Transportation Problem Variations

- **Maximization objective**
- **Total supply not equal to total demand**
- **Unacceptable transportation routes (arcs)**

Maximization Objective

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply
Row-1	4	3	3	1	8
Row-2	3	2	4	8	11
Row-3	5	4	6	3	16
Demand	4	9	9	13	

NORTHWEST STARTING SOLUTION

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply &u_i
Row-1	**4** 4	**4** 3	3 -2	1 -1	**8** (0)
Row-2	3 0	**5** 2	**6** 4	8 **7**	**11** (-1)
Row-3	5 0	4 0	**3** 6	**13** 3	**16** (1)
Demand &v_j	**4** (4)	**9** (3)	**9** (5)	**13** (2)	

The value of the solution =119

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply &u_i
Row-1	**4** 4	**4** 3	3 -2	1 -1	8 (0)
Row-2	3 0	**5** 2	**6** 4	8 **7**	11 (-1)
Row-3	5 0	4 0	**3** 6	**13** 3	16 (1)
Demand &v_j	4 (4)	9 (3)	9 (5)	13 (2)	

$\alpha = 6$

The new bfs

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	3		3 -9		1 -8	8 (0)
Row-2		3 0	**5**	2		4 -7	**6**	8	11 (-1)
Row-3		5 7		4 7	**9**	6	**7**	3	16 (-6)
Demand &v_j	4 (4)		9 (3)		9 (12)		13 (9)		

The value of the solution = 161
Arbitrarily choose one of two

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	3		3 -9		1 -8	8 (0)
Row-2		3 0	**5**	2		4 -7	**6**	8	11 (-1)
Row-3		5 7		4 7	**9**	6	**7**	3	16 (-6)
Demand &v_j	4 (4)		9 (3)		9 (12)		13 (9)		

$\alpha = 4$

the new bfs

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1		4 -7	**8**	3		3 -9		1 -8	8 (0)
Row-2		3 0	**1**	2		4 -7	**10**	8	11 (-1)
Row-3	**4**	5		4 7	**9**	6	**3**	3	16 (-6)
Demand &v_j	4 (11)		9 (3)		9 (12)		13 (9)		

The value of the solution = 189

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1		4 -7	**8**	3		3 -9		1 -8	8 (0)
Row-2		3 0	**1**	2		4 -7	**10**	8	11 (-1)
Row-3	**4**	5		4 7	**9**	6	**3**	3	16 (-6)
Demand &v_j	4 (11)		9 (3)		9 (12)		13 (9)		

α=1

The new bfs

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1		4 0	**8**	3		3 -2		1 -1	8 (0)
Row-2		3 -7		2 -7		4 -7	**11**	8	11 (6)
Row-3	**4**	5	**1**	4	**9**	6	**2**	3	16 (1)
Demand &v_j	4 (4)		9 (3)		9 (5)		13 (2)		

The value of the solution = 196 (with multiple optimals)

Alternative Optima

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	3		3 -2		1 -1	8 (0)
Row-2		3 -7		2 -7		4 -7	**11**	8	11 (6)
Row-3		5 0	**5**	4	**9**	6	**2**	3	16 (1)
Demand &v_j	4 (4)		9 (3)		9 (5)		13 (2)		

The value of the solution = 196

Maximization Objective
Alternative Approach

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply
Row-1	4	3	3	1	8
Row-2	3	2	4	8	11
Row-3	5	4	6	3	16
Demand	4	9	9	13	

Alternative Approach

$c_{ij} = Max\{c_{ij}\}-c_{ij}$

Minimization objective with the new c_{ij}'s

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply
Row-1	4	5	5	7	8
Row-2	5	6	4	0	11
Row-3	3	4	2	5	16
Demand	4	9	9	13	

NORTHWEST STARTING SOLUTION

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	5		5 2		7 1	8 (0)
Row-2		5 0	**5**	6	**6**	4		0 **-7**	11 (1)
Row-3		3 0		4 0	**3**	2	**13**	5	16 (-1)
Demand &v_i	4 (4)		9 (5)		9 (3)		13 (6)		

The value of the solution = 161

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	5		5 2		7 1	8 (0)
Row-2		5 0	**5**	6	**6**	4		0 **-7**	11 (1)
Row-3		3 0		4 0	**3**	2	**13**	5	16 (-1)
Demand &v_i	4 (4)		9 (5)		9 (3)		13 (6)		

α=6

The new bfs

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	5		5 9		7 8	8 (0)
Row-2		5 0	**5**	6		4 7	**6**	0	11 (1)
Row-3		3 -7		4 -7	**9**	2	**7**	5	16 (6)
Demand &v_j	4 (4)		9 (5)		9 (-4)		13 (-1)		

The value of the solution = 119

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1	**4**	4	**4**	5		5 9		7 8	8 (0)
Row-2		5 0	**5**	6		4 7	**6**	0	11 (1)
Row-3		3 -7		4 -7	**9**	2	**7**	5	16 (6)
Demand &v_j	4 (4)		9 (5)		9 (-4)		13 (-1)		

$\alpha=4$

The new bfs

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1		4 7	**8**	5		5 9		7 8	8 (0)
Row-2		5 7	**1**	6		4 7	**10**	0	11 (1)
Row-3	**4**	3		4 **-7**	**9**	2	**3**	5	16 (6)
Demand &v_j	4 (4)		9 (5)		9 (-4)		13 (-1)		

The value of the solution = 91

From/To	Col.-1		Col.-2		Col.-3		Col-4		Supply &u_i
Row-1		4 7	**8**	5		5 9		7 8	8 (0)
Row-2		5 7	**1**	6		4 7	**10**	0	11 (1)
Row-3	**4**	3		4 **-7**	**9**	2	**3**	5	16 (6)
Demand &v_j	4 (4)		9 (5)		9 (-4)		13 (-1)		

$\alpha=1$

The new bfs

From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply &u_i
Row-1	4 0	**8** 5	5 5	7 7	8 (0)
Row-2	5 7	6 7	4 7	**11** 0	11 (-6)
Row-3	**4** 3	**1** 4	**9** 2	**2** 5	16 (-1)
Demand &v_j	4 (4)	9 (5)	9 (3)	13 (6)	

The value of the solution = 84 Min

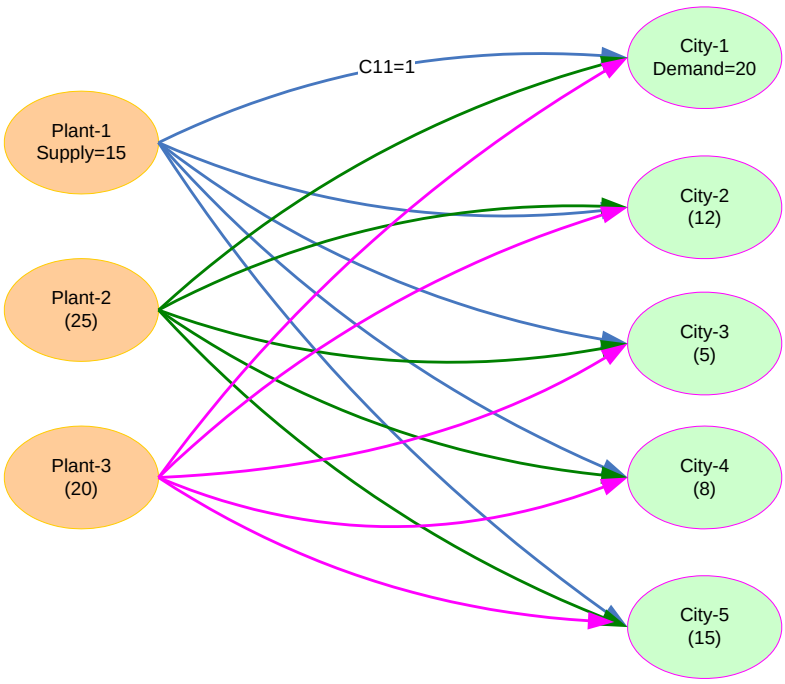
Alternate optima

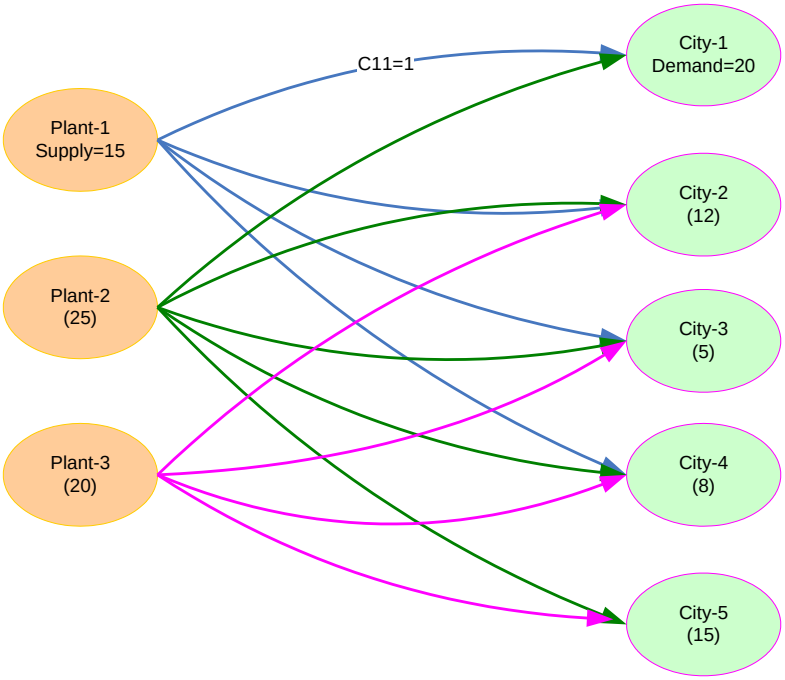
From/To	Col.-1	Col.-2	Col.-3	Col-4	Supply &u_i
Row-1	**4** 4 0	**4** 5	5 5	7 7	8 (0)
Row-2	5 7	6 7	4 7	**11** 0	11 (-6)
Row-3	3	**5** 4	**9** 2	**2** 5	16 (-1)
Demand &v_j	4 (4)	9 (5)	9 (3)	13 (6)	

Compare the solution with the maximization problem

Total supply not equal to total demand
Unacceptable transportation routes (arcs)

Power co Problem





Unacceptable routes

From/To	City-1	City-2	City-3	City-4	City-5	Supply
Plant-1	1	0	3	4	*	15
Plant-2	5	1	2	3	3	25
Plant-3	*	8	1	4	3	30
Demand	20	12	5	8	15	

Solution of the problem

From/To	City-1	City-2	City-3	City-4	City-5	Supply
Plant-1	1	0	3	4	*	15
Plant-2	5	1	2	3	3	25
Plant-3	*	8	1	4	3	30
Demand	20	12	5	8	15	

Put M for minimization (positive integer larger than the biggest cost) instead of *

Put 0 for maximization instead of *

From/to	City-1	City-2	City-3	City-4	City-5	Dummy	Supply
Plant-1	1	0	3	4	10	0	15
Plant-2	5	1	2	3	3	0	25
Plant-3	10	8	1	4	3	0	30
Demand	20	12	5	8	15	10	

Vogel's starting solution

From/to	City-1		City-2		City-3		City-4		City-5		Dummy		Supply &vj
Plant-1	**15**	1		0		3		4		10		0	15
Plant-2	**5**	5	**12**	1		2		3		3	**8**	0	25
Plant-3		10		8	5	1	**8**	4	**15**	3	**2**	0	30
Demand &ui	20		12		5		8		15		10		

From/to	City-1		City-2		City-3		City-4		City-5		Dummy		Supply &vj
Plant-1	**15**	1		0		3		4		10		0	15 (0)
Plant-2	**5**	5	**12**	1		2		3 -1		3	**8**	0	25 (4)
Plant-3		10		8	**5**	1	**8**	4	**15**	3	**2**	0	30 (4)
Demand &ui	20 (1)		12 (-3)		5 (-3)		8 (0)		15 (-1)		10 (-4)		

The value of the solution = 134 (not optimal)

From/to	City-1		City-2		City-3		City-4		City-5		Dummy		Supply &vj
Plant-1	**15**	1		0		3		4		10		0	15 (0)
Plant-2	**5**	5	**12**	1		2		3 -1		3	**8**	0	25 (4)
Plant-3		10		8	**5**	1	**8**	4	**15**	3	**2**	0	30 (4)
Demand &ui	20 (1)		12 (-3)		5 (-3)		8 (0)		15 (-1)		10 (-4)		

α=8

The new bfs

From/to	City-1		City-2		City-3		City-4		City-5		Dummy		Supply &vj
Plant-1	**15**	1		0 4		3 7		4 5		10 12		0 5	15 (0)
Plant-2	**5**	5	**12**	1		2 2	**8**	3		3 1		0 1	25 (4)
Plant-3		10		8	**5**	1	**0**	4	**15**	3	**10**	0	30 (5)
Demand &ui	20 (1)		12 (-3)		5 (-4)		8 (-1)		15 (-2)		10 (-5)		

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The value of the solution = 126 (optimum bfs solution)