

EXAMPLE (Multi Period Investment)

Finco Investment Corporation must determine investment strategy for the firm during the next three years. Currently (time 0) \$100,000 is available for investment. Investments A, B, C, D and E are available. The cash flow associated with investing \$1 in each investment is given in the following Table.

CASH FLOW (\$) AT TIME

| | 0 | 1 | 2 | 3 |
|---|----|------|------|------|
| A | -1 | +0.5 | +1 | 0 |
| B | 0 | -1 | +0.5 | +1 |
| C | -1 | +1.2 | 0 | 0 |
| D | -1 | 0 | 0 | +1.9 |
| E | 0 | 0 | -1 | +1.5 |

- Finco requires that at most \$75,000 be placed in any single investment.
- Finco can earn 8% per year by keeping uninvested cash in money market funds.
- Company's portfolio is diversified.
- Returns from investments may be immediately reinvested.
- Company cannot borrow funds, so the cash available for investment at any time is limited to cash on hand.

Formulate an LP that will maximize cash on hand at time 3.

We define the following decision variables.

A= dollars invested in investment A,

B= dollars invested in investment B,

C= dollars invested in investment C,

D= dollars invested in investment D,

E= dollars invested in investment E,

**S_t =dollars invested in money market funds at time t
($t=0,1,2$)**

Time 3 cash on hand = $B+1.9D+1.5E+1.08S_2$

Objective Function : $B+1.9D+1.5E+1.08S_2$

Cash available at time t= Cash invested at time t

For $t=0$

$$100000 = A + C + D + S_0$$

For $t=1$

$$0.5A + 1.2C + 1.08S_0 = B + S_1$$

For $t=2$

$$A + 0.5B + 1.08S_1 = E + S_2$$

Don't forget that at most \$75000 can be placed in any of investments A-E.

$$A \leq 75000, B \leq 75000, C \leq 75000, D \leq 75000, E \leq 75000,$$

Combining all with the sign restrictions yields the following LP:

$$\begin{aligned} \text{Max } Z &= B + 1.9D + 1.5E + 1.08S_2 \\ \text{s.t} \quad & A + C + D + S_0 = 100000 \\ & 0.5A + 1.2C + 1.08S_0 = B + S_1 \\ & A + 0.5B + 1.08S_1 = E + S_2 \\ & A \leq 75000 \\ & B \leq 75000 \\ & C \leq 75000 \\ & D \leq 75000 \\ & E \leq 75000 \\ & A, B, C, D, E, S_0, S_1, S_2 \geq 0 \end{aligned}$$

Solution : We find the optimal solution to be

$$z = \$218500$$

$$A = 60000$$

$$B = 30000$$

$$D = 40000$$

$$E = 75000$$

$$C = S_0 = S_1 = S_2 = 0$$

- Thus, Finco should not invest in money market funds.
- Finco should invest \$60000 in A and \$40000 in D.
- Then, at time 1, the \$30000 cash inflow from A should be invested in B.
- At time 2, the \$60000 cash inflow from A and the \$15000 cash inflow from B should be invested in E.
- At time 3, Finco's \$100000 will have grown to \$218500.

EXAMPLE

Two investments with varying cash flows (in thousands of dollars) are available as shown in the Table.

CASH FLOW (IN THOUSANDS AT TIME)

| INVESTMENT/TIME | 0 | 1 | 2 | 3 |
|-----------------|------|------|-----|-----|
| INVESTMENT-1 | -\$6 | -\$5 | \$7 | \$9 |
| INVESTMENT-2 | -\$8 | -\$3 | \$9 | \$7 |

- At time 0, \$10000 is available for investment, and
- At time 1, \$7000 is available for investment.

Assuming that $r = 0.10$ (We call r the annual interest rate), set up an **LINEAR PPROGRAMMING** whose solution maximizes the **NET PRESENT VALUE (NPV)** obtained from these investments.

- Assume that any fraction of an investment may be purchased.

NPV of investment 1

$$-6 - 5/1.1 + 7/(1.1)^2 + 9/(1.1)^3 = \$2.00$$

NPV of investment2

$$-8 - 3/1.1 + 9/(1.1)^2 + 7/(1.1)^3 = \$1.97$$

Let

x_1 = Fraction of investment 1 that is undertaken

and

x_2 = Fraction of investment 2 that is undertaken.

If we measure NPV in thousands of dollars we wish to solve the following LP.

$$\text{Max } Z = 2x_1 + 1.97 x_2$$

s.t

$$6x_1 + 8 x_2 \leq 10$$

$$5x_1 + 3 x_2 \leq 7$$

$$x_1 \leq 1$$

$$x_2 \leq 1$$

$$\text{All variables } \geq 0$$

Another Form (Canonical Form)

$$-Z = -2x_1 - 1.97x_2$$

$$Z - 2x_1 - 1.97x_2 = 0$$

$$6x_1 + 8x_2 + x_3 = 10$$

$$5x_1 + 3x_2 + x_4 = 7$$

$$x_1 + x_5 = 1$$

$$x_2 + x_6 = 1$$

Non Basic Variables: NBV(x_1, x_2)

Basic Variables $BV(x_3, x_4, x_5, x_6) = 10, 7, 1, 1$

EXAMPLE OF A SIMPLEX TABLEAU

| BASIS | x ₁ | x ₂ | x ₃ | x ₄ | x ₅ | x ₆ | RHS | RATIO |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|--------|
| x ₃ | 6 | 8 | 1 | 0 | 0 | 0 | 10 | 1.6667 |
| x ₄ | 5 | 3 | 0 | 1 | 0 | 0 | 7 | 1.4 |
| x ₅ | 1 < | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| x ₆ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | ∞ |
| Z | -2 < | -1.97 | 0 | 0 | 0 | 0 | 0 | |
| x ₃ | 0 | 8 < | 1 | 0 | -6 | 0 | 4 | 0.5 |
| x ₄ | 0 | 3 | 0 | 1 | -5 | 0 | 2 | 0.6667 |
| x ₁ | 1 | 0 | 0 | 0 | 1 | 0 | 1 | ∞ |
| x ₆ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| Z | 0 | -1.97 < | 0 | 0 | 2 | 0 | 2 | |
| x ₂ | 0 | 1 | 0.125 | 0 | -0.75 | 0 | 0.5 | |
| x ₄ | 0 | 0 | -0.375 | 1 | -2.75 | 0 | 0.5 | |
| x ₁ | 1 | 0 | 0 | 0 | 1 | 0 | 1 | |
| x ₆ | 0 | 0 | -0.125 | 0 | 0.75 | 1 | 0.5 | |
| Z | 0 | 0 | 0.2463 | 0 | 0.5275 | 0 | 2.985 | |

SOLUTION

$$x_1=1 \quad x_2=0.5 \quad x_3=0.0 \quad x_4=0.5 \quad x_5=0.0 \quad x_6=0.5$$

$$Z_{\text{MAX}}=2.985$$

NO ALTERNATIVE OPTIMA