ALTERNATIVE OPTIMAL SOLUTIONS

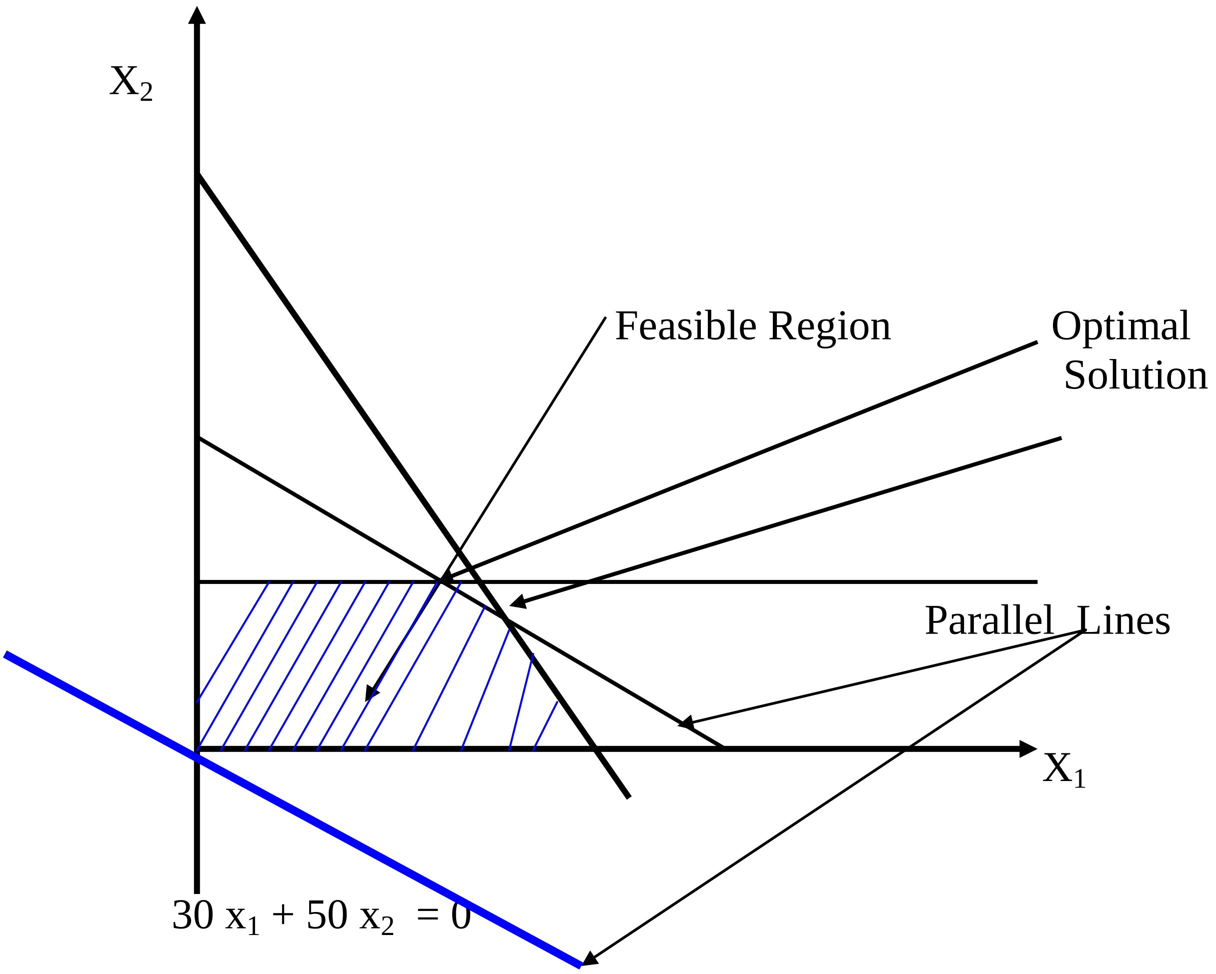
Max $Z = 30\, x_1 + 50\, x_2$

s.t

$$3\, x_1 + 5\, x_2 \le 150$$
$$x_2 \le 20$$
$$8\, x_1 + 5\, x_2 \le 300$$

$$x_1 , x_2 \ge 0$$

$x_1=50/3$, $x_2=20$ Optimal Solution
$x_1=30$, $x_2= 12$ Alternative Optima



Standard Form

$$Z - 30\,x_1 - 50\,x_2 = 0$$

$$\begin{aligned} 3\,x_1 + 5\,x_2 + x_3 \quad\quad\quad &= 150 \\ x_2 \quad + x_4 \quad\quad &= 20 \\ 8\,x_1 + 5\,x_2 \quad\quad + x_5 &= 300 \end{aligned}$$

BV(x_3,x_4,x_5)=150,20,300 NBV(x_1,x_2)=0

Initial Tableau

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_3	3	5	1	0	0	**150**	30
x_4	0	**1<<**	0	1	0	**20**	**20**<<
x_5	8	5	0	0	1	**300**	60
Z	**-30**	**-50**<	**0**	**0**	**0**	**0**	

Entering Variable : x_2

Leaving Variable : x_4

The first tableau

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_3	3<<	0	1	-5	0	**50**	16.66<
x_2	0	1	0	1	0	**20**	
x_5	8	0	0	-5	1	**200**	25
Z	**-30**	**0**	**0**	**50**	**0**	**1000**	

Entering Variable : x_1

Leaving Variable : x_3

The second tableau (Optimal tableau)

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_1	1	0	1/3	-5/3	0	**50/3**	
x_2	0	1	0	1	0	**20**	20
x_5	0	0	-8/3	25/3	1	**200/3**	8<<<
Z	0	0	10	0	0	**1500**	

?

X_1=50/3, X_2=20 Z_{MAX}= 1500 optimal solution

RULE :

When using simplex method we can recognize alternative optima Z_j equals zero for one or more of the non-basic variables in the final simplex tableau.

X_4 non-basic variable and has zero coefficient in Z_j for the final tableau.

BASIS	x_1	x_2	x_3	x_4	x_5	RHS	RATIO
x_1	1	0	-5/25	0	5/25	**30**	
x_2	0	1	8/25	0	-3/25	**12**	
x_4	0	0	-8/25	1	3/25	**8**	
Z	**0**	**0**	**10**	**0**	**0**	**1500**	

Unchanged

$X_1=30$ $X_2=12$ $Z_{MAX}= 1500$ alternative optimal solution

EXAMPLE

Two investments with varying cash flows (in thousands of dollars) are available as shown in the Table.

CASH FLOW (IN THOUSANDS AT TIME)

	0	1	2	3
INVESTMENT-1	-$6	-$5	$7	$9
INVESTMENT-2	-$8	-$3	$9	$7

- At time 0, $10000 is available for investment, and
- At time 1, $7000 is available for investment.

Assuming that r = 0.10 (We call r the annual interest rate), set up an LINEAR PPROGRAMMING whose solution maximizes the NET PRESENT VALUE (NPV) obtained from these investments.

- Assume that any fraction of an investment may be purchased.

NPV of investment 1

$$-6 - 5/1.1 + 7/(1.1)^2 + 9/(1.1)^3 = \$2.00$$

NPV of investment2

$$-8 - 3/1.1 + 9/(1.1)^2 + 7/(1.1)^3 = \$1.97$$

Let

x_1 = Fraction of investment 1 that is undertaken

and

x_2 = Fraction of investment 2 that is undertaken.

If we measure NPV in thousands of dollars we wish to solve the following LP.

$$\begin{aligned}
&\text{Max } Z = 2x_1 + 1.97\,x_2 \\
&\text{s.t} \\
&\quad 6x_1 + 8\,x_2 \le 10 \\
&\quad 5x_1 + 3\,x_2 \le 7 \\
&\quad x_1 \le 1 \\
&\quad x_2 \le 1 \\
&\quad \text{All variables} \ge 0
\end{aligned}$$

$$-Z = -2x_1 - 1.97x_2$$
$$Z - 2x_1 - 1.97x_2 = 0$$

$$\begin{aligned}
6x_1 + 8x_2 + x_3 \quad &= 10 \\
5x_1 + 3x_2 + \quad x_4 \quad &= 7 \\
x_1 \quad + x_5 \quad &= 1 \\
x_2 \quad + x_6 &= 1
\end{aligned}$$

$NBV(x_1,x_2)=0$ $\quad$ $BV(x_2, x_4, x_5, x_6)= 10,7,1,1$

SIMPLEX TABLEAU

BASIS	x_1	x_2	x_3	x_4	x_5	x_6	RHS	RATIO
x_3	6	8	1	0	0	0	**10**	1.6667
x_4	5	3	0	1	0	0	**7**	1.4
x_5	**1**<	0	0	0	1	0	**1**	1
x_6	0	1	0	0	0	1	**1**	∞
Z	**-2**<	**-1.97**	**0**	**0**	**0**	**0**	**0**	
x_3	0	**8<**	1	0	-6	0	**4**	0.5
x_4	0	3	0	1	-5	0	**2**	0.6667
x_1	1	0	0	0	1	0	**1**	∞
x_6	0	1	0	0	0	1	**1**	1
Z	**0**	**-1.97<**	**0**	**0**	**2**	**0**	**2**	
x_2	0	1	0.125	0	-0.75	0	**0.5**	
x_4	0	0	-0.375	1	-2.75	0	**0.5**	
x_1	1	0	0	0	1	0	**1**	
x_6	0	0	-0.125	0	0.75	1	**0.5**	
Z	0	0	0.2463	0	0.5275	0	**2.985**	

$x_1=1$ $x_2=0.5$ $x_3=0.0$ $x_4=0.5$ $x_5=0.0$ $x_6=0.5$ $Z_{MAX}=2.985$

NO ALTERNATIVE OPTIMA