Assignment3 – Decisions

Given: May 11 Due: May 21

Problem 3.1 (Decision Network) 0 pt

You try to decide on whether to take an umbrella (boolean variable M) to Uni. Obviously, it is useful (numeric variable U for the utility) to do so if it rains (boolean variable W) when you go back home, but it is annoying to carry around if it does not even rain. You decide based on whether the weather forecast predicts rain (boolean variable F).

1. Draw the decision network for bringing/leaving an umbrella depending on the weather forecast and the actual weather. Explain the four variables M, U, W, and F and what to store in their probability tables.
2. Explain formally how to compute whether or not to take an umbrella, assuming you know the probability that the forecast is correct.

Problem 3.2 (Expected Utility) 30 pt

1. State the formal definition of *expected utility* of an action in the current state of an agent? Explain the meaning of every variable in the defining equation.
2. How do we use expected utility to make decisions?

Problem 3.3 (Decision Theory) 30 pt

You are offered the following game: You pay x dollars to play. A fair coin is then tossed repeatedly until it comes up heads for the first time. Your payout is 2^n, where n is the number of tosses that occurred.

1. Assume your utility function is exactly the monetary value. How much should you, as a rational agent, be willing to pay to play? Use the formal definition of "expected utility" from the lecture.
2. Assume now, that your utility function for having k dollars is $U(k) = m \log_n k$ for some $m, n \in \mathbb{N}^+$. How does this change the result?
3. What is wrong with the result from the first exercise? Which implicit assumption leads to the apparently nonsensical result? How could it be fixed?

Hint: The series $\sum_{k=1}^{\infty} \frac{k}{2^k}$ is convergent with limit 2.

Problem 3.4 (Decision Network) 40 pt

You need a new car. Your local dealership has two models on offer

- C_1 for \$1500 with market value \$2000
- C_2 for \$1150 with market value \$1400

Either car can be of good quality or bad quality, and you have no information about that. If C_1 is of bad quality, repairing it will cost \$700, if C_2 is of bad quality repairing it will cost \$150.

You have the choice between two tests:

1. $Test_1$ at cost \$50: This will confirm that C_1 is of good quality (if it is) with certainty 85% probability, and that it is of bad quality (if it is) with certainty 65%.

2. $Test_2$ at cost \$20: This will confirm that C_2 is of good quality (if it is) with 75% certainty, and that it is of bad quality (if it is) with certainty 70%.

The a priori probability (without any tests) that a car is of good quality is 70% for C_1 and 65% for C_2.

The utility function is the monetary value, i.e., the difference of the market value of the acquired car and the amount of money spent on test, car, and repair.

1. Decision networks in general have three kinds of nodes. Explain the differences regarding the probability tables of the three kinds.

2. Now regarding the concrete network used here, explain the random variables of all nodes and their domains.

3. Draw the decision network for which test to apply and which car to buy in either case. This should include:
   - an action node for the test decision (no test, Test1, or Test2)
   - an action node for which car to buy
   - utility nodes for each of the two cars
   - chance nodes for the quality of the cars and the outcomes of the tests

4. Assume we have chosen to do $Test_1$ and the outcome was good. Compute which car to buy.