DEGENERACY

A Linear program is said to be degenerate if one or more of the basic variables have a value of zero.

Degeneracy does not cause any particular difficulties for the graphical solution procedure; however degeneracy can theoretically cause difficulties when the simplex method is used to solve a linear programming problem.

Definition ***: An LP is degenerate if it has one bfs in which a basic variable is equal to zero.***

Example: DEGENERACY

Max $Z = 50x_1 + 40\ x_2$
s.t.

$$3\ x_1 + 5\ x_2 \leq 175$$
$$x_2 \leq 120$$
$$8\ x_1 + 5\ x_2 \leq 300$$

All variables ≥ 0

$Z - 50\ x_1 - 40\ x_2 = 0$

$3\ x_1 + 5\ x_2 + x_3 = 175$
$x_2 + x_4 = 20$
$8\ x_1 + 5\ x_2 + x_5 = 300$

Initial Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_3	3	5	1	0	0	**175**	58.33
x_4	0	1	0	1	0	**20**	-
x_5	**8**<	5	0	0	1	**300**	37.5
Z	**-50**<	**-40**	**0**	**0**	**0**	**0**	

Entering variable: x_1

Leaving variable: x_5

First tableau

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_3	0	25/8<	1	0	-3/8	125/2	20
x_4	0	1	0	1	0	20	20
x_1	1	5/8	0	0	1/8	75/2	60
Z	0	-70/8<	0	0	50/8	1875	

Entering variable: x_2

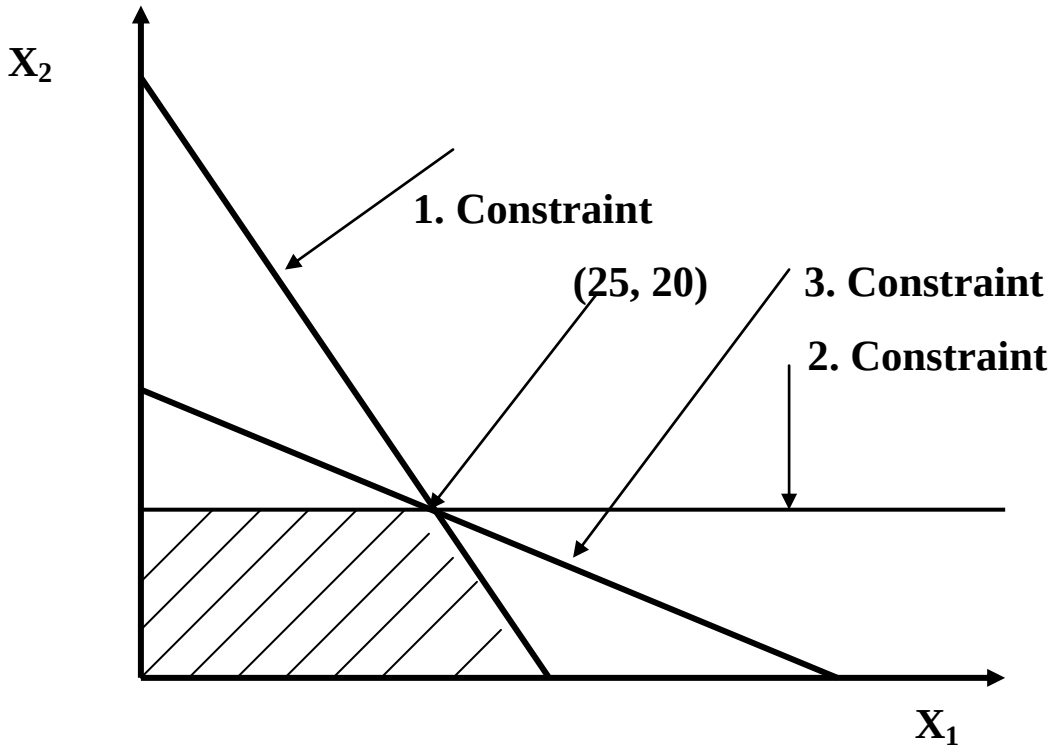
Leaving variable: x_3

Second tableau

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_2	0	1	8/25	0	-3/25	20	
x_4	0	0	-8/25	1	3/25	0	<<degeneracy
x_1	1	0	-5/25	0	5/25	25	
Z	0	0	70/25	0	130/25	2050	

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_3	3	5	1	0	0	**175**	58.33
x_4	0	1	0	1	0	**20**	-
x_5	**8**<	5	0	0	1	**300**	37.5
Z	**-50**<	**-40**	**0**	**0**	**0**	**0**	
x_3	0	**25/8**<	1	0	-3/8	**125/2**	20
x_4	0	1	0	1	0	**20**	20
x_1	1	5/8	0	0	1/8	**75/2**	60
Z	**0**	**-70/8**<	**0**	**0**	**50/8**	**1875**	
x_2	0	1	8/25	0	-3/25	**20**	
x_4	0	0	-8/25	1	3/25	**0**	<<degeneracy
x_1	1	0	-5/25	0	5/25	**25**	
Z	0	0	70/25	0	130/25	**2050**	

x_1=25, x_2= 20 x_4=0 Z_{max}=2050



Example: DEGENERACY

Max $Z = 5x_1 + 2x_2$

s.t.

$$x_1 + x_2 \leq 6$$
$$x_1 - x_2 \leq 0$$

All variables ≥ 0

$$\mathbf{Z - 5x_1 - 2x_2 = 0}$$

$$\mathbf{x_1 + x_2 + x_3 = 6}$$
$$\mathbf{x_1 - x_2 + x_4 = 0}$$

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	**1**	1	1	0	**6**	6
x_4	**1**<	-1	0	1	**0**	0<
Z	**-5**<	**-2**	**0**	**0**	**0**	
x_3	0	**2**	1	-1	**6**	3
x_1	1	-1	0	1	**0**	-
Z	**0**	**-7**<	**0**	**5**	**0**	
x_2	0	1	0.5	-0.5	**3**	
x_1	1	0	0.5	0.5	**3**	
Z	0	0	3.5	1.5	**21**	