CPCS 222

Relations

Introduction

Dr. Muhammad Umair Ramzan

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  - Let $A = \{0,1,2\}$ and $B = \{a,b\}$
  - $\{(0, a), (0, b), (1,a), (2, b)\}$ is a relation from *A* to *B*
  - **Relations** are **more general** than functions. A function is a relation where exactly one element of *B* is related to each element of *A*

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- **Representation:**
  - Let $A = \{0,1,2\}$ and $B = \{a,b\}$
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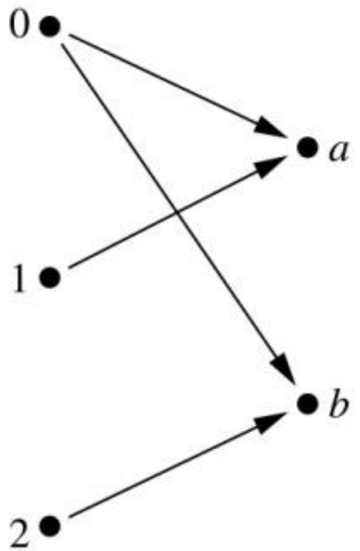

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  - **Table Representation**

R	a	b
0	×	×
1	×	
2		×

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  - $R = \{(1,1), (1, 2), (1,3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}$

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- **# of elements** in a Relation *R* on A

$$2^{|A|^2} = 2^{n^2}$$

Relation – Examples

- **Example 01:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\},$

$R_2 = \{(a,b) \mid a > b\},$

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$

$R_4 = \{(a,b) \mid a = b\},$

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- Which of these relations contains (1, 1)
  - R1, R3, R4, R6

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Relation – Examples

- **Example 02:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\}$,
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- Which of these relations contains (1, 2)

Relation – Examples

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$R_1 = \{(a,b) \mid a \leq b\},$ $R_4 = \{(a,b) \mid a = b\},$

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- Which of these relations contains (1, 2)

    - R1, R6

Relation – Examples

- **Example 03:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\},$

$R_2 = \{(a,b) \mid a > b\},$

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$

$R_4 = \{(a,b) \mid a = b\},$

$R_5 = \{(a,b) \mid a = b + 1\},$

$R_6 = \{(a,b) \mid a + b \leq 3\}.$

- Which of these relations contains (2, 1)

Relation – Examples

- **Example 03:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\},$

$R_2 = \{(a,b) \mid a > b\},$

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$R_4 = \{(a,b) \mid a = b\},$

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$R_6 = \{(a,b) \mid a + b \leq 3\}.$

- Which of these relations contains (2, 1)
  - R2, R5, R6

Relation – Examples

- **Example 04:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\},$

$R_2 = \{(a,b) \mid a > b\},$

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- Which of these relations contains (1, -1)

Relation – Examples

- **Example 04:** Consider these relations on the set of integers:

$R_1 = \{(a,b) \mid a \leq b\},$

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- Which of these relations contains (1, -1)

  - R2, R3, R6

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- **Example 05:** Consider these relations on the set of integers:

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- Which of these relations contains (2, 2)

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- Which of these relations contains (2, 2)
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- Reflexive Relation:

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$$\forall x \forall y\, [(x,y) \in R \land (y,x) \in R \longrightarrow x = y]$$

$R_1 = \{(a,b) \mid a \leq b\}$, ✔

$R_2 = \{(a,b) \mid a > b\}$, ✔

$R_4 = \{(a,b) \mid a = b\}$, ✔

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Relation – Types

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Relation – Operation

- **Union:** A relation *R* consist of all elements of two relations **R1** and **R2** i.e. $R = R1 \cup R2$

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Relation – Operation

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  - R_1={(1,1),(2,2),(3,3)}
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Relation – Operation

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Relation – Operation

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Relation – Operation

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Relation – Operation

- **Composition:** Suppose R_1 is a relation from a set A to a set B, R_2 is a relation from ***set B*** to a ***set C***, Then the *composition* (or *composite*) of R_2 with R_1, is a relation from *set A* to *set C*
  - if **(*x,y*)** is a member of $\boldsymbol{R_1}$ and **(*y,z*)** is a member of $\boldsymbol{R_2}$, then **(*x,z*)** is a member of $\boldsymbol{R_2 \circ R_1}$.

Relation – Operation

- if (x,y) is a member of R_1 and (y,z) is a member of R_2, then (x,z) is a member of $R_2 \circ R_1$.
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  - $R_2 \circ R_1 =\{(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)\}$

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Relation – Representation

- A relation *R* can be represented in TWO ways

Relation – Representation

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  - Using **Matrices**
  - Using **Diagraph**

Relation – Representation

- **Matrices:** A relation ***R*** between finite sets can be represented using a **zero-one matrix**

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- $A = \{a_1, a_2, \ldots, a_m\}$
- $B = \{b_1, b_2, \ldots, b_n\}$
- ***R*** is a relation from *A to B*

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- $B = \{b_1, b_2, \ldots, b_n\}$
- ***R*** is a relation from *A to B*

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

Relation – Representation

- **Example 01:** Suppose that $A = \{1,2,3\}$ and $B = \{1,2\}$. Let R be the relation from A to B containing (a,b) if $a \in A$, $b \in B$, and $a > b$. What is the matrix representing R (assuming the ordering of elements is the same as the increasing numerical order)?

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- $R = \{(2,1), (3,1),(3,2)\}$

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- ***R*** = {(2,1), (3,1),(3,2)}

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Relation – Representation

- **Example 02:** Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. Which ordered pairs are in the relation R represented by the matrix

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- ***R*** **= {(a_1, b_2), (a_2, b_1),(a_2, b_3), (a_2, b_4),(a_3, b_1), {(a_3, b_3), (a_3, b_5)}**

Relation – Matrices vs Types

- Reflexive:

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Relation – Matrices vs Types

- **Reflexive:** If R is a reflexive relation, all the elements on the main diagonal of M_R are equal to 1

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Relation – Matrices vs Types

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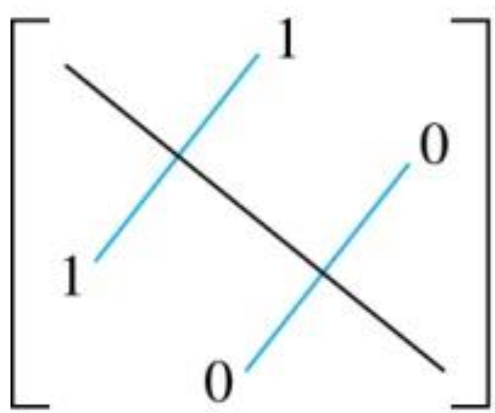

(a) Symmetric

C
P
C
S

2
2
2

Relation – Matrices vs Types

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Relation – Matrices vs Types

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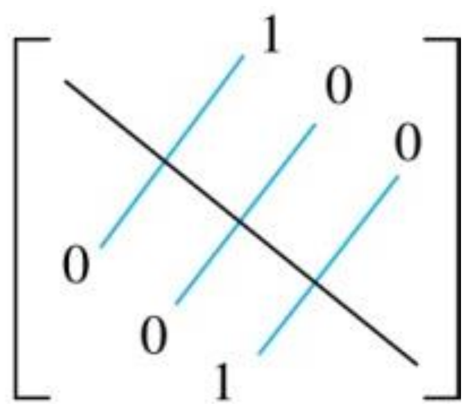

(b) Antisymmetric

Relation – Matrices vs Types

- **Example 01:** Suppose that the relation *R* on a set is represented by the matrix

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- **Type of Relation**

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- **Type of Relation**
  - **R** is **reflexive**
  - **R** is **Symmetric**
  - **R** is **not antisymmetric**

Relation – Representation

- **Diagraph:**

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Relation – Representation

- **Diagraph:** A *directed graph*, or *digraph*, consists of a **set** ***V*** of *vertices* (or *nodes*) together with a **set** ***E*** of ordered pairs of elements of *V* called *edges* (or *arcs*). The **vertex** ***a*** is called the ***initial vertex*** of the **edge (*a*,*b*)**, and the **vertex** ***b*** is called the ***terminal vertex*** of this edge

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- An **edge** of the form **(*a*,*a*)** is called a ***loop***

Relation – Representation

- **Example 01:** Draw a directed graph with vertices ***a*, *b*, *c*,** and ***d***, and edges **(*a*, *b*), (*a*, *d*), (*b*, *b*), (*b*, *d*), (*c*, a), (*c*, *b*),** and **(*d*, *b*)**

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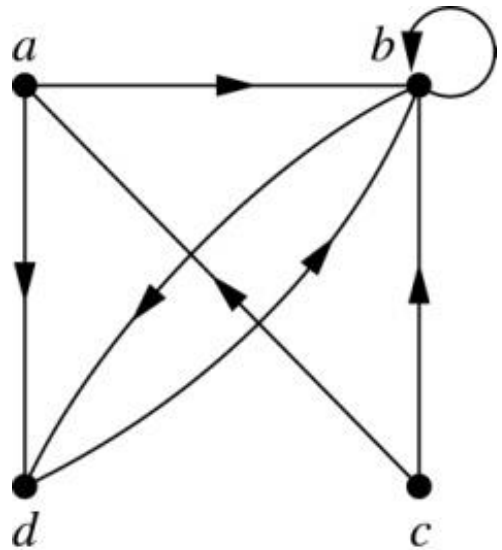
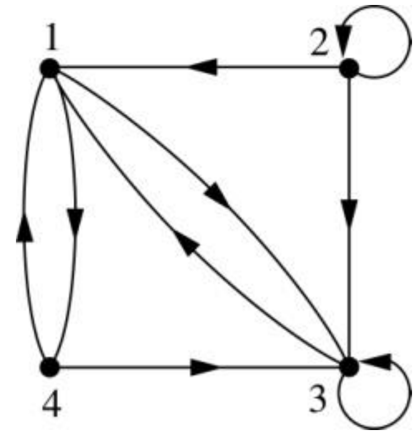


Relation – Representation

- **Example 02:** What are the ordered pairs in the relation represented by this directed graph?



Relation – Representation

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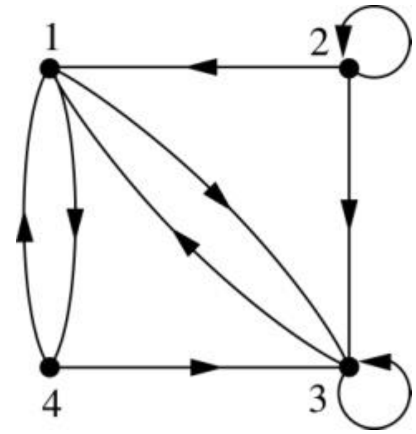

- **(1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 3)**

Relation – Diagraph vs Types

- **Reflexive:** A loop must be present at all vertices in the graph
- **Symmetric:** If (x, y) is an edge, then (y, x) must be an edge too
- **Antisymmetric:** If (x, y) with $x \neq y$ is an edge, then (y, x) is not an edge
- **Transitive:** If (x, y) and (y, z) are edges, then so is (x, z)

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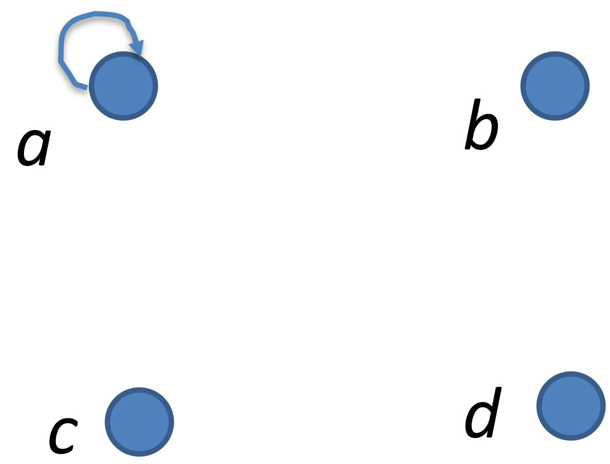
Relation – Diagraph vs Types

- Example 01:



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Relation – Diagraph vs Types

- Example 01:
- Reflexive
- Symmetric
- Antisymmetric
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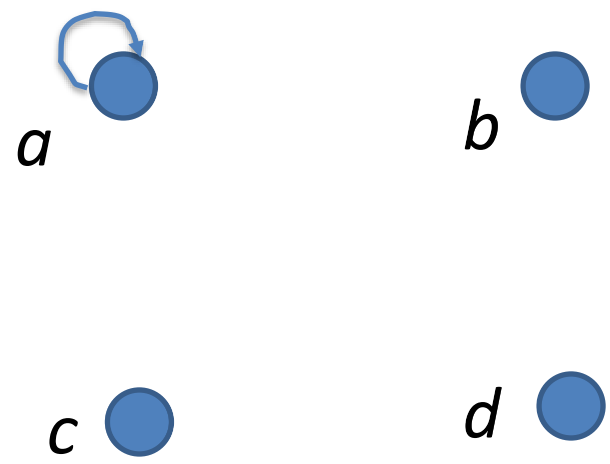

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Relation – Diagraph vs Types

- Example 01:
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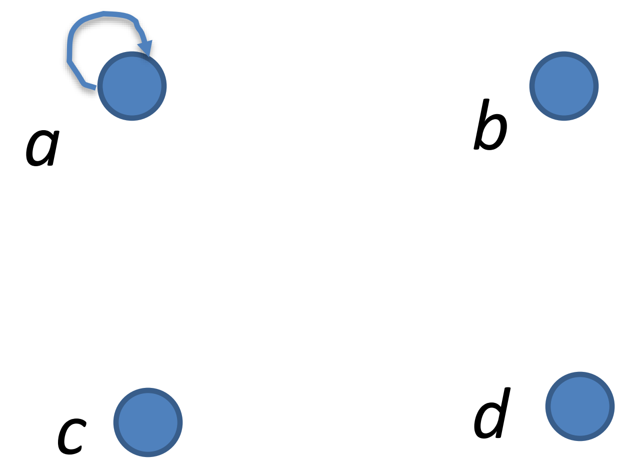
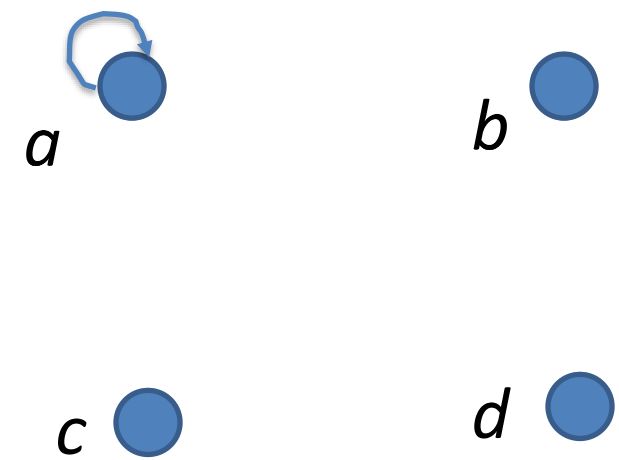


Relation – Diagraph vs Types

- Example 01:

- Reflexive
  - No loop
- Symmetric
  - No edges
- Antisymmetric

- Transitive

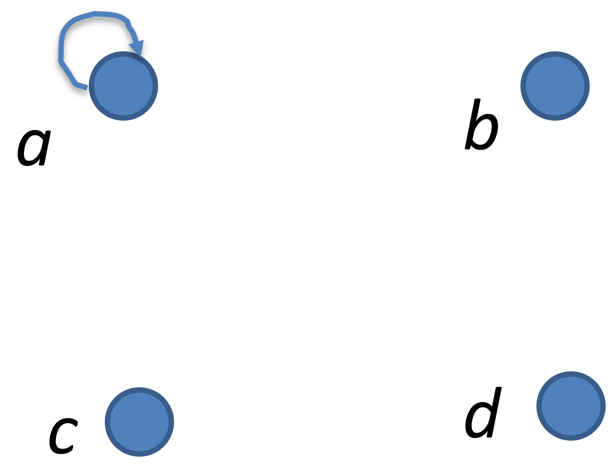


C P C S 2 2 2

Relation – Diagraph vs Types

- Example 01:

- Reflexive
  - No loop
- Symmetric
  - No edges
- Antisymmetric
  - No edges
- Transitive



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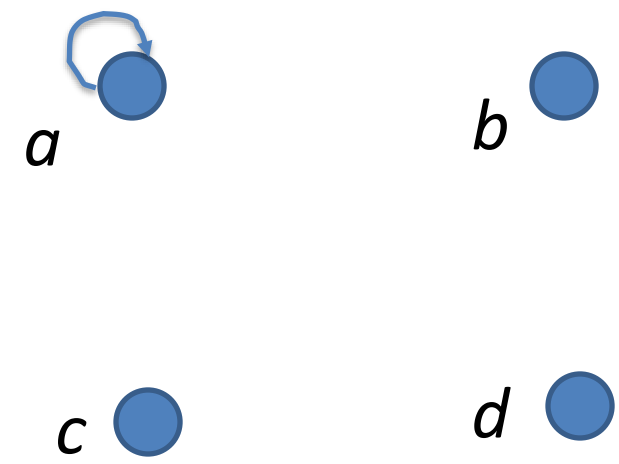
Relation – Diagraph vs Types

- Example 01:

- Reflexive ✗
  - No loop
- Symmetric ✓
  - No edges
- Antisymmetric ✓
  - No edges
- Transitive ✓
  - No edges



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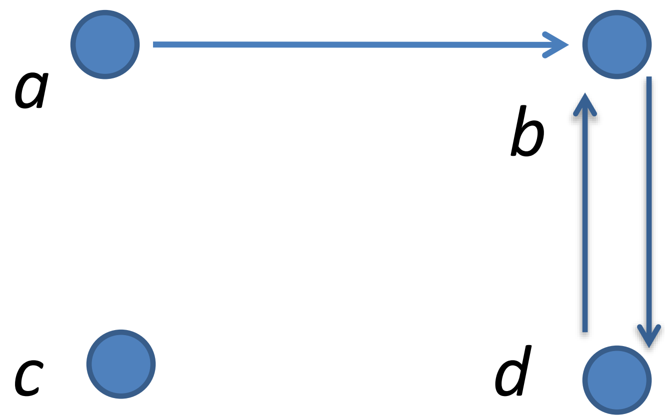
Relation – Diagraph vs Types

- Example 02:

- Reflexive
- Symmetric
- Antisymmetric
- Transitive



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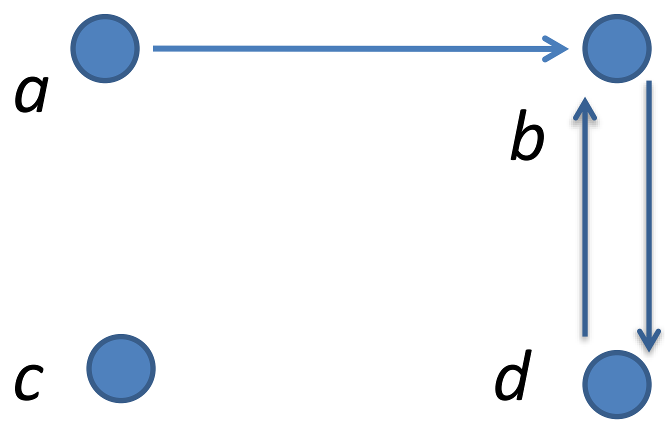
Relation – Diagraph vs Types

- Example 02:

- Reflexive ✖
    - No loop
- Symmetric
- Antisymmetric
- Transitive



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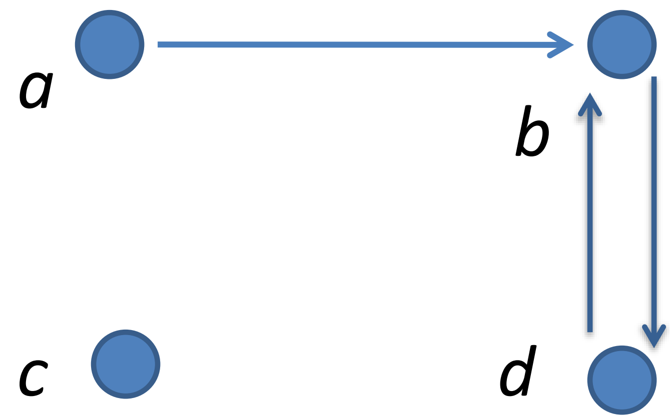
Relation – Diagraph vs Types

- Example 02:

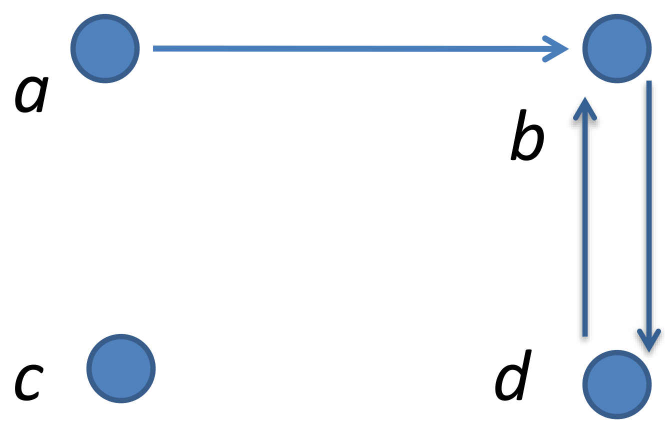
- Reflexive ✖
  - No loop
- Symmetric ✖
  - No edge from b to a
- Antisymmetric
- Transitive



Relation – Diagraph vs Types

- Example 02:

- Reflexive ✖
  - No loop
- Symmetric ✖
  - No edge from b to a
- Antisymmetric ✖
  - Edge b to d & d to b
- Transitive



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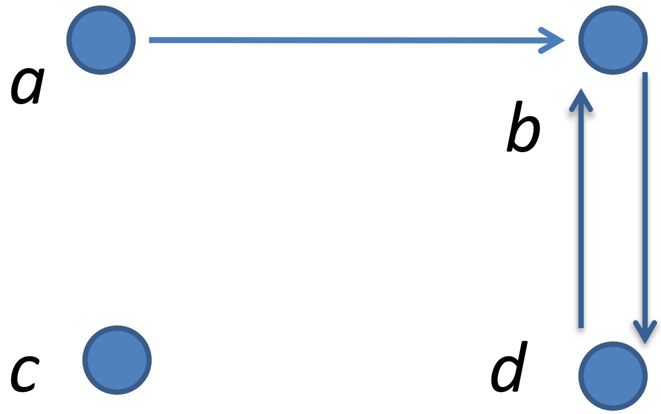
Relation – Diagraph vs Types

- Example 02:

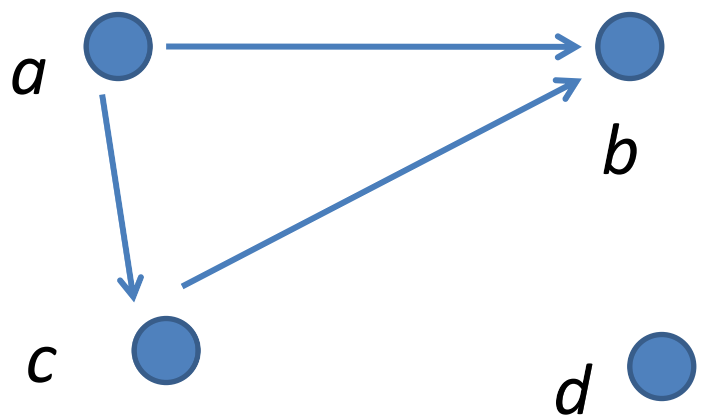
- Reflexive ✖
  - No loop
- Symmetric ✖
  - No edge from b to a
- Antisymmetric ✖
  - Edge b to d & d to b
- Transitive ✖
  - Edge a to b, b to d but no edge from a to d



Relation – Diagraph vs Types

- Example 03:

- Reflexive
- Symmetric
- Antisymmetric
- Transitive



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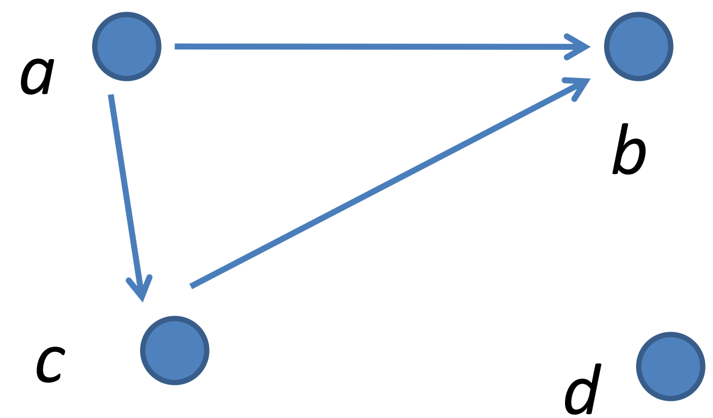
Relation – Diagraph vs Types

- Example 03:

- Reflexive ✖
  - No loop
- Symmetric
- Antisymmetric
- Transitive



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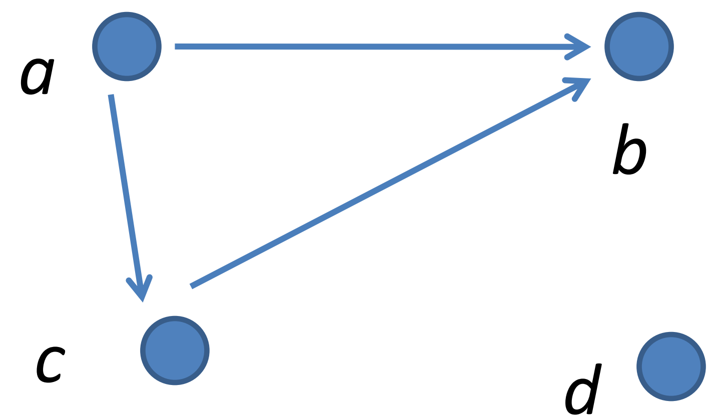
Relation – Diagraph vs Types

- Example 03:

- Reflexive ✖
    - No loop
- Symmetric ✖
    - No edge from c to a
- Antisymmetric
- Transitive



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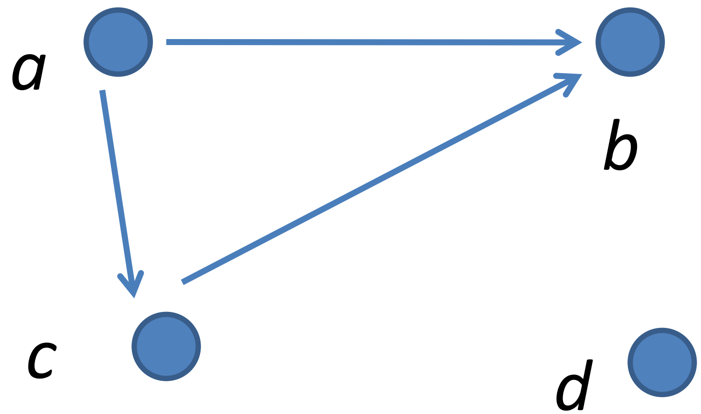
Relation – Diagraph vs Types

- Example 03:

- Reflexive ✖
  - No loop
- Symmetric ✖
  - No edge from c to a
- Antisymmetric ✔
  - No two way edges
- Transitive



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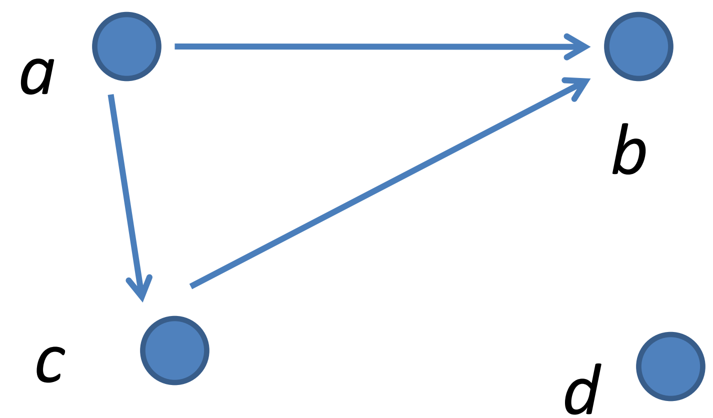
Relation – Diagraph vs Types

- Example 03:

- Reflexive
  - No loop
- Symmetric
  - No edge from c to a
- Antisymmetric
  - No two way edges
- Transitive
  - Edge a to c, c to b and also an edge from a to b



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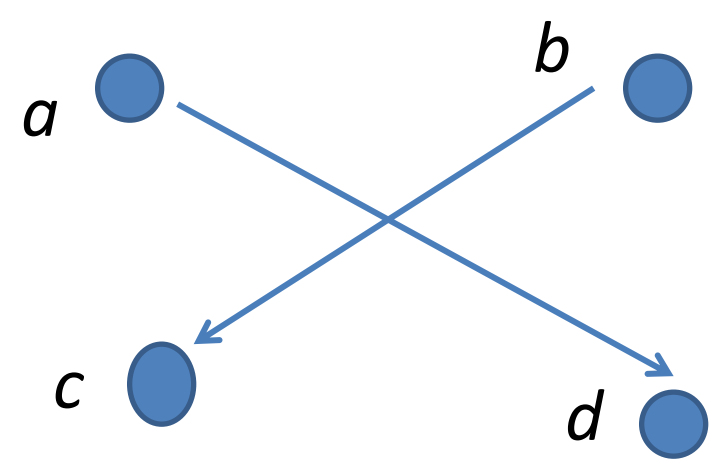
Relation – Diagraph vs Types

- Example 04:

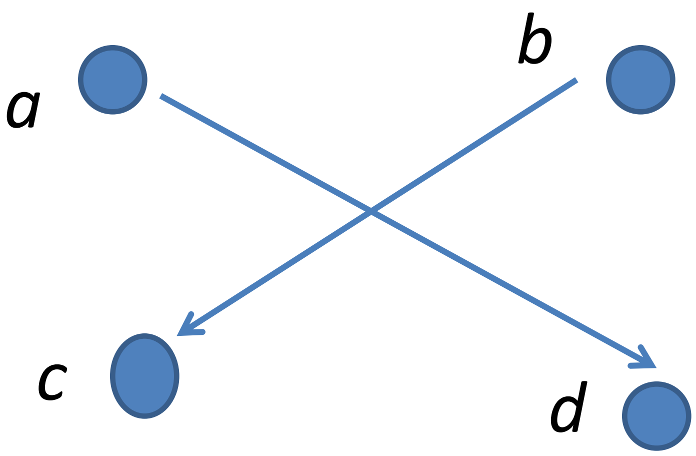
- Reflexive
- Symmetric
- Antisymmetric
- Transitive



Relation – Diagraph vs Types

- Example 04:

- Reflexive ✖
    - No loop
- Symmetric
- Antisymmetric
- Transitive

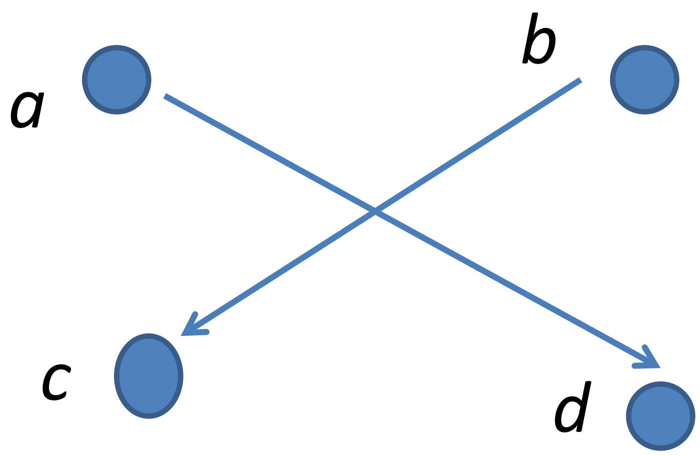


C P C S 2 2 2

Relation – Diagraph vs Types

- Example 04:

- Reflexive ✖
  - No loop
- Symmetric ✖
  - No edge from d to a
- Antisymmetric
- Transitive



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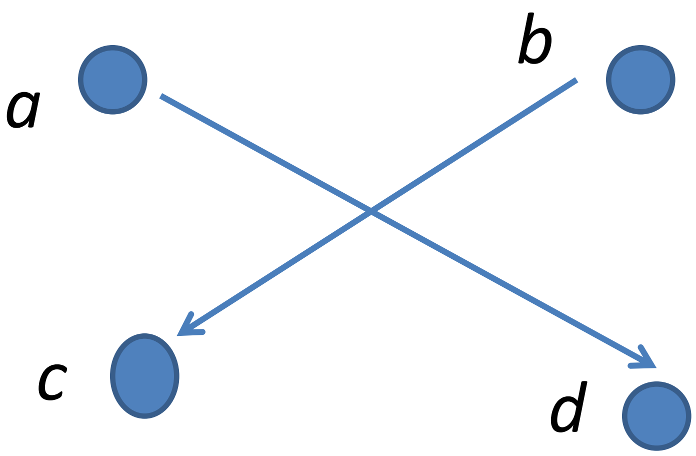
Relation – Diagraph vs Types

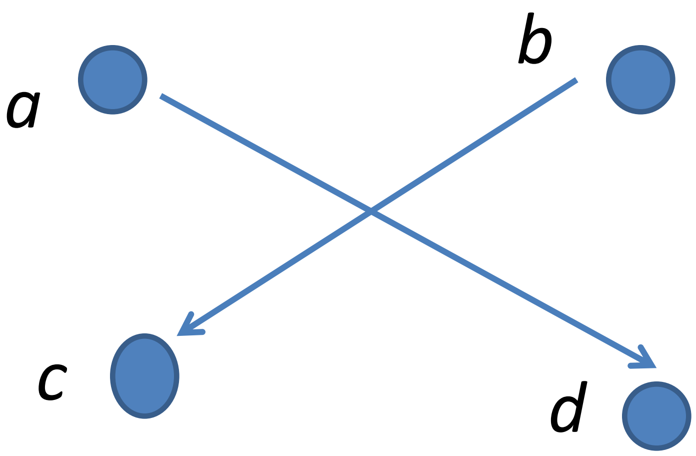
- Example 04:

- Reflexive ✖
    - No loop
- Symmetric ✖
    - No edge from d to a
- Antisymmetric ✔
    - No two way edges
- Transitive



Relation – Diagraph vs Types

- **Example 04:**



- **Reflexive** ✗
  - No loop
- **Symmetric** ✗
  - No edge from d to a
- **Antisymmetric** ✓
  - No two way edges
- **Transitive** ✓
  - there are no two edges where the first edge ends at the vertex where the second edge begins

Dr. Muhammad Umair Ramzan

Relation – Equivalence Relations

- **Definition:** A relation on a set A is called an ***equivalence***; if it is **reflexive**, **symmetric**, and **transitive**

Relation – Equivalence Relations

- **Definition:** A relation on a set *A* is called an ***equivalence***; if it is **reflexive**, **symmetric**, and **transitive**
- **Definition:** Two elements *a*, and *b* that are related by an equivalence relation are called *equivalent.* The notation $a \sim b$ is often used to denote that *a* and *b* are equivalent elements with respect to a particular equivalence relation

Relation – Equivalence Relations

- **Example01:** Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation?

Relation – Equivalence Relations

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- **Reflexive:**
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Relation – Equivalence Relations

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- **Symmetric:**
- **Transitive:**

Relation – Equivalence Relations

- **Example01:** Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation?

- **Reflexive:** Because $l(a) = l(a)$, it follows that aRa for all strings a
- **Symmetric:** Suppose that aRb. Since l(a) = l(b), l(b) = l(a) also holds and bRa
- **Transitive:**

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Relation – Equivalence Relations

- **Example01:** Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation?

Reflexive: Because $l(a) = l(a)$, it follows that aRa for all strings a

Symmetric: Suppose that aRb. Since l(a) = l(b), l(b) = l(a) also holds and bRa

Transitive: Suppose that aRb and bRc. Since l(a) = l(b),and l(b) = l(c), l(a) = l(c) also holds and aRc

Relation – Equivalence Relations

- **Example02:** Let m be an integer with $m > 1$. Show that the relation $R = \{(a,b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on the set of integers

Recall that $a \equiv b \pmod{m}$ if and only if m divides $a - b$

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Relation – Equivalence Relations

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- **Reflexive:**

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Reflexive: $a \equiv a \pmod{m}$ since $a - a = 0$ is divisible by m since

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Recall that $a \equiv b \pmod{m}$ if and only if m divides $a - b$

Reflexive: $a \equiv a \pmod{m}$ since $a - a = 0$ is divisible by m since

Symmetric: Suppose that $a \equiv b \pmod{m}$. Then $a - b$ is divisible by m, and so $a - b = km$, where k is an integer. It follows that $b - a = (-k)\, m$, so $b \equiv a \pmod{m}$

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Relation – Equivalence Relations

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Relation – Equivalence Relations

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Transitive: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

Relation – Equivalence Relations

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Transitive: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

Then m divides both a − b and b − c. Hence, there are integers k and l

Relation – Equivalence Relations

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Transitive: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

Then m divides both a − b and b − c. Hence, there are integers k and l with $a - b = km$ and $b - c = lm$. We obtain by adding the equations:

Relation – Equivalence Relations

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Transitive: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

Then m divides both a − b and b − c. Hence, there are integers k and l with $a - b = km$ *and* $b - c = lm$. We obtain by adding the equations:

$$a - c = (a - b) + (b - c) = km + lm = (k + l)\,m$$

Relation – Equivalence Relations

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Recall that $a \equiv b \pmod{m}$ if and only if m divides $a - b$

Transitive: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

Then m divides both a − b and b − c. Hence, there are integers k and l with $a - b = km$ and $b - c = lm$. We obtain by adding the equations:

$$a - c = (a - b) + (b - c) = km + lm = (k + l)\, m$$

Therefore, $a \equiv c \pmod{m}$

Relation – Equivalence Relations

- **Example03:** Show that the "divides" relation on the set of positive integers is not an equivalence relation

- **Reflexive:**
- **Symmetric:**
- **Transitive:**

Relation – Equivalence Relations

- **Example03:** Show that the "divides" relation on the set of positive integers is not an equivalence relation

- **Reflexive:** a / a for all a
- **Symmetric:**
- **Transitive:**

Relation – Equivalence Relations

- **Example03:** Show that the “divides” relation on the set of positive integers is not an equivalence relation

✔ **Reflexive:** $a \mid a$ for all a

✖ **Symmetric:** For example, $4 \mid 2$, but $2 \nmid 4$. Hence, the relation is not symmetric

- **Transitive:**

Relation – Equivalence Relations

- **Example03:** Show that the "divides" relation on the set of positive integers is not an equivalence relation

✔ **Reflexive:** a / a for all a

✖ **Symmetric:** For example, 4 / 2, but 2 ∤ 4. Hence, the relation is not symmetric

✔ **Transitive:** Suppose that a divides b and b divides c. Then there are positive integers k and l such that $b = ak$ and $c = bl$. Hence, $c = a(kl)$, so a divides c. Therefore, the relation is transitive