Activation Functions and Convolutional Neural Networks

A. Maier, V. Christlein, K. Breininger, Z. Yang, L. Rist, M. Nau, S. Jaganathan, C. Liu, N. Maul, L. Folle, K. Packhäuser, M. Zinnen
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
April 24, 2023

Outline

Activation Functions

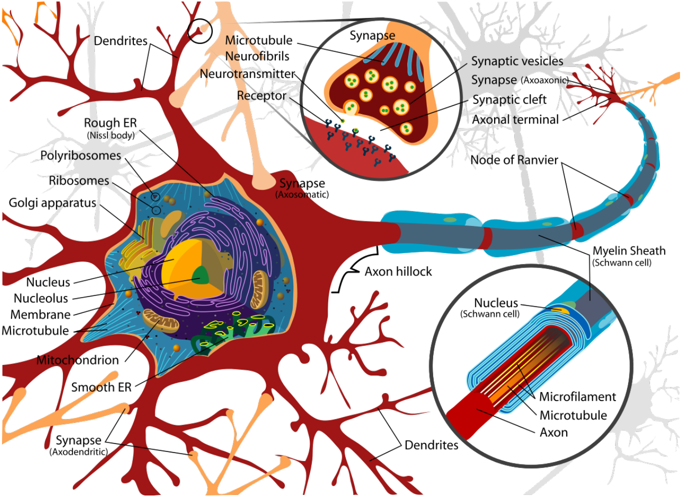
Convolutional Neural Networks

Convolutional Layers

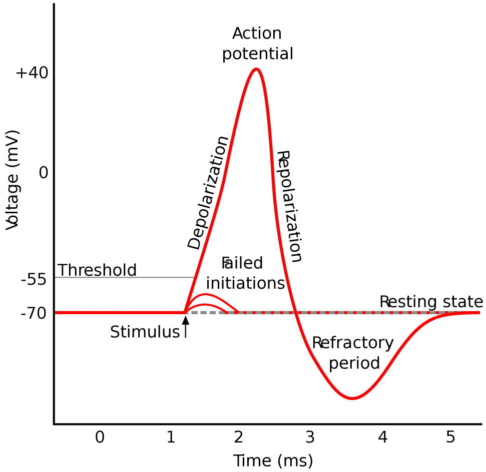
Pooling Layers

Activation Functions

Biological Activation



Biological Activation



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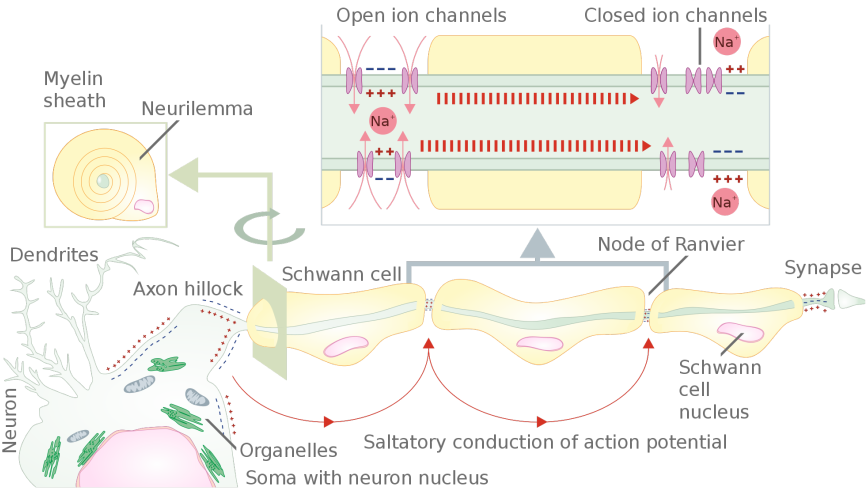

Summary - Activations in Biological Neurons

- The knowledge lies in the **connections**
- Both **inhibitory** and **excitatory** connections exist
- The synapses anatomically enforce **feed forward** processing
- However, the connections can be in any direction
- The **sum** of activations is crucial
- Activations are electric spikes
  - With **specified intensity**
  - Which also encode **information over time**

Activations in Artificial Neural Networks so far

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  ➔ Highly important for a powerful network

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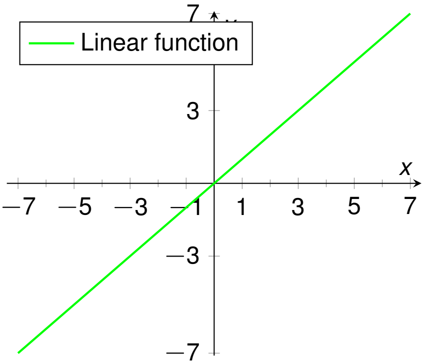
- Can we do better?

Linear Activation Function

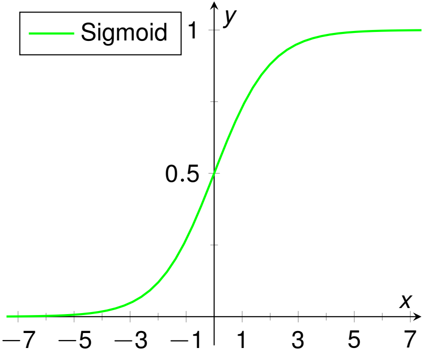


Linear function with parameter α:

$$f(x) = \alpha x$$
$$f'(x) = \alpha$$

+ Simple
− Does not introduce non-linearity
+ Therefore, it renders the optimization problem convex
- Listed here mainly for completeness

Sigmoid Activation Function



Sigmoid (logistic function)

$$f(x) = \frac{1}{1 + exp(-x)}$$

$$f'(x) = f(x)(1 - f(x))$$

- Close to biological model, but differentiable

+ Probabilistic output

− Saturates for $x \ll 0$ and $x \gg 0$

− Not zero-centered

Zero-Centering

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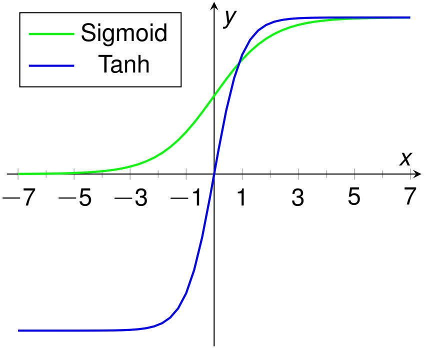
  - ➔ $\nabla_{\mathbf{w}}$ will either be all $+$ or all $-$
- A mean $\mu = 0$ of the input distribution will always be shifted to $\mu > 0$
  - ➔ **co-variate shift** of successive layers
  - ➔ layers **constantly** have to **adapt** to the shifting distribution
- Batch learning reduces the variance σ of the updates

Tanh Activation Function



Tanh

$$f(x) = tanh(x)$$
$$f'(x) = 1 - f(x)^2$$

+ Zero-centered (LeCun '91)
– Shifted version of sigmoid σ: $\mathtt{tanh}(x) = 2\sigma(2x) - 1$
    - ➔ Still saturates
    - ➔ Still causes vanishing gradients

Vanishing and Exploding Gradients

- Essence of learning: How does **x** affect **y**?
- Sigmoid/tanh map **large regions** of **X** to a **small range** in **Y**

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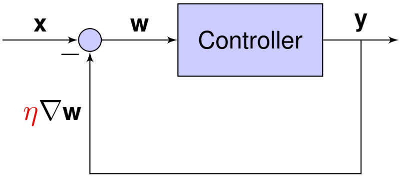
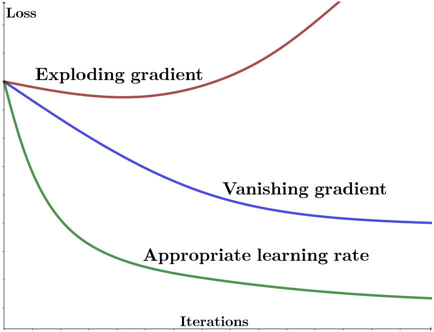
- Related problem: Exploding gradients

Recap: Feedback Loop - Vanishing and Exploding Gradients



Analogy to control theory



- If η is to high $\mapsto$ **positive feedback** $\mapsto$ loss grows **without bounds**
- If η is to small $\mapsto$ **negative feedback** $\mapsto$ **gradient vanishes**
- Choice of η is **critical** for learning

NEXT TIME

ON DEEP LEARNING

Activation Functions and Convolutional Neural Networks - Part 2

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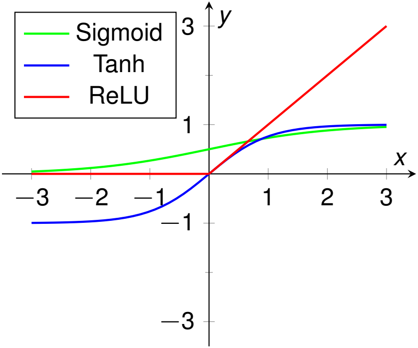
Rectified Linear Units (ReLU)



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$$f(x) = \max(0, x)$$

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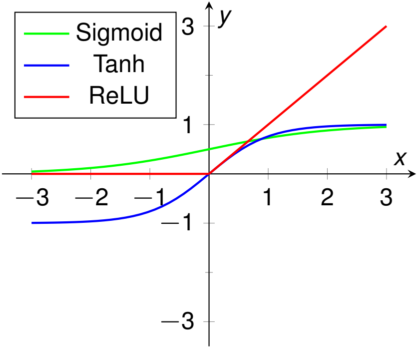

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+ Good generalization due to piece-wise linearity
+ Speed up during learning ($\approx$ 6x (Krizhevsky ’12))
+ No vanishing gradient problem
− Not zero-centered

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More on ReLUs

- ReLUs were a **big step forward**!
- ReLUs enable training of **deep** supervised neural networks **without unsupervised pre-training**
- First derivative is 1 if the unit is active, second derivative is 0 almost anywhere
  - → No second-order effects

Dying ReLUs

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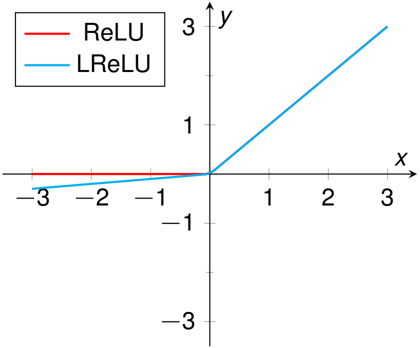
- Often related to a (too) high learning rate

Activation Function



Leaky ReLU / Parametric ReLU

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+ Fixes dying ReLU problem
- Leaky ReLU: $\alpha = 0.01$ [5]

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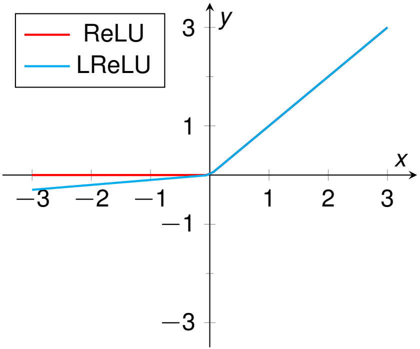

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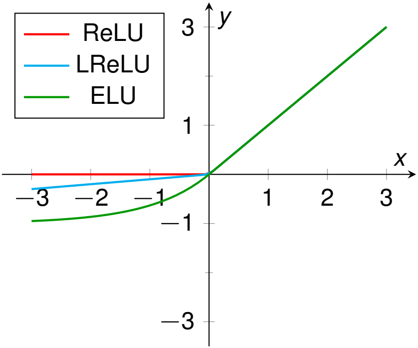
- Leaky ReLU: $\alpha = 0.01$ [5]
- Parametric ReLU (PReLU): learn α [2]

Exponential Linear Units (ELU)



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$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha(\exp(x) - 1) & \text{else} \end{cases}$$

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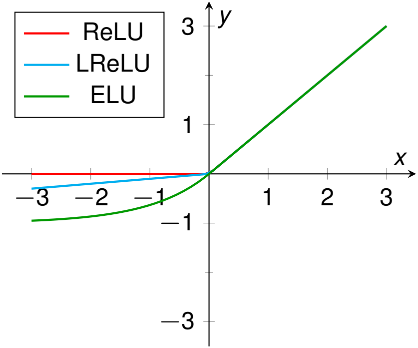

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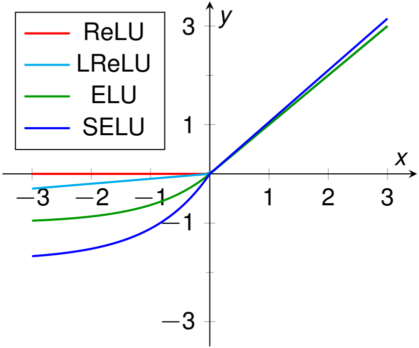
+ Also no vanishing gradient
+ Reduces shift in activations

Scaled ELU (SELU)



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$$\lambda_{01} = 1.0507$$

$$\alpha_{01} = 1.6733$$

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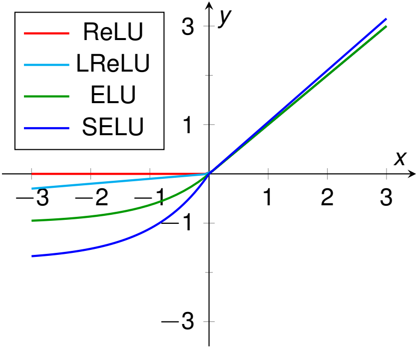

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- Alternative variant of ReLU [3]
- Idea: Self-normalizing - $\mu = 0, \sigma = 1 \mapsto \lambda_{01}, \alpha_{01}$
- Alternative to Batch Normalization? (see next lecture)

Other Activation Functions

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- Maxout: Learns the activation function [1]

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This is getting ridiculous - what should we use?

Finding the Optimal Activation Function

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- **Reinforcement learning/search** problem

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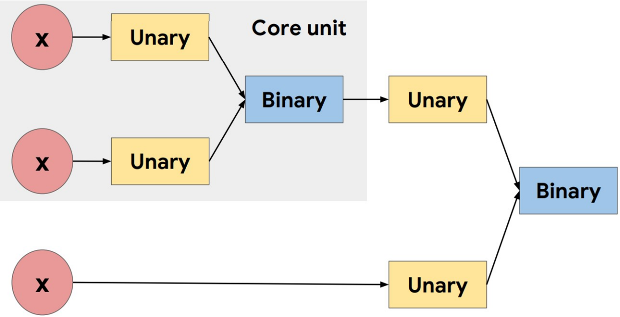
Strategy

1. Define a search-space
2. Perform the search using a RNN with reinforcement learning
3. Use the best result

Searching for Activation Functions [6]



Search space for activation functions

Source: https://arxiv.org/pdf/1710.05941.pdf

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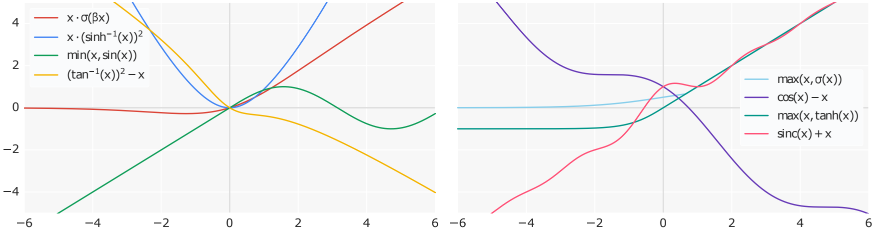

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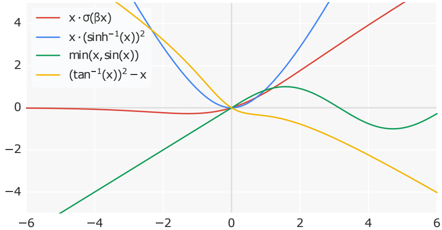

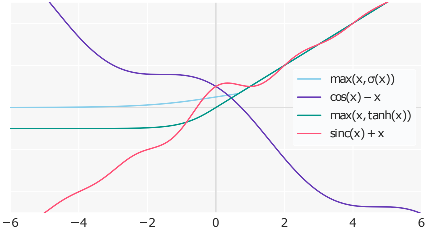

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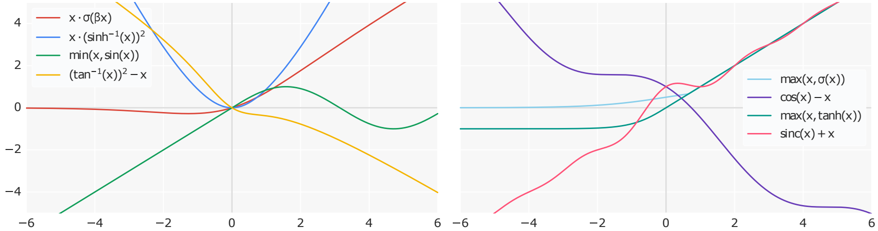

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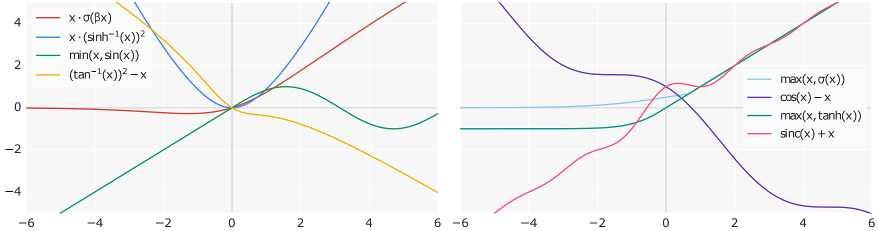

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- **Complicated** activation functions **didn't perform well**
- They now call $x \cdot \sigma(\beta x)$ the "Swish" function
- Has been proposed before as "Sigmoid-weighted Linear Unit" [7]

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Let's look into their results in detail

Disclaimer

Never show tables in you slides. Try hard to find a better representation.

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Let's look into their results in detail

Model	Top-1 Acc. (%)		
LReLU	79.5	79.5	79.6
PReLU	79.7	79.8	80.1
Softplus	80.1	80.2	80.4
ELU	75.8	79.9	80.0
SELU	79.0	79.2	79.2
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Inception-Resnet-V2 architecture trained on ImageNet.

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- Are any of these differences significant?

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Summary Activation Functions

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- Best activation function is a **difficult, expensive optimization problem**.

What we know about good activation functions:

- They have almost linear areas to prevent **vanishing gradients**.
- They have **saturating areas** to provide **non-linearity**.
- They should be **monotonic**.

NEXT TIME

ON DEEP LEARNING

Activation Functions and Convolutional Neural Networks - Part 3

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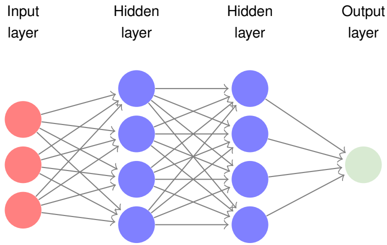
Convolutional Neural Networks

Motivation

- So far: Fully connected layers - each input is connected to each node
- Very powerful: Can represent any kind of (linear) relationship between inputs



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- Large part of machine learning: images/videos/sounds
- Assume we have:
  - An image with size 512×512 pixels
  - One hidden layer with 8 neurons
    - ➔ $(512^2 + 1) \cdot 8 > 2$ million trainable weights!

Motivation (cont.)

So the size is a problem. Is there something else?

Source: https://news.nationalgeographic.com

Motivation (cont.)

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  - Highly correlated
  - Scale dependent
  - Intensity variations
  - ...

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- Example: Classify between cat and dog
- Pixels are **bad** features!
  - Highly correlated
  - Scale dependent
  - Intensity variations
  - ...
- Pixels are a bad representation from a machine learning point of view

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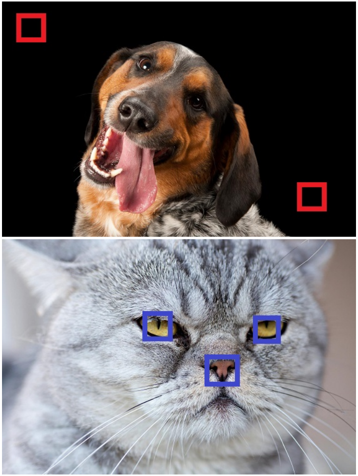
Motivation (cont.)

Can we find a better representation?



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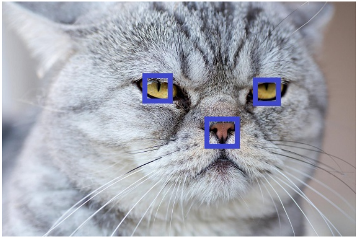
Motivation (cont.)

Can we find a better representation?

- We have a certain degree of the locality in an image



Source: https://news.nationalgeographic.com

Motivation (cont.)

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- We have a certain degree of the locality in an image
- We can find the same "macro features" at different locations
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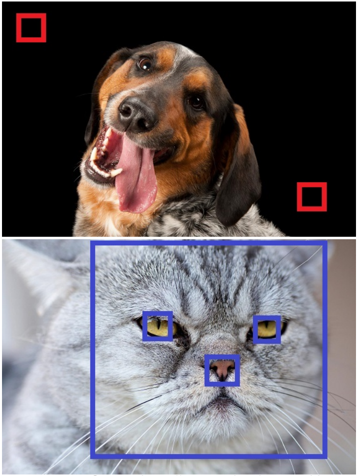
  - edges + corners $\mapsto$ eyes
  - eyes + nose + ears $\mapsto$ face
  - face + body + legs $\mapsto$ animal



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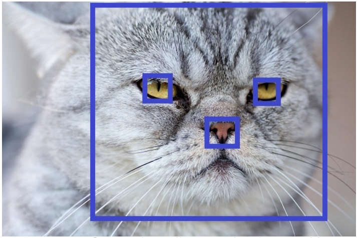
- Composition matters!

→ Learn better representation, then classify!



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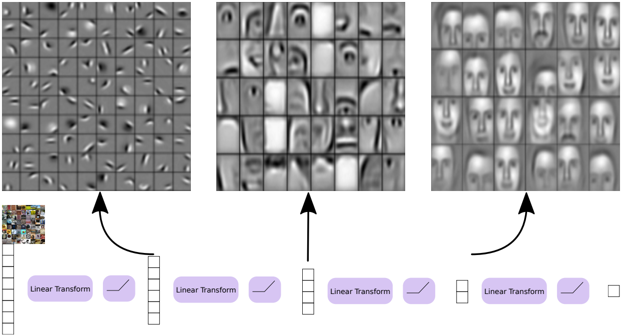
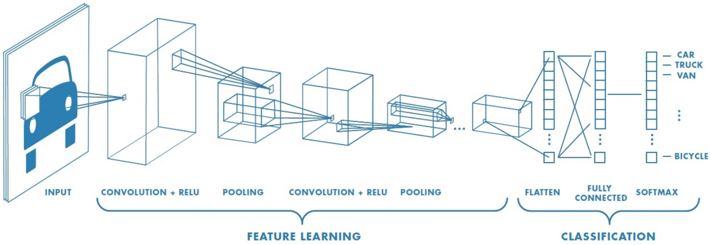
Convolutional Neural Networks



- **Local** connectivity ➔ filters
- Use **same filters** over the whole image ➔ translational equivariance
- Hierarchy of filters working on **different scales**
- + **learning** = **Convolutional Neural Networks**

Convolutional Neural Networks - Architecture



Four essential building blocks:

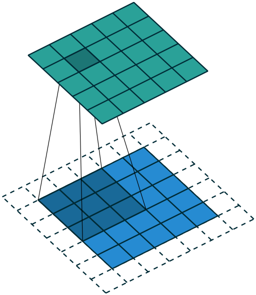
- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress and aggregate information, save parameters
- Last layer: Fully-connected for classification

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Convolutional Layers

Convolutional Layer - Local Connectivity

- Exploit spacial structure by only connecting pixels in a neighborhood
- Can be expressed as fully connected layer: **Except** for local connections, each entry in **W** is 0



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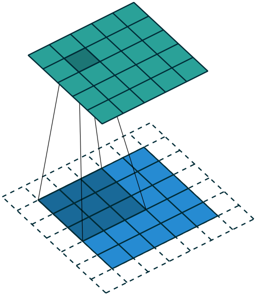

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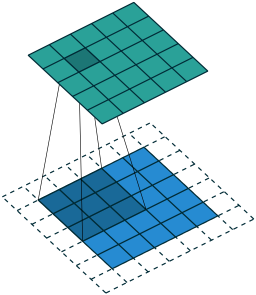

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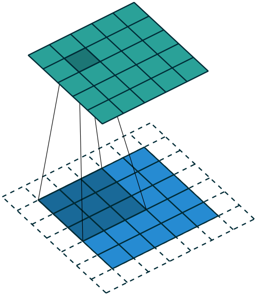
➔ **Convolution with trainable filters**



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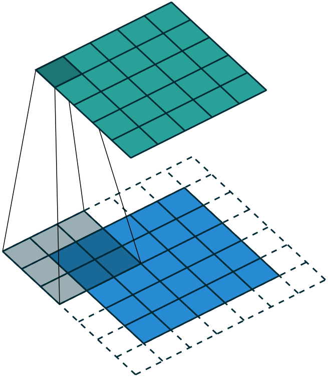
Recap: Convolution



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- Convolution:

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- Cross-correlation is convolution with a flipped kernel g – and vice versa!

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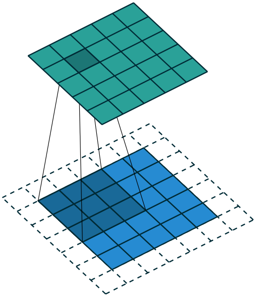
- Cross-correlation is convolution with a flipped kernel g – and vice versa!
- Implementation: Cross-correlation is frequently used in the forward pass - the weights are initialized randomly anyway

Padding

- Convolution reduces image size by $2 \cdot \left\lfloor \frac{n}{2} \right\rfloor$ pixels (n: kernel size).
- Necessary to pay attention to the borders:
- 'Same' padding (usually zero padding):
  - ➔ Input and output have the same size
- 'Valid'/no padding:
  - ➔ The output is smaller than the input



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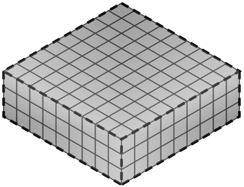
Forward Pass: Multi-channel convolution

- Input of size $X \times Y \times S$, where S is the number of input channels



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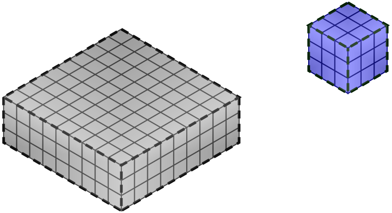

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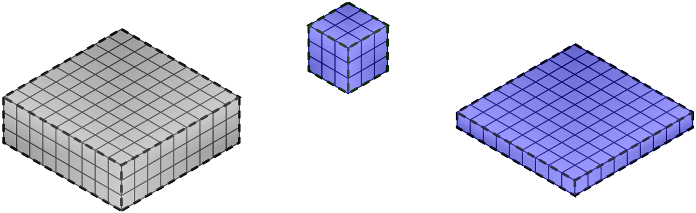

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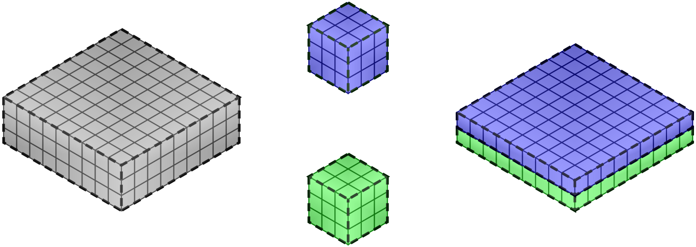

Backward pass: Multi-channel convolution

- Convolution can be expressed as matrix multiplication with matrix **W**: using a **Toeplitz matrix**
- We can use the **same formulas** as for the fully connected layer!

$$\mathbf{E}_{l-1} = \mathbf{W}^T \mathbf{E}_l$$
$$\nabla \mathbf{W} = \mathbf{E}_l \mathbf{X}^T$$

where

- $\mathbf{X}$ is the input and
- $\mathbf{E}_l/\mathbf{E}_{l-1}$ is the error in layer $l/l-1$

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  where
  - $\mathbf{X}$ is the input and
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- Backward pass can also be expressed as convolutions $\mapsto$ exercise

Convolutional Layer - What have we gained?

- Stack multiple filters to a get a trainable filter bank.
- Layer with 8 filters (nodes) with 5×5 neighborhood
  - ➔ $5^2 \cdot 8 = \underline{200}$ weights

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- Stack multiple filters to a get a trainable filter bank.
- Layer with 8 filters (nodes) with 5×5 neighborhood
  - ➔ $5^2 \cdot 8 = \underline{200}$ weights
- Convolution: **Independent** of image size!
- Much more training data for one weight!

Strided Convolutions

- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
- **Stride** *s* describes the offset

Source: https://en.wikipedia.org, Monty Python's Flying Circus episode, Dinsdale

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- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
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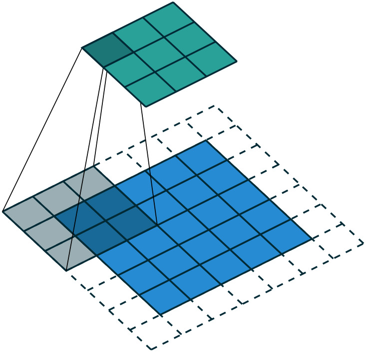
Strided Convolutions

- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
- **Stride** *s* describes the offset
- **Reduces** the **size** of the output by a factor of *s*
- **Mathematically: Convolution + subsampling**

Source: https://en.wikipedia.org, Monty Python's Flying Circus episode, Dinsdale

Strided Convolutions

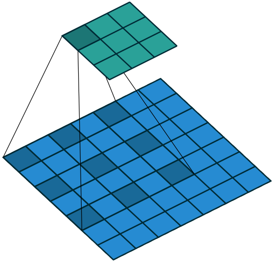


Strided Convolution with stride $s = 2$

Source: https://github.com/vdumoulin/conv_arithmetic

Dilated/Atrous Convolutions

- Additional variant of convolution in neural networks
- Dilate convolution kernel: Skip certain pixels
- Goal: Wider receptive field with less parameters/weights



Dilated Convolution

Source: https://github.com/vdumoulin/conv_arithmetic

1×1 Convolution Concept

- So far: H filters with neighborhood $3 \times 3, 5 \times 5, \ldots$ and 'depth' S
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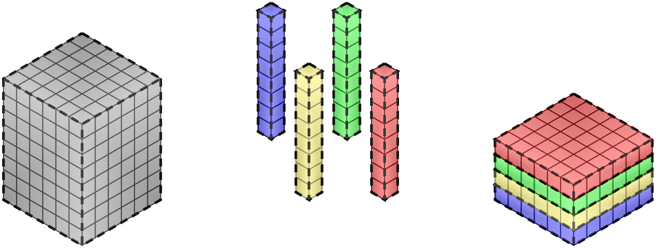
- And **just** use the fully connected property in the depth dimension



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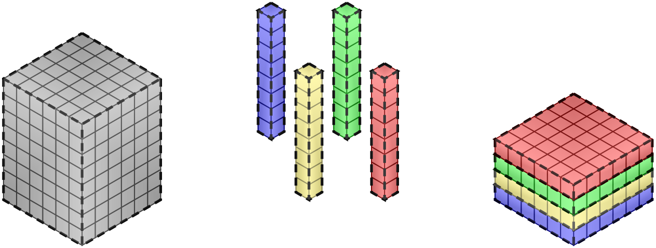

→ Dimensionality reduction/expansion from S channels to H channels

→ **If we flatten the input,** 1×1 **convolutions are fully connected layer!**

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1×1 Convolution Concept (cont.)

- First described in “Network in Network” by Lin et al. [4]
- 1×1 convolutions simply calculate **inner products** at each position
- Simple and efficient method to **decrease** the **size** of a network
- **Learns** dimensionality reduction, e.g., can reduce redundancy in your feature maps

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- Equivalent but more flexible: $N \times N$ convolution

NEXT TIME

ON DEEP LEARNING

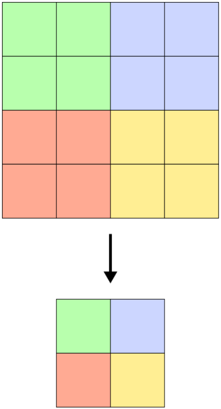
Activation Functions and Convolutional Neural Networks - Part 4

A. Maier, V. Christlein, K. Breininger, Z. Yang, L. Rist, M. Nau, S. Jaganathan, C. Liu, N. Maul, L. Folle, K. Packhäuser, M. Zinnen
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
April 24, 2023

Pooling Layers

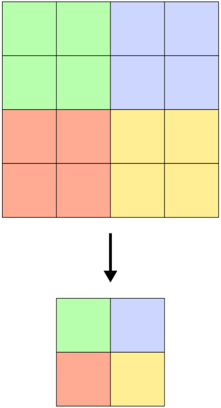
Idea behind Pooling Layers

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting



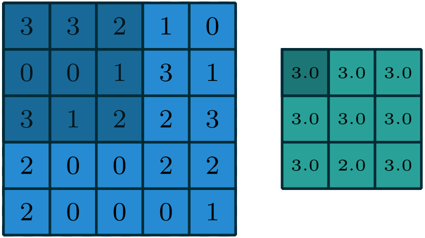
Idea behind Pooling Layers

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting
- Assumptions:
  - Features are hierarchically structured
  - Creates “summaries” of regions
  - Provides translational invariance
  - Exact location of a feature is not important



Max Pooling – Forward Pass

- Propagate maximum value in a neighborhood to next layer
- Typical choices: 2×2 or 3×3 neighborhood
- “Stride” of pooling usually equals the neighborhood size
- Maximum propagation adds additional non-linearity



Max pooling concept. Note that usually a stride > 1 is used for pooling.

Max Pooling – Backward Pass

Max Pooling – Backward Pass

- ➔ Only one value contributes to error
- ➔ Error is propagated only along the path of the maximum value

Average Pooling

- Propagate average of the neighborhood
- Does not consistently perform better than max pooling
- Backward pass: Error is shared to equal parts

Additional Pooling Strategies

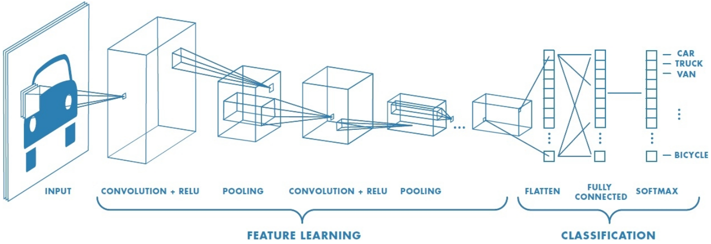
- Fractional max pooling
- L_p pooling
- Stochastic pooling
- Spacial pyramid pooling
- Generalized pooling
- ...

Alternative: Strided Convolution

- Historically, max pooling was the most frequently used pooling strategies due to additional non-linearity
- More recently, convolution with stride $s > 1$ has become more common

➔ Allows for trainable downsampling strategy

Recap: Convolutional Neural Networks - Architecture



Four essential building blocks:

- Convolutional layer: Feature extraction
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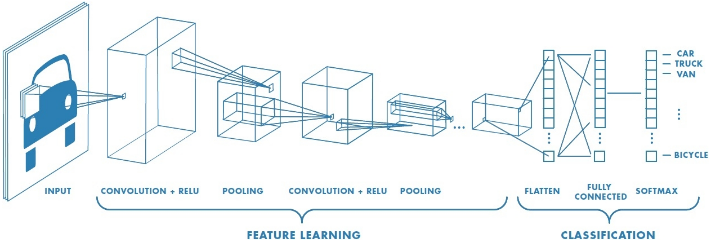

~~Four~~Three essential building blocks:

- Convolutional layer: Feature extraction
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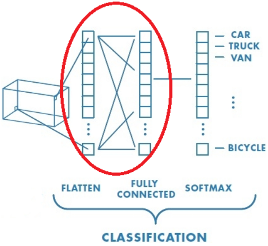
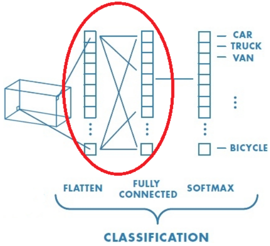
Replacing the Fully Connected Layer



- Conv and pooling layers generate better representation ➔ better features
- Fully connected layers for classification

Replacing the Fully Connected Layer



- Conv and pooling layers generate better representation ➔ better features
- Fully connected layers for classification
- **Alternatively and equivalently**: Use flatten & 1×1 convolution or $N \times N$ convolution
- Enables **arbitrary input sizes** in combination with global average pooling!

Inception model

- Szegedy et al. (2014): Going Deeper With Convolutions [8]
  - Very influencial publication: > 17000 citations (Nov. ’19)
  - Won ImageNet Large-Scale Visual Recognition Challenge 2014
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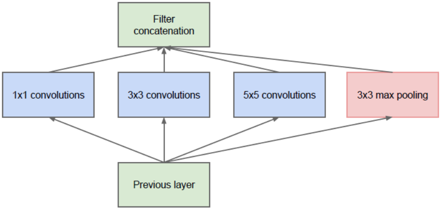
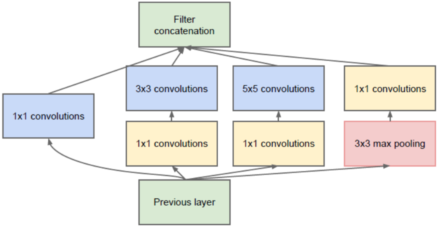
Source: http://knowyourmeme.com/photos/531557-we-need-to-go-deeper

Inception model

- **Idea**: Why use only one type of filter in one layer?
  Why not combine different neighborhoods/pooling/etc.?
- Construction of ‘inception modules’ that are stacked to form a large network.



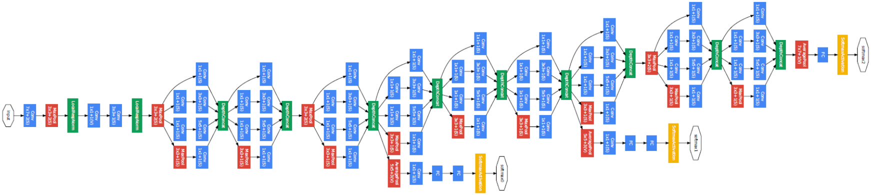
Naïve inception module



Inception module with dimensionality reduction

Source: [8]

Inception model – GoogLeNet



GoogLeNet architecture with inception modules.

Source: [8]

NEXT TIME

ON DEEP LEARNING

Coming up

- How to prevent networks from just memorizing the training data?
- Is there a way to force features to be independent?
- How can we make sure our network also recognizes cats in different poses?
- Can we fix the covariate shift problem?

Comprehensive Questions

- Name five activation functions.
- Discuss those 5 activation functions.
- What is the zero-centering problem?
- Why does ReLU as activation function perform much better than sigmoid/tanh in a large number of tasks?
- Why are convolutional networks well suited for image and audio processing?
- Write down a mathematical description of strided convolution.
- What is the connection between 1×1 convolutions and fully connected layers?
- How would you implement a classifier which operates on image patches?
- What is a pooling layer?
- Why do we use pooling layers?
- On what data would CNNs probably perform bad?

Further Reading

- Link - [3] for a paper about Self Normalizing Networks
- Link - [4] for a creative Network in Network paper
- Link - [6] for details on learned activation functions
- Link - [8] if everything so far was not deep enough for you

Questions?

References

References I

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References II

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