EXAMPLE

MULTIPERIOD WORK SCHEDULING

ABC is a chain of computer service stores. The number of hours of skilled repair time that ABC requires during the next five months is as follows:

Month 1 (January) : 6000 hours
Month 2 (February): 7000 hours
Month 3 (March) : 8000 hours
Month 4 (April) : 9500 hours
Month 5 (May) : 11000 hours

- At the beginning of January, **50** skilled engineers work for ABC.
- Each skilled engineer can work up to **160** hours per month.
- In order to meet future demands, new engineers must be trained.
- It takes one month to train a new engineer.
- During the month of training, a trainee must be supervised for **50** hours by an experienced engineer.
- Each experienced engineer is paid **$2000** a month (even he or she does not work the full 160 hours).
- During the month of training, a trainee is paid **$1000** a month.
- At the end of each month, **%5** of ABC's experienced engineers quit to join DEF computers.

Formulate an LP whose solution will enable ABC to minimize the labor cost incurred in meeting the service requirements for the next five months.

Solution

ABC must determine the number of engineers that should be trained during month t (t = 1,2,3,4,5). Thus, we define

x_t =number of engineers trained during month t (t = 1,2,3,4,5)

- **ABC wants to minimize total labor cost during the next five months.**

Total labor cost = Cost of paying trainees + cost of paying experienced engineers

To express the cost of paying experienced engineers, we need to define, for t = 1, 2,3,4,5,

y_t = number of experienced engineers at the beginning of month t

Then total labor cost = $1000(x_1 + x_2 + x_3 + x_4 + x_5)$
$+ \quad 2000(y_1 + y_2 + y_3 + y_4 + y_5)$

What constraints does ABC face?

Note that we are given $y_1 = 50$
And that for t = 1,2,3,4,5 ABC must ensure that

$160y_1 - 50x_1 \geq 6000$ (month 1 constraint)
$160y_2 - 50x_2 \geq 7000$ (month 2 constraint)
$160y_3 - 50x_3 \geq 8000$ (month 3 constraint)
$160y_4 - 50x_4 \geq 9500$ (month 4 constraint)
$160y_5 - 50x_5 \geq 11000$ (month 5 constraint)

It is important to realize that the number of skilled engineers available at the beginning of any month is determined by the number of skilled engineers available during the previous month and the number of engineers trained during the previous month:

For February

$$\mathbf{y_2 = y_1 + x_1 - 0.05y_1 \quad or \quad y_2 = 0.95\, y_1 + x_1}$$

For March,

$$\mathbf{y_3 = 0.95\, y_2 + x_2}$$

For April,

$$\mathbf{y_4 = 0.95\, y_3 + x_3}$$

For May,

$$\mathbf{y_5 = 0.95\, y_4 + x_3}$$

Adding the sign restrictions

$x_t \geq 0$ and $y_t \geq 0$ ($t = 1,2,3,4,5$)

$$\text{Min } Z = 1000x_1 + 1000x_2 + 1000x_3 + 1000\, x_4 + 1000x_5 + 2000y_1 + 2000\, y_2 + 2000y_3 + 2000y_4 + 2000y_5$$

$$
\begin{aligned}
\text{s.t.} \quad & 160\, y_1 - 50\, x_1 \geq 6000 \\
& 160\, y_2 - 50\, x_2 \geq 7000 \\
& 160\, y_3 - 50\, x_3 \geq 8000 \\
& 160\, y_4 - 50\, x_4 \geq 9500 \\
& 160\, y_5 - 50\, x_5 \geq 11000 \\
& y_1 = 50 \\
& 0.95y_1 + x_1 = y_2 \quad (0.95\, y_1 - y_2 + x_1 = 0) \\
& 0.95y_2 + x_2 = y_3 \\
& 0.95y_3 + x_3 = y_4 \\
& 0.95y_4 + x_4 = y_5 \\
& y_t,\, x_t \geq 0 \quad (t = 1,2,3,4,5)
\end{aligned}
$$

The optimal solution is

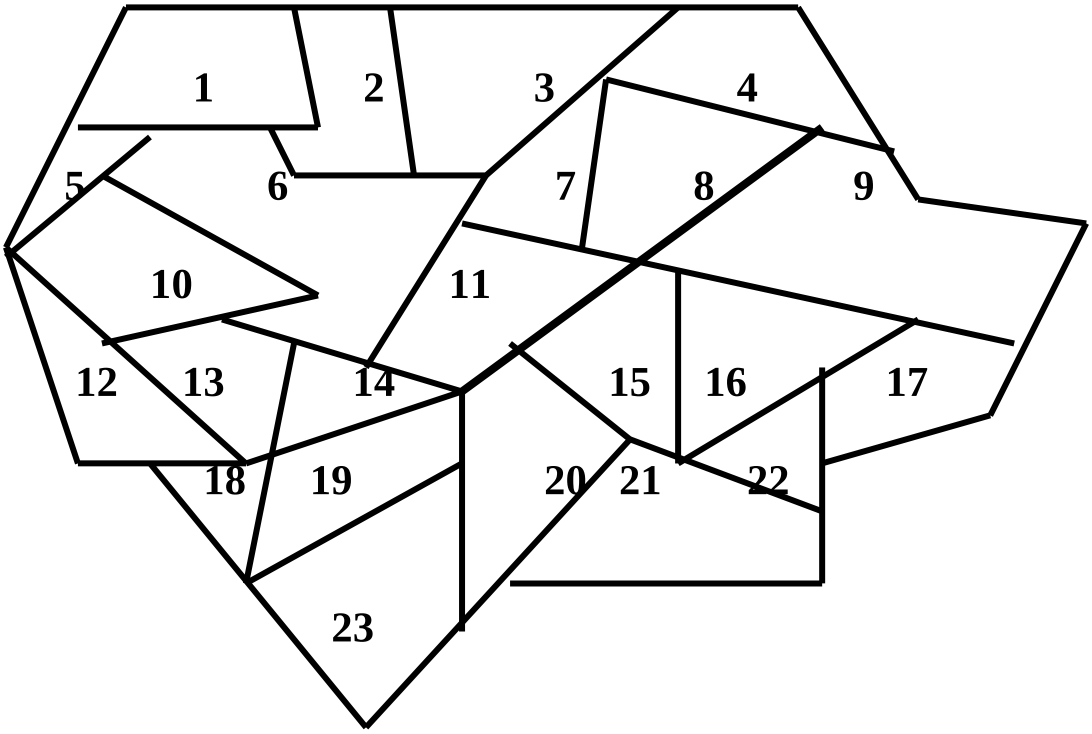
z_{min}= 593777
x_1 = 0,
x_2 = 8.45 ,
x_3 =11.45,
x_4 = 9.52 ,
x_5 = 0,
y_1= 50 ,
y_2 = 47.5,
y_3 = 53.58,
y_4 = 62.34
y_5 = 68.75

- **In reality, the x_t's must be integers , so this solution is difficult to interpret.**

- **Of course, we could obtain a feasible integer solution by rounding x_2 up to 9, x_3 up to 12, x_4 up to 10.**

- **But there is no guarantee that this solution is the OPTIMAL INTEGER SOLUTION.**

EXAMPLE (Binary Programming)



The police department of the city of Flint, Michigan, has divided the city into 23 patrol sectors, such that the response time of a patrol unit (squad car) will be less than three minutes between any two points within the sector.

However, severe budget cuts have forced the city to eliminate some patrols. The chief of police has mandated that each sector be covered by at least one unit located either within that sector or in an adjacent sector.

$$\min \sum_{i=0}^{23} x_i$$

1. $x_1 + x_2 + x_5 + x_6 \geq 1$
2. $x_1 + x_2 + x_3 + x_6 \geq 1$
3. $x_2 + x_3 + x_4 + x_6 + x_7 + x_8 \geq 1$
4. ..
5. ..
6. ..

23. $x_{19} + x_{20} + x_{21} + x_{23} \geq 1$

$$x_i = 0{,}1 Binary$$