Known Operator Learning for Modelling Prior Knowledge in Deep Networks

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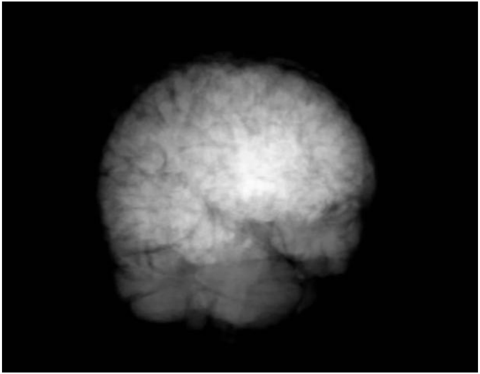
April 24, 2023

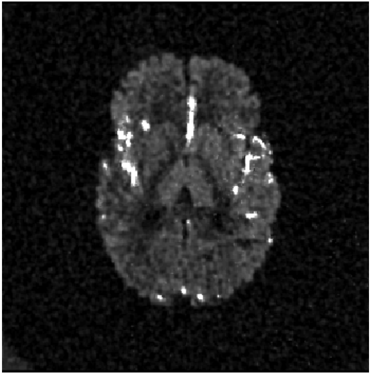
Pattern Recognition Lab

- One Professors
- One Lecturer
- Two Post-Docs
- $\approx$50 PhD Students
- Three Admins

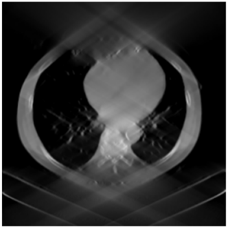
- Biomedical Image and Data Processing
- Computer Vision
- Speech Recognition
- Digital Humanities

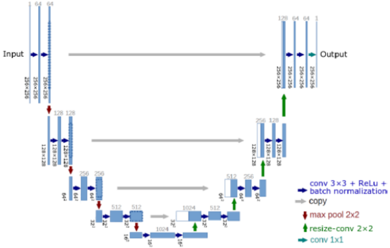
Image Reconstruction

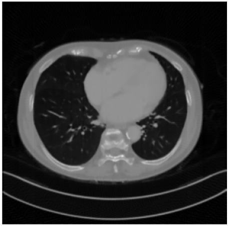




Deep Learning in Image Reconstruction?

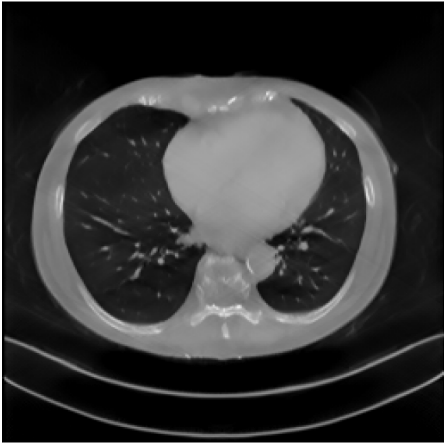






Source: Huang et al. [1]

Deep Learning in Image Reconstruction?



Source: Huang et al. [1]

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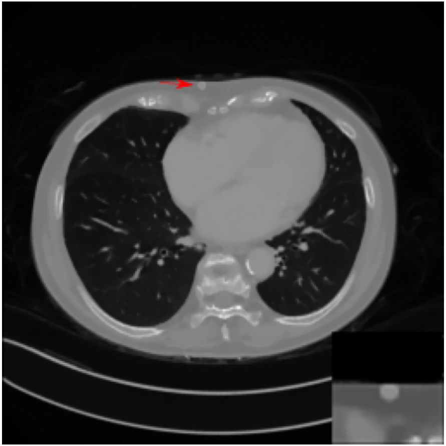
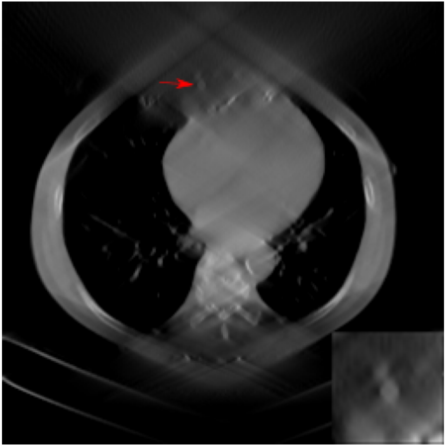


Source: Huang et al. [1]

Deep Learning in Image Reconstruction?



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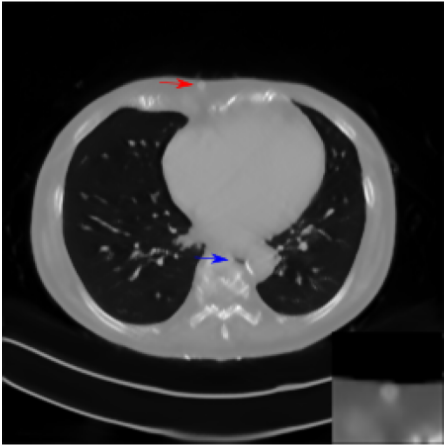

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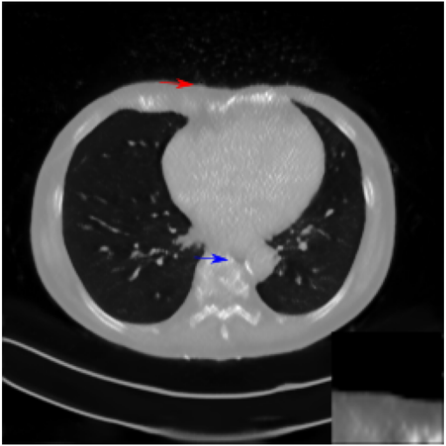
Deep Learning in Image Reconstruction?



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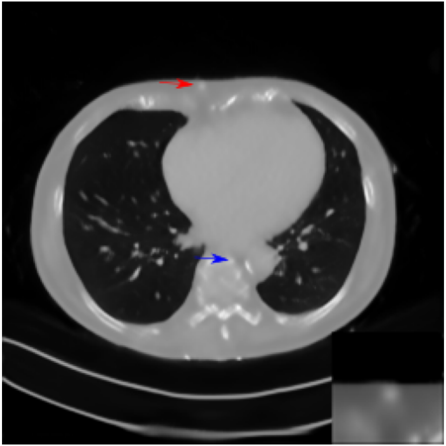

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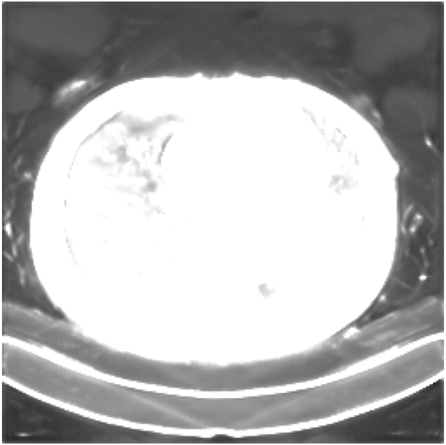
Deep Learning in Image Reconstruction?



Source: Huang et al. [1]

Known Operators in Neural Networks

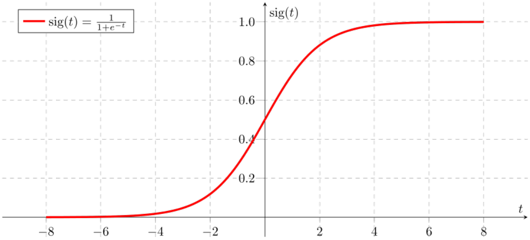
Universal Approximation Theorem

- Any continuous function can be approximated by Neural Net

$$u(\mathbf{x}) \approx U(\mathbf{x}) = \sum_i u_i s(\mathbf{w}_i^\top \mathbf{x} + w_{i,0}) \, ,$$

- The error is bound by

$$|U(\mathbf{x}) - u(\mathbf{x})| \leq \epsilon_u$$



Known Operator Learning Concept

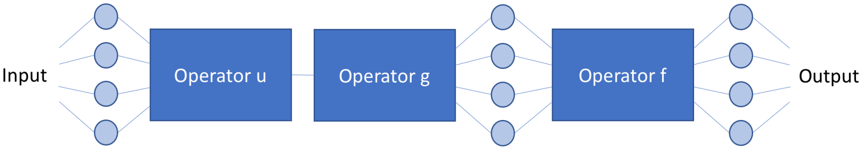


Figure: Known operators embedded in a neural network.

- Consider using known operators in learned mappings
- Specifically consider the use of two operators in sequence

$$f(\mathbf{x}) = g(u(\mathbf{x}))$$

Source: Maier et al. [5]

Approximation Sequences

- Sequential operations

$$f(\mathbf{x}) = g(u(\mathbf{x}))$$

- Can be approximated:

$$F_u(\mathbf{x}) = g(U(\mathbf{x})) = f(\mathbf{x}) - e_u$$
$$F_g(\mathbf{x}) = G(u(\mathbf{x})) = f(\mathbf{x}) - e_g$$
$$F(\mathbf{x}) = G(U(\mathbf{x})) = f(\mathbf{x}) - e_f$$

Source: Maier et al. [5]

Error of Approximation Sequences

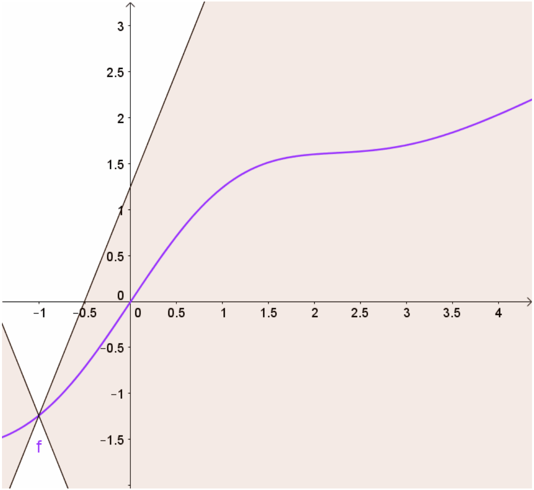
- Approximation introduces error

$$
\begin{aligned}
f(\mathbf{x}) = g(u(\mathbf{x})) &= G(u(\mathbf{x})) + e_g \\
&= \sum_j g_j s\left(u_j(\mathbf{x})\right) + g_0 + e_g \\
&= \sum_j g_j s\left(U_j(\mathbf{x}) + e_{u_j}\right) + g_0 + e_g
\end{aligned}
$$

- Can we find bounds on this error?

Source: Maier et al. [5]

Lipschitz Continous Functions



Observation on Bounds

- Bound on Error:

$$|e_f| \leq \underbrace{\sum_j |g_j| \cdot l_s \cdot |e_{u_j}|}_{\text{error } U(\mathbf{x})} + \underbrace{\epsilon_g}_{\text{error } G(x)}$$

- Observations
  - Error in $U(\mathbf{x})$ and $G(\mathbf{x})$ additive
  - Error of $U(\mathbf{x})$ amplified by $g(\mathbf{x})$
  - → Features are more important than classifiers
  - Requires Lipschitz continuity

Source: Maier et al. [5]

Computed Tomography

- Efficient solution via filtered back-projection:

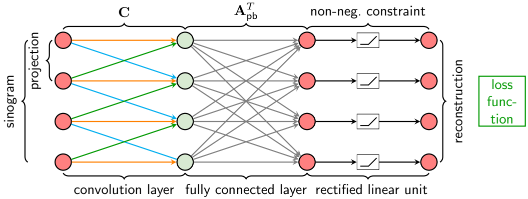
$$f(x, y) = \int_0^{\pi} p(s, \theta) * h(s)\big|_{s=x\cos\theta+y\sin\theta} d\theta$$

- Can also be derived in matrix notation:

$$\begin{aligned} \mathbf{Ax} &= \mathbf{p} \\ \mathbf{x} &= \mathbf{A}^{+}\mathbf{p} \\ \mathbf{x} &= \mathbf{A}^{T}(\mathbf{AA}^{T})^{-1}\mathbf{p} \end{aligned}$$

Reconstruction Networks

- All three steps can be modeled as a neural network:



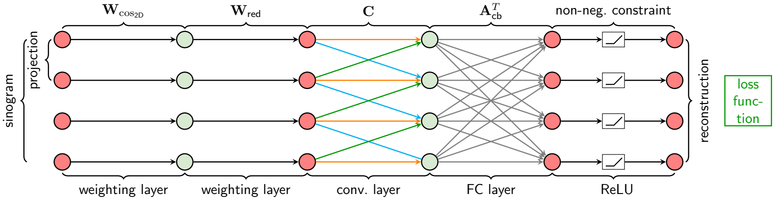
- All weights are known from FBP
- Operator **A** does not fit into memory

➔ compute on-the-fly

Source: Würfl et al. [2]

Reconstruction Networks

- Reconstruction Networks can be expanded



- Embedding of "heuristics" for artifact reduction possible

Source: Würfl et al. [3]

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Application to Incomplete Scans

Figure: Reconstruction with 360°

Application to Incomplete Scans

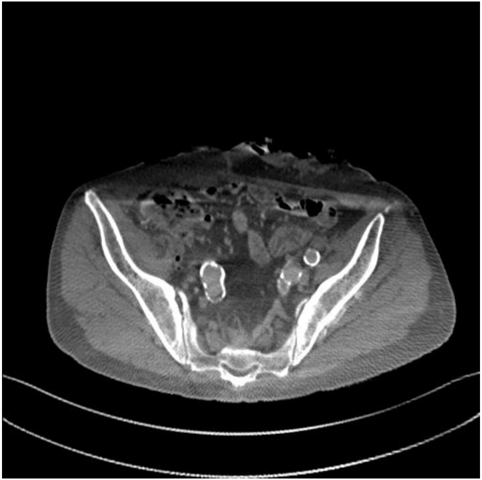


Figure: Reconstruction with 180° (FBP)

Application to Incomplete Scans

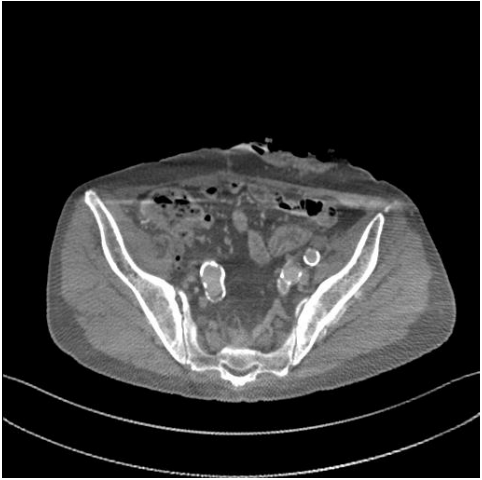


Figure: Reconstruction with 180° (NN)

Learned Weights

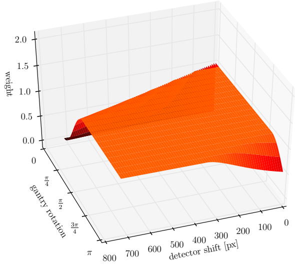


Figure: Parker et al. 1996

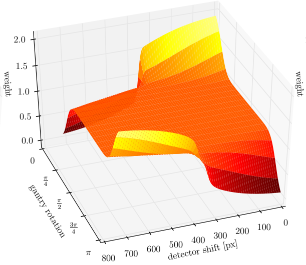


Figure: Schäfer et al. 2017

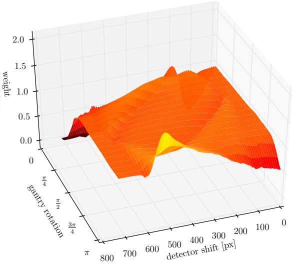


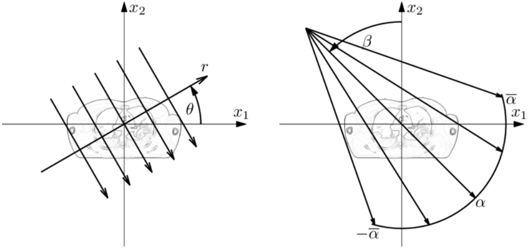
Figure: Neural Network 2018

Source: Würfl et al. [3]

Can we "derive" networks?

$$\mathbf{A}_{\angle}\mathbf{x} = \mathbf{p}_{\angle}$$

$$\mathbf{A}_{||}\mathbf{x} = \mathbf{p}_{||}$$



Source: Syben et al. [7]

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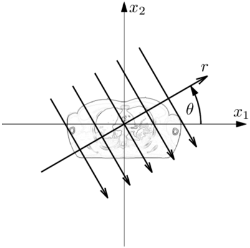
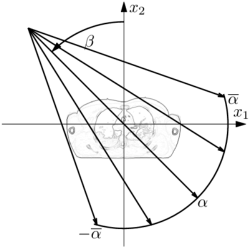
$$\mathbf{x} = \underbrace{\mathbf{A}_\angle^T (\mathbf{A}_\angle \mathbf{A}_\angle^T)^{-1}}_{\text{ill-posed } \angle \text{ reconstruction}} \mathbf{p}_\angle$$





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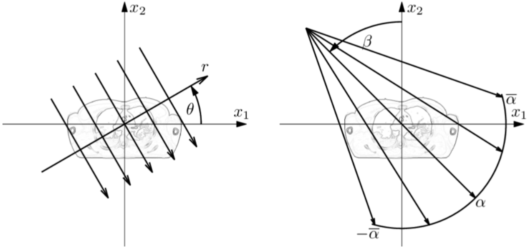
$$\mathbf{p}_{||} = \mathbf{A}_{||}\mathbf{A}_{\angle}^{T} \underbrace{(\mathbf{A}_{\angle}\mathbf{A}_{\angle}^{T})^{-1}}_{\text{No efficient form}} \mathbf{p}_{\angle}$$



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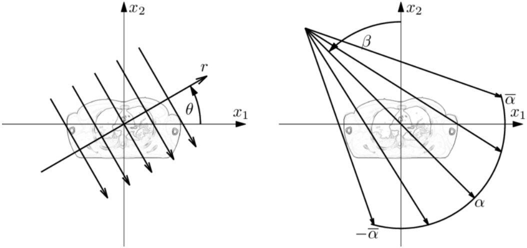
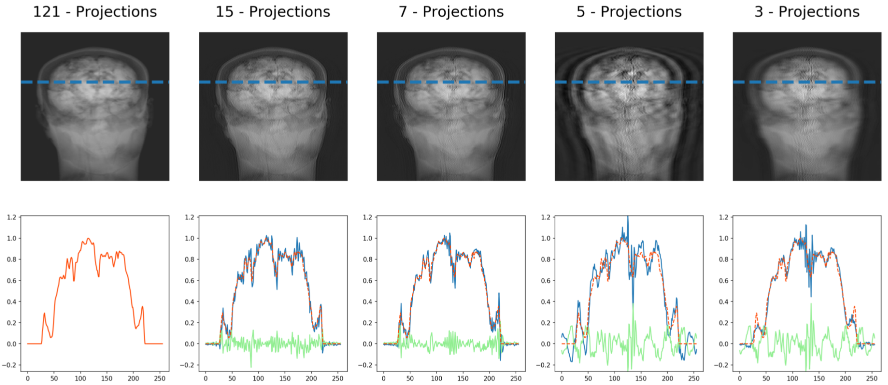
$$\mathbf{p}_{||} = \mathbf{A}_{||} \mathbf{A}_\angle^T \underbrace{(\mathbf{A}_\angle \mathbf{A}_\angle^T)^{-1}}_{\text{No efficient form}} \mathbf{p}_\angle$$

$$\mathbf{p}_{||} = \mathbf{A}_{||} \mathbf{A}_\angle^T \mathcal{F}^H \underbrace{\mathbf{K}}_{\text{model}} \mathcal{F} \mathbf{p}_\angle$$



Source: Syben et al. [7]

Learned Algorithm in Action



Source: Syben et al. [7]

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Known Operator Learning

- Many traditional approaches **equivalent** to neural networks and vice versa
- Learned algorithms are **again** traditional algorithms
- Learned parameters can be **interpreted**
- Virtually all state-of-the art methods can also be **integrated**
- Methods **efficient** and interpretable

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References

References I

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[2] T. Würfl, F. C. Ghesu, V. Christlein, and A. Maier, “Deep Learning Computed Tomography”, in MICCAI 2016, vol. 3, Athen, 2016, pp. 432–440.

[3] T. Würfl, M. Hoffmann, V. Christlein, K. Breininger, Y. Huang, M. Unberath, and A. Maier, “Deep Learning Computed Tomography: Learning Projection-Domain Weights from Image Domain in Limited Angle Problems”, IEEE Transactions on Medical Imaging, vol. 37, no. 6, pp. 1454–1463, 2018.

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[5] A. K. Maier, C. Syben, B. Stimpel, T. Würfl, M. Hoffmann, F. Schebesch, W. Fu, L. Mill, L. Kling, and S. Christiansen, "Learning with known operators reduces maximum error bounds", Nature Machine Intelligence, vol. 1, no. 8, pp. 373–380, 2019.

[6] C. Syben, M. Michen, B. Stimpel, S. Seitz, S. Ploner, and A. K. Maier, "Technical note: Pyro-nn: Python reconstruction operators in neural networks", Medical Physics, vol. 0, no. 0, eprint: https://aapm.onlinelibrary.wiley.com/doi/pdf/10.1002/mp.13753.

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[7] C. Syben, B. Stimpel, J. Lommen, T. Würfl, A. Dörfler, and A. Maier, “Deriving neural network architectures using precision learning: Parallel-to-fan beam conversion”, in German Conference on Pattern Recognition, Springer, 2018, pp. 503–517.

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[9] W. Fu, K. Breininger, R. Schaffert, N. Ravikumar, and A. Maier, “A divide-and-conquer approach towards understanding deep networks”, arXiv preprint arXiv:1907.06194, 2019.