Assignment - 3

MC-405 ; Graph Theory

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Ans ① The complement of a graph G(V,E) denoted as $\bar{G}$. A vertex v in a connected graph G is called a cut-vertex if G-v is disconnected.

Let G-v consists of connected components $C_1, C_2, \dots C_k$ $(k \geq 2)$. Then the subgraph $\bar{G}-v$ of $\bar{G}$ induced on $C_1 \cup C_2 \cup \dots C_k$ is connected, since for any $i, j \in [1,k]$ $(i \neq j)$ every vertex in C_i is adjacent in $\bar{G}$ to every vertex in C_j. In addition for any $i \in [1,k]$, if x' and x'' are two vertices in C_i then we select $y \in C_j$, where $j \neq i$ and path $x'yx''$ which connects x' with x'', Thus $\bar{G}-v$ is a connected graph.

Ans ② In a proper k-coloring of a graph there exists a vertex which is adjacent to k-1 colors, else we can rearrange and use less number of colors. Since no vertex in graph is adjacent to two different colors $\chi(G) < 3$. So $\chi(G) = 2$, since a vertex is joined to a vertex with different color or we can say in simple words it is so because there is an edge.

Ans ③ (i) The law of conservation holds at a because :-

$\sum_v f_{va} = f_{sa} = 2$ and $\sum_v f_{av} = f_{ac} + f_{ae} = 2+0 = 2$

At e $\sum_v f_{ve} = f_{ae} + f_{be} = 0+1 = 1$

and $\sum_v f_{ev} = f_{ec} + f_{ed} = 0+1 = 1$

It holds at d because $\sum_v f_{vd} = f_{bd} + f_{ed} = 3+1 = 4$

and $\sum_v f_{dv} = f_{dt} = 4$.

ii) The value of the flow is 6.

iii) The capacity of the cut is $C_{ac} + C_{ae} + C_{be} + C_{bd}$
$= 3+1+4+3 = 11$

iv) The flow is not maximum. Ex:- It can be increased by adding 1 to the flow in the arcs along sact.

Ans (4) A graph is called Non separable if it does not contain cut vertex and subset E' of E is called cut set if deletion of all the edges from E' makes G disconnected. Let $u_i, u_{i+1} \cdots u_n$ be the vertices adjacent to a vertex v. Let us remove edges $u_i v, u_{i+1} v, \ldots u_n v$. The graph now becomes disconnected into two components. Similarly if we remove other edges, graph will decompose into many components. Hence the removal of these edges makes graph disconnected. Hence the set of these edges is responsible for disconnected graph which is known as cut-set.

Ans (5) Let G be a ~~bicolo~~ bicolorable. Let V_1 denote the set of all the vertices for which first color is assigned and V_2 be the set of all vertices for which second color is assigned. Then $V = V_1 \cup V_2$ is a partition of V in G. Otherwise at least two vertices in V_1 or V_2 have the same color. Therefore Graph G is bipartite.

Conversely, let us assume that G is bipartite. Let (V_1, V_2) be the partition of V in G. Then a 2-coloring for G can be given by coloring the vertices in V_1 by one color and the remaining vertices in V_2 by another color. Hence G is bicolorable.

Ans ⑥ ~~Verte~~ Every vertex is connected to every other vertex in a perfect graph, ∴ degree of each vertex is $2n-1$. Out of these $2n-1$ edges we have to choose 1 edge to include two vertices, similarly we continue :-

$$a_n = a_{2n-1} \cdot a_{2n-3} \cdot \ldots 5 \cdot 3 \cdot 1$$

$$a_n = \frac{((2n-1)(2n-3) \ldots 3 \times 1) \times (2n \times (2n-2) \times \ldots 4 \times 2)}{((2n) \times (2n-2) \ldots 4 \times 2)}$$

$$= \frac{(2n)!}{(2^n \times n \times (n-1) \times \ldots 2 \times 1)}$$

$$\boxed{a_n = \frac{(2n)!}{2^n \times n!}}$$

Ans ⑦ From theorem :- G has each vertex with $d \geq \frac{1}{2}|V|$ then G is hamiltonian. G has Hamiltonian cycle. Taking every second edge of this cycle yields a perfect matching. Hence G is a graph and if $|V|$ is even and each vertex has degree $d \geq \frac{1}{2}|V|$, g has a perfect matching.