Analysis of Algorithms

Chaper 1.1, 1.2, 1.3

Syllabus

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Syllabus

Web Page

http://efe.ege.edu.tr/~bulut/courses/451/451.htm

Textbook

- Anany Levitin. Introduction to The Design and Analysis of Algorithms, Addison Wesley, 3rd edition.

Other Books

- Introduction to Algorithms, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, The MIT Press; 3rd edition, 2009 ISBN-10: 0262033844 ISBN-13: 978-0262033848

Prerequisites:

- BİL 107 & BİL 112 Algorithms and Programming I & II
- BİL 111 Discrete Mathematics
- BİL 204 Data Structures

Syllabus-Catalog Description:

- Basic definitions and data structures.
- Introduction to analysis of algorithms.
- Standard algorithm design techniques;
  - divide-and-conquer,
  - greedy,
  - dynamic programming,
  - branch-and-bound,
  - backtracking,
  - etc.
- Basic algorithms;
  - sorting and searching,
  - graph algorithms,
  - etc.
- Introduction to complexity classes.

Syllabus - Educational Objectives:

This course introduces basic algorithms, algorithm design and analysis techniques which can be used in designing solutions to real life problems. After this course, you will

- able to design a new algorithms for a problem using the methods discussed in the class
- able to analyze an algorithm with respect to various performance criteria such as memory use and running time
- able to choose the most suitable algorithm for a problem to be solved,
- able to implement an algorithm efficiently.

Syllabus - Attendance Policy:

- Class attendance is advised but will not be a part of your final grade. However, a minimum of 70% attendance is required. DO NOT SIGN FOR OTHER STUDENTS.
- Please be considerate of your classmates during class. Students are expected to show courtesy and respect toward their classmates.
- Please do not carry on side discussions with other students during lecture time – when you have a question, please raise your hand and ask the question so that everyone may benefit from it.
- Also, please try to make sure that your cellular phone and/or pager does not interrupt during lecture time, and especially during exams.

Syllabus - Assessment:

Term Learning Activities	Count	Weight %	Contribution to Assesment %
Midterm	1	50	**30**
Quiz	3	30	**18**
Project	1	20	**12**
TOTAL		100	**60**
Contribution of Term Learning Activities to Success Grade		60	
Contribution of Final Exam to Success Grade		40	**40**
	TOTAL	**100**	**100**

ROAD MAP

- **Introduction**
  - Definition and Properties of Algorithm
  - Fundamentals of Algorithmic Problem Solving
  - Important Problem Types
- Mathematical Background

Introduction

Algorithms are “methods for solving problems which are suited for computed implementation..” [Sedgewick]

An algorithm is “a finite sequence of instructions, each of which has a clear meaning and can be performed with a finite amount of effort in a finite length of time.” [Aho, Hopcroft, & Ulman]

Introduction

"*Algorithmics* [defined as the study of algorithms -- A.L.] is more than a branch of computer science. <u>It is the core of computer science</u>, and, in all fairness, can be said to be relevant to most of science, business, and technology." [David Harel, "Algorithmics: The Spirit of Computing"]

What is an Algorithm ?

An *algorithm* is a finite, clearly specified sequence of instructions to be followed to solve a problem or compute a function

An *algorithm* generally

- takes some input
- carries out a number of effective instructions in a finite amount of time
- produces some output.

An effective instruction is an operation so basic that it is possible to carry it out using pen and paper.

Two main issues related to algorithms

- How to design algorithms

- How to analyze algorithm efficiency

Expressing Algorithms

Algorithms can be expressed in

- natural languages
  - verbose and ambiguous
  - rarely used for complex or technical algorithms
- pseudocode, flowcharts
  - structured ways to express algorithms
  - avoid ambiguities in natural language statements
  - independent of a particular implementation language
- programming languages
  - intended for expressing algorithms in a form that can be executed by a computer
  - can be used to document algorithms

Example:

Problem: Find the largest number in an (unsorted) list of numbers.

Idea: Look at every number in the list, one at a time.

Natural Language:

- Assume the first item is largest.
- Look at each of the remaining items in the list and if it is larger than the largest item so far, make a note of it.
- The last noted item is the largest in the list when the process is complete.

Example:

Pseudocode:

Algorithm LargestNumber

Input: A non-empty list of numbers L.

Output: The *largest* number in the list L.

largest ← L_0

for each *item* **in** the list $L_{i \geq 1}$, **do**

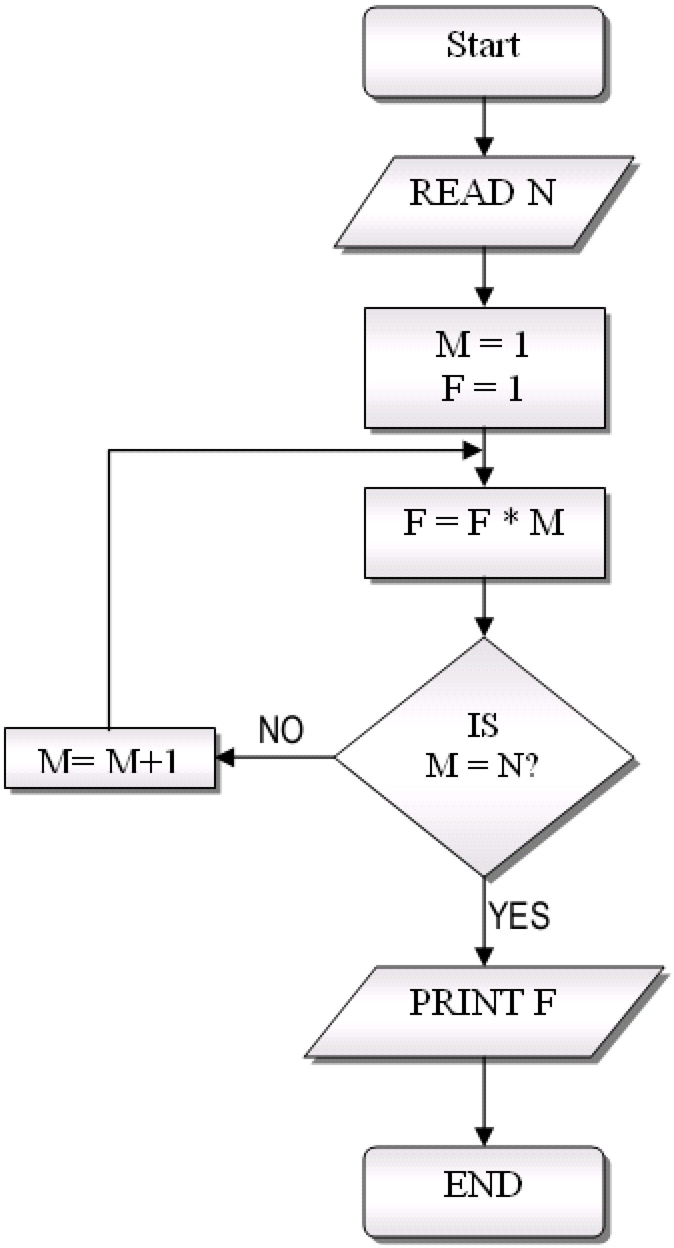
    if the *item* > *largest*, **then**

        largest ← the *item*

return *largest*

Example:

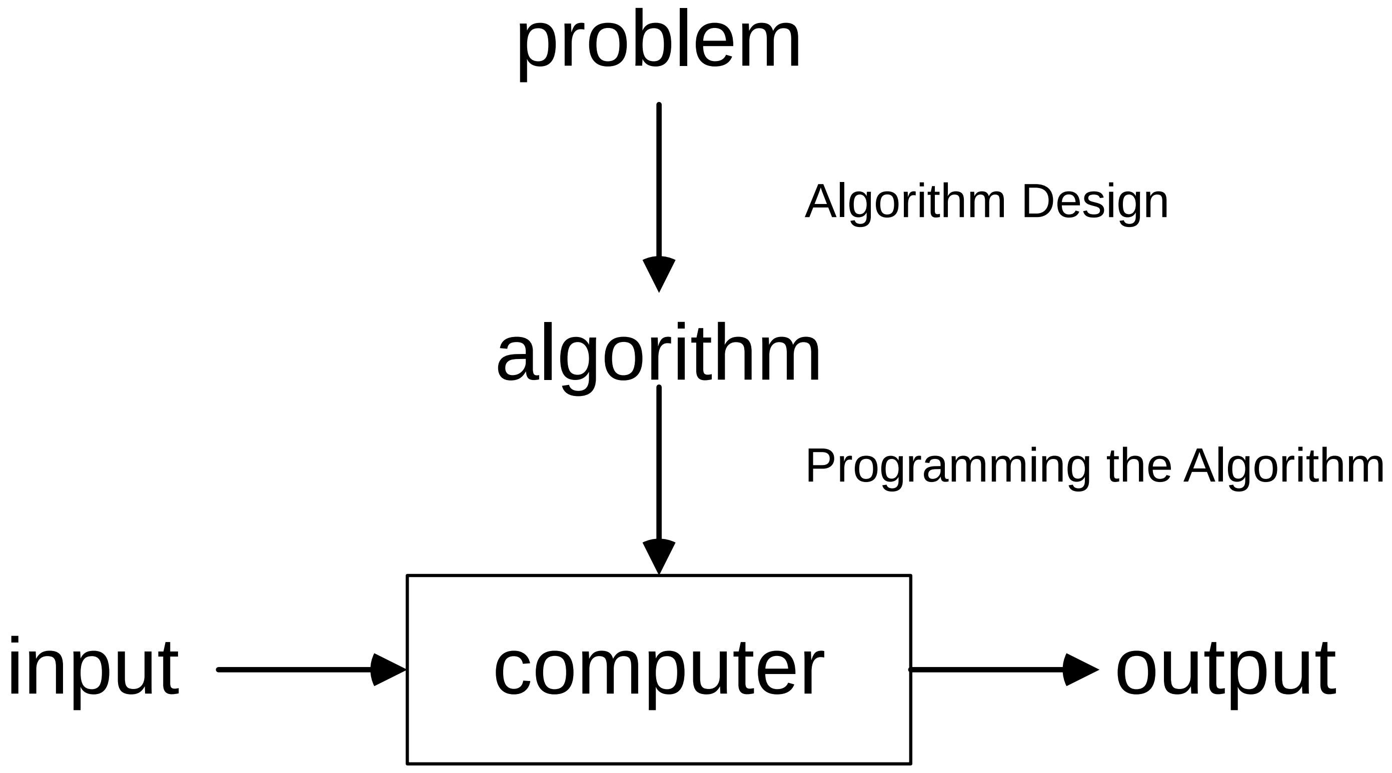
Flowchart:



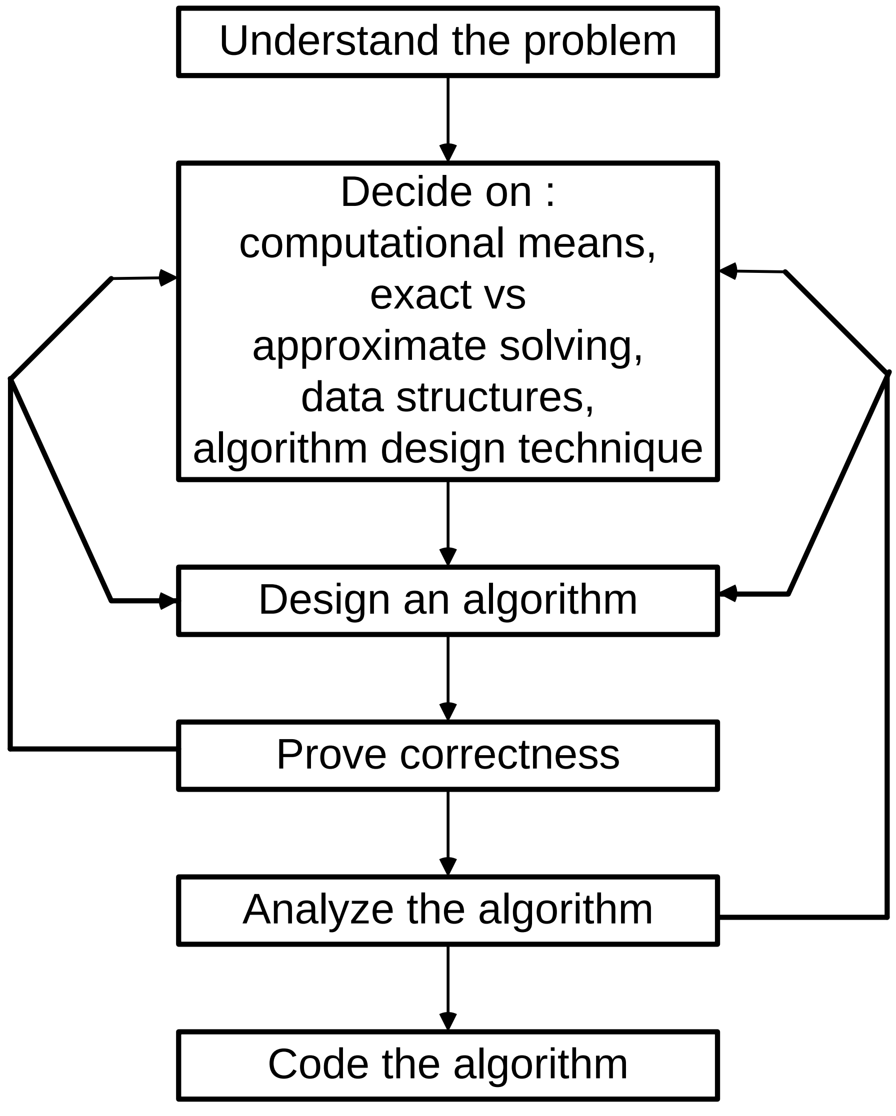
Properties of an Algorithm

- **Effectiveness**
  - Instructions are simple
    - can be carried out by pen and paper
- **Definiteness**
  - Instructions are clear
    - meaning is unique
- **Correctness**
  - Algorithm gives the right answer
    - for all possible cases
- **Finiteness**
  - Algorithm stops in reasonable time
    - produces an output

Notion of an Algorithm



Algorithm Design Process



Deciding on Appropriate Data Structures

Algorithms + Data Structures = Programs

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Euclid's Algorithm

- **Problem:** Find gcd(m,n), the greatest common divisor of two nonnegative, not both zero integers m and n

- Examples: gcd(60,24) = 12, gcd(60,0) = 60, gcd(0,0) = ?

- Euclid's algorithm is based on repeated application of equality

  gcd(m,n) = gcd(n, m mod n)

- until the second number becomes 0, which makes the problem trivial.

  Example: gcd(60,24) = gcd(24,12) = gcd(12,0) = 12

Structured Description of Euclid's Algorithm

- **Step 1** If $n = 0$, return m and stop; otherwise go to Step 2
- **Step 2** Divide m by n and assign the value to the remainder to r
- **Step 3** Assign the value of n to m and the value of r to n. Go to Step 1.

Euclid's Algorithm (Pseudocode)

ALGORITHM *Euclid*(m, n)

//Computes gcd(m, n) by Euclid's algorithm
//Input: Two nonnegative, not-both-zero integers m and n
//Output: Greatest common divisor of m and n
while $n \neq 0$ **do**
$r \leftarrow m \bmod n$
$m \leftarrow n$
$n \leftarrow r$
return m

Consecutive integer checking algorithm

Step 1 Assign the value of min{*m,n*} to *t*

Step 2 Divide *m* by *t.* If the remainder is 0, go to Step 3; otherwise, go to Step 4

Step 3 Divide *n* by *t.* If the remainder is 0, return *t* and stop; otherwise, go to Step 4

Step 4 Decrease *t* by 1 and go to Step 2

Middle-school procedure for computing gcd(m, n)

Step 1 Find the prime factors of m.

Step 2 Find the prime factors of n.

Step 3 Find all the common prime factors

Step 4 Compute the product of all the common prime factors and return it as gcd(*m,n*)

60 = 2 x 2 x 3 x 5

24 = 2 x 2 x 3

gcd(60, 24) = 2 x 2 x 3 = 12

- *Is this an algorithm?*

Sieve of Eratosthenes

- **A simple Algorithm Generating Consecutive Primes Not Exceeding Any Given Integer n: Sieve of Eratosthenes**
- Example:

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
2	3	x	5	x	7	x	9	x	11	x	13	x	15	x	17	x	19	x	21	x	23	x	**25**
2	**3**		5		7		x		11		13		x		17		19		x		23		**25**
2	**3**		**5**		**7**				**11**		**13**				**17**		**19**				**23**		**x**

Sieve of Eratosthenes

```
ALGORITHM  Sieve(n)
    //Implements the sieve of Eratosthenes
    //Input: An integer n ≥ 2
    //Output: Array L of all prime numbers less than or equal to n
    for p ← 2 to n do A[p] ← p
    for p ← 2 to ⌊√n⌋ do    //see note before pseudocode
        if A[p] ≠ 0   //p hasn't been eliminated on previous passes
            j ← p * p
            while j ≤ n do
                A[j] ← 0    //mark element as eliminated
                j ← j + p
    //copy the remaining elements of A to array L of the primes
    i ← 0
    for p ← 2 to n do
        if A[p] ≠ 0
            L[i] ← A[p]
            i ← i + 1
    return L
```

Algorithm design techniques/strategies

- Brute force
- Divide and conquer
- Decrease and conquer
- Transform and conquer
- Space and time tradeoffs
- Greedy approach
- Dynamic programming
- Iterative improvement
- Backtracking
- Branch and bound

Analysis of algorithms

- How good is the algorithm?
  - time efficiency
  - space efficiency

- Does there exist a better algorithm?
  - lower bounds
  - optimality

Important problem types

- sorting
- searching
- string processing
- graph problems
- combinatorial problems
- geometric problems
- numerical problems

Fundamental data structures

- list
  - array
  - linked list
  - string
- stack
- queue
- priority queue
- graph
- tree
- set and dictionary

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- **Mathematical Background**

Mathematical Background

- Functions
- Logarithm
- Summation
- Probability
- Asymptotic Notations
- Recursion
  - Recurrence equation

Properties of Logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a x^y = y \log_a x$
4. $\log_a xy = \log_a x + \log_a y$
5. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
6. $a^{\log_b x} = x^{\log_b a}$
7. $\log_a x = \dfrac{\log_b x}{\log_b a} = \log_a b \log_b x$

Important Summation Formulas

1. $\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$ (l, u are integer limits, $l \le u$); $\sum_{i=1}^{n} 1 = n$

2. $\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2} \approx \frac{1}{2}n^2$

3. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{1}{3}n^3$

4. $\sum_{i=1}^{n} i^k = 1^k + 2^k + \cdots + n^k \approx \frac{1}{k+1}n^{k+1}$

Important Summation Formulas

5. $\sum_{i=0}^{n} a^i = 1 + a + \cdots + a^n = \frac{a^{n+1} - 1}{a - 1} \; (a \neq 1); \quad \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

6. $\sum_{i=1}^{n} i2^i = 1 \cdot 2 + 2 \cdot 2^2 + \cdots + n2^n = (n-1)2^{n+1} + 2$

7. $\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \cdots + \frac{1}{n} \approx \ln n + \gamma$, where $\gamma \approx 0.5772 \ldots$ (Euler's constant)

8. $\sum_{i=1}^{n} \lg i \approx n \lg n$

Sum Manipulation Rules

1. $$\sum_{i=l}^{u} ca_i = c\sum_{i=l}^{u} a_i$$

2. $$\sum_{i=l}^{u} (a_i \pm b_i) = \sum_{i=l}^{u} a_i \pm \sum_{i=l}^{u} b_i$$

3. $$\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i, \text{ where } l \le m < u$$

4. $$\sum_{i=l}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$$