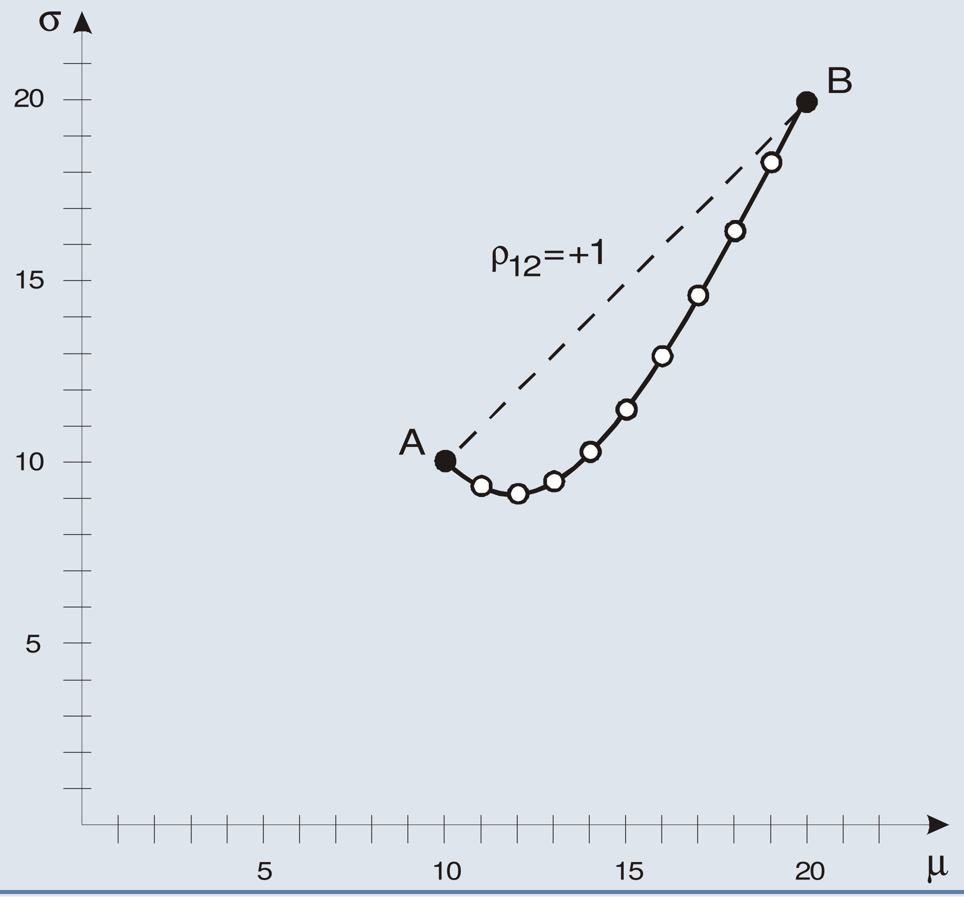
Decision Theory

Correlation

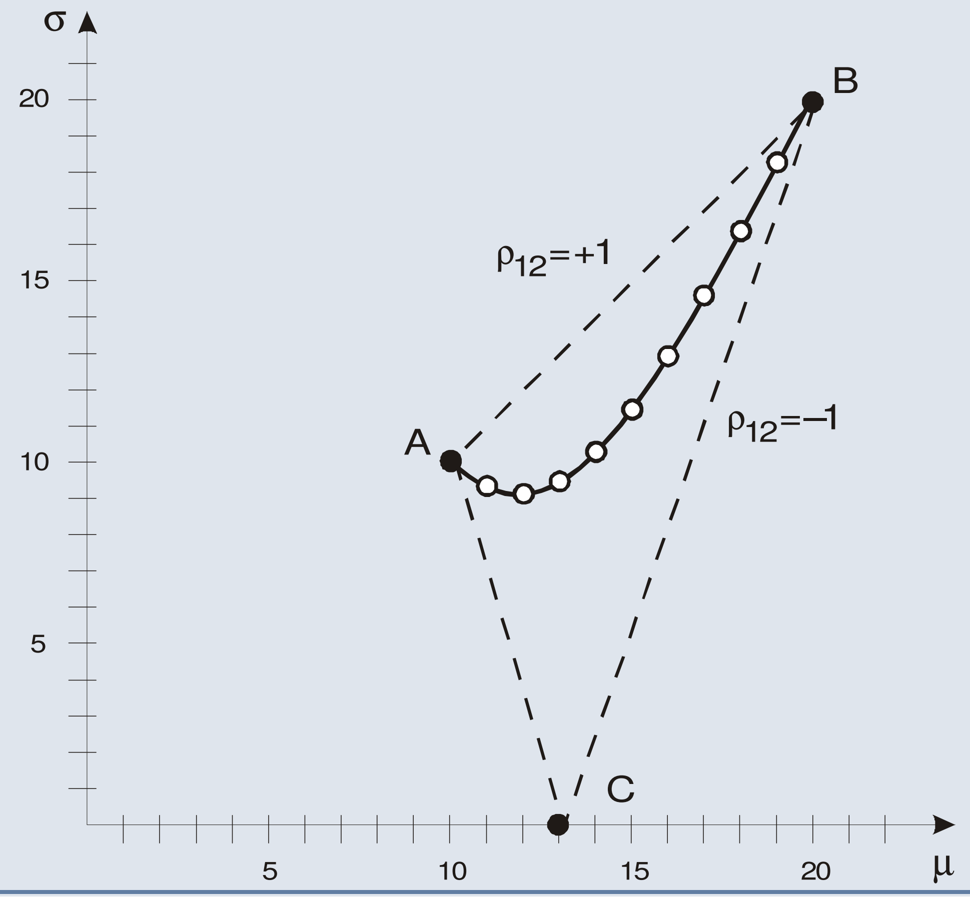


- μ-σ diagram:
- Suppose we have $\rho = -1$

$$\begin{aligned}\sigma &= \left|x_1\sigma_1 - x_2\sigma_2\right| \\ &= \left|(\sigma_1 + \sigma_2)x_1 - \sigma_2\right|\end{aligned}$$

- For $x_1 = \sigma_2/(\sigma_1 + \sigma_2)$, we get $\sigma = 0$

Correlation



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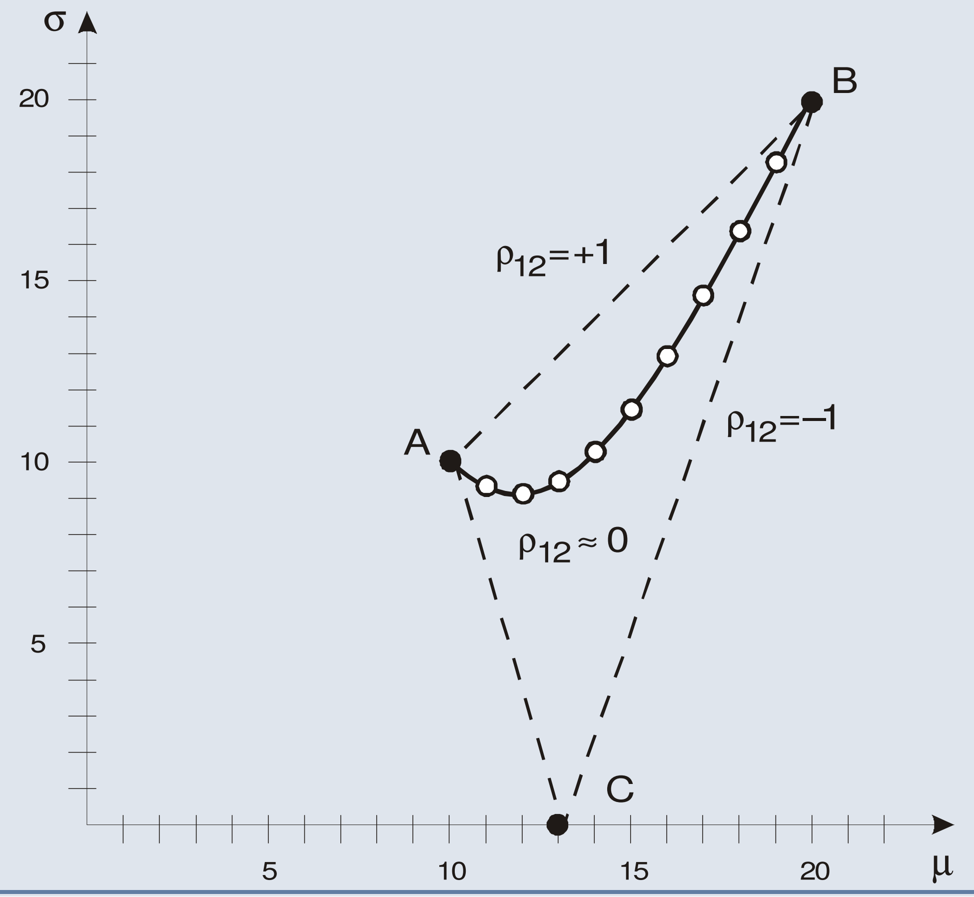
Correlation



- μ-σ diagram:
- Suppose ρ is between -1 and +1
- Get a curve in triangle ABC
- For ρ close to -1: convex, close to C
- For ρ close to +1: steep, close to line AB

Decision Theory

Example

- We continue to have Stock 1 with $\mu_1 = 10$ and $\sigma_1 = 10$ as an option
- Also, there are government bonds with a one-year maturity
- Guaranteed to have no variance
- $\mu_0 = 6$ and $\sigma_0 = 0$
- Proportions x_0 and x_1
- Expected value?

$$\mu = 6x_0 + 10x_1$$

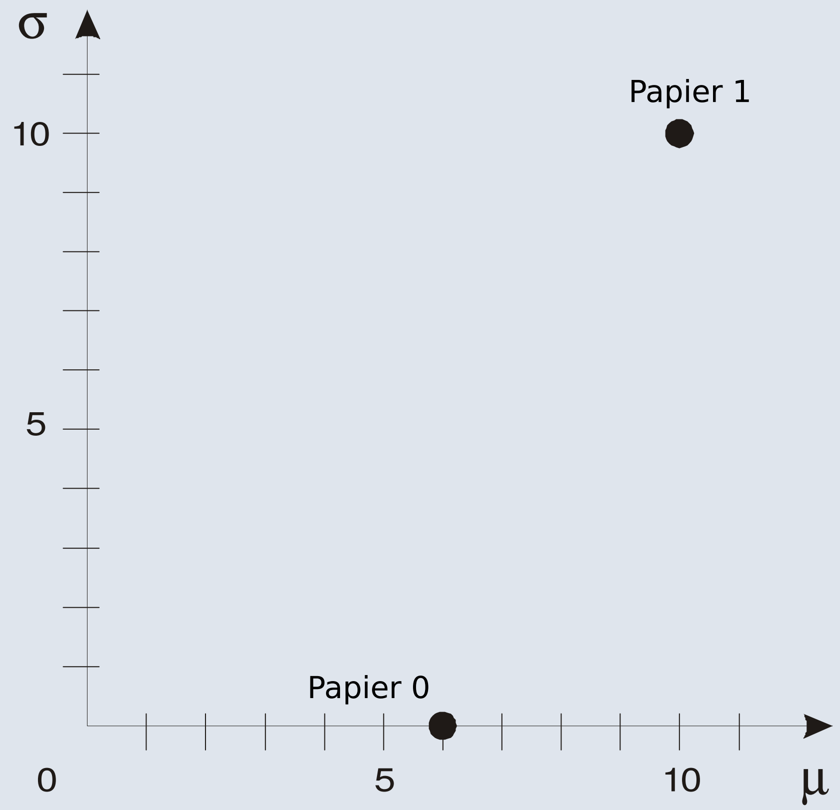
- Variance?

$$\sigma^2 = \sigma_0^2 x_0^2 + \sigma_1^2 x_1^2 + 2x_0 x_1 \sigma_0 \sigma_1 \rho_{12} = \sigma_1^2 x_1^2 = 100x_1^2$$

Decision Theory

Portfolio Line

- What does the portfolio line look like?



Decision Theory

Portfolio Line

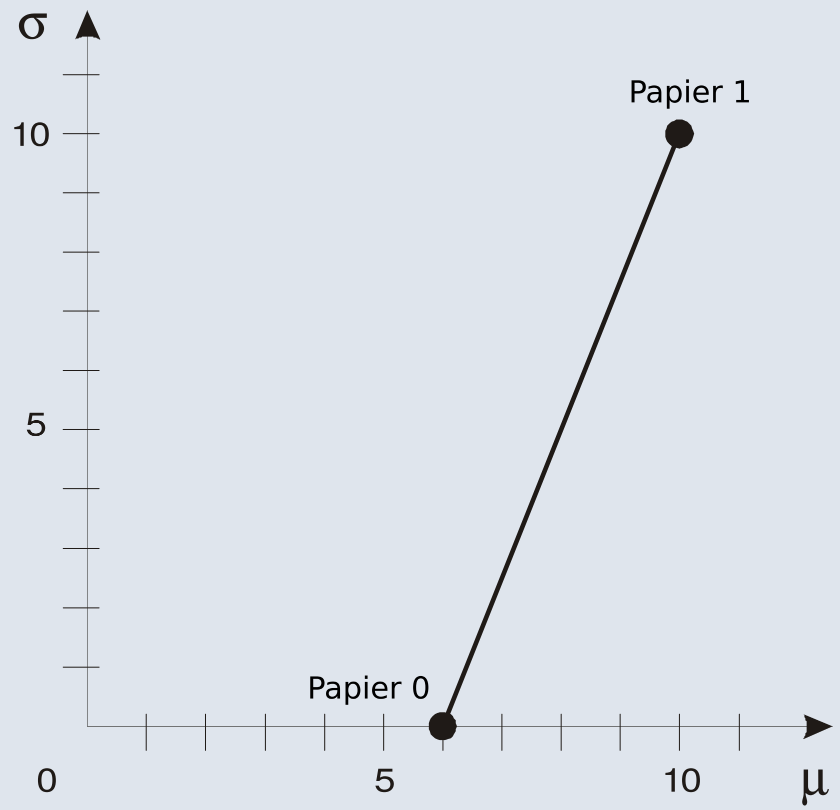
- $x_0 + x_1 = 1$
- $\mu = \mu_0 x_0 + \mu_1 x_1 = \mu_0(1 - x_1) + \mu_1 x_1 = (\mu_1 - \mu_0)x_1 + \mu_0$
- $x_1 = \frac{\mu - \mu_0}{\mu_1 - \mu_0}$
- $\sigma = \sigma_1 x_1 = \sigma_1 \cdot \frac{\mu - \mu_0}{\mu_1 - \mu_0} = \frac{\sigma_1}{\mu_1 - \mu_0}\mu - \frac{\mu_0 \sigma_1}{\mu_1 - \mu_0}$
- With $\mu_0 = 6$, $\mu_1 = 10$, and $\sigma_1 = 10$, we have:

$$\sigma = 2.5\mu - 15$$

Decision Theory

Visualization

$$\sigma = 2.5\mu - 15$$



Decision Theory

Portfolio Optimization

- Which portfolio is the best?
- Must know the preference function $\Phi(\mu, \sigma)$
- Substitution: function in (x_1, x_2), then only x_1
- Take the derivative with respect to x_1 and set it equal to zero

Decision Theory

Portfolio Optimization

- Which portfolio is the best?
- Must know the preference function $\Phi(\mu, \sigma)$
- Substitution: function in (x_1, x_2), then only x_1
- Take the derivative with respect to x_1 and set it equal to zero

In the Example

$$\Phi(\mu, \sigma) = \mu - \alpha\sigma^2 = (\mu_1 - \mu_0)x_1 + \mu_0 - \alpha\sigma_1^2 x_1^2$$

$$\Phi'(x_1) = \mu_1 - \mu_0 - \alpha\sigma_1^2 x_1$$

$$\Phi'(x_1) = 0 \Leftrightarrow x_1 = \frac{\mu_1 - \mu_0}{\alpha\sigma_1^2}$$

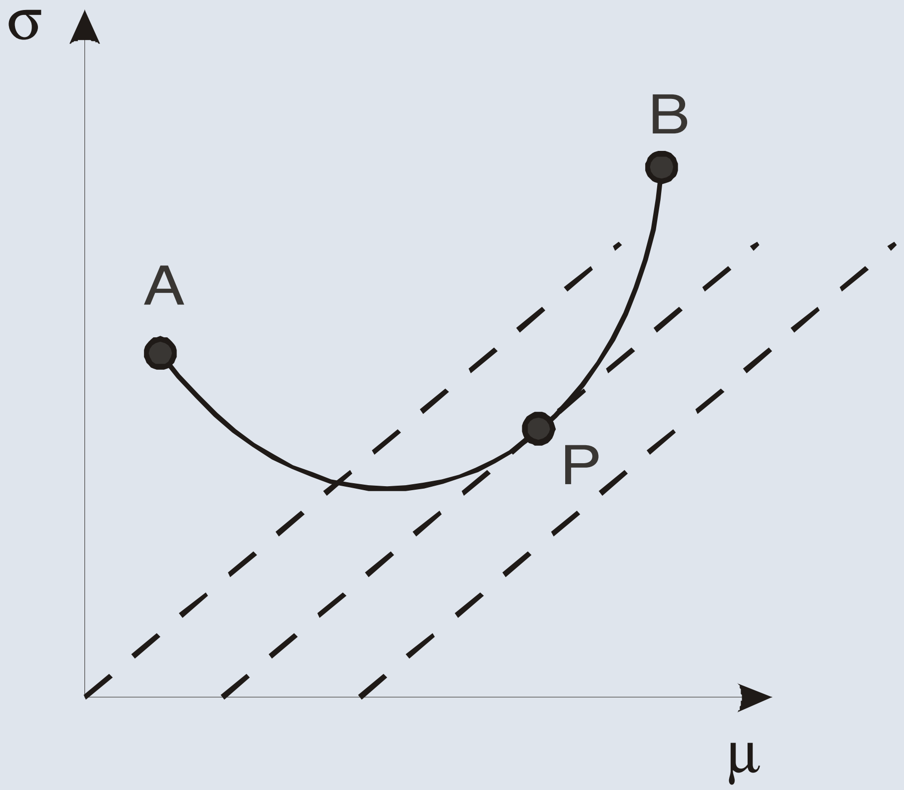
Decision Theory

Portfolio Optimization

- Can also use a portfolio line
- $\Phi(\mu, \sigma) = \mu - \alpha\sigma$, $\alpha > 0$: linear level curves
- $\Phi(\mu, \sigma) = \mu - \alpha\sigma^2$, $\alpha > 0$: concave level curves
- Find the intersection point of the highest level curve with the portfolio line

Decision Theory

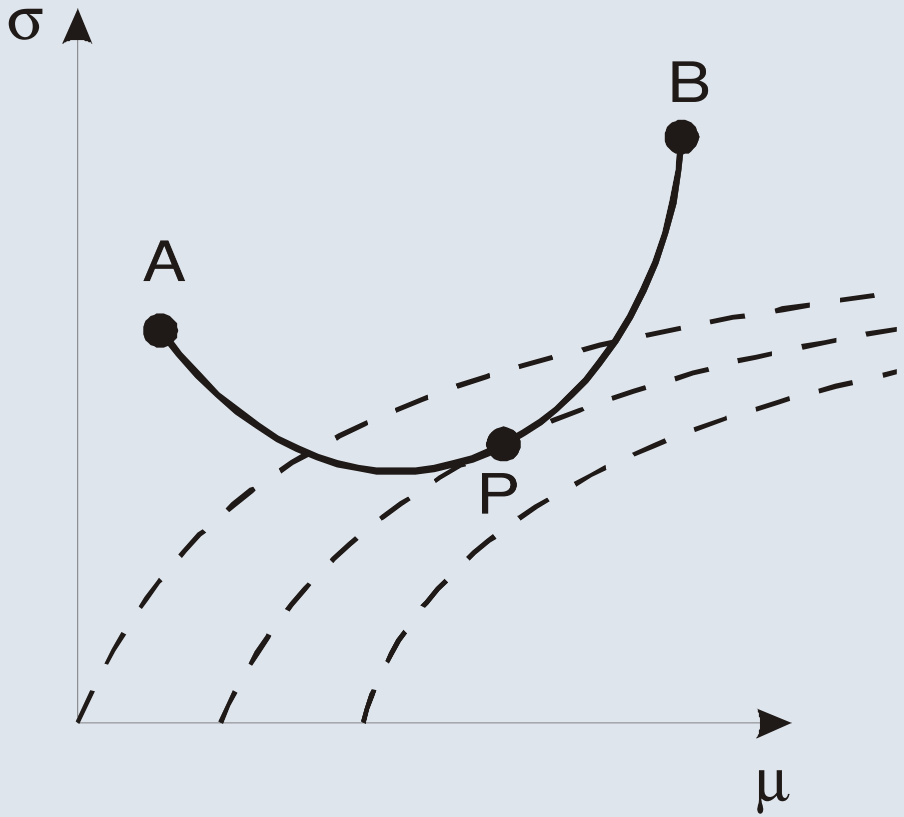
Case $\Phi(\mu, \sigma) = \mu - \alpha\sigma$ (Linear Level Curves)



- The best portfolio is at point P

Decision Theory

Case $\Phi(\mu, \sigma) = \mu - \alpha\sigma^2$ (Concave Level Curves)



- The best portfolio is at point P

Decision Theory

Relationship

- Smaller α:
  - We are more risk-friendly
  - Level curves are steeper
  - Intersection point is further to the right
- Larger α:
  - We are more risk-averse
  - Level curves are flatter
  - Intersection point is further to the left
- With more risk, we can achieve portfolios with a better expected value

Decision Theory

Bernoulli Principle – What is it about?

(Daniel) Bernoulli, 1738:

- Evaluation of gambling games
- Not the expected value of the win
- ”Moral expectation” = expected value of utility
- Utility not linear in the win

Bernoulli Principle

1. Use utility function $u_{ij} = u(e_{ij})$
2. Preference $\Phi(a_i)$ is the expected value of the utility values
3. ”Risk-Utility Function”

Decision Theory

Objective Function

$$\max_{i\in[m]} \Phi(a_i) \text{ with } \Phi(a_i) = \sum_{j\in[n]} p_j u(e_{ij})$$

Example

Choose an alternative. Decision matrix:

p	0.1	0.5	0.4
a_1	0	9	36
a_2	49	25	4

Assume utility is $u(e) = \sqrt{e}$. Utility matrix:

p	0.1	0.5	0.4
a_1	0	3	6
a_2	7	5	2

- $\Phi(a_1) = 0 \cdot 0.1 + 3 \cdot 0.5 + 6 \cdot 0.4 = 3.9$
- $\Phi(a_2) = 7 \cdot 0.1 + 5 \cdot 0.5 + 2 \cdot 0.4 = 4.0$

St. Petersburg Paradox

- Utility function circumvents paradox
- For example, for $u(x) = \log_2 x$:

	s_1	s_2	s_3	$\dots$	s_n
$p(s_i)$	$1/2$	$1/4$	$1/8$	$\dots$	2^{-n}
Win	2	4	8	$\dots$	2^n
Utility	1	2	3	$\dots$	n

- How is the game assessed?

$$\Phi = \sum_{i=1}^{\infty} p_i u(e_i) = \sum_{i=1}^{\infty} 2^{-i} i = 2$$

Decision Theory

Properties

- Assume replacing $u(e)$ with a positive-linear transformation

$$\hat{u}(e) = \alpha + \beta \cdot u(e), \qquad \beta > 0$$

- Preference functions:

$$\begin{aligned}
\Phi(a_i) &= \sum_{j \in [n]} p_j u(e_{ij}) \\
\hat{\Phi}(a_i) &= \sum_{j \in [n]} p_j \hat{u}(e_{ij}) = \sum_{j \in [n]} p_j \left(\alpha + \beta u(e_{ij})\right) \\
&= \alpha \sum_{j \in [n]} p_j + \beta \sum_{j \in [n]} p_j u(e_{ij}) = \alpha + \beta \Phi(a_i)
\end{aligned}$$

- $\hat{\Phi}$ is a positive-linear transformation of Φ

Decision Theory

Properties

- Preference functions can be positively linearly transformed without changing the decision
- Arbitrariness of the zero point and unit of measurement
- Sign of Φ is not an indicator of whether the result is pleasant or unpleasant

Decision Theory

Determination of Φ

- Comparison between hypothetical choices of:
  - Certain income
  - Simple chance

Simple Chance

- Exactly two possible outcomes e' and e''
- Probabilities p' and $p'' = 1 - p'$
- Abbreviated as $(e'; p'; e'')$
- e' is the favorable outcome
- p': Probability of success

Decision Theory

Example

- Lottery:
  - 99% lose
  - 1% win 1000 euros
- $e' = 1000$, $p' = 0.01$
- $e'' = 0$, $p'' = 0.99$
- Notation: $(1000; 0.01; 0)$

Utility Elicitation – First Step

- All possible outcomes e_{ij} should lie in the range $[e'', e']$
- Assign arbitrary utilities $1 = u' = u(e') > u'' = u(e'') = 0$

Example

- All outcomes lie in the range between 0 and 100
- Define:
  - $e' = 100$, $u' = u(100) = 1$
  - $e'' = 0$, $u'' = u(0) = 0$

Decision Theory

Utility Elicitation – Second Step

- Decision maker is faced with the choice between
  - A certain alternative with outcome $\overline{e}$
  - A simple chance $(e'; p'; e'')$

Example

- Let's assume choosing $\overline{e} = 25$
- Compare between:
  - Certain outcome of 25
  - Simple chance $(100; p'; 0)$

Decision Theory

Utility Elicitation – Third Step

- p' is determined so that both alternatives are considered equally good
- Critical value: $p^*(\overline{e})$

Example

- p' close to 1: $(100; p'; 0)$ better than $\overline{e} = 25$
- p' close to 0: $(100; p'; 0)$ worse than $\overline{e} = 25$
- By iterative questioning, $p^*(25)$ is determined
- Let's assume $p^*(25) = 0.5$

Utility Elicitation – Fourth Step

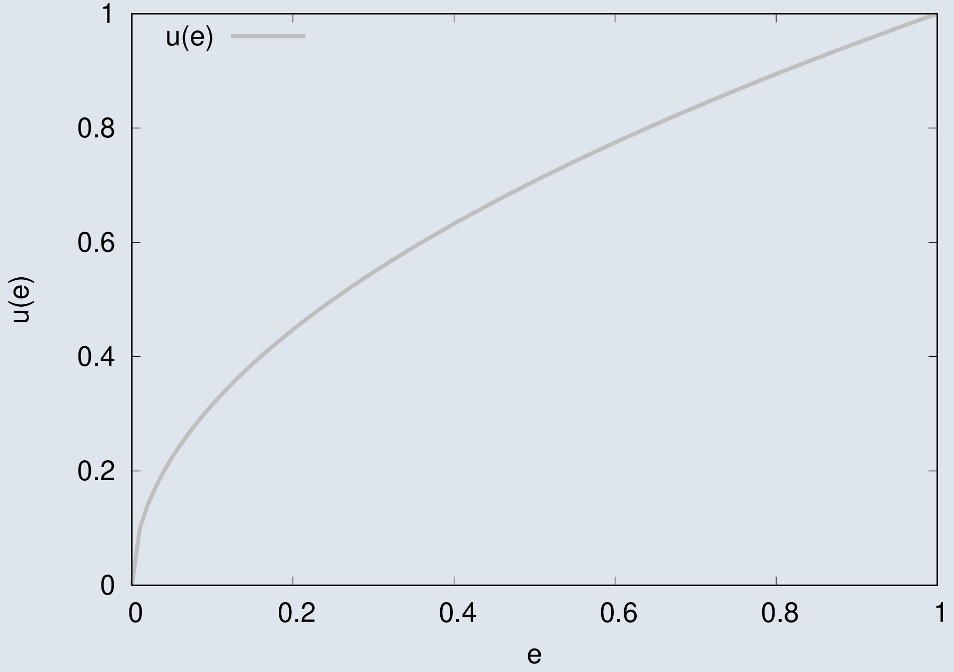
- $\overline{e}$ is as good as $(e'; p^*(\overline{e}); e'')$
- Therefore,
$$u(\overline{e}) = p^*(\overline{e})u(e') + (1 - p^*(\overline{e}))u(e'')$$
- Through normalization: $u(\overline{e}) = p^*(\overline{e})$

Example

$$u(25) = 0.5u(100) + (1 - 0.5)u(0) = 0.5 \cdot 1 = 0.5$$

Course of the Risk-Utility Function

Example from the beginning: $u(e) = \sqrt{e}$



- Larger outcome yields greater utility: $\frac{du}{de} > 0$
- What about the curvature?
  - $\frac{d^2u}{de^2} < 0$: concave increasing
  - $\frac{d^2u}{de^2} > 0$: convex increasing
  - $\frac{d^2u}{de^2} = 0$: linear increasing

Decision Theory

Course of the Risk-Utility Function

- Concave course:
  - Certainty equivalent less than expected value
  - Risk-averse
- Convex course:
  - Certainty equivalent greater than expected value
  - Risk-seeking
- Linear course:
  - Certainty equivalent equal to expected value
  - Risk-neutral

Decision Theory

Quiz

Question 1

	s_1	s_2
1	8	-2
2	-4	1

What combination (x_1, x_2) yields a portfolio with the minimum variance?

Question 2

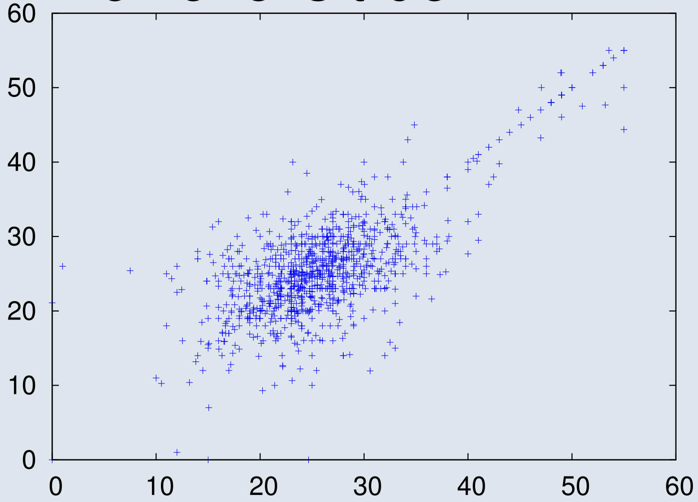
Let u be a linear utility function with $u(8) = 0$ and $u(18) = 1$. What is the expected utility?

p	0.2	0.6	0.2
a_1	18	10	8

1. $\Phi(a_1) = 0.20$
2. $\Phi(a_1) = 0.32$
3. $\Phi(a_1) = 0.41$

Question 3

Which one is true?



1. positive correlation
2. no correlation
3. negative correlation

Decision Theory

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For $x_1 = 1/3$ and $x_2 = 2/3$, all values are zero $\rightarrow$ variance is zero.

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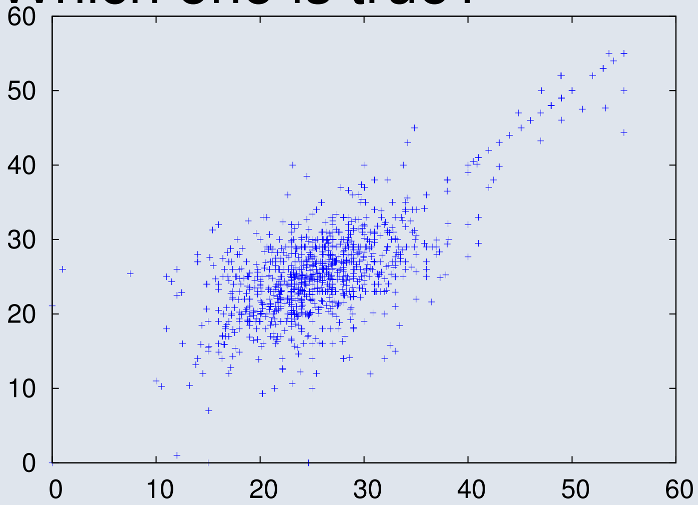

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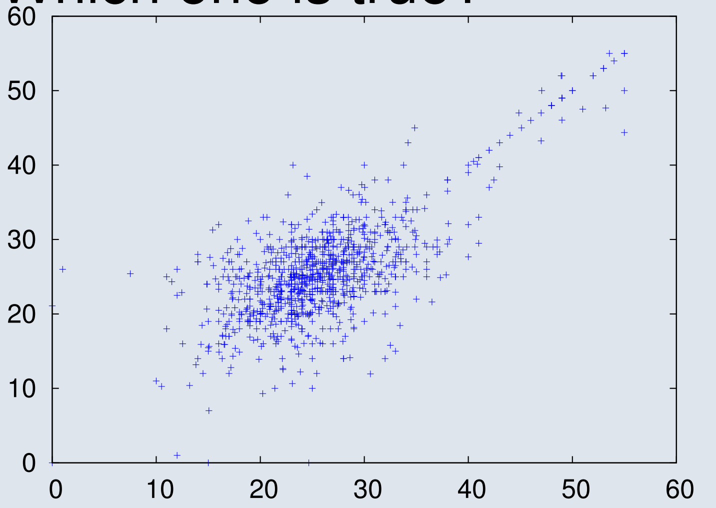

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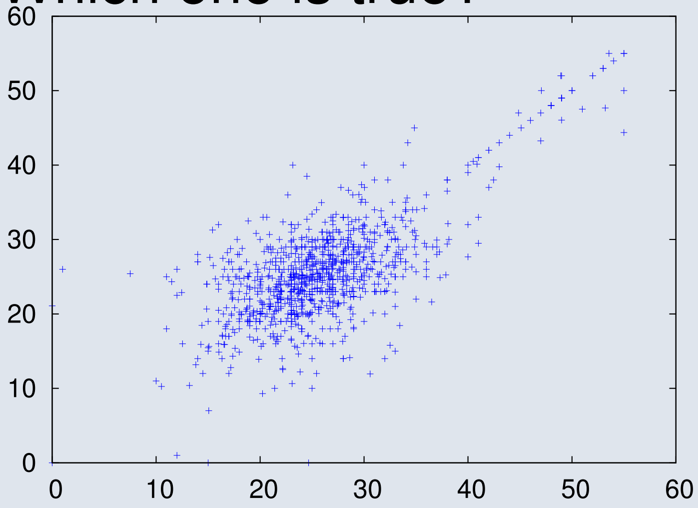

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