$\underbrace{(p \to r) \land (q \to r)}_{x} \stackrel{?}{\equiv} (p \lor q) \to r$ $\quad 1 \to 0 \Rightarrow 0$ $\quad 4$

p	q	r	$p \to r$	$q \to r$	x	$p \lor q$	$(p \lor q) \to r$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	0	1	0
0	1	1	1	1	1	1	1
1	0	0	0	1	0	1	0
1	0	1	1	1	1	1	1
1	1	0	0	0	0	1	0
1	1	1	1	1	1	1	1

✓

$\overset{x}{(p \to q) \to (r \to s)} \stackrel{?}{\equiv} \overset{y}{(p \to r) \to (q \to s)}$

p	q	r	s	$p \to q$	$r \to s$	x	$p \to r$	$q \to s$	y
0	0	0	0	1	1				
0	0	0	1	1	1				
0	0	1	0	1	0	0			
0	0	1	1	1	1				
0	1	0	0	1	1			0	0
0	1	0	1	1	1				
0	1	1	0	1	0	0		0	0
0	1	1	1	1	1				
1	0	0	0	0	1		0		
1	0	0	1	0	1		0		
1	0	1	0	0	0				
1	0	1	1	0	1				
1	1	0	0	1	1		0	0	
1	1	0	1	1	1		0		
1	1	1	0	1	0	0		0	0
1	1	1	1	1	1				

4

1a)

For $(p \rightarrow r) \wedge (q \rightarrow r)$ to be false,
one of the two conditional statements must be false, which happens exactly when r is false and at least one of p and q is true. But these are precisely the cases in which $p \vee q$ is true and r is false, which is precisely when $(p \vee q) \rightarrow r$ is false. Because the two propositions are false in exactly the same situations, they are logically equivalent.

b)

Many answers are possible. If we let r be true and p, q, and s be false, then $(p \rightarrow q) \rightarrow (r \rightarrow s)$ will be false, but $(p \rightarrow r) \rightarrow (q \rightarrow s)$ will be true.

c)

$\neg(p \wedge q)$ is true when ei ther p or q, or both, are false,
and is false when both p and q are
true. Because this was the definition of $p \mid q$, the two compound propositions are logically equivalent.

d)

2a)

Big-Omega and Big-Theta Notation

Big-O notation is used extensively to describe the growth of functions, but it has limitations. In particular, when f (x) is O(g(x)), we have an upper bound, in terms of g(x), for the size of f(x) for large values of x. However, big-O notation does not provide a lower bound for the sizeof f(x) for large x. For this, we use big-Omega (big- Ω) notation.When we want to give both an upper and a lower bound on the size of a function f (x), relative to a reference function g(x),we use big-Theta (big-Θ) notation. Both big-Omega and big-Theta notation were introduced by Donald Knuth in the 1970s. His motivation for introducing these notations was the common misuse of big-O notation when both an upper and a lower bound on the size of a function are needed.

There is a strong connection between big-O and big-Omega notation. In particular, f (x) is (g(x)) if and only if g(x) is O(f (x)).

b)

$3x^4 + 1 \le 4x^4 = 8(x^4/2)$ for all $x > 1$, so $3x^4+1$ is $O(x^4/2)$, with witnesses C = 8, k=1.

Also $x^4/2 \leq 3x^4+1$ for all $x > 0$, so $x^4/2$ is $O(3x^4+1)$, with witnesses C=1, k=0

c) $O(n^3.n!)$

3. Linear

4. a) Let m = c = 2, a = 0, and b = 1. Then $0 = ac \equiv bc = 2 \pmod 2$, but $0 = a \not\equiv b$ 1 (mod 2).

b) Let m = 5, a = b = 3, c = 1, and d = 6. Then $3 \equiv 3 \pmod 5$ and $1 \equiv 6 \pmod 5$,

but $3^1 = 3 \not\equiv 4 \equiv 729 = 3^6 \pmod 5$.

5. gcd (92928, 123552) = 1056; lcm(92928, 123552) = 10,872,576; both products are 11,481,440,256.

6. a) 1010 1011 1100 1101 1110 1111

b) $(11101110010101101 0001)_2$, $(1273)_8$

7 a) $A^n (A^{-1})^n = A(A \ldots (A(AA^{-1})A^{-1}) \ldots A^{-1})A^{-1}$ by the associative law.

Because $AA^{-1} = I$, working from the inside shows that $A^n (A^{-1})^n = I$. Similarly $(A^{-1})^n A^n = I$.

Therefore $(A^n)^{-1} = (A^{-1})^n$

b)

a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

8 a) Let P(n) be "1.2 + 2.3 + … + n(n-1) = n(n+1)(n+2)/3."

Basis step: P(1) is true because 1.2 = 2 = 1(1+1)(1+2)/3.

Inductive step: Assume that P(k) is true.

Then 1.2 + 2.3 + k(k+1) + (k+1)(k+2) = [k(k-1)(k+2)/3] + (k+1)(k+2) =

(k+1)(k+2)[(k/3)+1] = (k+1)(k+2)(k+3)/3

b)

Let P(n) be "n^5-n is divisible by 5."

Basis step: P(0) is true because $0^5 - 0 = 0$ is divisible by 5.

Inductive step: Assume that P(k) is true, that is,

$k^5 - \cancel{5}$ is divisible by 5. Then

k

$(k+1)^5 - (k+1) = (k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) - (k+1) = (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)$ is also divisible by 5, because both terms in this sum are divisible by 5.

9. $a_{n+1} = a_n + 2$ for $n \geq 1$ and $a_1 = 3$

10.

$$b^n \textbf{ mod } m = (b \cdot (b^{n-1} \textbf{ mod } m)) \textbf{ mod } m,$$

initial condition $b^0 \textbf{ mod } m = 1$.

$$b^n \textbf{ mod } m = (b^{n/2} \textbf{ mod } m)^2 \textbf{ mod } m$$

when n is even and

$$b^n \textbf{ mod } m = ((b^{\lfloor n/2 \rfloor} \textbf{ mod } m)^2 \textbf{ mod } m \cdot b \textbf{ mod } m) \textbf{ mod } m$$

procedure *mpower*(b, n, m: integers with $m \geq 2$, $n \geq 0$)
if $n = 0$ **then**
    mpower$(b, n, m) = 1$
else if n is even **then**
    mpower$(b, n, m) =$ *mpower*$(b, n/2, m)^2$ **mod** m
else
    mpower$(b, n, m) = ($*mpower*$(b, \lfloor n/2 \rfloor, m)^2$ **mod** $m \cdot b$ **mod** $m)$ **mod** m
{*mpower*$(b, n, m) = b^n$ **mod** m}