# UNBOUNDED LP

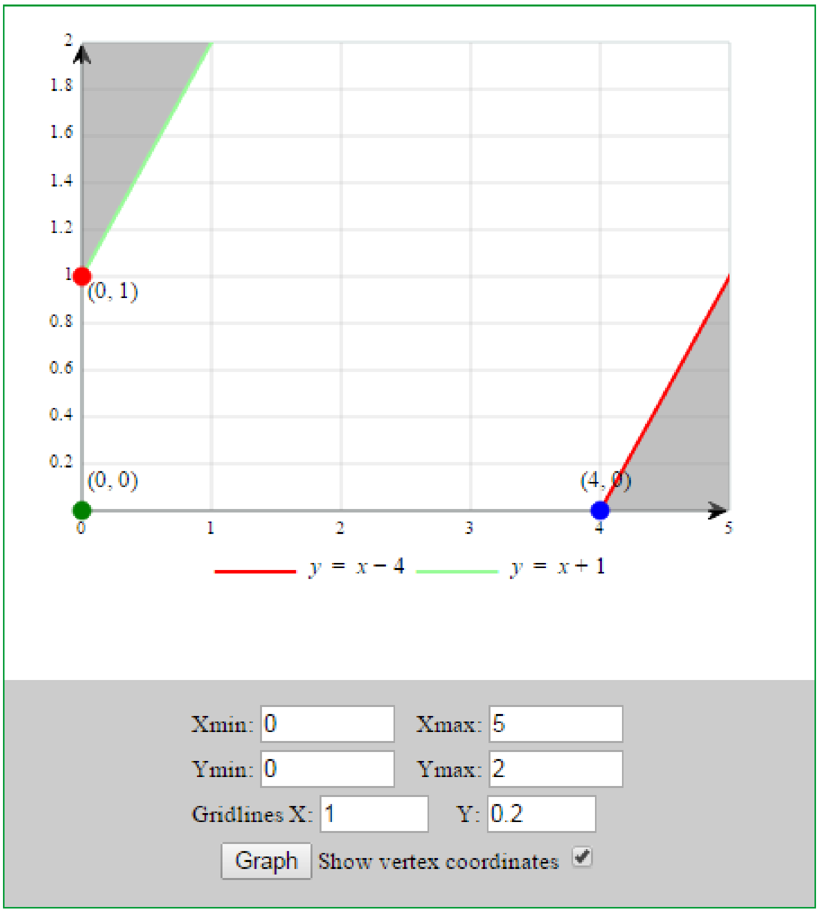
For a **max problem**, an unbounded LP occurs if it is possible to find points in the feasible region with arbitrarily large z values corresponding to a decision maker earning arbitrarily large revenues or profits.

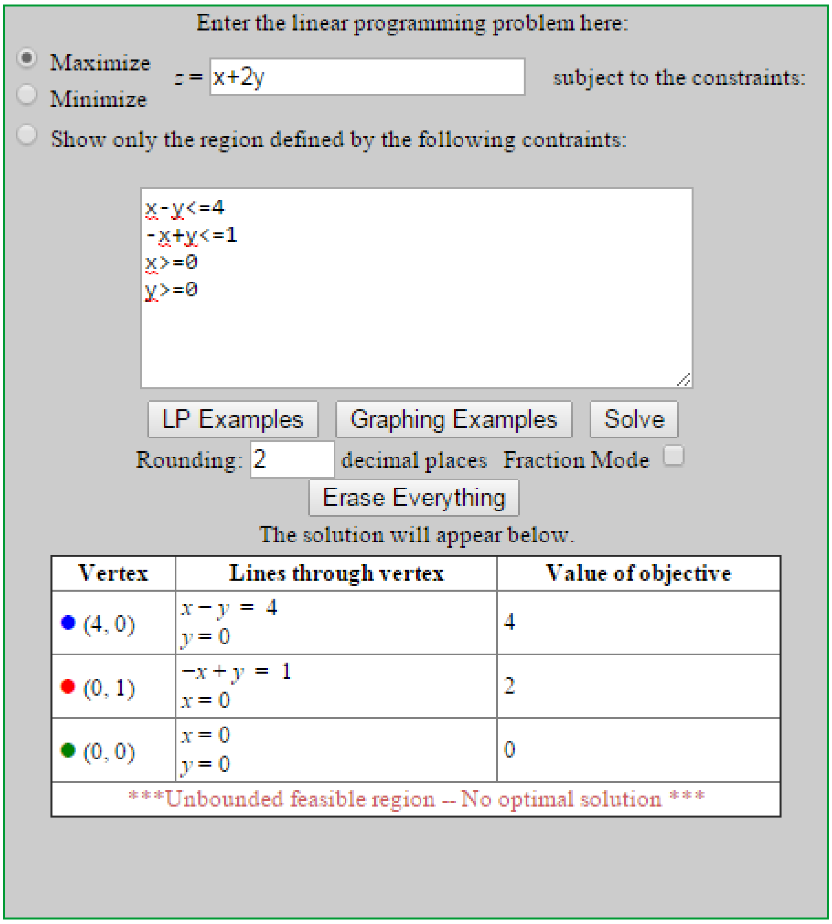
This would indicate that an unbounded optimal solution **should not occur in a correctly formulated LP.**

For a **min problem** an LP is unbounded if there are points in a feasible region with arbitrarily small z values.

**Example (UNBOUNDED LP)**

$$
\begin{aligned}
&\text{Max } Z = x_1 + 2x_2 \\
&\text{s.t} \\
&\qquad x_1 - x_2 \le 4 \\
&\qquad -x_1 + x_2 \le 1 \\
&\qquad x_1, x_2 \ge 0
\end{aligned}
$$



Enter the linear programming problem here:

Maximize

Minimize

$z$ = x+2y subject to the constraints:

Show only the region defined by the following contraints:

```
x-y<=4
-x+y<=1
x>=0
y>=0
```

LP Examples | Graphing Examples | Solve

Rounding: 2 decimal places Fraction Mode

Erase Everything

The solution will appear below.

| Vertex | Lines through vertex | Value of objective |
|---|---|---|
| ● (4, 0) | $x - y = 4$<br>$y = 0$ | 4 |
| ● (0, 1) | $-x + y = 1$<br>$x = 0$ | 2 |
| ● (0, 0) | $x = 0$<br>$y = 0$ | 0 |
| ***Unbounded feasible region -- No optimal solution *** | | |

## Standard Form

$$Z - x_1 - 2x_2 = 0$$

$$\begin{aligned} x_1 - x_2 + x_3 \quad &= 4 \\ -x_1 + x_2 \quad + x_4 &= 1 \end{aligned}$$

## Initial Simplex Tableau

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | RHS | RATIO |
|---|---|---|---|---|---|---|
| $x_3$ | 1 | -1 | 1 | 0 | **4** | |
| $x_4$ | -1 | 1<< | 0 | 1 | **1** | 1< |
| **Z** | **-1** | **-2<** | **0** | **0** | **0** | |

## Entering Variable: $x_2$

## Leaving Variable: $x_4$

## The First Improved Simplex Tableau

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | RHS | RATIO |
|---|---|---|---|---|---|---|
| $x_3$ | 0 | 0 | 1 | 1 | **5** | NONE |
| $x_2$ | -1 | 1 | 0 | 1 | **1** | |
| **Z** | **-3** | **0** | **0** | **2** | **2** | |

**Unbounded (Pivot element not available)**

## UNBOUNDED SIMPLEX TABLEAU

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | RHS | RATIO |
|---|---|---|---|---|---|---|
| $x_3$ | 1 | -1 | 1 | 0 | **4** | |
| $x_4$ | -1 | 1<< | 0 | 1 | **1** | 1< |
| **Z** | **-1** | **-2<** | **0** | **0** | **0** | |
| $x_3$ | 0 | 0 | 1 | 1 | **5** | NONE |
| $x_2$ | -1 | 1 | 0 | 1 | **1** | |
| **Z** | **-3** | **0** | **0** | **2** | **2** | |

**An unbounded LP occurs when a variable with a negative coefficient in row Z has a non-positive coefficient in each constraint (row).**

## Example (UNBOUNDED LP)

$$\text{Max } Z = 36\,x_1 + 30\,x_2 - 3\,x_3 - 4\,x_4$$
s.t
$$x_1 + x_2 - x_3 \leq 5$$
$$6\,x_1 + 5\,x_2 - x_4 \leq 10$$
$$x_i \geq 0$$

$$\mathbf{Z - 36\,x_1 - 30\,x_2 + 3\,x_3 + 4\,x_4 = 0}$$

$$\mathbf{x_1 + x_2 - x_3 + x_5 = 5}$$
$$\mathbf{6\,x_1 + 5\,x_2 - x_4 + x_6 = 10}$$

$BV=(x_5,x_6)=5,10$ $NBV=(x_1,x_2,x_3,x_4)=0$

## Initial Tableau

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | RATIO |
|---|---|---|---|---|---|---|---|---|
| $x_5$ | 1 | 1 | -1 | 0 | 1 | 0 | **5** | 5.0 |
| $x_6$ | **6<<** | 5 | 0 | -1 | 0 | 1 | **10** | 1.666< |
| **Z** | **-36<** | **-30** | **3** | **4** | **0** | 0 | **0** | |

Entering Variable : $x_1$
Leaving Variable : $x_6$

## The first tableau

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | RATIO |
|---|---|---|---|---|---|---|---|---|
| $x_5$ | 0 | 1/6 | -1 | 1/6 | 1 | -1/6 | **10/3** | 20.0<< |
| $x_1$ | 1 | 5/6 | 0 | -1/6 | 0 | 1/6 | **5/3** | |
| **Z** | **0** | **0** | **3** | **-2<** | **0** | 6 | **60** | |

**Entering Variable : $x_4$**
**Leaving Variable : $x_5$**

## The second tableau

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | RATIO |
|---|---|---|---|---|---|---|---|---|
| $x_4$ | 0 | 1 | -6 | 1 | 6 | -1 | **20** | NONE |
| $x_1$ | 1 | 1 | -1 | 0 | 1 | 0 | **20** | |
| **Z** | 0 | 2 | -9 | 0 | 12 | **4** | **100** | |

UNBOUNDED

# UNBOUNDED SIMPLEX TABLEAU

| BASIS | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | RATIO |
|---|---|---|---|---|---|---|---|---|
| $x_5$ | 1 | 1 | -1 | 0 | 1 | 0 | **5** | 5.0 |
| $x_6$ | **6<<** | 5 | 0 | -1 | 0 | 1 | **10** | 1.666< |
| **Z** | **-36<** | **-30<** | **3** | **4** | **0** | 0 | **0** | |
| $x_5$ | 0 | 1/6 | -1 | 1/6 | 1 | -1/6 | **10/3** | 20.0< |
| $x_1$ | 1 | 5/6 | 0 | -1/6 | 0 | 1/6 | **5/3** | |
| **Z** | **0** | **0** | **3** | **-2** | **0** | 6 | **60** | |
| $x_4$ | 0 | 1 | -6 | 1 | 6 | -1 | **20** | NONE |
| $x_1$ | 1 | 1 | -1 | 0 | 1 | 0 | **20** | |
| **Z** | 0 | 2 | -9 | 0 | 12 | **4** | **100** | |