

THE CAPACITATED TRANSPORTATION PROBLEM

A variation of the basic transportation problem in which there are capacities on some or all of the arcs.

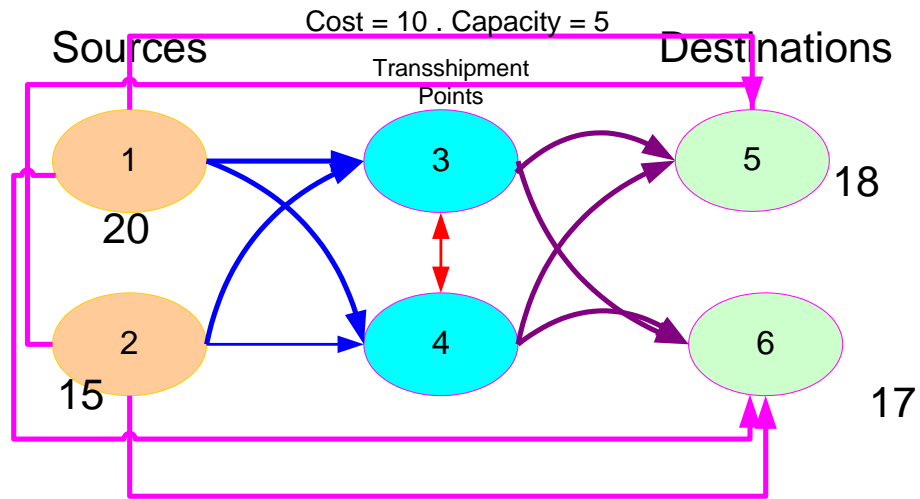
$$\begin{aligned} \min \quad & \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n x_{ij} = s_i \quad (i = 1, 2, \dots, m) \\ & \sum_{i=1}^m x_{ij} = d_j \quad (j = 1, 2, \dots, n) \\ & 0 \leq x_{ij} \leq U_{ij} \quad (i = 1, \dots, m; j = 1, \dots, n) \end{aligned}$$

THE CAPACITATED TRANSSHIPMENT PROBLEM

A variation of the basic transshipment problem in which there are capacities on some or all of the arcs.

We know sometimes shipments take place by first transporting goods one or more of several **transshipment** nodes before reaching their final destination. Such problems are known as **transshipment problems**. If, in addition, an upper limit is placed on the amount of flow along one or more arcs in the network, the problem is called a **capacitated transshipment** or a **general network model** (The Out of Kilter Algorithm).

Example:



From/To	Node-3	Node-4	Node-5	Node-6	Supply
Node-1	4(20)	2(10)	10(5)	3(10)	20
Node-2	3(10)	5(12)	6(10)	4(12)	15
Node-3	0(0)	7(14)	5(12)	8(7)	35
Node-4	4(9)	0(0)	4(10)	7(8)	35
Demand	35	35	18	17	

COST

CAPACITY

Capacitated transshipment or a general network model

Linear Programming Model

$$\begin{aligned}\text{Min } Z = & 4x_{13} + 2x_{14} + 10x_{15} + 3x_{16} + 3x_{23} + 5x_{24} + 6x_{25} + 4x_{26} \\ & + 7x_{34} + 5x_{35} + 8x_{36} + 4x_{43} + 4x_{45} + 7x_{46}\end{aligned}$$

Transshipment Constraints

$$\begin{aligned}\text{node-1:} & x_{13} + x_{14} + x_{15} + x_{16} = 20 \\ \text{node-2:} & x_{23} + x_{24} + x_{25} + x_{26} = 15 \\ \text{node-3:} & x_{13} + x_{23} + x_{43} - x_{34} - x_{35} - x_{36} = 0 \\ \text{node-4:} & x_{14} + x_{24} + x_{34} - x_{43} - x_{45} - x_{46} = 0 \\ \text{node-5:} & x_{15} + x_{25} + x_{35} + x_{45} = 18 \\ \text{node-6:} & x_{16} + x_{26} + x_{36} + x_{46} = 17\end{aligned}$$

Capacity Constraints

$$\begin{aligned}x_{13} & \leq 20 \\ x_{14} & \leq 10 \\ x_{15} & \leq 5 \\ x_{16} & \leq 10 \\ x_{23} & \leq 10 \\ x_{24} & \leq 12 \\ x_{25} & \leq 10 \\ x_{26} & \leq 12 \\ x_{34} & \leq 14 \\ x_{35} & \leq 12 \\ x_{36} & \leq 7 \\ x_{43} & \leq 9 \\ x_{45} & \leq 10 \\ x_{46} & \leq 8\end{aligned}$$

$$x_{ij} \geq 0, \text{ all } x_{ij} \text{ integer}$$