# Continuous Uniform Distribution

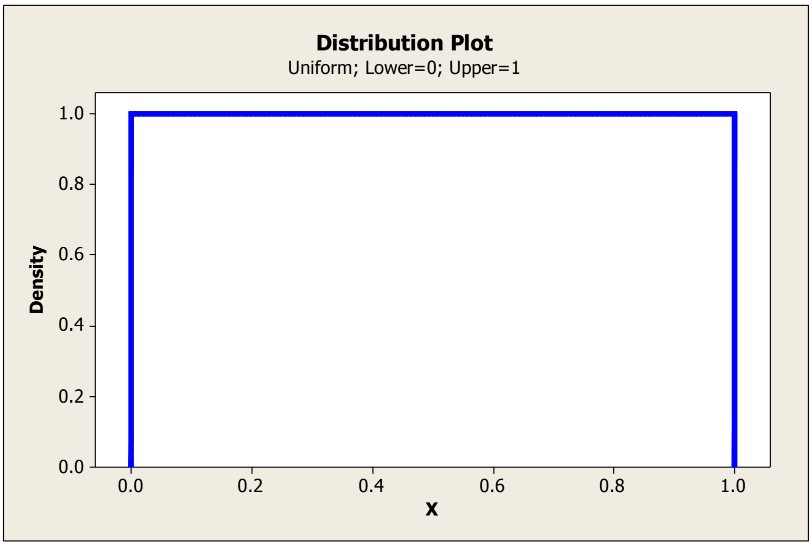
**The simplest continuous distribution is analogous to its discrete counterpart.**

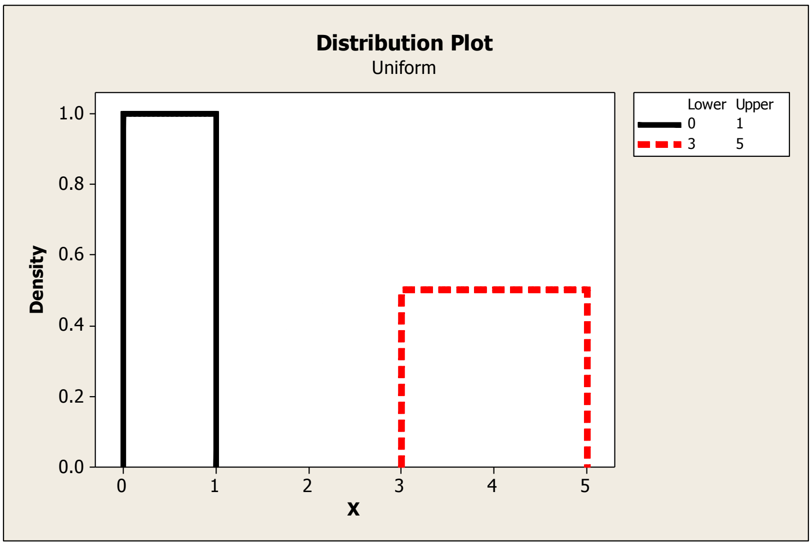
## Definition:

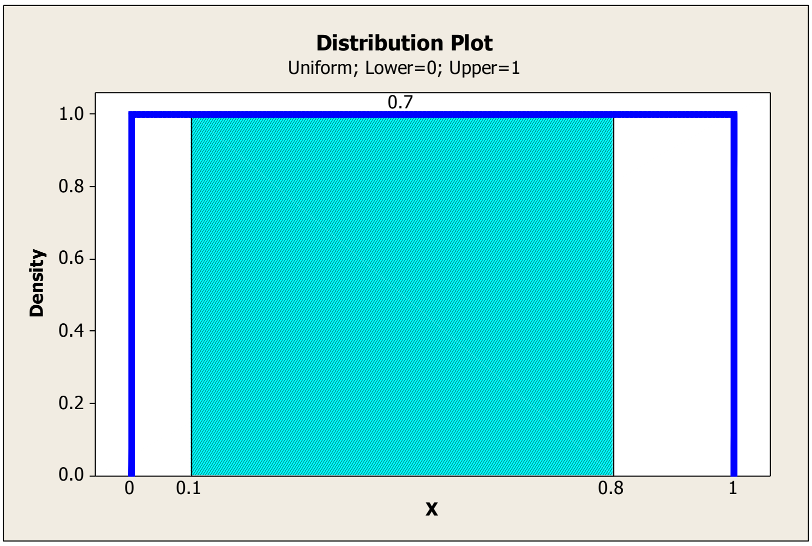
***A continuous random variable X with probability density function***

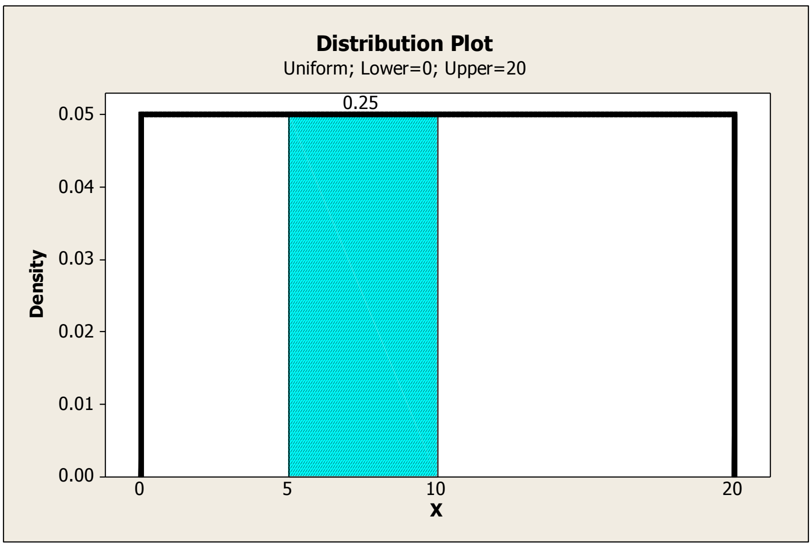
$$f(x) = 1/(b-a), \qquad a \le x \le b$$

***is a continuous uniform random variable.***




Distribution Plot
Uniform
Lower Upper
0 1
3 5
Density
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
X


Distribution Plot
Uniform; Lower=0; Upper=1
0.7
Density
1.0
0.8
0.6
0.4
0.2
0.0
0
0.1
0.8
1
X


Distribution Plot
Uniform; Lower=0; Upper=20
0.25
Density
0.05
0.04
0.03
0.02
0.01
0.00
0
5
10
20
X

# The Mean and Variance of Continuous Uniform Distribution

***The mean of the continuous uniform random variable X is***

$$E(X) = \int_a^b \frac{x}{b-a} dx = \left. \frac{0.5x^2}{b-a} \right|_a^b = \frac{a+b}{2}$$

***The variance of X is***

$$V(X) = \int_a^b \frac{\left(x - \left(\frac{a+b}{2}\right)\right)^2}{b-a} dx = \left. \frac{\left(x - \frac{a+b}{2}\right)^3}{3(b-a)} \right|_a^b = \frac{(b-a)^2}{12}$$

***These results are summarized as follows.***

***If X is a continuous uniform random variable over*** $a \le x \le b$,

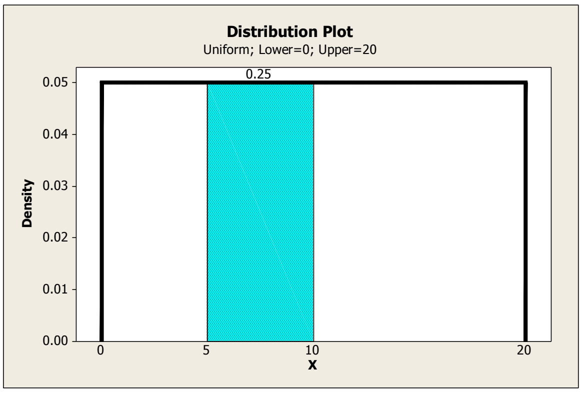
$$\mu = E(X) = \frac{(a+b)}{2}$$

***and***

$$\sigma^2 = V(X) = \frac{(b-a)^2}{12}$$

**Example:** Let the continuous random variable X denotes the current measured in a thin copper wire in milliamperes. Assume that the range of X is [0,20mA], and assume that the probability density function of X is

$f(x) = 0.05,\ 0 \le x \le 20$.



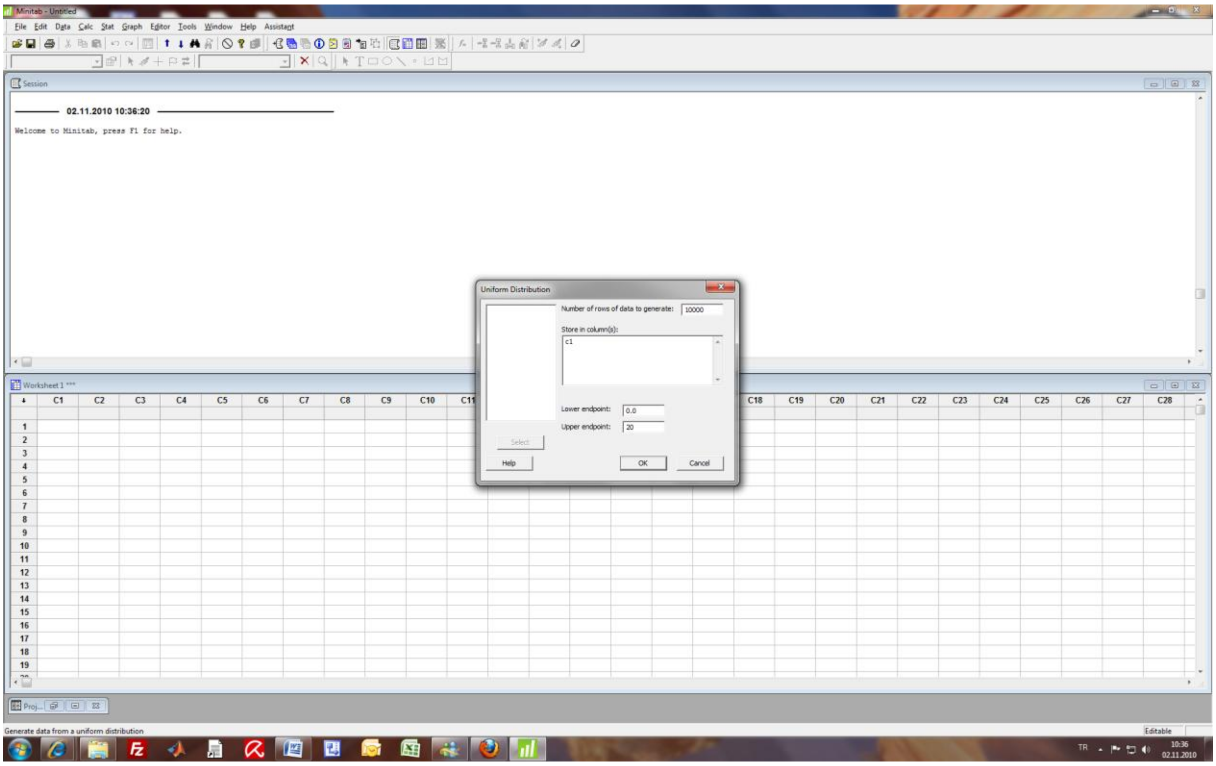

**i. What is the probability that a measurement of current is between 5 and 10 milliamperes?**

$$P(5 < X < 10) = \int_{5}^{10} f(x)dx = 5(0.05) = 0.25$$

**The mean and variance formulas can be applied with a=0 and b=20. Therefore**

$$\mu = E(X) = \frac{(a+b)}{2} = 10mA$$

$$\sigma^2 = V(X) = \frac{(b-a)^2}{12} = 20^2/12 = 33.33mA^2$$

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MTB > Describe C1;
SUBC>   Mean;
SUBC>   Variance.
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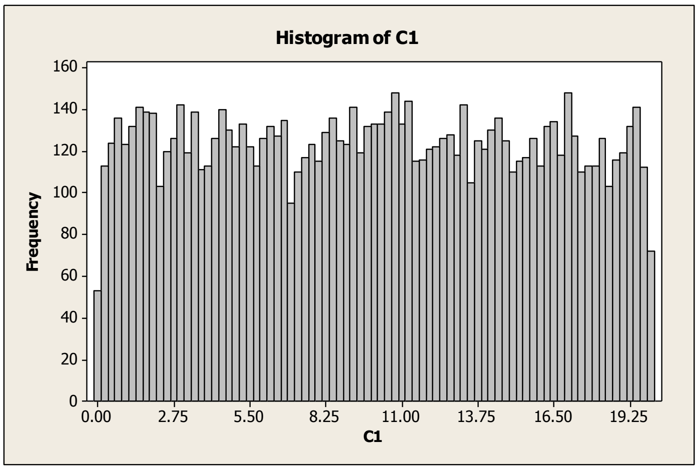
**Descriptive Statistics: C1**

| Variable | Mean | Variance |
|---|---|---|
| C1 | 9.9517 | 33.1478 |



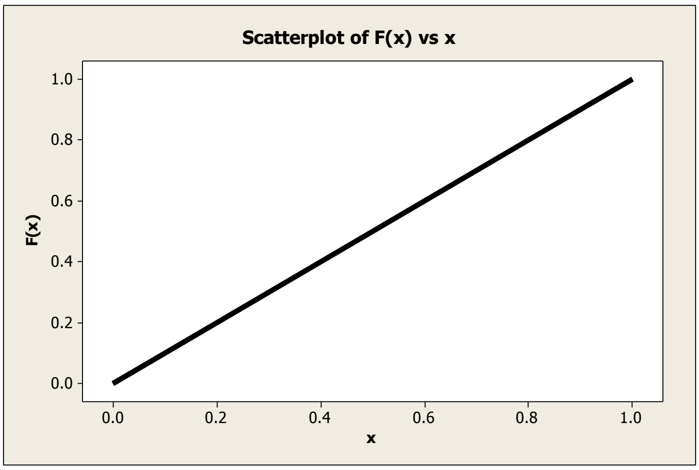
## The cumulative uniform distribution function

**The cumulative distribution function of a continuous uniform random variable is obtained by integrating. If a<X<b.**

$$F(x)=\int_a^x 1/(b-a)du = x/(b-a)-a/(b-a)$$

**Therefore, the complete description of the cumulative distribution function of a continuous uniform random variable is**

$$F(x)=\begin{cases}0 & x<a \\ (x-a)/(b-a) & a\le x<b \\ 1 & b=x\end{cases}$$




**Cumulative Distribution Function of Uniform (0, 1)**

**If U~(0,1) Uniform**

$$\frac{(x-a)}{(b-a)} = u$$

**then**

$$(x-a) = u(b-a)$$

$$x = a + u(b-a)$$

**Try this in MINITAB.**