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Research Article

Research on Optimization of Customized Bus Routes Based on Uncertainty Theory

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In the optimization process of the routes of customized buses, there are numerous uncertainties in the route planning and setting. In this study, the uncertainty theory is introduced into the optimization problem of a customized bus route, and an uncertain customized bus route optimization model is established, which aims at the minimizing the total mileage of vehicle operation. An improved genetic algorithm is used to solve the model, whose feasibility is verified by a case study. The results show that the optimization model based on the uncertainty theory can yield a reasonable customized bus route optimization scheme, and the total mileage reduced from 35.6 kilometers to 32.2 kilometers. This research provides the theoretical support for the optimization of customized bus routes.

1. Introduction

Public transport priority is an important approach to alleviate traffic congestion. Customized public transport is a public transport service mode between regular public transport and taxis, which takes the form of multiperson transportation. Its objective is to customize a public transport service for people located in similar areas with similar travel times and similar travel needs [1]. Scientific and reasonable driving routes are crucial for customized bus operation. Customized reasonable and appropriate bus routes can not only save the travel costs and travel times of the passengers but also improve the economic benefits of customized bus companies, reduce carbon emissions, and protect the environment.

With the promotion of customized public transport in various cities, there is an increasing number of studies on the subject. To ensure the punctuality of customized public transport, some scholars set a time window to reach each stop and built the relevant model as constraints. Wang [2] adopted numerous customized bus routes as objects of study and considered the limitations of the passenger travel starting point and stop time window. Accordingly, the optimal vehicle scheduling scheme was formulated, and the optimal solution of the model was obtained by combining a greedy algorithm with a genetic algorithm. Li [3] established a mixed-load custom bus routing model that satisfies the requirements of the time window and used the Cplex software to solve the model. Carol Tong [4] established an optimization model that satisfies the time window and passenger needs, to optimize vehicle paths and allocate vehicles. Finally, an example analysis verified the effectiveness of the algorithm and the sensitivity under different actual operating conditions. Guo [5] built a mixed-integer programming model for custom bus route optimization problems with time window constraints. Branch-cut, genetic, and tabu search algorithms were used to solve the problem and were compared separately. The above papers all set the time window for the customized bus to arrive at each station and use it as a constraint in the establishment of the model, so as to regulate that the customized bus can arrive at the station more on time and improve the satisfaction of passengers on the customized bus.

In addition, Guo [6] established a mixed-integer programming model to describe the problem of customized bus routing and proposed the location and route of a bus stop.

Similar to the studies of Rongge Guo, there are numerous others on the design and optimization of customized bus routes. Huang [7] divided the operation of customized buses into dynamic and static stages and used two-stage optimization models to optimize the design of customized bus routes. Wang [8] proposed a solution method for real-time customized bus route optimization problems under a random user demand and employed a two-stage method to solve the model. Yan [9] proposed a comprehensive planning method to plan customized bus routes. This model can meet the needs of passengers, minimize the number of stops, and maximize the profit by the walking distance, and was empirically verified. Han [10] introduced a customized bus network planning method to balance the interests of operators, society, and passengers and discussed the influence of the customized bus model, fixed operating cost, and weight of each subcost. These articles are aimed at optimizing the design of customized bus routes. Some optimization goals are to reduce the travel time of customized buses, some are to save bus company operating costs, and some are to optimize for comprehensive goals.

The relevant models and algorithms commonly used for customized public transport include bi-level programming models and genetic algorithm. For example, Lei [11] constructed a customized dynamic network dispatching model for public transportation on the Internet. The objectives of this model are to maximize the demand service rate and minimize the cost. Moreover, its constraints are factors such as maximum passenger capacity and passenger time threshold. He [12] established a bus line design model that simultaneously considers the minimum cost of responsive customized bus operators and the requirements of passenger travel reliability and comfort and used genetic algorithm to solve the model. Xue [13] considered the uncertain factor of number of passengers at a bus stop in reality, introduced the uncertainty theory to construct an uncertain bi-level programming model of bus line allocation, and solved it using MATLAB. Other scholars [14–18] analyzed customized buses from the aspects of passenger flow prediction and influencing factors. However, in real life, the time for a vehicle to arrive at each stop is not fixed, and few studies regard it as an uncertain variable. Most studies assume that a vehicle travels on a road at a predetermined speed and therefore arrives at its stop at a fixed point in time. However, this is different from the actual scenario. Simultaneously, most articles on the optimization of customized buses regard the needs of passengers as known conditions. However, after a period of actual operation, some customized buses, especially companies and schools, may no longer collect passenger demand every day or every week. Thus, passenger demand should be analyzed as an uncertain event, which is rarely mentioned in existing research.

To quantitatively analyze the abovementioned uncertain factors, Liu [19] proposed the uncertainty theory in 2007 and improved it in the following years. Accordingly, the author ensured the uncertainty theory satisfies normative axioms, duality axioms, axiomatized mathematical systems with secondary axioms, and product axioms. Subsequently, numerous scholars have conducted in-depth research on the uncertainty theory and applied it to logic, transportation, logistics, finance, and other fields. In transportation, the uncertainty theory is extensively used in logistics distribution and vehicle scheduling. For example, Huang [20] described the delivery time of a third-party logistics supplier as an uncertain variable, subsequently converted the model into an equivalent deterministic model, and designed several improved genetic algorithms to solve it. Hua [21] used the time of logistics project development activities as an uncertain variable to establish a model, designed an intelligent algorithm based on simulated annealing and conducted a logistics project as an example to illustrate the effectiveness of the model and algorithm. Zhang [22] used the quantities of available resources as uncertain variables and the minimum total cost as the objective. Accordingly, a project scheduling model with uncertain resource availability was established and subsequently solved by genetic algorithm. Jiao [23] considered stop demand and vehicle travel time as two uncertain factors and proposed a new uncertain planning model for vehicle scheduling. It can be seen from the related literature the uncertainty theory is applied in logistics distribution and location selection, vehicle scheduling, and transportation; however, it is rarely applied for customized bus optimization.

Uncertainty theory is a method of studying uncertain events in real life. It can help us build customized bus route planning models that are more in line with actual conditions. In the existing research, few studies consider both vehicle operating times and passenger demands as uncertain variables simultaneously. By the analysis of previous survey data, it is found that the arrival of vehicles and the passenger demand conform to an uncertain distribution [13]. Therefore, we consider applying the uncertainty theory to the optimization of customized bus routes and provide new concepts for the same. The structure of this paper is organized as follows: Section 2 introduces the uncertainty theory. Section 3 establishes the customized bus route optimization model that uses vehicle arrival time and passenger demand as uncertain variables. Section 4 introduces the model solution algorithms, and Section 5 presents a case study. The Section 6 summarizes the research conclusion.

2. Uncertainty Theory

The uncertainty theory was proposed and established by Liu Baoding, and numerous researchers have subsequently studied it. At present, the uncertainty theory is a branch of axiomatic mathematics. The uncertainty theory is essentially a measurement theory with four axioms: normative, duality, subadditivity, and product measurement [19].

Let Γ be a nonempty set, and L is the $\sigma-$ algebra of Γ. Each element $\Lambda \in L$ is called as an event, and an aggregate function M: $L \longrightarrow [0,1]$ is called as an uncertain measure, whose properties are as follows.

Property 1: for an uncertain measure M, for any event $\Lambda_1 \subset \Lambda_2$, $M\{\Lambda_1\} \leq M\{\Lambda_2\}$, where M is a monotonically increasing function.

Property 2: the uncertain measure, M, of the empty set, $\varnothing$, is zero, i.e., $M\{\varnothing\} = 0$.

Property 3: If M is an uncertain measure, then for any event Λ, $0 \leq M\{\Lambda\} \leq 1$.

In addition, the uncertain distribution and the inverse uncertain distribution are defined.

Definition 1. For an uncertain variable ξ, its uncertain distribution Φ is defined as $\Phi(x) = M\{\xi \leq x\}$, where x is any integer.

Definition 2. For an uncertain variable ξ, if its uncertain distribution Φ has an inverse function $\Phi^{-1}(\alpha)$, which exists and is unique for any $\alpha \in (0,1)$, then Φ^{-1} is called as the inverse uncertain distribution of ξ.

Commonly used uncertain distributions include linear and normal uncertain distributions, which are introduced in Section 3.1.

3. Customized Bus Route Optimization Model

3.1. Problem Description. In actual operation, due to factors such as the operating speed of the vehicle, the road conditions during operation, and the difference in actual conditions, the operating time of the vehicle between the two stations is an uncertain variable. This is typically ignored when optimizing a route, so that the actual arrival time at each stop does not meet the specified time, and the punctuality of the customized buses is reduced, which leads to passenger dissatisfaction with the route optimization. In addition, due to the influence of practical factors, there will be a certain error between the number of people getting on and off at each stop during the actual operation of these customized buses and the number of passengers initially collected. Specifically, the number of passengers on and off each stop is uncertain. Based on the information provided by the passengers, the condition of uncertain passenger demand can be added to the optimization objective, to obtain better optimization results. When optimizing this type of a problem, the minimum operating cost of the public transport enterprise is generally chosen as the optimization goal, and the operating cost includes fuel consumption, salary of the driver, and vehicle wear and tear. This study directly adopts the shortest total mileage of the customized public transport vehicles as the optimization objective because a shorter mileage will reduce the operating costs, and the two of them are consistent. A shorter mileage can also shorten the travel time of passengers and improve their travel satisfaction. At the same time, shortening the mileage of vehicles will also reduce environmental pollution. Therefore, this article chooses the shortest vehicle mileage as the optimization goal.

Therefore, the examination of the problems in this study can be summarized as the optimization of the direction of a customized bus route considering the uncertain running time of the vehicle and the uncertain demand of the passengers. Moreover, the minimum total mileage of all the vehicles of the customized bus route is considered as the optimization objective.

3.2. Model Construction. First, define the symbols mentioned below. The variables and their definitions involved in the model are shown in Table 1.

Suppose there are$K$customized buses in a certain area, and each bus can transport Q passengers. Let there be a total of Ncustomized bus stops in the area. All the customized buses have different routes and pass through different stops. Suppose the n-th stop passed by the k-th car is x_{k_n}, then the driving path of the k-th bus is$x_{k_1} \longrightarrow x_{k_2} \longrightarrow x_{k_3} \longrightarrow \ldots \longrightarrow x_{k_n}$. Specifically, this bus first reaches the first stop, then passes through other stops of the bus, and finally reaches the terminal stop. At the same time, we set that one stop can only be served by one customized bus, and each customized bus will be sent out from the parking lot as the starting point. When designing the route, we must include all stops.

Owing to the randomness of the actual traffic conditions, it is difficult for buses to arrive at stops accurately based on the time requirements of their passengers. Therefore, there is a certain time range fluctuation in the allowed bus pick-up times of the passengers, which is called as the time window in the paper. Let the actual time that the k-th bus arrives at stop x_{k_n} be $f_{x_{k_n}}$, and the time window for the passengers to allow the buses to arrive at a stop be $[a_{x_{k_n}}, b_{x_{k_n}}]$. For example, let the expected departure time of the passengers be $7:10$ in the morning, and the bus be allowed to arrive early or late by 5 min. Therefore, the window is $[7:05, 7:15]$, and the actual arrival time of the vehicle may be $7:13$.

3.2.1. Vehicle Arrival Time. Because each stop has a time window restriction on the arrival of the customized buses, the time for each customized bus to reach the different stops needs to be discussed. Suppose the time that the k-th bus departs from a parking lot is t_k and the running time from the parking lot to the first stop is T_{0xk_1}, then the time that the k-th bus arrives at the stop, x_{k_1}, is

$$f_{x_{k_1}} = t_k + T_{0x_{k_1}}. \tag{1}$$

In an actual operation, some customized buses may arrive at a stop outside the time window; therefore, we stipulate the following:

(1) The customized buses arrive at a stop earlier than the time specified in the time window and must wait for the earliest time allowed to the passengers before they depart

(2) The customized buses arrive at a stop within the time specified in the time window, or arrive at a stop later than the time specified in the time window, and can pick up and drop off passengers directly from the stop

Thus, the time that the k-th bus arrives at stop x_{k_n} is

$$f_{x_{k_n}} = \max\left\{f_{x_{k_{n-1}}}, a_{x_{k_{n-1}}}\right\} + T_{x_{k_{n-1}}x_{k_n}}. \tag{2}$$

Therefore, for the k-th vehicle, because its running time $T_{x_{k_{n-1}}x_{k_n}}$between stops $x_{k_{n-1}}$ and x_{k_n} is an uncertain variable,

Table 1: Definition of each symbol in model.

Symbol	Definition
K	Number of customized buses
Q	Rated passenger capacity of customized bus
N	Total number of bus stops in area
x_{k_n}	n-th stop passed by k-th bus
$f_{x_{k_n}}$	Time when k-th vehicle arrives at stop x_{k_n}
$[a_{x_{k_n}}, b_{x_{k_n}}]$	Time window for passengers to allow bus to arrive at stop x_{k_n}
t_k	Time when the $\underline{k}$-th bus left the parking lot
$T_{x_{k_{n-1}}x_{k_n}}$	The running time of the k-th vehicle between station $x_{k_{n-1}}$ and station x_{k_n}
$\Psi^{-1}_{x_{k_n}}$	The inverse uncertainty distribution corresponding to $f_{x_{k_n}}$
$\Phi^{-1}_{x_{k_{n-1}}x_{k_n}}$	The inverse uncertainty distribution corresponding to $T_{x_{k_{n-1}}x_{k_n}}$
$q_{x_{k_1}}$	Customize number of passengers on bus when bus departs from stop x_{k_n}
$\psi^{-1}_{x_{k_n}}$	The inverse uncertainty distribution corresponding to $q_{x_{k_n}}$
$\phi^{-1}_{x_{k_n}}$	The inverse uncertainty distribution corresponding to $h_{x_{k_n}}$
$l_{x_{k_{n-1}}x_{k_n}}$	Mileage between stops $x_{k_{n-1}}$ and x_{k_n}
l_k	Total mileage of k-th bus, $l_k = l_{x_0x_{k_1}} + \sum_{j=1}^{n} l_{x_{k_j}x_{k_{j+1}}}$
L	Total mileage of all vehicles, $L = \sum_{k=1}^{K} l_k$

time $f_{x_{k_1}}$ it takes to arrive at stop x_{k_1} is also an uncertain variable, and the corresponding inverse uncertain distribution is

$$\Psi^{-1}_{x_{k_1}} = t_k + \Phi^{-1}_{0x_{k_1}}. \tag{3}$$

The inverse uncertain distribution corresponding to time $f_{x_{k_n}}$ of arrival at stop x_{k_n} is

$$\Psi^{-1}_{x_{k_n}} = \max\left\{\Psi^{-1}_{x_{k_{n-1}}}, a_{x_{k_n}}\right\} + \Phi^{-1}_{x_{k_{n-1}}x_{k_n}}. \tag{4}$$

The times for each customized bus to reach different stops can be calculated according to the above formula.

3.2.2. Passenger Demand. In order to ensure the comfort of passengers, we must ensure that each passenger has a seat. Because of the uncertainty of the passengers getting on and off at each stop, we restrict the total number of passengers on the bus. When the k-th bus is at stop x_{k_1}, the difference between the number of passengers getting on the bus and that getting off is $h_{x_{k_1}}$. If the number of passengers in the bus when departing from stop $x_{k_1}$is $q_{x_{k_1}}$, then

$$q_{x_{k_1}} = h_{x_{k_1}}. \tag{5}$$

The corresponding inverse uncertain distribution is

$$\psi^{-1}_{x_{k_1}} = \phi^{-1}_{x_{k_1}}. \tag{6}$$

Similarly, the number of passengers in the bus when departing from stop x_{k_n} is

$$q_{x_{k_n}} = q_{x_{k_{n-1}}} + h_{x_{k_n}}. \tag{7}$$

The corresponding inverse uncertain distribution is

$$\psi^{-1}_{x_{k_n}} = \psi^{-1}_{x_{k_{n-1}}} + \phi^{-1}_{x_{k_n}}. \tag{8}$$

The number of passengers in each customized bus can be calculated according to the above formula.

3.2.3. Model Building. Let the mileage between stops $x_{k_{n-1}}$ and x_{k_n} be $l_{x_{k_{n-1}}x_{k_n}}$, and the total mileage of the k-th bus be

$$l_k = l_{x_0x_{k_1}} + \sum_{j=1}^{n} l_{x_{k_j}x_{k_{j+1}}} \tag{9}$$

Thus, the total mileage of all the vehicles is $L = \sum_{k=1}^{K} l_k$, and the objective of the optimization model is that L is minimum.

To ensure the punctuality of the customized bus operation, we expect that the customer of each stop receives the service within the time window determined by the customer with a reliability of α. Specifically, the customized bus arrives at stop x_{k_n} within time window $[a_{x_{k_n}}, b_{x_{k_n}}]$ with a reliability of α, which can be expressed as

$$M\left\{a_{x_{k_n}} \le f_{x_{k_n}} \le b_{x_{k_n}}\right\} \ge \alpha. \tag{10}$$

Similarly, to ensure the comfort of the customized bus, we expect the reliability of the passengers within the rated passenger number of the vehicle to be β. Specifically, when the customized bus, x_{k_n}, departs from the stop, $q_{x_{k_1}}$, the number of passengers on the vehicle is less than the rated passenger number, Q, and this can be expressed as

$$M\left\{q_{x_{k_n}} \le Q\right\} \ge \beta. \tag{11}$$

In addition, each stop is only served by one customized bus, i.e., the n-th stop passed by the k-th vehicle is not the same as the m-th stop passed by the j-th vehicle, which can be expressed as

$$x_{k_n} \ne x_{j_m}. \tag{12}$$

Subsequently, the model expression is as follows.

$$\min L,$$
$$\begin{cases} M\left\{a_{x_{k_n}} \leq f_{x_{k_n}} \leq b_{x_{k_n}}\right\} \geq \alpha, \\ M\left\{q_{x_{k_n}} \leq Q\right\} \geq \beta, \\ 1 \leq n \leq N, \\ x_{k_n} \neq x_{j_m}. \end{cases} \tag{13}$$

In order to facilitate subsequent calculations [23], we convert the constraints expressed by the uncertain measures in the above model into the form of inverse uncertain distribution:

$$\min L$$
$$\begin{cases} b_{x_{k_n}} \leq \Psi^{-1}_{x_{k_n}} \leq a_{x_{k_n}}, \\ \psi^{-1}_{x_{k_n}} \geq Q, \\ 1 \leq n \leq N, \\ x_{k_n} \neq x_{j_m}. \end{cases} \tag{14}$$

4. Model Solution

4.1. Uncertain Distribution. Because the model contains uncertain variables, to solve it, first the distribution of the uncertain variables should be determined. Subsequently, the uncertainty theory is used to obtain its inverse uncertain distribution. To simplify the calculation process, this study assumes that the vehicle arrival time and the passenger demand follow normal and linear uncertain distributions, respectively.

Linear uncertain variables: if an uncertain variable ξ follows a linear uncertain distribution, then its uncertain distribution is as follows:

$$\Phi(x') = \begin{cases} 0, & x' \leq a, \\ \dfrac{x' - a}{b - a}, & a \leq x' \leq b, \\ 1, & x' \geq b. \end{cases} \tag{15}$$

Among them, a and b are constants and $\mathrm{a} < \mathrm{b}$, and the above distribution is recorded as $L(\mathrm{a}, \mathrm{b})$. The inverse uncertain distribution of the linear uncertain distribution $L(\mathrm{a}, \mathrm{b})$ is

$$\Phi^{-1}(\alpha') = (1 - \alpha')a + \alpha' b \tag{16}$$

Normal uncertain variable: if an uncertain variable ξ follows a normal uncertain distribution, then its uncertain distribution is as follows:

$$\Phi(x') = \left[1 + \exp\left(\frac{\pi(e - x')}{\sqrt{3}\sigma}\right)\right]^{-1}, \quad x' \in R. \tag{17}$$

Among them, e and σ are constants and $\sigma > 0$, and the above distribution is referred to as $N(e, \sigma)$. The inverse uncertain distribution of the normal uncertain distribution, $N(e, \sigma)$, is

$$\Phi^{-1}(\alpha') = e + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha'}{1 - \alpha'}. \tag{18}$$

By giving the expected confidence, α and β, and transforming the uncertain distribution into the inverse uncertain distribution, the distribution of the uncertain variables under this confidence can be obtained.

4.2. Simulated Annealing Genetic Algorithm. At present, the solution method of this type of problem model is mainly a heuristic algorithm, and a genetic algorithm is a type of heuristic algorithm. It is a random search method that evolves from the genetic mechanism of the survival of the fittest in the biological world. It has a high calculation speed and high global optimization ability. Because genetic algorithms are prone to premature maturity, resulting in failure to obtain the optimal solutions, and considering that simulated annealing algorithms have strong local search capabilities, genetic and simulated annealing algorithms are combined. Moreover, the solution mechanism Metropolis criteria of simulated annealing algorithms are adopted, accept the difference with a certain probability, and improve the genetic algorithm. In this study, a customized bus route is used as a chromosome in the genetic algorithm. The specific steps and processes are shown in Figure 1.

Step 1: the basic data of the customized bus route optimization are input, and the control parameters are initialized.

Step 2: N chromosomes are randomly generated to form the initial population, the initialization of the population is completed, and the steps for generating the initial solution are as follows:

(i) The stop with the earliest departure time is selected among all the stops as the starting point of the first vehicle
(ii) The next stop is randomly selected from the remaining stops, and it is determined whether the time window limit is met.
(iii) If the time window limit is satisfied, determination of whether the passenger demand limit is met is continued.
(iv) If the time window restriction is not met, the site is not being regarded as the next site, and Step ② is returned.
(v) If the time window and passenger demand restrictions are met, the current stop is taken as the next stop. Steps ②–⑥ and the search for subsequent stops is continued.
(vi) If the time window limit is met but the passenger demand limit is not met, that is, the sum of the passengers on the bus and the passengers who are about to board the bus exceeds the approved passenger capacity of the customized bus. Then,

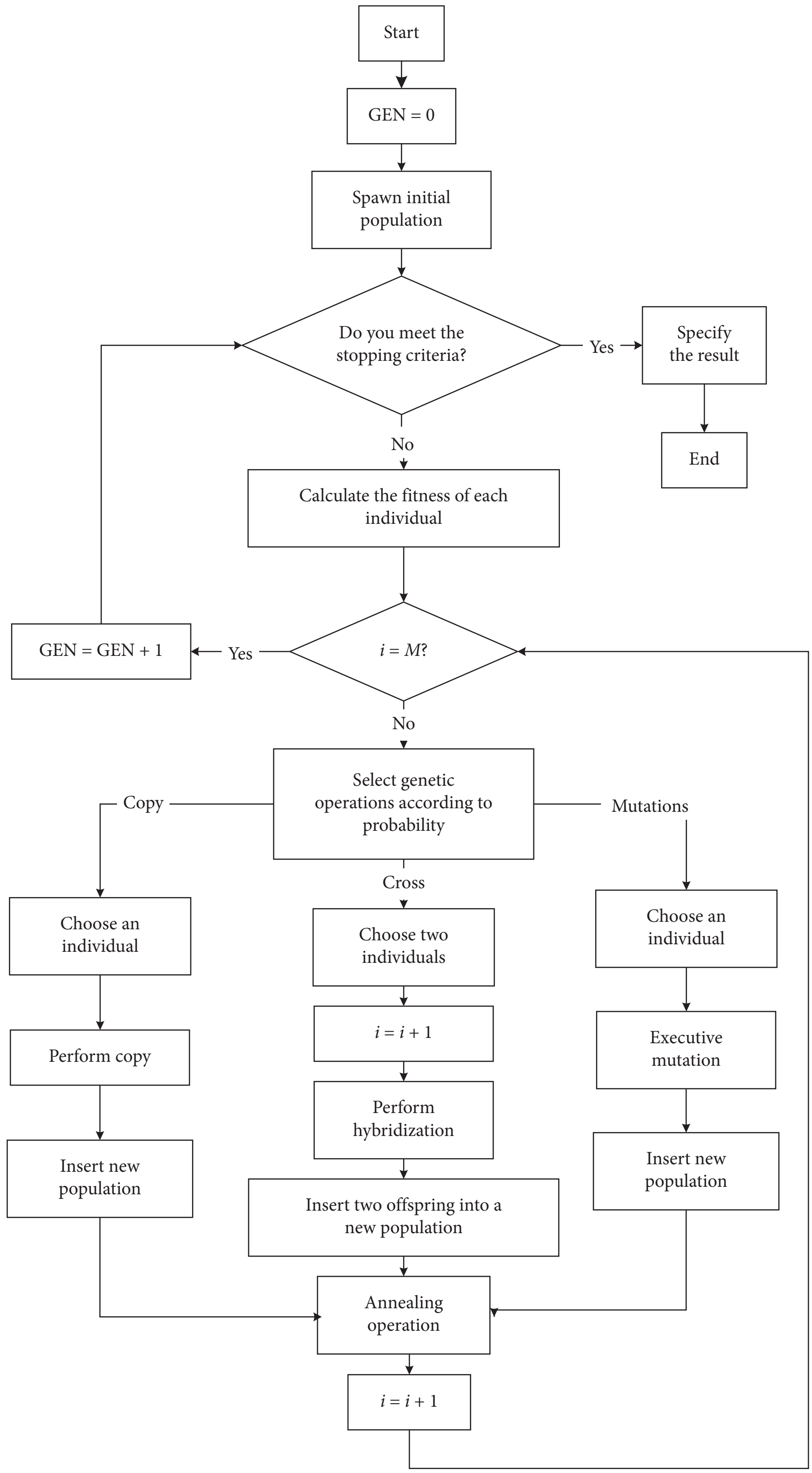

Figure 1: Flow chart of improved genetic algorithm.

Table 2: Time windows and requirements of passengers at different stops.

Bus stop	Time window limit	Uncertain distribution of demand	Bus stop	Time window limit	Uncertain distribution of demand
1	[8 : 10, 8 : 20]	L (6, 5)	6	[8 : 30, 8 : 40]	L (3, 9)
2	[8 : 05, 8 : 15]	L (1, 7)	7	[8 : 25, 8 : 35]	L (5, 8)
3	[8 : 20, 8 : 30]	L (7, 6)	8	[8 : 35, 8 : 45]	L (3, 6)
4	[8 : 15, 8 : 25]	L (2, 8)	9	[8 : 40, 8 : 50]	L (4, 9)
5	[8 : 20, 8 : 30]	L (4, 7)	10	[8 : 35, 8 : 45]	L (1, 9)

Table 3: Distances between stops (km).

Stop	Starting point	1	2	3	4	5	6	7	8	9	10	Destination
Starting point	0	2.2	2.1	1.9	3.1	3.4	5.7	5.4	3.6	5.2	6.1	8.5
1	2.2	0	2.6	3.1	1.9	2.2	2.7	3	2.5	1.8	2.1	7.6
2	2.1	2.6	0	3.3	2.6	1.7	2.4	2.7	2.2	3.1	1.7	6.4
3	1.9	3.1	3.3	0	2.6	3.3	3.7	2.5	1.6	3.4	2.8	5.7
4	3.1	1.9	2.6	2.6	0	2.1	1.5	2.4	0.7	3.1	2.9	4.9
5	3.4	2.2	1.7	3.3	2.1	0	2.5	2.8	2.2	1.2	3.4	4.8
6	5.7	2.7	2.4	3.7	1.5	2.5	0	3	2	1.9	2.2	5.2
7	5.4	3	2.7	2.5	2.4	2.8	3	0	1.7	2.2	2.4	5.1
8	3.6	2.5	2.2	1.6	0.7	2.2	2	1.7	0	2.6	2.1	4.6
9	5.2	1.8	3.1	3.4	3.1	1.2	1.9	2.2	2.6	0	2.8	3.2
10	6.1	2.1	1.7	2.8	2.9	3.4	2.2	2.4	2.1	2.8	0	2.8
Destination	8.5	7.6	6.4	5.7	4.9	4.8	5.2	5.1	4.6	3.2	2.8	0

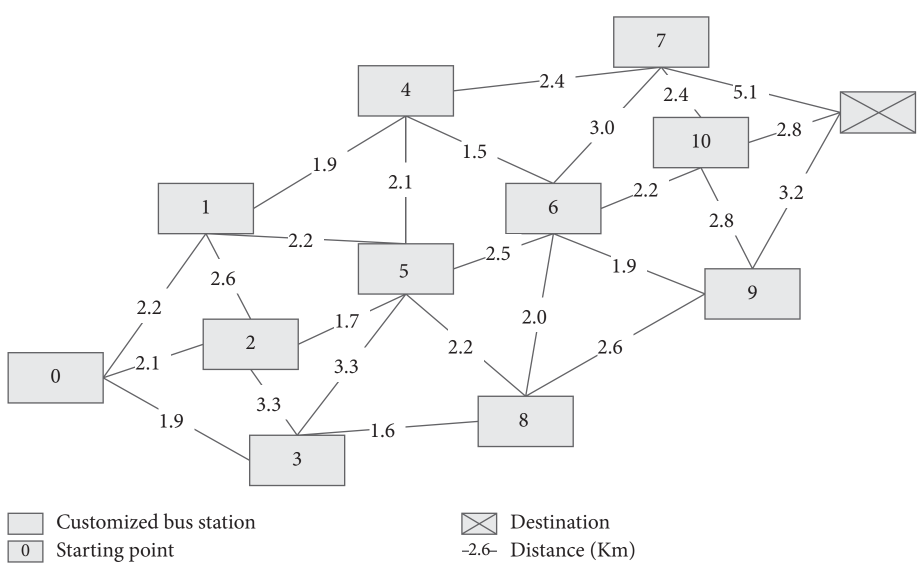

Figure 2: Schematic of stop distances.

Table 4: Comparison of optimization schemes obtained by different algorithms.

Comparison index	Conventional genetic algorithm	Improved genetic algorithm
Optimization	Customized bus 1: 0 ⟶ 1 ⟶ 3 ⟶ 6⟶destination Customized bus 2: 0⟶2⟶5⟶7⟶10⟶destination Customized bus 3: 0⟶4⟶8⟶9⟶destination	Customized bus 1: 0 ⟶ 1 ⟶ 4 ⟶ 7 ⟶destination Customized bus 2: 0⟶2⟶5⟶6⟶10⟶destination Customized bus 3: 0⟶3⟶8⟶9⟶destination
Total operating mileage (km)	35.6	32.2

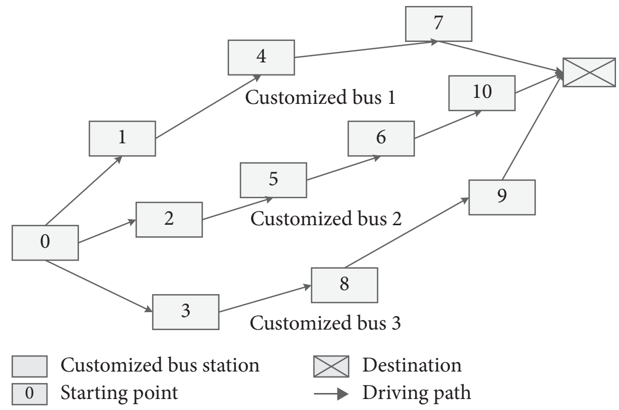

Figure 3: Customized bus optimization plan.

the stop will not be regarded as the next stop, and the destination is directly reached.

(vii) After the route of a customized bus is generated, the stop with the earliest departure time is selected from the remaining stops as the starting point of the second bus, and Steps ②–⑥ are repeated to generate other customized bus routes.

Step 3: each chromosome in the i-th generation population is decoded, and the fitness value of each chromosome is calculated by the simulated annealing stretching method.

Step 4: a roulette is used to select the new populations.

Step 5: the crossover operation is performed on the chromosomes according to crossover probability P_c.

Step 6: the mutation operation is performed on the chromosomes according to mutation probability P_m.

Step 7: the simulated annealing process is conducted on the generated chromosomes, and the replication strategy is used based on the Metropolis criterion to generate the next generation of population. First, the optimal retention strategy is implemented, and then a new individual j is randomly generated in the neighborhood of chromosome i, and chromosomes i and j compete in the next generation.

Step 8: if the convergence condition is met and the predetermined evolution algebra, M, is reached, then Step 9 is entered; otherwise, the selection operation is continued to the iteration.

Step 9: the calculation is stopped, and the individuals in the population that minimize the objective function are output as the final calculation result.

5. Case Study

To test the practicality of the model, we conduct a simple case analysis. The case scenario is set as the operating scene of a shuttle bus of a company at work. The passengers take customized buses from different stops to the same company, i.e., each customized bus starts from the same starting point, passes through different routes, and finally reaches the same destination. Assume that the bus company has three customized buses and there are 12 bus stops in the area including the starting stop and the terminal stop. Each customized bus has a rated capacity of 30 people. The passenger demand at the different stops satisfies different uncertain distributions. We set the scene as a company shuttle bus of a company in Nanchang, measured the distance between bus stops around the company, and used this as the data for the calculation example. The corresponding time window restrictions and the uncertain distribution of the passenger demand are listed in Table 2.

The reliability of the passengers of the bus stops in the designated time window is 0.9. The reliability of each customized bus during the operation does not exceed its maximum rated capacity of 0.95. For simplicity, we assume that the time that each bus arrives at the first stop is the earliest time specified by the time window limit for the passengers. We also assume that the distance between two bus stops $x_{k_{n-1}}$ and x_{k_n} is $l_{x_{k_{n-1}}x_{k_n}}$, where $k = 1, 2$ and $N = 1, 2, 3, 4$. Furthermore, the transit time of the customized bus between the two stops follows a normal uncertain distribution, $T_{x_{k_{n-1}}x_{k_n}} \sim N(2l_{x_{k_{n-1}}x_{k_n}}, 1)$, where $k = 1, 2, 3$ and $n = 1, 2, 3, 4$. The distances between two stops are listed in Table 3. A schematic of the distances between the stops is shown in Figure 2.

We set the population size as 100, crossover probability $Pc = 0.25$, mutation probability $Pm = 0.1$, and the maximum number of iterations = 500. The solutions using the conventional genetic algorithm and the improved genetic algorithm are listed in Table 4. The bus driving schematic diagram is shown in Figure 3, since the calculation content of this example is relatively small, the calculation time of the two algorithms is very fast, so only the calculation results are compared.

By setting the vehicle arrival time and the passenger demand as the uncertain factors, optimizing the customized bus route can be closer to reality and can make the designed route more user-friendly, and this optimization method has been proved to be feasible by calculation. Simultaneously, solving the calculation example with an improved genetic algorithm can reduce the total operating mileage by 9.55%

compared to the conventional genetic algorithm, reducing the mileage will result in a reduction in fuel costs, depreciation fees, etc., resulting in a reduction in the operating costs of the bus company.

6. Conclusions

This study considers the uncertain factors encountered in the operation of customized buses, analyzes the vehicle running time and passenger demand as uncertain variables, and introduces the uncertainty theory. Moreover, it establishes the uncertainty theory with the objective of minimizing the total mileage of the running vehicles. The customized bus route optimization model and the feasibility of the model are demonstrated by calculation examples. The established model comprehensively considers the two factors of vehicle punctuality and passenger capacity and can output a scientific customized bus line design plan, which provides theoretical support to bus operators in the design of bus lines. Nanchang customized public transportation is in the development stage. Correlating the research concepts presented in this paper with the actual scenario in Nanchang can provide new approaches for the formulation and future development of Nanchang customized public transportation routes, which requires further research.

In reality, there are numerous different restrictions or objectives to be achieved in the customized bus route optimization problem, such as vehicle configuration and route design for multiple models, and customized bus route optimization considering other uncertain factors. In addition, further optimization of the algorithm is also an important research direction in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] C. Wang and C. Ma, "Optimization of parking lot and multi-vehicle route of customized buses based on genetic algorithm," *Journal of Transport Information and Safety*, vol. 37, no. 3, pp. 109–117+127, 2019.

[2] J. Wang, Y. Cao, and Y. Wang, "Customized bus route vehicle schedule method considering travel time windows," *China Journal of Highway and Transport*, vol. 31, no. 5, pp. 143–150, 2018.

[3] Z. J. Li, R. Song, S. He, and M. Bi, "Methodology of mixed load customized bus lines and adjustment based on time windows," *PLoS One*, vol. 13, no. 1, Article ID e0189763, 2018.

[4] L. Carol, L. Z. Tong, J. Liu, and X. Zhou, "Customized bus service design for jointly optimizing passenger-to-vehicle assignment and vehicle routing," *Transportation Research Part C*, vol. 85, no. 85, pp. 451–475, 2017.

[5] R. Guo, W. Guan, W. Zhang, F. Meng, and Z. Zhang, "Customized bus routing problem with time window restrictions: model and case study," *Transportmetrica A: Transport Science*, vol. 15, no. 2, pp. 1804–1824, 2019.

[6] R. Guo, W. Guan, and W. Zhang, "Route design problem of customized buses: mixed integer programming model and case study," *Journal of Transportation Engineering, Part A: Systems*, vol. 144, no. 11, Article ID 04018069, 2018.

[7] D. Huang, Y. Gu, S. Wang, Z. Liu, and W. Zhang, "A two-phase optimization model for the demand-responsive customized bus network design," *Transportation Research Part C: Emerging Technologies*, vol. 111, pp. 1–21, 2020.

[8] C. Wang, C. Ma, and X. Xu, X. Xuecai, Multi-objective optimization of real-time customized bus routes based on two-stage method," *Physica A: Statistical Mechanics and Its Applications*, vol. 537, Article ID 122774, 2020.

[9] L. Yan, C. Chi-Yin, V. C. S. Lee et al., "CB-Planner: A bus line planning framework for customized bus systems," *Transportation Research Part C*, vol. 101, pp. 233–253, 2019.

[10] Z. Han, Y. Chen, H. Li, and Y. Zhang, "Customized bus network design based on individual reservation demands," *Sustainability*, vol. 11, no. 19, p. 5535, 2019.

[11] Y. Lei, P. Lin, and K. Yao, "The network scheduling model and solution algorithm of Internet customized public transportation," *Transportation System Engineering and Information*, vol. 17, no. 1, pp. 157–163, 2017.

[12] M. He, M. Li, S. Wenbing, and H. Qian, "Influence of reliability and comfort on responsive custom bus route design," *Journal of Highway and Transportation Research and Development*, vol. 36, no. 5, pp. 145–151, 2019.

[13] Y. Xue, J. Guo, J. An, and S. Zi, "Uncertain bi-level programming model for vehicle allocation problem of bus lines," *Journal of Transportation Systems Engineering and Information Technology*, vol. 20, no. 2, pp. 145–150, 2020.

[14] M. Li, L. Wang, J. Yang et al., "Passenger flow forecast for customized bus based on time series fuzzy clustering algorithm," *Interaction Studies*, vol. 20, no. 1, pp. 42–60, 2019.

[15] J. Zhang, D. Z. W. Wang, and M. Meng, "Analyzing customized bus service on a multimodal travel corridor: an analytical modeling approach[J]," *Journal of Transportation Engineering, Part A: Systems*, vol. 143, no. 11, Article ID 04017057, 2017.

[16] J. Wang, T. Yamamoto, K. Liu et al., "Role of customized bus services in the transportation system: insight from actual performance," *Journal of Advanced Transportation*, vol. 2019, Article ID 6171532, 14 pages, 2019.

[17] L. Jing, Y. Lv, J. Ma et al., "Methodology for extracting potential customized bus routes based on bus smart card data," *Energies*, vol. 11, no. 9, p. 2224, 2018.

[18] X. Fu, Y. Gu, and Z. Liu, "Scheduling activity and travel patterns in multi-modal transit networks with customized bus services," *Journal of Transportation Systems Engineering and Information Technology*, vol. 19, no. 4, pp. 20–27, 2019.

[19] B. Liu, *Uncertainty theory*, Springer-Verlag, Berlin, Germany, 2nd edition, 2007.

[20] M. Huang, L. Ren, L. H. Lee, X. Wang, H. Kuang, and H. Shi, "Model and algorithm for 4PLRP with uncertain delivery time," *Information Sciences*, vol. 330, pp. 211–225, 2016.
[21] K. Hua, L. Wang, and H. Huang, "An uncertain model for RCPSP with solution robustness focusing on logistics project schedule[J]," *International Journal of E-Navigation and Maritime Economy*, vol. 3, 2015.
[22] Q. Zhang, X. Wang, W. Li, and H. Hu, "An uncertain resource constrained scheduling model based on uncertainty theory," *Mathematics and Computer Science*, vol. 4, no. 6, pp. 138–141, 2019.
[23] D. Jiao, *Uncertain Programming Model for Vehicle Routing problem*, Tsinghua University, Beijing, China, 2015.