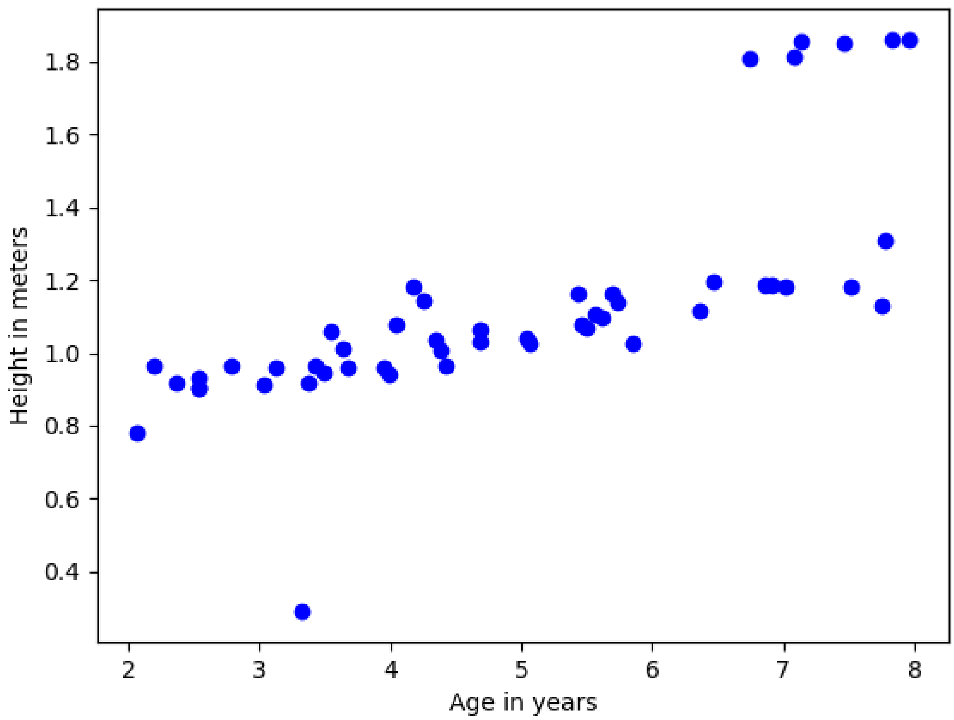
Machine Learning Assignment 1

Name: Anh Quoc Do
NetID: aqd14

Part A

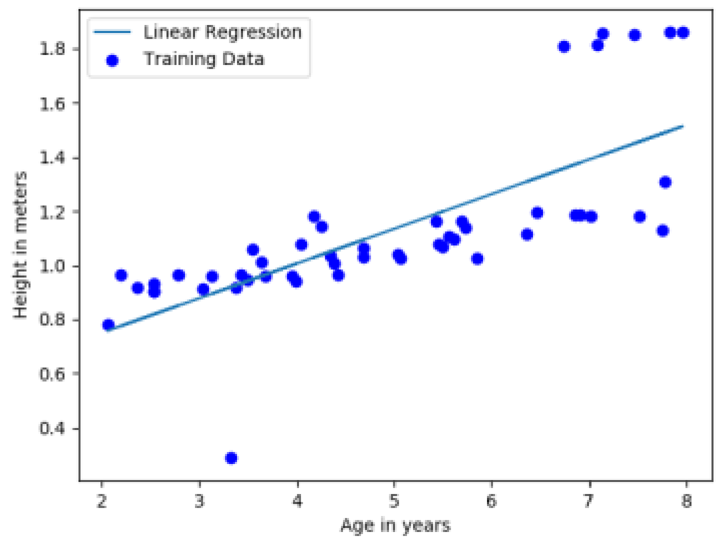
1. Plot of raw data



2. Converged value of θ

- First iteration:
  - θ = [0.0787538, 0.41494108]
- Converged after 803 iterations (set error threshold 1e-5)
  - θ = [0.49311086, 0.12820581]

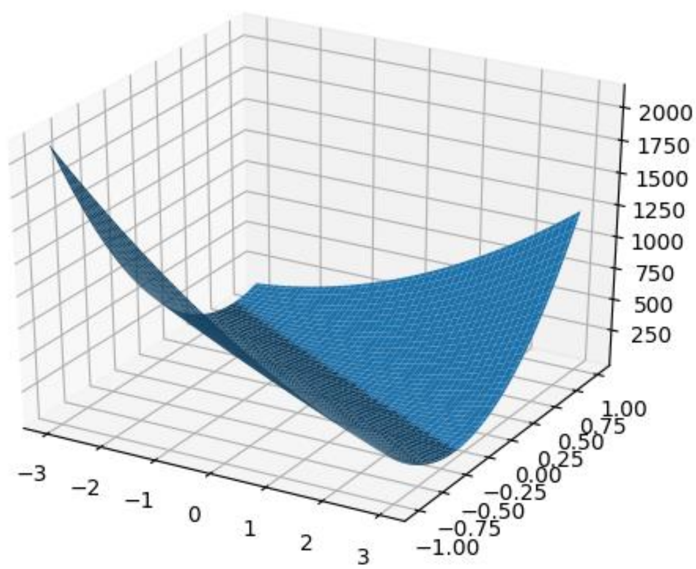
3. Plot of line that you fit



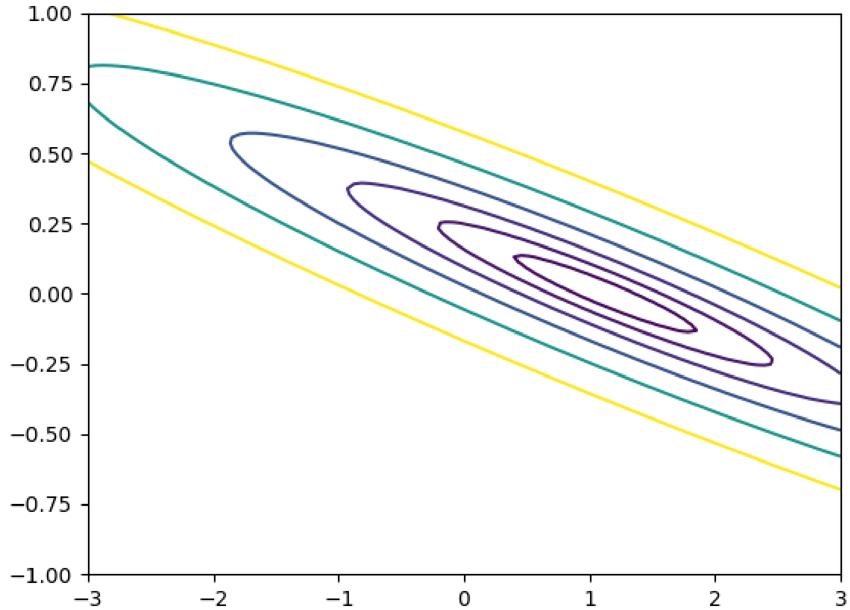
4. Prediction of your model for the height of two boys, age 3.5 and 7

- prediction = [0.94183121, 1.39055157]

5. Plot of 3D surface of J



6. Plot of contour of J



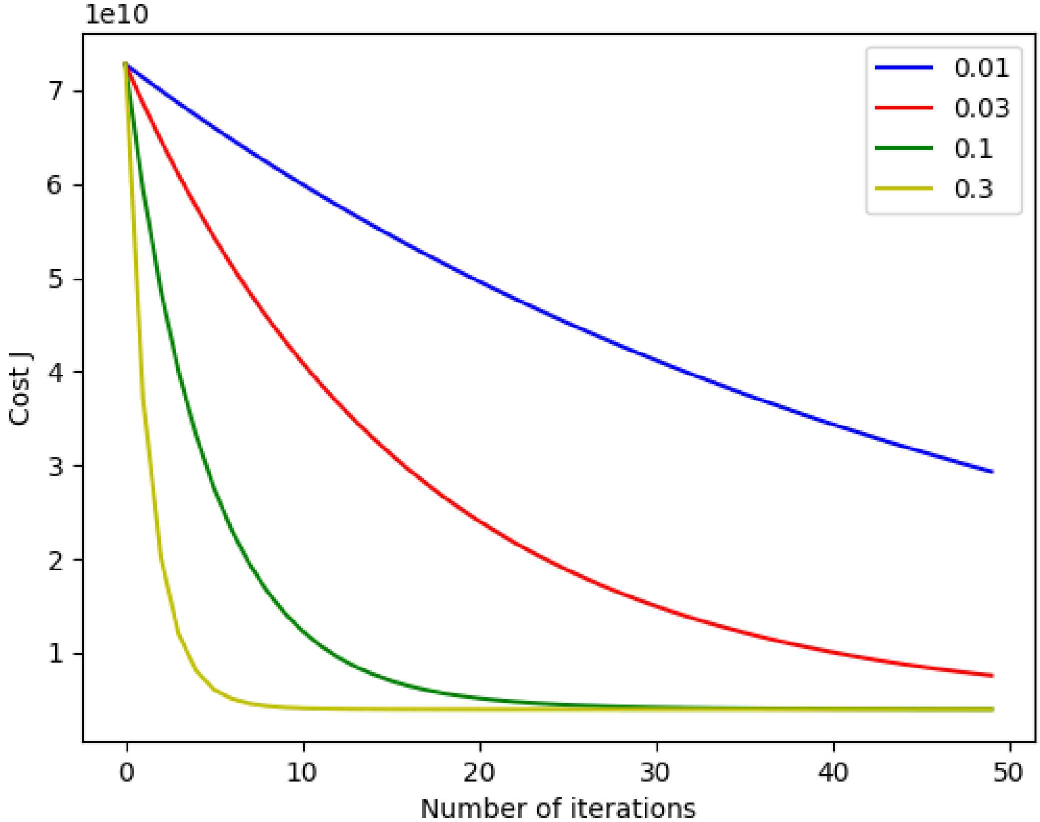
7. Answer to question about relationship between these plots and θ value found by your algorithm.

- These plots offer a visual view of how different values of θ affect to the value of cost function.
- There is exist an optimal vector θ that helps the cost function reaches minimum value. With gradient descent, in each step we update theta values that move toward optimal values

8. Include your code solution to Task A in your pdf submission
See appendix

Part B

1. Plot(s) comparing different learning rates



2. Converged value of θ for best learning rate
   - θ = [353178.61702128, 114973.94457945, -3702.28587976] with alpha = 0.3

3. Prediction of your model for the price of a house with 1650 square feet and 3 bedrooms
   - Predicted price = $303,614

4. Value of θ computed using normal equations
   - θ = [109185.1639846, 123.62723839, -3185.43247293]
   - These values are different from those using gradient descent

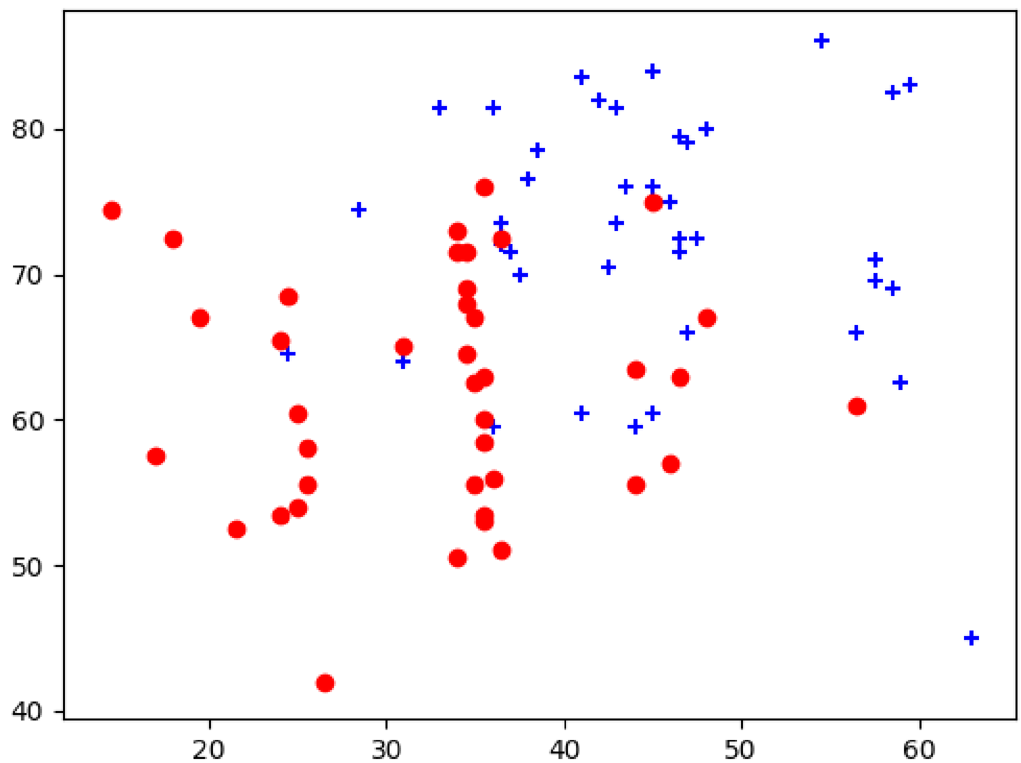
5. Prediction of your normal equations θ on a house with 1650 square feet and 3 bedrooms.
   - Predicted price = $303,614

6. Include your code solution to Task b in your pdf submission

See Appendix

Part C

1. Plot of raw data



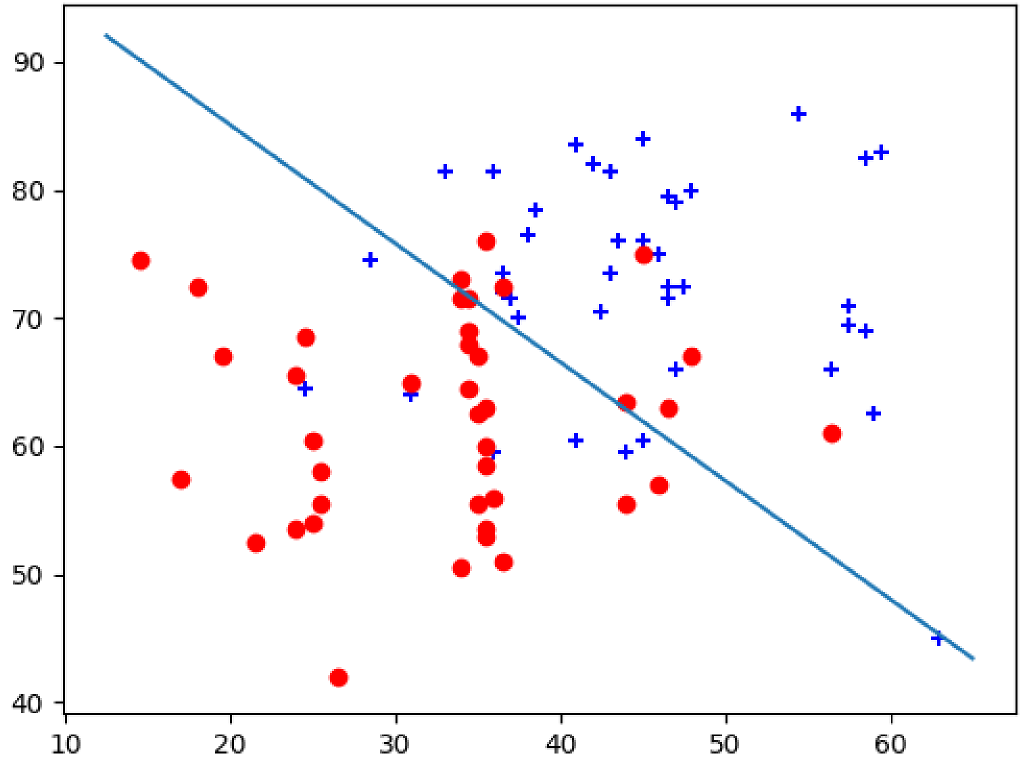
2. What values of θ did you get? How many iterations were required for convergence?

- Convergered at epoch 6
- θ = [-15.76822471, 0.14087783, 0.15219571]

3. What is the probability that a student with a score of 20 on Exam 1 and a score of 80 on Exam 2 will not be admitted?

- p = 0.684604

4. Plot of final decision boundary found



Appendix

Part A & B

```
# Implmentation of linear regession algorithm for only one feature dataset
# by using gradient descent to find optimum weights to the classification

'''
The data files ax.dat and ay.dat contain some example measurements of
heights for various boys between the ages of two and eights. The y-values
are the heights measured in meters, and the x-values are the ages of the boys
corresponding to the heights.
```

```
There are 50 training examples in total
'''

# usage python linear_regression.py

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from numpy.linalg import inv

class LinearRegression(object):
        """ Linear regression model

        Parameters
        ----------
        alpha : float
                Learning rate

        iter : integer
                Number of iteration

        check_conv: boolean
                Check if model is check_conv or not to stop iterations
        """

        def __init__(self, alpha=0.01, iter=2000, check_conv=True):
                self.alpha = alpha
                self.iter = iter
                self.check_conv = check_conv
                # self.normalized = normalized

        def fit(self, X, y):
                """ Fit traning dataset to the model.
                Update theta simultaneously by using gradient descent.
                For simplicity, ignore error handling.


                X : integer
                        training input
                y :     integer
                        label output
                """

                self.theta = np.zeros((X.shape[1], 1)) # Column vector
```

```
            err_threshold = 1e-5 # Stop when the cost function is smaller than threshold
            J = []
            for i in range(self.iter):
                cost = cost_function(X, y, self.theta)
                J.append(cost)

                gra, _ = self.gradient(X, y)
                temp = self.theta - self.alpha * gra
                # if np.array_equal(temp, self.theta):
                if self.check_conv is True and np.sum(abs(self.theta - temp)) < err_threshold:
                    break
                # Simultaneously update theta
                self.theta = temp

            # print('Not convergered!')
            return self.theta, J

        def hypothesis(self, X):
            # return np.dot(X, self.theta[1:]) + self.theta[0]
            return np.dot(X, self.theta)

        def gradient(self, X, y):
            """Calculate gradient descent

            Parameters
            ----------
            X : array-like
                training data

            Y : vector
                label output

            Returns
            -------
            gradient : float
                gradient value of given data

            mse : float
                mean squared errors
            """
            err = y - self.hypothesis(X)
            mse = (1.0/X.shape[0]) * np.sum(np.power(err, 2))
```

```python
            gradient = -(1.0/X.shape[0]) * X.T.dot(err)
            return gradient, mse

    def predict(self, X):
            return self.hypothesis(X)

def cost_function(X, y, theta):
        """
        Calculate cost function
        """
        m = X.shape[0] # Training size
        err = y - np.dot(X, theta)
        J = 1.0/(2*m) * np.sum(np.power(err, 2))

        return J

# ----------------------- PART A ----------------------- #
def part_a():

        # load data
        X = np.loadtxt('data/ax.dat') # input data
        y = np.loadtxt('data/ay.dat') # output data

        # plot dataset
        plt.scatter(X, y, facecolors='blue')
        plt.xlabel('Age in years')
        plt.ylabel('Height in meters')
        # plt.show()

        # Data preprocessing
        m = X.shape[0] # Training size
        X = np.stack((np.ones(m), X), axis=-1)
        y = y.reshape(m, 1)

        model = LinearRegression(alpha=0.07)
        theta, J = model.fit(X, y)

        # Plot straight line
        plt.plot(X[:,1],np.dot(X,theta))
        plt.legend(['Linear Regression', 'Training Data'])
        plt.show()
        # plt.savefig('plot1.png')

        # Prediction
        test_data = np.array([[1, 3.5], [1, 7]])
```

```python
        prediction = model.predict(test_data)
        print("Predicted height of kids age 3.5 and 7: ", prediction)

        plot_surface_contour(X, y)

def plot_surface_contour(X, y):
        # Display Surface Plot of J
        t0 = np.linspace(-3, 3, 100).reshape(100, 1)
        t1 = np.linspace(-1, 1, 100).reshape(100, 1)

        T0, T1 = np.meshgrid(t0, t1)

        J_vals = np.zeros((len(t0), len(t1)))

        for i in range(len(t0)):
                for j in range(len(t1)):
                        t = np.hstack([t0[i], t1[j]])
                        J_vals[i, j] = cost_function(X, y, t)

        #Because of the way meshgrids work with plotting surfaces
        #we need to transpose J to show it correctly
        J_vals = J_vals.T

        # print(J_vals)

        fig = plt.figure()
        ax = fig.gca(projection='3d')
        ax.plot_surface(T0,T1,J_vals)
        plt.show()
        plt.close()
        #Display Contour Plot of J
        plt.contour(T0,T1,J_vals, np.logspace(-2,2,15))
        plt.show()


# ----------------------- END PART A --------------------- #


# ----------------------- PART B ------------------------- #
def part_b():
        # Load data from files
        X = np.loadtxt('data/bx.dat')
        y = np.loadtxt('data/by.dat')

        sigma = np.std(X, axis=0) # std
```

```
        mu = np.mean(X, axis=0) # mean

        # Data preprocessing
        m = X.shape[0]
        y = y.reshape(m, 1)


        X_scaled = (X-mu) / sigma # normalize(X)
        intercept = np.ones((m, 1))
        X_scaled = np.column_stack((intercept, X_scaled))
        # print(X_scaled)
        # y_scaled = normalize(y)

        iterations = 50

        alphas = [0.01, 0.03, 0.1, 0.3]
        colors = ['b', 'r', 'g', 'y']

        test_data = np.array([[1, 1650], [1, 3]])

        for i in range(len(alphas)):
                model = LinearRegression(alpha=alphas[i], iter=iterations,
check_conv=False)
                theta, J = model.fit(X_scaled, y)

                # print('Learning rate %f' % alphas[i])
                # print('Theta: %d -- %d' % (theta[0], theta[1]))
                # print('Predict for the price of a house with 1650 square
feet and 3 bedrooms: ', model.predict(test_data))

                # Now plot J
                plt.plot(range(iterations), J, color=colors[i],
label=str(alphas[i]))

        plt.xlabel('Number of iterations')
        plt.ylabel('Cost J')
        plt.legend(loc='upper right')
        plt.show()

        # Find optimal theta vector with best learning rate
        # Assume we already know that alpha = 0.3 is the best learning rate
        model = LinearRegression(alpha=0.3, check_conv=True)
        theta, _ = model.fit(X_scaled, y)
        # print(theta)
```

```
        test_data = np.array([1650, 3])
        test_data_scaled = (test_data - mu) / sigma
        # print(test_data_scaled)
        test_data_scaled = np.append([1], test_data_scaled)
        # print(test_data_scaled)
        print('Predicted price using gradient descent: ',
model.predict(test_data_scaled))

        # test_data_scaled.shape = (3, 1)
        # print(theta.T.dot(test_data_scaled))

        X = np.column_stack((intercept, X))
        test_data = np.append([1], test_data)
        theta_normal_equation = normal_equation(X, y)
        print('Predicted price using normal equation: ',
theta_normal_equation.T.dot(test_data))

def normalize(X):
        # Preprocess data to give std of 1 and mean of 0
        sigma = np.std(X, axis=0) # std
        mu = np.mean(X, axis=0) # mean
        X = (X-mu) / sigma # adjustment
        return X

def normal_equation(X, y):
        theta = inv(X.T.dot(X)).dot(X.T).dot(y)
        return theta

# ----------------------- END PART B ------------------------- #

part_a()
part_b()
```

Part C

```
import numpy as np
import matplotlib.pyplot as plt
from math import log
from numpy.linalg import inv

def newton_method(X, y, theta, tolerance=1e-5, max_iters=15):
        MAX_ITERS = 1
        epoch = 1
        for _ in range(max_iters):
```

```python
                H = hessian(X, theta)
                # print(hes)
                g = gradient(X, y, theta)
                # print('Gradient shape: ', g.shape)
                temp = theta - np.dot(inv(H), g)
                if np.sum(abs(theta - temp)) < tolerance:
                        print('Convergered at epoch %d' % epoch)
                        break
                theta = temp
                epoch += 1
        return theta

def gradient(X, y, theta):
        m = X.shape[0]
        h = hypothesis(X, theta)
        g = (1.0/m) * X.T.dot(h-y)
        return g

def hessian(X, theta):
        m = X.shape[0]
        h = hypothesis(X, theta)
        h.shape = (len(h),)
        H = (1.0/m) * np.dot(np.dot(X.T, np.diag(h)), np.dot(np.diag(1-h), X))
        return H

def sigmoid(z):
        result = 1.0/(1.0+np.exp(-z))
        return result

def cost_function(X, y, theta):
        m = X.shape[0]
        h = hypothesis(X, theta)
        J = (1.0/m) * (-y.dot(np.log(h)) - (1-y).dot(np.log(1-h)))
        return J

def hypothesis(X, theta):
        # print('Hypothesis: ', h.shape)
        return sigmoid(X.dot(theta))

def main():
        X = np.loadtxt('data/cx.dat')
        y = np.loadtxt('data/cy.dat')

        # Get positive and negative indices
        pos = np.nonzero(y)
```

```
        neg = np.where(y==0)

        # Plot
        plt.scatter(X[pos,0], X[pos,1], color='b', marker='+')
        plt.scatter(X[neg,0], X[neg,1], color='r', marker='o')
        # plt.show()

        m = X.shape[0]
        X = np.column_stack((np.ones((m, 1)), X))
        y.shape = (y.shape[0], 1)
        theta = np.zeros((X.shape[1], 1))

        theta = newton_method(X, y, theta)
        print(theta)

        # predict if the student got 20 in exam 1 and 80 in exam 2
        test_data = np.array([1, 20, 80])
        p = 1 - hypothesis(test_data, theta)
        print('Probability that student is not admitted with a score of 20 on
Exam 1 and a score of 80 on Exam 2 is %f' % p)

        # Plot decision boundary
        min_X = np.min(X[:,1:3])
        max_X = np.max(X[:,1:3])
        plot_x = [min_X-2, max_X+2]
        plot_y = (-1/theta[2])*(theta[1]*plot_x) + theta[0]
        plt.plot(plot_x, plot_y)
        plt.show()

main()
```

Appendix

Part A: