

Example

$$\text{Max } Z = 100 x_1 + 80 x_2$$

s.t

$$6 x_1 + 3 x_2 \leq 120$$

$$8 x_1 + 12 x_2 \leq 240$$

$$10 x_1 \leq 170$$

$$12 x_2 \leq 180$$

$$x_1, x_2 \geq 0$$

$$Z - 100 x_1 - 80 x_2 = 0$$

$$6 x_1 + 3 x_2 + x_3 = 120$$

$$8 x_1 + 12 x_2 + x_4 = 240$$

$$10 x_1 + x_5 = 170$$

$$12 x_2 + x_6 = 180$$

$$\text{NBV} = (x_1, x_2) \quad \text{BV} = (x_3, x_4, x_5, x_6) = 120, 240, 170, 180$$

Initial Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_3	6	3	1	0	0	0	120	20
x_4	8	12	0	1	0	0	240	30
x_5	10<<	0	0	0	1	0	170	17<
x_6	0	12	0	0	0	1	180	
Z	-100<	-80	0	0	0	0	0	

Entering variable: x_1

Leaving variable: x_5

The Initial and First Improved Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_3	6	3	1	0	0	0	120	20
x_4	8	12	0	1	0	0	240	30
x_5	10<<	0	0	0	1	0	170	17<
x_6	0	12	0	0	0	1	180	
Z	-100<	-80	0	0	0	0	0	
x_3	0	3<<	1	0	-0.6	0	18	6<
x_4	0	12	0	1	-0.8	0	104	8.67
x_1	1	0	0	0	0.1	0	17	-
x_6	0	12	0	0	0	1	180	15
Z	0	-80<	0	0	10	0	1700	

Entering variable: x_2

Leaving variable: x_3

The Second Improved Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_3	6	3	1	0	0	0	120	20
x_4	8	12	0	1	0	0	240	30
x_5	10<<	0	0	0	1	0	170	17<
x_6	0	12	0	0	0	1	180	
Z	-100<	-80	0	0	0	0	0	
x_3	0	3<<	1	0	-0.6	0	18	6<
x_4	0	12	0	1	-0.8	0	104	8.67
x_1	1	0	0	0	0.1	0	17	-
x_6	0	12	0	0	0	1	180	15
Z	0	-80<	0	0	10	0	1700	
x_2	0	1	0.33	0	-0.2	0	6	
x_4	0	0	-4	1	1.6	0	32	20<<
x_1	1	0	0	0	0.1	0	17	170
x_6	0	0	-4	0	2.4	1	108	45
Z	0	0	26.67	0	-6	0	2180	

Entering variable: x_5

Leaving variable: x_4

The Third Improved Simplex Tableau (Optimal Tableau)

BASIS	x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	RHS	RATIO
x ₃	6	3	1	0	0	0	120	20
x ₄	8	12	0	1	0	0	240	30
x ₅	10<<	0	0	0	1	0	170	17<
x ₆	0	12	0	0	0	1	180	
Z	-100<	-80	0	0	0	0	0	
x ₃	0	3<<	1	0	-0.6	0	18	6<
x ₄	0	12	0	1	-0.8	0	104	8.67
x ₁	1	0	0	0	0.1	0	17	-
x ₆	0	12	0	0	0	1	180	15
Z	0	-80<	0	0	10	0	1700	
x ₂	0	1	0.33	0	-0.2	0	6	
x ₄	0	0	-4	1	1.6	0	32	20<<
x ₁	1	0	0	0	0.1	0	17	170
x ₆	0	0	-4	0	2.4	1	108	45
Z	0	0	26.67	0	-6	0	2180	
x ₂	0	1	-0.17	0.13	0	0	10	
x ₅	0	0	-2.5	0.63	1	0	20	
x ₁	1	0	0.25	-0.06	0	0	15	
x ₆	0	0	2	-1.5	0	1	60	
Z	0	0	11.67	2.75	0	0	2300	

Solution:

x₁=15, x₂= 10 ,x₃=x₄ =0, x₅=20, x₆=60, z_{max}= 2300

Using the Simplex Algorithm to solve minimization problems

Example

$$\text{Min } Z = 2x_1 - 3x_2$$

s.t

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 6$$

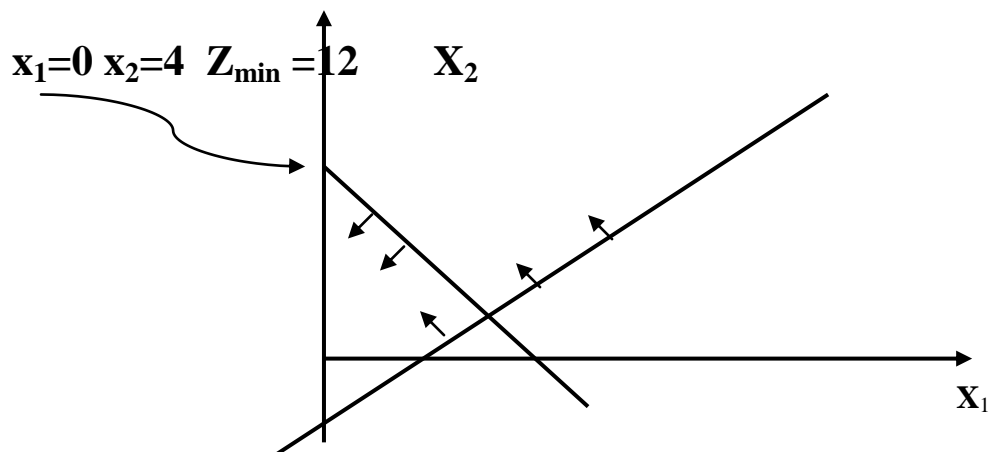
$$x_1, x_2 \geq 0$$

$$-Z + 2x_1 - 3x_2 = 0$$

$$x_1 + x_2 + x_3 = 4$$

$$x_1 - x_2 + x_4 = 6$$

$$\text{BV}=(x_3, x_4) = 4, 6 \quad \text{NBV}=(x_1, x_2)$$



Initial Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	1	$1 \ll$	1	0	4	4
x_4	1	-1	0	1	6	
-Z	2	-3<	0	0	0	

Entering variable: x_2

Leaving variable: x_3

The First Improved Simplex Tableau (Optimal)

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_2	1	1	1	0	4	
x_4	2	0	1	1	10	
-Z	5	0	3	0	12	

The Initial and First Simplex Tableau (Optimal)

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	1	$1 \ll$	1	0	4	4
x_4	1	-1	0	1	6	
Z	2	-3<	0	0	0	
x_2	1	1	1	0	4	
x_4	2	0	1	1	10	
-Z	5	0	3	0	12	

$$x_1=0 \quad x_2=4 \quad Z_{\min} = -12$$