SORU 1 $y = \dfrac{\ln x}{x}$ fonksiyonunun grafiği:

$1^\circ)$ $T.B = \{ x \mid 0 < x < \infty \}$

$2^\circ)$ $y = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$ $(1, 0)$ nok. dan geçer

$3^\circ)$ $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = \lim\limits_{x\to\infty} \dfrac{1/x}{1} = \lim\limits_{x\to\infty} \dfrac{1}{x} = 0 \Rightarrow y = 0$ Yatay Asmp.

$4^\circ)$ $x = 0 \Rightarrow \lim\limits_{x\to 0} \dfrac{\ln x}{x} = -\infty$ old. dan Düşey Asmp. YOK.

$5^\circ)$ $y' = \dfrac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x = 0$

$x = e$ $(e, 1/e)$ extr.

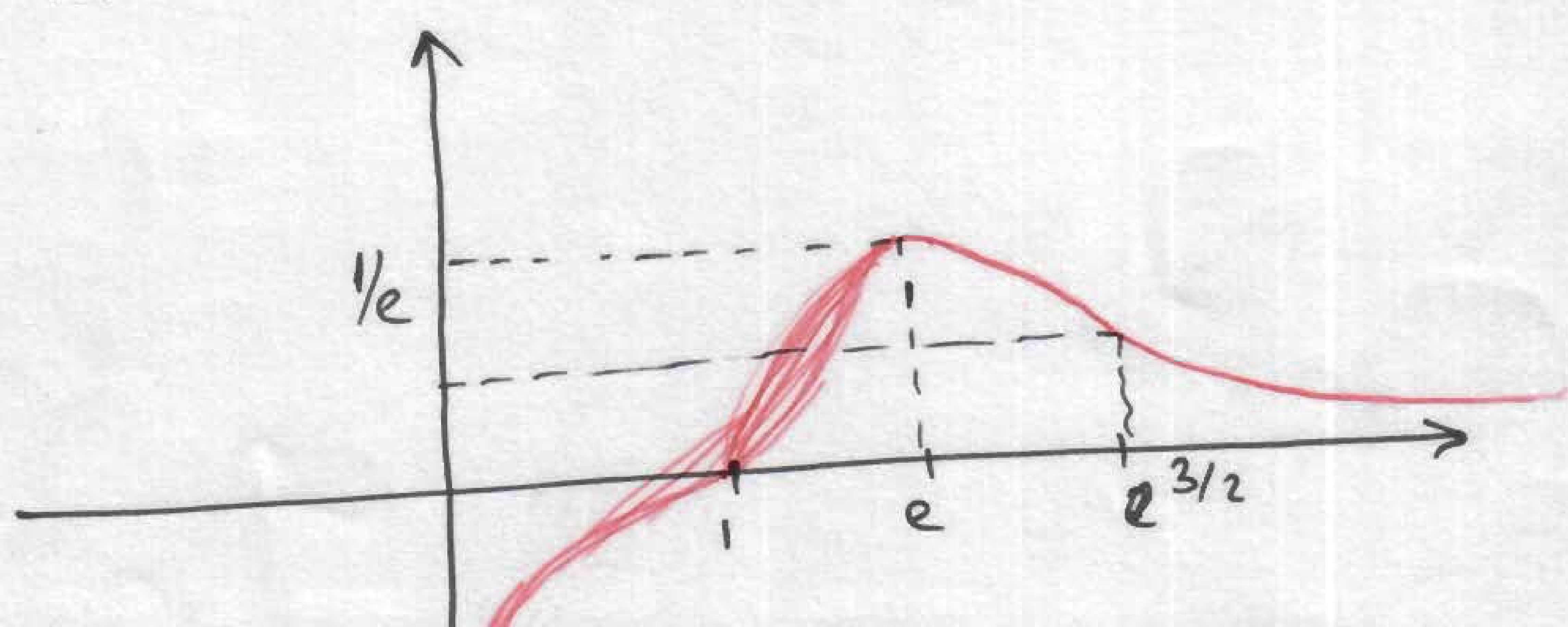
$$y'' = \frac{2\ln x - 3}{x^3}$$

$y''(e) = \dfrac{2 - 3}{e^3} = -1/e^3$ $(e, 1/e)$ yerel ext.

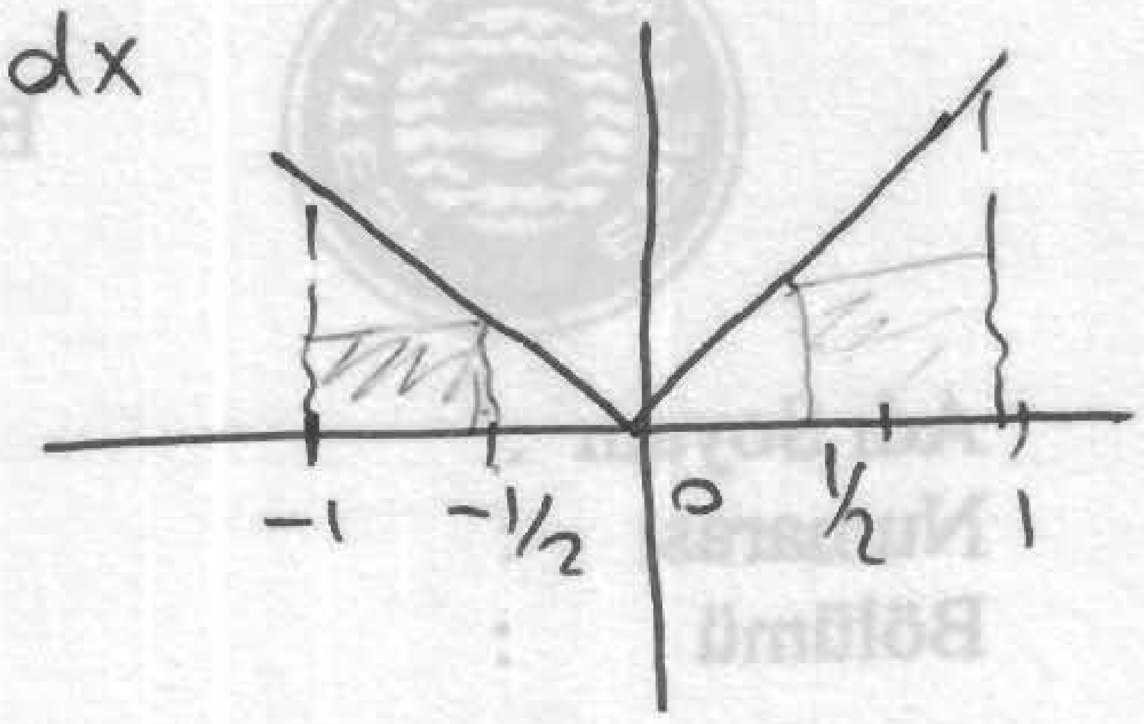
$y'' = 0 \Rightarrow 2\ln x - 3 = 0 \Rightarrow x = e^{3/2}$ Dönüm nok.

$$y(e^{3/2}) = \frac{3/2}{e^{3/2}} = \frac{3}{2e^{3/2}}$$

x	0	1	e	$e^{3/2}$	$+\infty$
y'					
y	$-\infty \nearrow$	$0 \nearrow$	$1/e \searrow$	$\frac{3}{2}e^{-3/2} \searrow$	0



2) a) $\int_{-1}^{1} |x|\, dx = -\int_{-1}^{0} x\, dx + \int_{0}^{1} x\, dx$



$$L_f(P) = \sum_{k=1}^{2} m_k \Delta x_k + \sum_{k=1}^{2} m_k \Delta_k x$$

$$= (-1)\frac{1}{2} + \left(-\frac{1}{2}\right)\cdot\frac{1}{2} + 0\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}$$

$$= -\frac{1}{2} - \frac{1}{4} + \frac{1}{4} = -1/2$$

$$U_f(P) = \sum_{k=1}^{2} M_k \Delta_k x + \sum_{k=1}^{2} M_k \Delta_k x$$

$$= \left(-\frac{1}{2}\right)\frac{1}{2} + 0\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + 1\cdot\frac{1}{2}$$

$$= -\frac{1}{4} + \frac{1}{4} + \frac{1}{2} = 1/2$$

b) $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{i}{n}} = \int_0^1 \sqrt{x}\, dx$$

$$= \frac{3}{2}\sqrt{x^3}\Big|_0^1 = \frac{3}{2}$$

3) a) $\displaystyle\int_0^1 x\, f(x)\, dx = -\int_1^0 (1-y)\, f(1-y)\, dy$

$x = 1-y$
$dx = -dy$

$$= \int_0^1 (1-y)\, f(y)\, dy$$

$$= \int_0^1 (1-x)\, f(x)\, dx$$

$$\int_0^1 x\, f(x)dx = \int_0^1 f(x)\,dx - \int_0^1 x\, f(x)\,dx$$

$$2\int_0^1 x\, f(x)\,dx = \int_0^1 f(x)\,dx$$

$$\int_0^1 x\, f(x)\,dx = \frac{1}{2}\int_0^1 f(x)\,dx$$

b) $$\lim_{x\to 0} \frac{\sin 2x + 3\int_x^0 e^{\sin t}\, dt}{2x} = \frac{0}{0} \quad \text{Hospital}$$

$$\lim_{x\to 0} \frac{2.\cos 2x - 3\, e^{\sin x} .\, 1}{2} = \frac{2-3}{2} = -\frac{1}{2} \,//$$

4) $$\left|\int_1^{\sqrt{3}} \frac{\sin x}{x^2+1}\,dx\right| \leq \int_1^{\sqrt{3}} \frac{|\sin x|}{x^2+1}\,dx \leq \int_1^{\sqrt{3}} \frac{1}{x^2+1}\,dx$$

$|\sin x| \leq 1$

$$\int_1^{\sqrt{3}} \frac{1}{x^2+1}\,dx = \arctan x \Big|_1^{\sqrt{3}} = \arctan\sqrt{3} - \arctan 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

$$\Rightarrow \left|\int_1^{\sqrt{3}} \frac{\sin x}{x^2+1}\,dx\right| \leq \frac{\pi}{12}$$

5) a) $$\int_a^b f(x)\,dx = f(c)\,(b-a) \quad \exists c \in (a,b) \ \underline{\text{ODT.}}$$

$$\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx = \frac{c}{\sqrt{c^2+9}} \cdot 4 \qquad \exists c \in (0,4)$$

$x^2+9 = u^2$

$2x\,dx = 2u\,du$

$$\int \frac{u\,du}{u} = \sqrt{x^2+9}\Big|_0^4 = 5-3 = 2$$

$$\Rightarrow 2 = \frac{c}{\sqrt{c^2+9}} \cdot 4 \Rightarrow \frac{c}{\sqrt{c^2+9}} = \frac{1}{2}$$

$$\frac{c^2}{c^2+9} = \frac{1}{4} \Rightarrow 4c^2 = c^2+9$$

$3c^2 = 9$

$c^2 = 3$

$c = \mp\sqrt{3}$

$$\underline{\underline{\exists c = \sqrt{3}}} \in (0,4)$$

5) b

$$\left(\int_0^1 \frac{\sqrt{x}\cos \pi x}{\sqrt{x^2+1}}\,dx\right)^2 \leq \int_0^1 \frac{x}{x^2+1}\,dx \cdot \int_0^1 \cos^2 \pi x\,dx$$

$$\cos^2 \pi x = \frac{1+\cos 2\pi x}{2}$$

$$f(x)=\frac{\sqrt{x}}{\sqrt{x^2+1}} \qquad g(x)=\cos \pi x$$

$$x^2+1=u$$
$$2x\,dx=du$$

$$\leq \frac{1}{2}\int \frac{du}{u} \cdot \frac{1}{2}\int_0^1 (1+\cos 2\pi x)\,dx$$

$$\leq \frac{1}{2}\ln(x^2+1)\Big|_0^1 \cdot \frac{1}{2}\left(x+\frac{\sin 2\pi x}{2\pi}\right)\Big|_0^1$$

$$= \frac{1}{2}\ln 2 \cdot \frac{1}{2}\cdot 1$$

$$= \frac{1}{4}\ln 2$$

$$\left(\int_0^1 \frac{\sqrt{x}\cos \pi x}{\sqrt{x^2+1}}\,dx\right)^2 \leq \frac{1}{4}\ln 2$$

$$\Rightarrow \left|\int_0^1 \frac{\sqrt{x}\cos \pi x}{\sqrt{x^2+1}}\,dx\right| \leq \frac{\sqrt{\ln 2}}{2}$$

6) a) $\displaystyle\int \frac{x^2}{e^{x^3}}\,dx = ?$

$x^3 = u$
$3x^2\,dx = du$

$$\Rightarrow \frac{1}{3}\int \frac{du}{e^u} = \frac{1}{3}\int e^{-u}\,du = -\frac{1}{3}\int e^{v}\,dv$$

$-u = v$
$-du = dv$

$$= -\frac{1}{3}e^{v} + C$$

$$= -\frac{1}{3}e^{-u} + C = -\frac{1}{3}e^{-x^3} + C$$

b) $\displaystyle\int \sin^4 x\,(1-\sin^2 x)^2 \cos x\,dx = ?$

$\sin x = t$
$\cos x\,dx = dt$

$$\Rightarrow \int t^4 (1-t^2)^2\,dt$$

$$= \int (t^4 - 2t^6 + t^8)\,dt$$

$$= \frac{t^5}{5} - 2\frac{t^7}{7} + \frac{t^9}{9} + C$$

$$= \frac{\sin^5 x}{5} - \frac{2\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$$

c) $\displaystyle\int \frac{x+8}{x^2+6x+12}\,dx = ?$

$$\int \frac{x+3+5}{x^2+6x+12}\,dx = \int \frac{x+3}{x^2+6x+12}\,dx + \int \frac{5}{x^2+6x+12}\,dx$$

$x^2+6x+12 = u$
$(2x+6)\,dx = du$
$(x+3)\,dx = \frac{du}{2}$

$$= \frac{1}{2}\int \frac{du}{u} + 5\int \frac{dx}{(x+3)^2+3}$$

$x+3 = t$
$dx = dt$

$$= \frac{1}{2}\ln(x^2+6x+12) + 5\int \frac{dt}{t^2+(\sqrt{3})^2}$$

$$= \frac{1}{2}\ln(x^2+6x+12) + 5\,\frac{1}{\ldots}\arctan\frac{x+3}{\ldots} + C$$

6) d) $\int (\ln x)^2 \, dx = ?$

$(\ln x)^2 = u$ $\qquad\qquad$ $dx = dv$

$2 \ln x \cdot \frac{1}{x} \, dx = du$ $\qquad\qquad$ $x = v$

$$\int u \, dv = uv - \int v \, du$$

$$= x(\ln x)^2 - 2\int \cancel{x} \cdot \ln x \cdot \frac{1}{\cancel{x}} \, dx$$

$$= x(\ln x)^2 - 2\underbrace{\int \ln x \, dx}_{\Downarrow \text{ kısmi}}$$

$\ln x = u_1$ $\qquad\qquad$ $dx = dv_1$

$\frac{1}{x} \, dx = du_1$ $\qquad\qquad$ $x = v_1$

$$= x(\ln x)^2 - 2\left[x \ln x - \int \frac{1}{x} \cdot x \, dx\right]$$

$$= x(\ln x)^2 - 2(x \ln x - x) + C$$