UNBOUNDED LP

For a **max problem**, an unbounded LP occurs if it is possible to find points in the feasible region with arbitrarily large z values corresponding to a decision maker earning arbitrarily large revenues or profits.

This would indicate that an unbounded optimal solution **should not occur in a correctly formulated LP.**

For a **min problem** an LP is unbounded if there are points in a feasible region with arbitrarily small z values.

Example (UNBOUNDED LP)

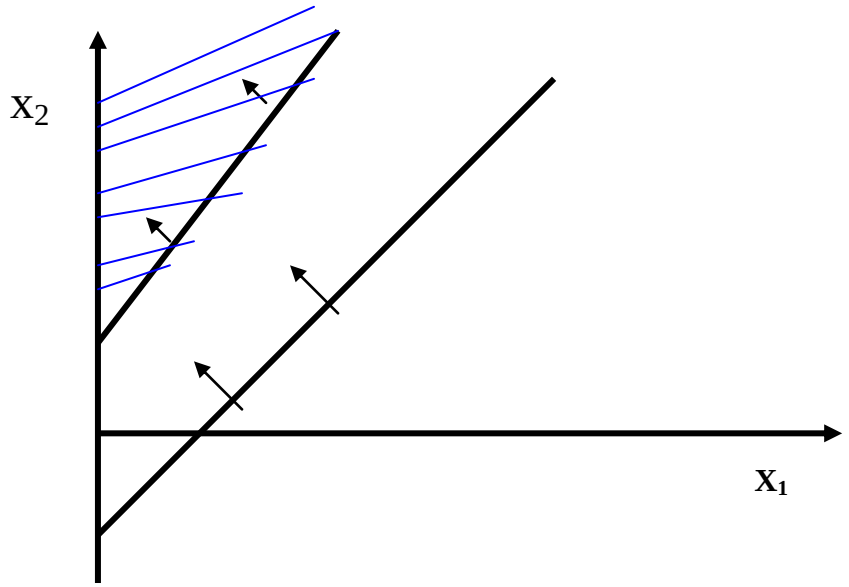
$$\text{Max } Z = x_1 + 2\,x_2$$

s.t

$$x_1 - x_2 \le 4$$

$$-x_1 + x_2 \le 1$$

$$x_1, x_2 \ge 0$$



Standard Form

$$\mathbf{Z - x_1 - 2\,x_2 = 0}$$

$$\mathbf{x_1 - x_2 + x_3 = 4}$$

$$\mathbf{-x_1 + x_2 + x_4 = 1}$$

Initial Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	1	-1	1	0	**4**	
x_4	-1	1<<	0	1	**1**	1<
Z	**-1**	**-2<**	**0**	**0**	**0**	

Entering Variable: x_2

Leaving Variable: x_4

The First Improved Simplex Tableau

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	0	0	1	1	**5**	NONE
x_2	-1	1	0	1	**1**	
Z	**-3**	**0**	**0**	**2**	**2**	

Unbounded (Pivot element not available)

UNBOUNDED SIMPLEX TABLEAU

BASIS	x_1	x_2	x_3	x_4	RHS	RATIO
x_3	1	-1	1	0	**4**	
x_4	-1	1<<	0	1	**1**	1<
Z	**-1**	**-2<**	**0**	**0**	**0**	
x_3	0	0	1	1	**5**	NONE
x_2	-1	1	0	1	**1**	
Z	**-3**	**0**	**0**	**2**	**2**	

An unbounded LP occurs when a variable with a negative coefficient in row Z has a non-positive coefficient in each constraint (row).

Example (UNBOUNDED LP)

$$\text{Max } Z = 36\,x_1 + 30\,x_2 - 3\,x_3 - 4\,x_4$$

s.t

$$x_1 + x_2 - x_3 \le 5$$
$$6\,x_1 + 5\,x_2 - x_4 \le 10$$
$$x_i \ge 0$$

$$\mathbf{Z - 36\,x_1 - 30\,x_2 + 3\,x_3 + 4\,x_4 = 0}$$

$$\mathbf{x_1 + x_2 - x_3 + x_5 = 5}$$
$$\mathbf{6\,x_1 + 5\,x_2 - x_4 + x_6 = 10}$$

BV=(x_5,x_6)=5,10 NBV=(x_1,x_2,x_3,x_4)=0

Initial Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_5	1	1	-1	0	1	0	**5**	5.0
x_6	**6<<**	5	0	-1	0	1	**10**	1.666<
Z	**-36<**	**-30**	**3**	**4**	**0**	0	**0**	

Entering Variable : x_1
Leaving Variable : x_6

The first tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_5	0	1/6	-1	1/6	1	-1/6	**10/3**	20.0<<
x_1	1	5/6	0	-1/6	0	1/6	**5/3**	
Z	**0**	**0**	**3**	**-2<**	**0**	6	**60**	

Entering Variable : x_4
Leaving Variable : x_5

The second tableau

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_4	0	1	-6	1	6	-1	**20**	NONE
x_1	1	1	-1	0	1	0	**20**	
Z	0	2	-9	0	12	**4**	**100**	

UNBOUNDED

UNBOUNDED SIMPLEX TABLEAU

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_5	1	1	-1	0	1	0	**5**	5.0
x_6	**6<<**	5	0	-1	0	1	**10**	1.666<
Z	**-36<**	**-30<**	**3**	**4**	**0**	0	**0**	
x_5	0	1/6	-1	1/6	1	-1/6	**10/3**	20.0<
x_1	1	5/6	0	-1/6	0	1/6	**5/3**	
Z	**0**	**0**	**3**	**-2**	**0**	6	**60**	
x_4	0	1	-6	1	6	-1	**20**	NONE
x_1	1	1	-1	0	1	0	**20**	
Z	0	2	-9	0	12	**4**	**100**	