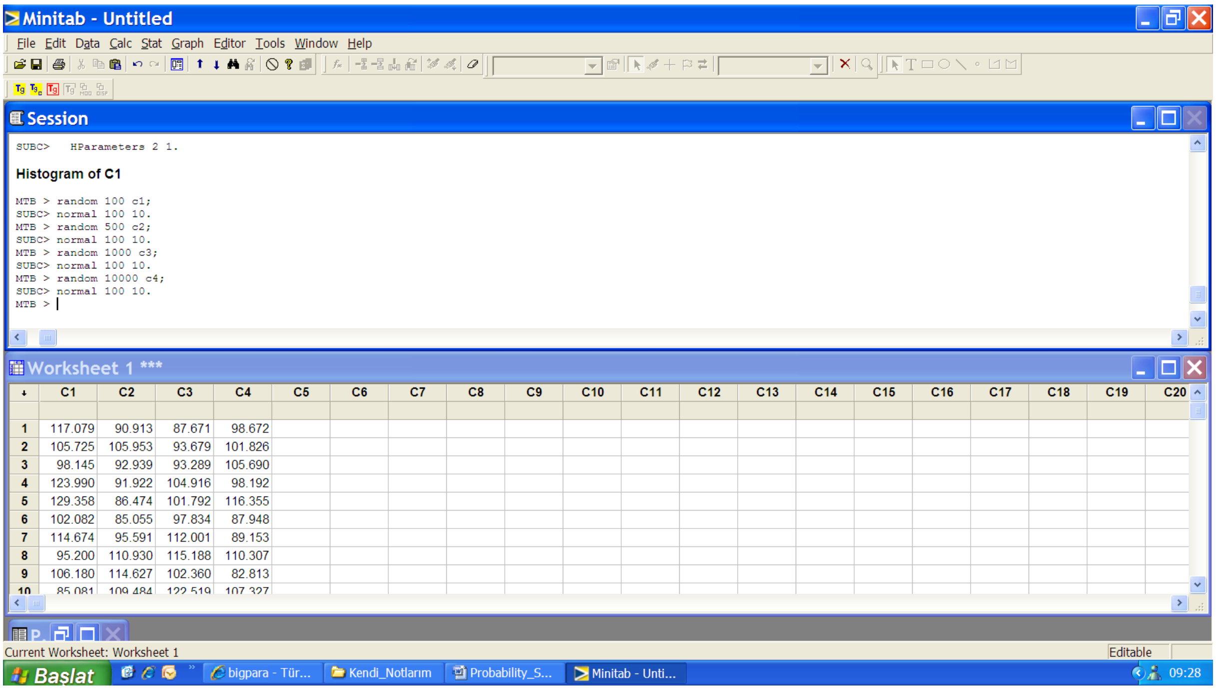
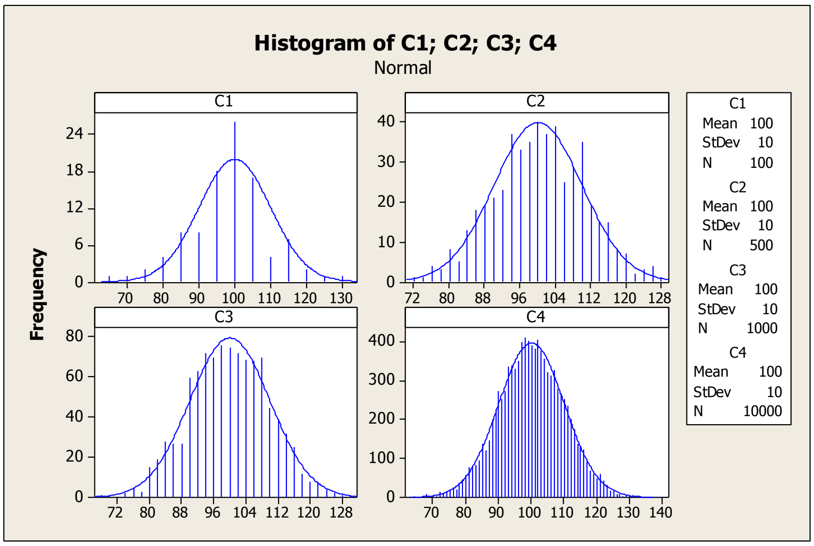
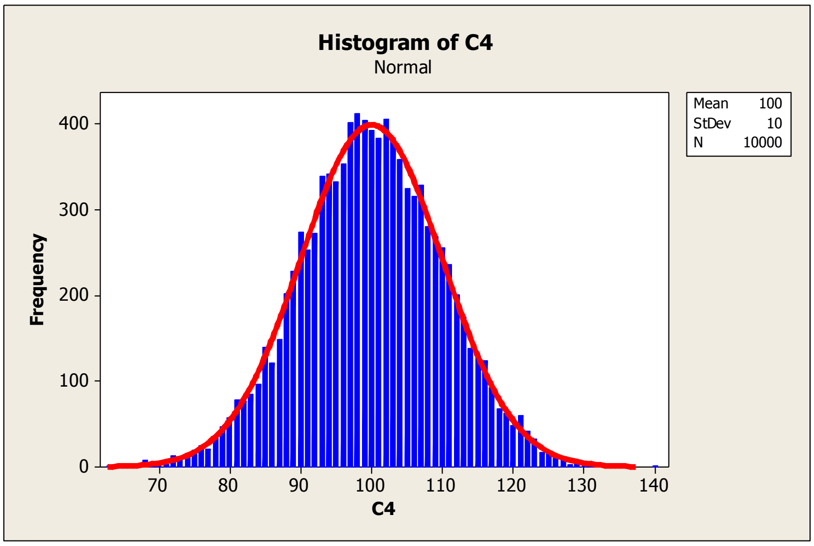
Continuous Random Variables And Probability Distributions

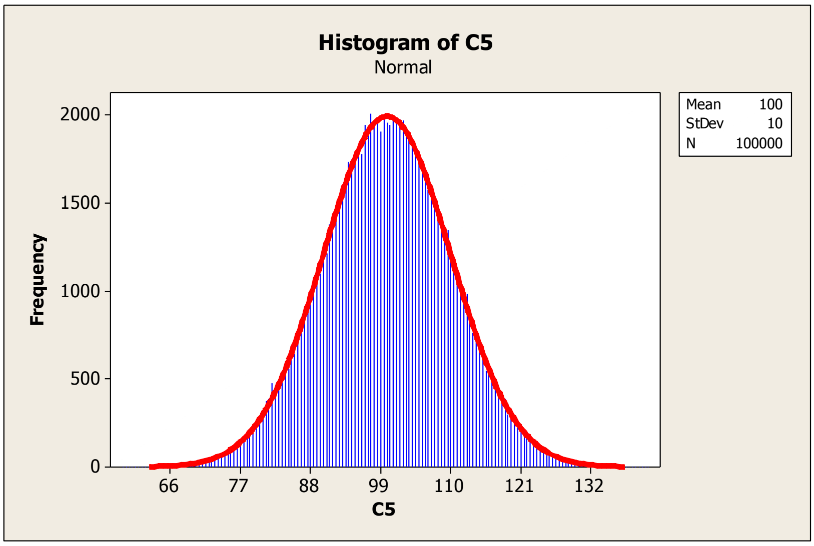
- **When a random variable x is discrete, we can assign a positive probability to each value that x can take and get the probability distribution for x. The sum of all the probabilities associated with the different values of x is 1.**
- **However, not all experiments result in random variables that are discrete. Continuous random variables, such as heights and weights, length of life of a particular product, or experimental error, can assume the infinitely many values corresponding to points on a line interval.**
- **Density functions are commonly used in engineering to describe physical systems.**
- **Suppose we have a set of measurements on a continuous random variable, and we create a relative frequency histogram to describe their distribution.**
- **For a small number of measurements, we could use a small number of classes.**
- **Then as more and more measurements are collected, we can use more classes and reduce the class width.**
- **The outline of the histogram will change slightly, for most part becoming less and less irregular.**

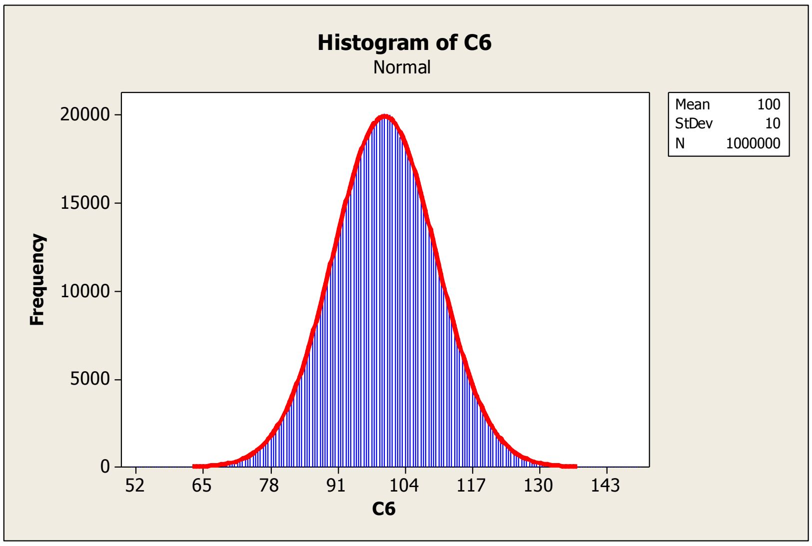

Minitab - Untitled
File Edit Data Calc Stat Graph Editor Tools Window Help
Session
SUBC> HParameters 2 1.
Histogram of C1
MTB > random 100 c1;
SUBC> normal 100 10.
MTB > random 500 c2;
SUBC> normal 100 10.
MTB > random 1000 c3;
SUBC> normal 100 10.
MTB > random 10000 c4;
SUBC> normal 100 10.
MTB >
Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11 C12 C13 C14 C15 C16 C17 C18 C19 C20
1 117.079 90.913 87.671 98.672
2 105.725 105.953 93.679 101.826
3 98.145 92.939 93.289 105.690
4 123.990 91.922 104.916 98.192
5 129.358 86.474 101.792 116.355
6 102.082 85.055 97.834 87.948
7 114.674 95.591 112.001 89.153
8 95.200 110.930 115.188 110.307
9 106.180 114.627 102.360 82.813
Current Worksheet: Worksheet 1
Editable
Başlat
bigpara - Tür...
Kendi_Notlarım
Probability_S...
Minitab - Unti...
09:28


Histogram of C1; C2; C3; C4
Normal
C1
C2
C3
C4
Frequency
C1
Mean 100
StDev 10
N 100
C2
Mean 100
StDev 10
N 500
C3
Mean 100
StDev 10
N 1000
C4
Mean 100
StDev 10
N 10000


Histogram of C4
Normal
Frequency
C4
Mean 100
StDev 10
N 10000

- **As the number of measurements becomes <u>very large and the class widths become very narrow</u>, the relative frequency histogram appears more and more like the smooth curve.**





This smooth curve describes the probability distribution of the continuous random variable.

(A histogram is an <u>approximation</u> to a probability density function.)

Probability Density Function

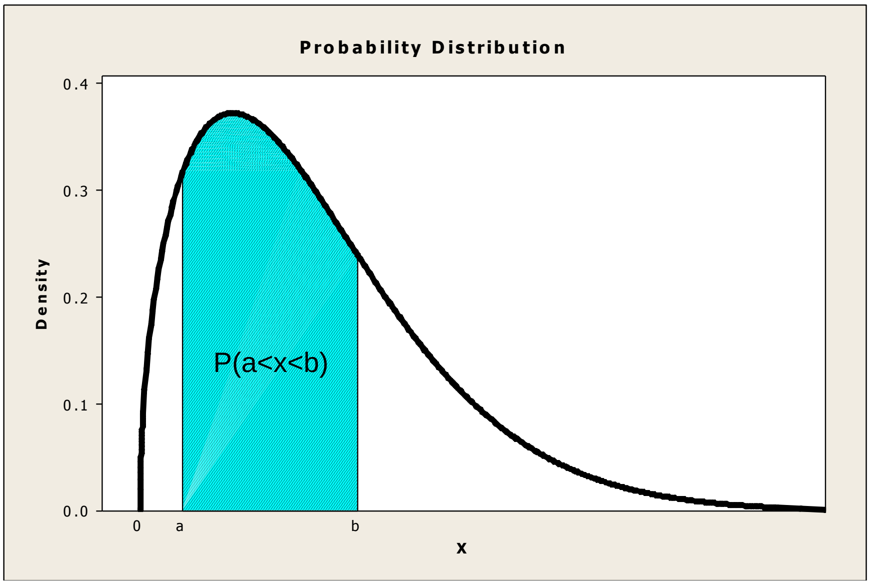
For a continuous random variable X, a probability density function is a function such that

1. $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x)dx = 1$

3. $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$

$= area\ under\ f(x)\ from\ a\ to\ b$



The important point is that f(x) is used to calculate an area that represents the probability that X assumes a value in [a, b].

Some Properties:

- **The area under a continuous probability distribution is equal to 1.**
- **The probability that x will fall into a particular interval – say, from a to b –is equal to the area under the curve between the two points a and b. This is the shaded area in the figure.**
- **P(X=a)=0 for continuous random variables.**
- **This implies that**

  $P(X \geq a) = P(x > a)$ and $P(X \leq a) = P(x < a)$.

  This is not true in general for discrete random variables.

If X is continuous random variable

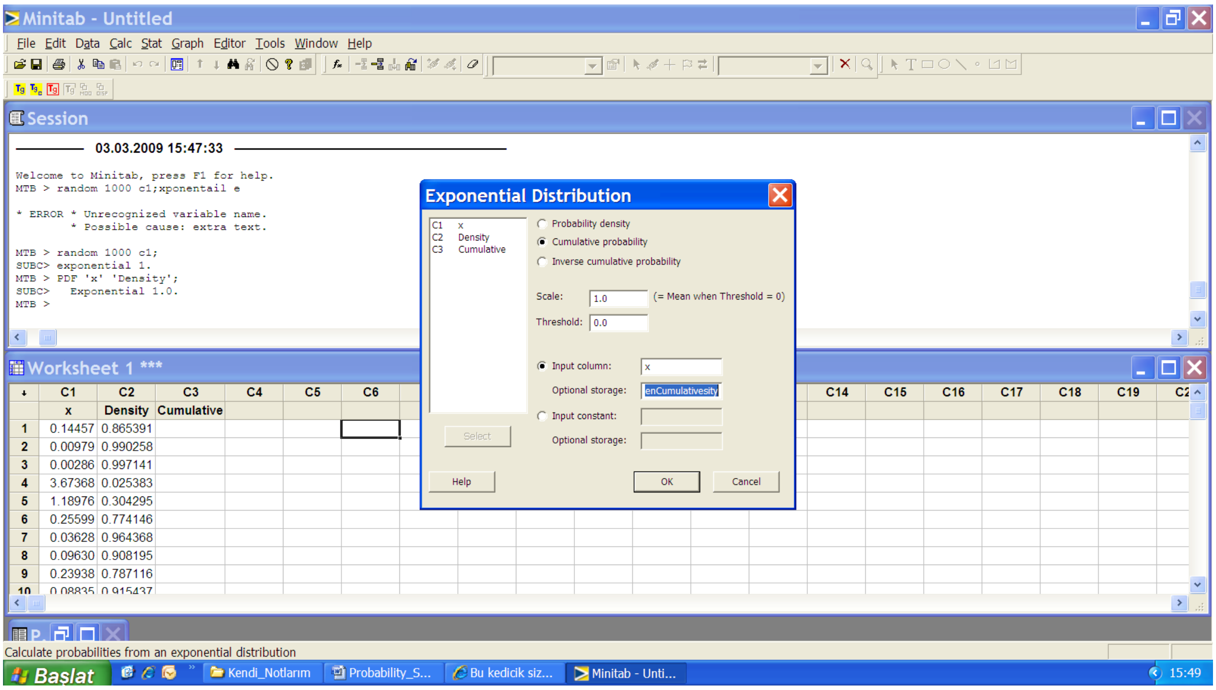
$$P(x_1 \le X \le x_2) = P(x_1 < X \le x_2) = P(x_1 \le X < x_2) = P(x_1 < X < x_2)$$

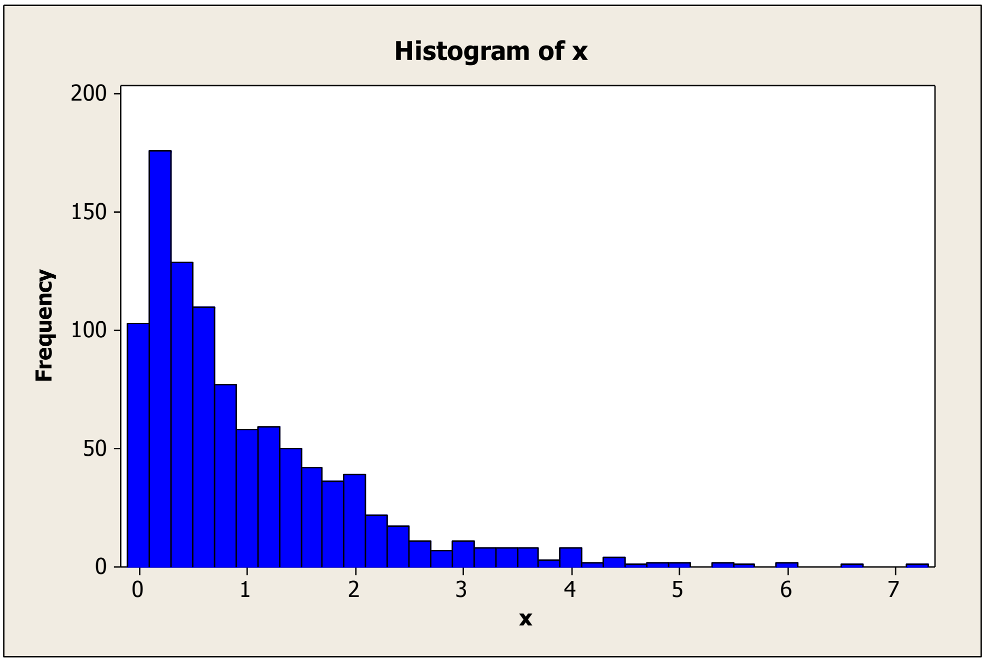
Cumulative Distribution Functions

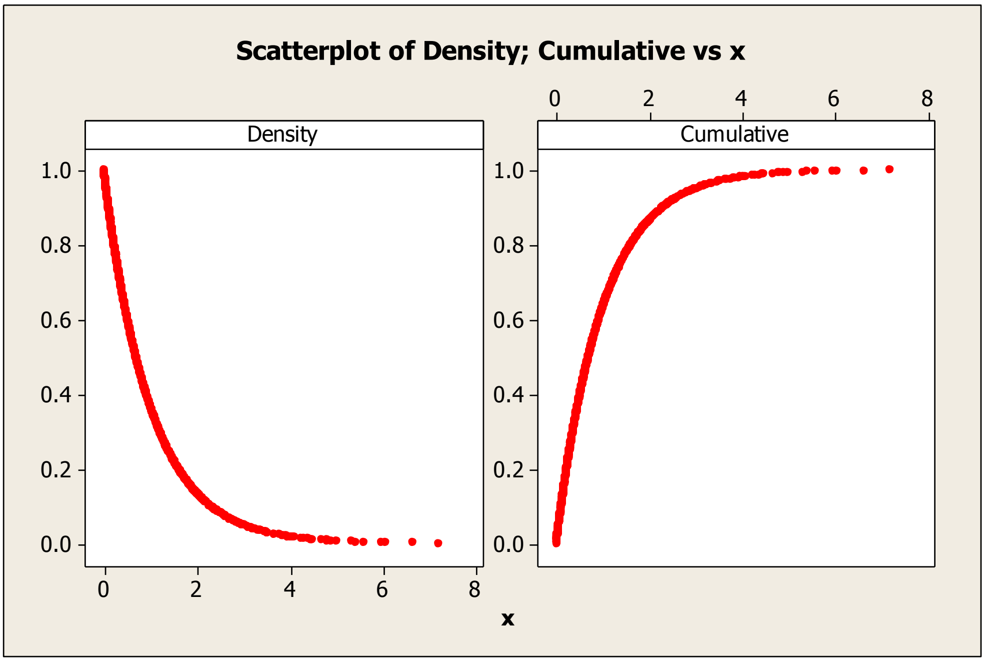
An alternative method to describe the distribution of a discrete random variable can also be used for continuous random variables.

The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du$$




Histogram of x
Frequency
200
150
100
50
0
0
1
2
3
4
5
6
7
x


Scatterplot of Density; Cumulative vs x
Density
Cumulative
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
x

Mean and Variance of a Continuous Random Variable

The mean and variance can also be defined for a continuous random variable. Integration replaces summation in the discrete distributions.

Definition:

Suppose X is a continuous random variable with probability density function f(x).

The mean or expected value of X, denoted as μ or E(X), is

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

The variance of X, denoted as σ^2 or V(X), is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

The standard deviation of X is

$$\sigma = \sqrt{\sigma^2}$$