Assignment1 – Probability

Given: Apr 27 Due: May 7

Problem 1.1 (Bayesian Rules) 0 pt

Give the formulas and a one-sentence explanation of the following basic rules in Bayesian inference:

1. Bayes rule
2. Product rule
3. Chain rule
4. Marginalization
5. Normalization

Problem 1.2 (Disjunctive Random Variables) 20 pt

We know that given Boolean random variables A and B we have

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

Extend this formula to the case of three random variables $P(A \vee B \vee C)$. Draw a Venn diagram to "prove" your formula.

Problem 1.3 (Basic Probability) 20 pt

Let A, B, C be Boolean random variables, and let a, b, c denote the atomic events that A, B, C, respectively, are true. Which of the following equalities are always true? Justify each of your answers in one sentence.

1. $P(b) = P(a, b) + P(\neg a, b)$
2. $P(a) = P(a|b) + P(a|\neg b)$
3. $P(a, b) = P(a) \cdot P(b)$
4. $P(a, b|c) \cdot P(c) = P(c, a|b) \cdot P(b)$
5. $P(a \vee b) = P(a) + P(b)$
6. $P(a, \neg b) = (1 - P(b|a)) \cdot P(a)$

Problem 1.4 (Chained Production Elements) 20 pt

An apparatus consists of six elements A, B, C, D, E, F. Assume the probabilities $P(b_X)$, that element X breaks down, are all stochastically independent, with $P(b_A) = 5\%$, $P(b_B) = 10\%$, $P(b_C) = 15\%$, $P(b_D) = 20\%$, $P(b_E) = 25\%$, and $P(b_F) = 30\%$.

1. Assume the apparatus works if and only if at least A and B are operational, C and D are operational, or E and F are operational. What is the probability the apparatus works?

2. Consider a different scenario, in which the elements A and C, D and F and B and E are pairwise linked; such that if either of them breaks down, then the linked element is not operational either. What is the probability that the apparatus works now?

Note that we deliberately differentiate between *not being operational* and *being broken*. If an element breaks, it is not operational; if an element is not operational, either it or the linked element broke.

Problem 1.5 (Probabilities in Python) 40 pt

Complete the partial implementation of probabilities at `https://kwarc.info/teaching/AI/resources/AI2/probabilities/`