EXAMPLE (Multi Period Investment)

Finco Investment Corporation must determine investment strategy for the firm during the next three years. Currently (time 0) $100,000 is available for investment. Investments A, B, C, D and E are available. The cash flow associated with investing $1 in each investment is given in the following Table.

CASH FLOW ($) AT TIME

	0	1	2	3
A	-1	+0.5	+1	0
B	0	-1	+0.5	+1
C	-1	+1.2	0	0
D	-1	0	0	+1.9
E	0	0	-1	+1.5

- **Finco requires that at most $75,000 be placed in any single investment.**
- **Finco can earn 8% per year by keeping uninvested cash in money market funds.**
- **Company's portfolio is diversified.**
- **Returns from investments may be immediately reinvested.**
- **Company cannot barrow funds, so the cash available for investment at any time is limited to cash on hand.**

Formulate an LP that will maximize cash on hand at time 3.

We define the following decision variables.

A= dollars invested in investment A,
B= dollars invested in investment B,
C= dollars invested in investment C,
D= dollars invested in investment D,
E= dollars invested in investment E,
S_t=dollars invested in money market funds at time t (t=0,1,2)

Time 3 cash on hand =$B+1.9D+1.5E+1.08S_2$

Objective Function : $B+1.9D+1.5E+1.08S_2$

Cash available at time t= Cash invested at time t

For t=0

$$100000= A + C + D + S_0$$

For t=1

$$0.5A + 1.2\,C + 1.08\,S_0 = B + S_1$$

For t=2

$$A + 0.5\,B + 1.08\,S_1 = E + S_2$$

Don't forget that at most $75000 can be placed in any of investments A-E.

$A\leq 75000$, $B\leq 75000$, $C\leq 75000$, $D\leq 75000$, $E\leq 75000$,

Combining all with the sign restrictions yields the following LP:

$$\text{Max } Z = B + 1.9D + 1.5E + 1.08S_2$$

s.t

$$A + C + D + S_0 = 100000$$

$$0.5A + 1.2C + 1.08S_0 = B + S_1$$

$$A + 0.5B + 1.08S_1 = E + S_2$$

$$A \leq 75000$$

$$B \leq 75000$$

$$C \leq 75000$$

$$D \leq 75000$$

$$E \leq 75000$$

$$A, B, C, D, E, S_0, S_1, S_2 \geq 0$$

Solution : **We find the optimal solution to be**

z=$218500

A=60000

B=30000

D=40000

E=75000

C= $S_0=S_1=S_2=0$

- **Thus, Finco should not invest in money market funds.**
- **Finco should invest $60000 in A and $40000 in D.**
- **Then, at time 1, the $30000 cash inflow from A should be invested in B.**
- **At time 2, the $60000 cash inflow from A and the $15000 cash inflow from B should be invested in E.**
- **At time 3, Finco's $100000 will have grown to $218500.**

EXAMPLE

Two investments with varying cash flows (in thousands of dollars) are available as shown in the Table.

CASH FLOW (IN THOUSANDS AT TIME)

INVESTMENT/TIME	0	1	2	3
INVESTMENT-1	-$6	-$5	$7	$9
INVESTMENT-2	-$8	-$3	$9	$7

- **At time 0, $10000 is available for investment, and**
- **At time 1, $7000 is available for investment.**

Assuming that r = 0.10 (We call r the annual interest rate), set up an LINEAR PPROGRAMMING whose solution maximizes the NET PRESENT VALUE (NPV) obtained from these investments.

- **Assume that any fraction of an investment may be purchased.**

NPV of investment 1

$$-6 - 5/1.1 + 7/(1.1)^2 + 9/(1.1)^3 = \$2.00$$

NPV of investment2

$$-8 - 3/1.1 + 9/(1.1)^2 + 7/(1.1)^3 = \$1.97$$

Let

x_1 = Fraction of investment 1 that is undertaken and

x_2 = Fraction of investment 2 that is undertaken.

If we measure NPV in thousands of dollars we wish to solve the following LP.

$$\text{Max } Z = 2x_1 + 1.97\,x_2$$

s.t

$$6x_1 + 8\,x_2 \le 10$$
$$5x_1 + 3\,x_2 \le 7$$
$$x_1 \le 1$$
$$x_2 \le 1$$
$$\text{All variables} \ge 0$$

Another Form (Canonical Form)

$$-Z = -2x_1 - 1.97x_2$$
$$Z - 2x_1 - 1.97x_2 = 0$$

$$6x_1 + 8x_2 + x_3 = 10$$
$$5x_1 + 3x_2 + x_4 = 7$$
$$x_1 + x_5 = 1$$
$$x_2 + x_6 = 1$$

Non Basic Variables: NBV(x_1, x_2)

Basic Variables BV(x_3, x_4, x_5, x_6) = 10,7,1,1

EXAMPLE OF A SIMPLEX TABLEAU

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_3	6	8	1	0	0	0	**10**	1.6667
x_4	5	3	0	1	0	0	**7**	1.4
x_5	**1<**	0	0	0	1	0	**1**	1
x_6	0	1	0	0	0	1	**1**	∞
Z	**-2<**	**-1.97**	**0**	**0**	**0**	**0**	**0**	
x_3	0	**8<**	1	0	-6	0	**4**	0.5
x_4	0	3	0	1	-5	0	**2**	0.6667
x_1	1	0	0	0	1	0	**1**	∞
x_6	0	1	0	0	0	1	**1**	1
Z	**0**	**-1.97<**	**0**	**0**	**2**	**0**	**2**	
x_2	0	1	0.125	0	-0.75	0	**0.5**	
x_4	0	0	-0.375	1	-2.75	0	**0.5**	
x_1	1	0	0	0	1	0	**1**	
x_6	0	0	-0.125	0	0.75	1	**0.5**	
Z	0	0	0.2463	0	0.5275	0	**2.985**	

SOLUTION

x_1=1 x_2=0.5 x_3=0.0 x_4=0.5 x_5=0.0 x_6=0.5
Z_{MAX}=2.985

NO ALTERNATIVE OPTIMA