Sensitivity Analysis For Transportation Problem

Changing the Objective Function Coefficient of a Nonbasic Variable

From/To	Col-1	Col-2	Col-3	Col-4	Supply &v_j
Row-1	8 **2**	**10** 6	**25** 10	9 **7**	**35** (0)
Row-2	**45** 9	12 **3**	**5** 13	7 **2**	**50** (3)
Row-3	14 **5**	**10** 9	16 **3**	**30** 5	**40** (3)
Demand &u_i	**45** (6)	**20** (6)	**30** (10)	**30** (2)	

Is this optimal solution unique?

All $c_{ij}' > 0$ Z_{min} = 1020 (Unique solution)

Question : For what values of the cost of shipping 1 million kwh of electricity from plant-1(row-1) to city-1(col-1) will the current basis remain optimal?

Suppose we change c_{11} from 8 to 8 + Δ.

For what values of Δ will the current basis remain optimal?

$$c'_{11} = c_{11} - u_1 - v_1 = (8 + \Delta) - 0 - 6 = 2 + \Delta$$

$$2 + \Delta \geq 0 \qquad \Delta \geq -2$$

Thus the current basis remains optimal for Δ≥ -2 and

$$c_{11} \geq 8 + (-2) = 6$$

$$c_{11} \geq 6$$

Changing the Objective Function Coefficient of a Basic Variable

Suppose we change c_{13} from 10 to 10 +Δ.

Then the equation $c'_{13} = 0$ changes from $u_1+v_3=10$ to $u_1+v_3=10+\Delta$.

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8	**10**	6	**25**	10+Δ		9	**35** (0)
Row-2	45	9		12	**5**	13		7	**50** (3-Δ)
Row-3		14	**10**	9		16	**30**	5	**40** (3)
Demand &u_i	45 (6+Δ)		**20** (6)		**30** (10+Δ)		**30** (2)		

We now price out each non-basic variable.

The current basis will remain optimal as long as each nonbasic variable has a positive (nonnegative) coefficient.

$$c'_{11} = c_{11} - u_1 - v_1 = 8 - 0 - (6 + \Delta) = 2 - \Delta \geq 0 \quad \Delta \leq 2$$
$$c'_{14} = c_{14} - u_1 - v_4 = 9 - 0 - 2 = 7$$
$$c'_{22} = c_{22} - u_2 - v_2 = 3 + \Delta \geq 0 \quad \Delta \geq -3$$
$$c'_{24} = c_{24} - u_2 - v_4 = 2 + \Delta \geq 0 \quad \Delta \geq -2$$
$$c'_{31} = c_{31} - u_3 - v_1 = 5 - \Delta \geq 0 \quad \Delta \leq 5$$
$$c'_{33} = c_{33} - u_3 - v_3 = 3 - \Delta \geq 0 \quad \Delta \leq 3$$

$$-2 \leq \Delta \leq 2$$

Thus the current basis remain optimal for

$$\mathbf{-2 \leq \Delta \leq 2,}$$

or

$$\mathbf{10\text{-}2 \leq c_{13} \leq 10 + 2;}$$

$$\mathbf{8 \leq c_{13} \leq 12}$$

Increasing both supply s_i and demand d_j by Δ

Suppose we increase s_1 and d_2 by 2. Since x_{12} is a basic solution, the new optimal solution will be the one in the following table.

New Z value =Old Z value + Δu_i +Δ v_j
= 1020+ 2 u_1 + 2 v_2
=1020 + 0 + 12 = 1032

New Solution

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**12**	6	**25**	10		9 7	**37** (0)
Row-2	**45**	9		12 3	**5**	13		7 2	50 (3)
Row-3		14 5	**10**	9		16 3	**30**	5	40 (3)
Demand &u_i	45 (6)		**22** (6)		30 (10)		30 (2)		

If x_{ij} is a nonbasic variable.

Suppose we increase both s_1 and d_1 by1.

x_{11} is a non-basic variable. Find the loop involving x_{ij} (nonbasic) and some of the basic variables. Find an odd cell in the loop that is in row i. Increase the value of this odd cell by Δ and go around the loop, alternately increasing and then decreasing current basic variables in the loop by Δ.

The odd cell in the loop and row 1 is x_{13}. Thus the new optimal solution will be obtained by increasing both x_{13} and x_{12} by 1 and decreasing x_{23} by 1.

OLD SOLUTION

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_i
Row-1	*	8 2	**10**	6	**25**	10		9 7	**35** (0)
Row-2	**45**	9		12 3	**5**	13		7 2	**50** (3)
Row-3		14 5	**10**	9		16 3	**30**	5	**40** (3)
Demand &u_i	**45** (6)		**20** (6)		**30** (10)		**30** (2)		

LOOP INVOLVING NONBASIC

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_i
Row-1		8 2	**10**	6	**25**	10		9 7	**35** (0)
Row-2	**45**	9		12 3	**5**	13		7 2	**50** (3)
Row-3		14 5	**10**	9		16 3	**30**	5	**40** (3)
Demand &u_i	**45** (6)		**20** (6)		**30** (10)		**30** (2)		

NEW SOLUTION

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_i
Row-1		8 2	**10**	6	**26**	10		9 7	**36** (0)
Row-2	**46**	9		12 3	**4**	13		7 2	**50** (3)
Row-3		14 5	**10**	9		16 3	**30**	5	**40** (3)
Demand &u_i	**46** (6)		**20** (6)		**30** (10)		**30** (2)		

Another Example

$s_2 \rightarrow 56$ $d_2 \rightarrow 22$ $d_3 \rightarrow 32$ $d_4 \rightarrow 32$

OLD

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**10**	6	**25**	10		9 7	**35** (0)
Row-2	**45**	9		12 3	**5**	13		7 2	**50** (3)
Row-3		14 5	**10**	9		16 3	**30**	5	**40** (3)
Demand &u_i	**45** (6)		**20** (6)		**30** (10)		**30** (2)		

Modified Table

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**12**	6	**23**	10		9 7	**35** (0)
Row-2	**45**	9		12 3	**7**	13		7 2	**52** (3)
Row-3		14 5	**10**	9		16 3	**30**	5	**40** (3)
Demand &u_i	**45** (6)		**22** (6)		**30** (10)		**30** (2)		

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**12**	6	**23**	10		9 7	35 (0)
Row-2	**45**	9		12 3	**9**	13		7 2	54 (3)
Row-3		14 5	**10**	9		16 3	**30**	5	40 (3)
Demand &u_i	45 (6)		22 (6)		32 (10)		30 (2)		

Modified Table

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**12**	6	**23**	10		9 7	35 (0)
Row-2	**45**	9		12 3	**9**	13		7 2	56 (3)
Row-3		14 5	**10**	9		16 3	**30**	5	40 (3)
Demand &u_i	45 (6)		22 (6)		32 (10)		32 (2)		

Modified Table <> New Solution

From/To	Col-1		Col-2		Col-3		Col-4		Supply &v_j
Row-1		8 2	**14**	6	**21**	10		9 7	35 (0)
Row-2	**45**	9		12 3	**11**	13		7 2	56 (3)
Row-3		14 5	**8**	9		16 3	**32**	5	40 (3)
Demand &u_i	45 (6)		22 (6)		32 (10)		32 (2)		

Z_{new}=1064+ 2(2) +2(3) =1074