Московский Физико-Технический Институт

Вычислительная математика

Компьютерное задание №1

Практическое применение явной схемы Эйлера и схемы Рунге-Кутты 4-го порядка

Модель свободных колебаний физического маятника с затуханием, описываемая уравнением:

$$lu''(t) + 2\lambda u'(t) + g \sin u(t) = 0$$

$$u(0) = u_0 \qquad u'(0) = v_0$$

Период колебаний считается равным:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Параметры l, λ, g, u_0, v_0 и период разбиения задаются внутри программы. Для расчетов используются явная схема Эйлера (EE) и схема Рунге-Кутты 4-го порядка (RK).

Для малых колебаний используется формула точного решения (AN):

$$u(t) = Ae^{-\gamma t} \sin(\omega t + \varphi)$$

$$\varphi = \arctan \frac{\omega u_0}{v_0 + \gamma u_0} \qquad A = \frac{u_0}{\sin \varphi} \qquad \gamma = \frac{\lambda}{l} \qquad \omega = \sqrt{\omega_0^2 - \gamma^2}$$

В работе рассматривается случай: $\omega_0^2 > \gamma^2$.
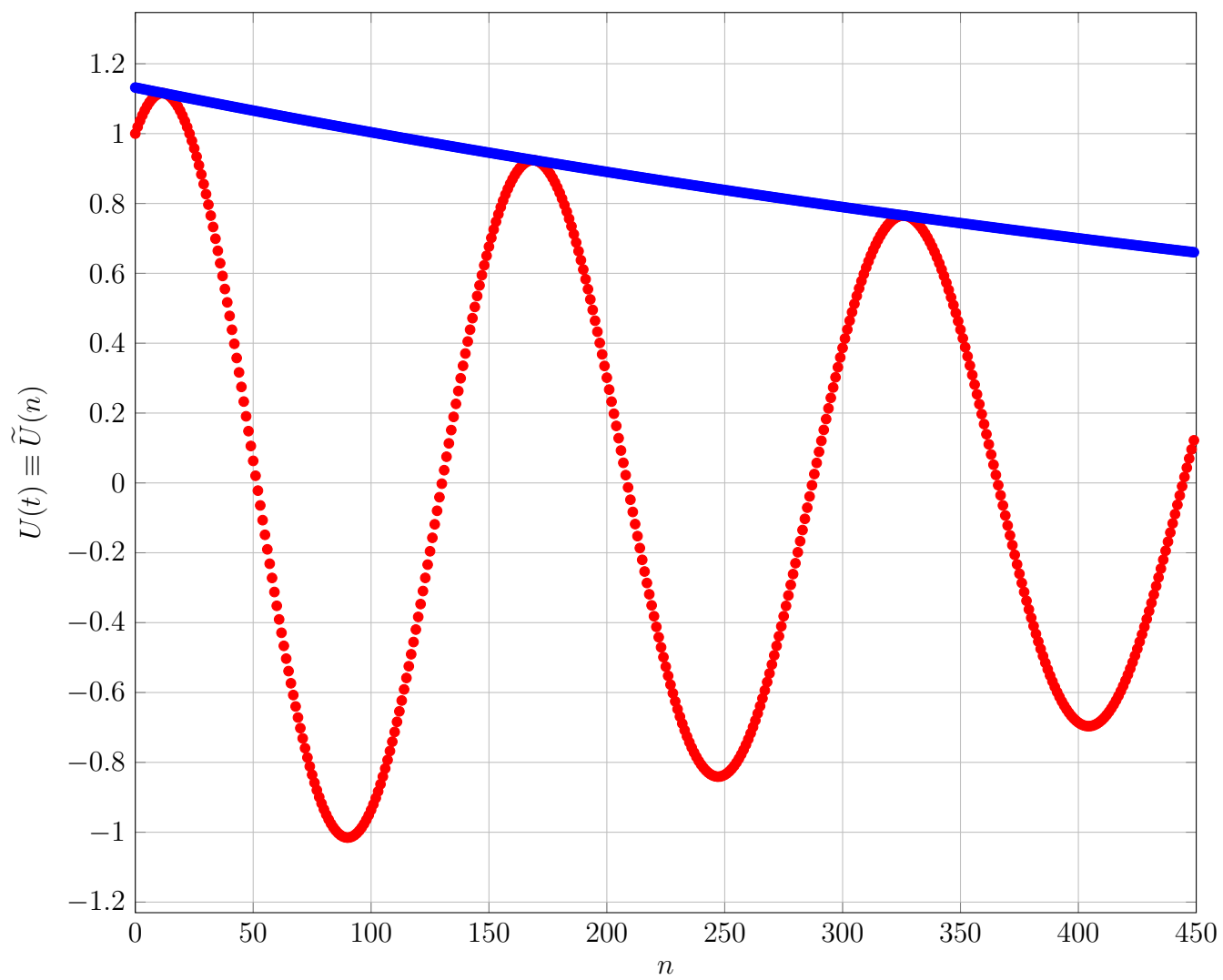
Выполнил:
Р.Р. Валиев, 715 гр.
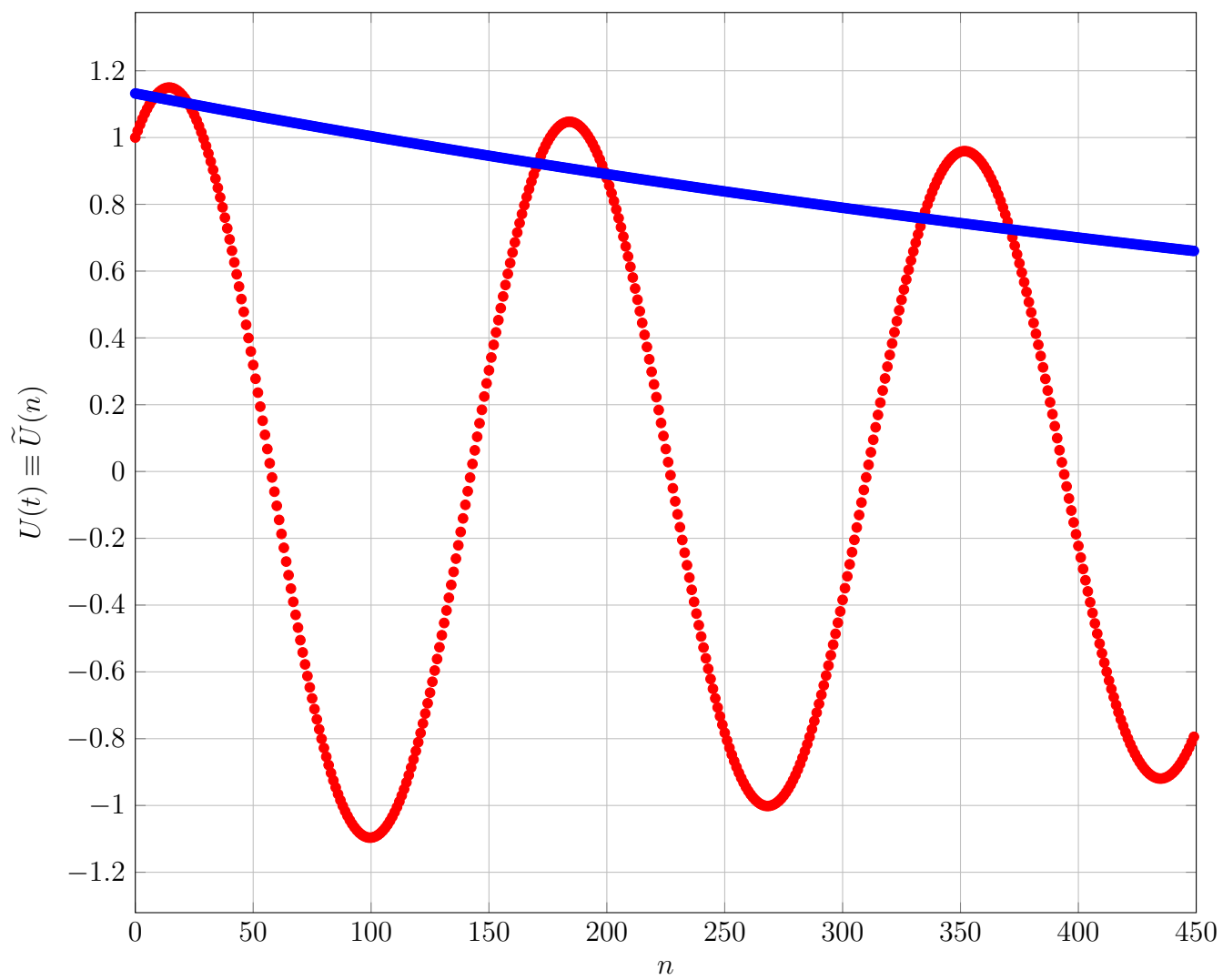

Проверил:
Н.Б. Явич

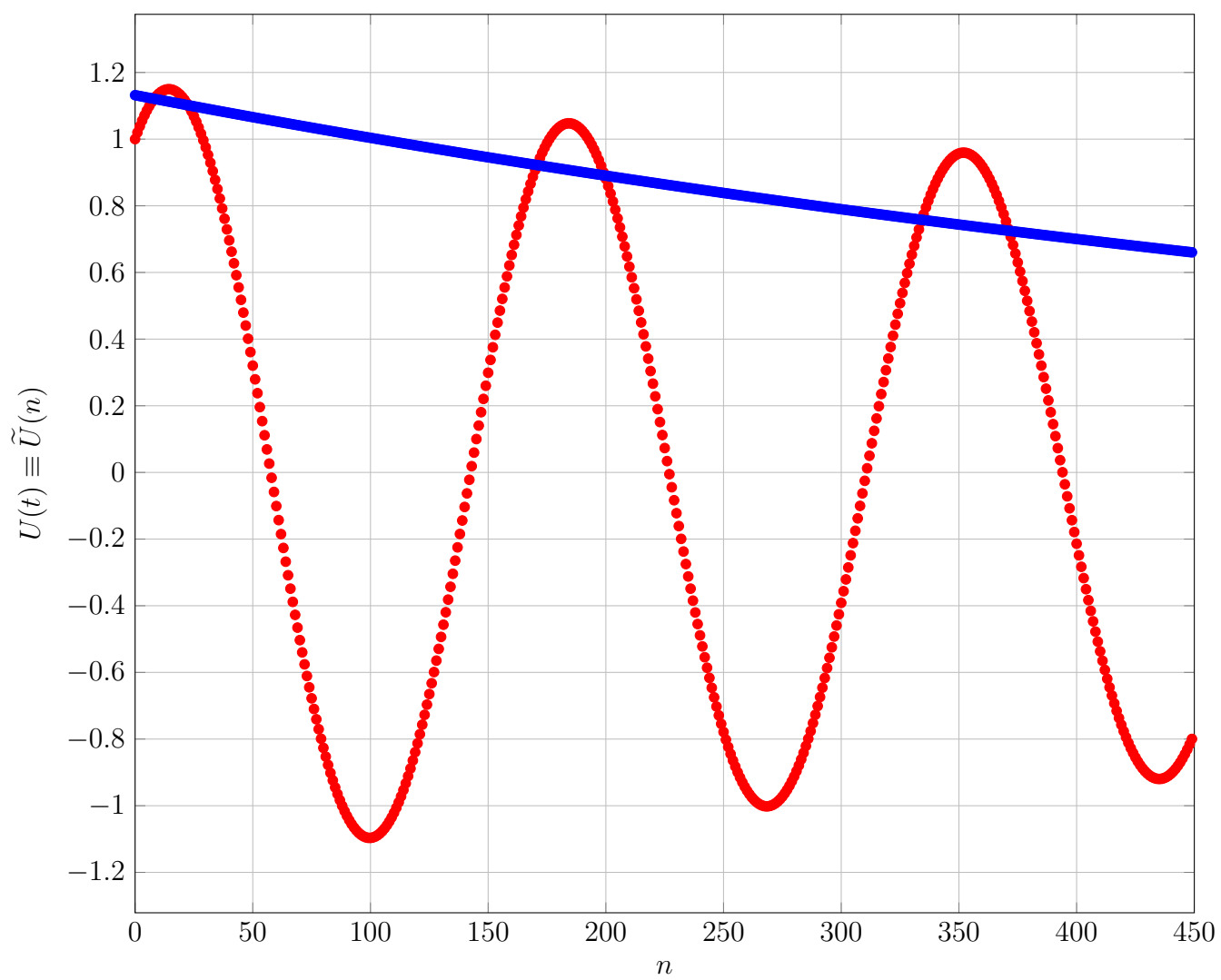Рис. 1: Exact solution (AN)

Рис. 2: Explicit Euler method (EE)

Рис. 3: Runge-Kutta 4th order method (RK)

n	AN	EE	RK
0	1	1	1
1	1.01917	1.02	1.02
2	1.03667	1.03861	1.03861
3	1.05247	1.0558	1.05581
4	1.06655	1.07158	1.07159
5	1.07889	1.08593	1.08595
6	1.08947	1.09884	1.09886
7	1.0983	1.1103	1.11033
8	1.10534	1.12031	1.12035
9	1.1106	1.12886	1.12892
10	1.11408	1.13595	1.13603
11	1.11576	1.14158	1.14167
12	1.11566	1.14574	1.14585
13	1.11378	1.14844	1.14857
14	1.11012	1.14967	1.14982
15	1.1047	1.14944	1.14962
16	1.09753	1.14775	1.14796
17	1.08861	1.14461	1.14484
18	1.07798	1.14001	1.14027
19	1.06566	1.13397	1.13426
20	1.05166	1.12649	1.12681
21	1.03601	1.11758	1.11793
22	1.01875	1.10724	1.10763
23	0.999894	1.09549	1.09591
24	0.97949	1.08234	1.0828
25	0.95757	1.0678	1.06829
26	0.934173	1.05188	1.05241
27	0.909339	1.03459	1.03517
28	0.883111	1.01596	1.01658
29	0.855535	0.995996	0.996654
30	0.826659	0.97472	0.975421
31	0.79653	0.952152	0.952898
32	0.765201	0.928315	0.929106
33	0.732725	0.903231	0.904068
34	0.699155	0.876926	0.87781
35	0.664549	0.849428	0.850359
36	0.628964	0.820766	0.821744
37	0.592459	0.790972	0.791997
38	0.555096	0.760078	0.761151
39	0.516934	0.728121	0.729241
40	0.478038	0.695138	0.696306
41	0.438472	0.661169	0.662384
42	0.3983	0.626257	0.627519
43	0.357588	0.590447	0.591755
44	0.316401	0.553784	0.555138
45	0.274808	0.516319	0.517718
46	0.232876	0.478102	0.479545
47	0.190672	0.439188	0.440673
48	0.148265	0.39963	0.401156
49	0.105722	0.359487	0.361052

Таблица 1: Point comparison

n	AN	EE	RK
50	0.0631123	0.318818	0.320421
51	0.020504	0.277683	0.279323
52	-0.022035	0.236146	0.237819
53	-0.0644368	0.19427	0.195975
54	-0.106634	0.15212	0.153854
55	-0.14856	0.109762	0.111523
56	-0.190147	0.0672638	0.0690492
57	-0.231332	0.024692	0.0264989
58	-0.272048	-0.0178851	-0.0160597
59	-0.312231	-0.0603996	-0.0585585
60	-0.35182	-0.102783	-0.10093
61	-0.390751	-0.144969	-0.143106
62	-0.428964	-0.186889	-0.18502
63	-0.466401	-0.228477	-0.226606
64	-0.503003	-0.269669	-0.267798
65	-0.538713	-0.310399	-0.308532
66	-0.573477	-0.350605	-0.348746
67	-0.607241	-0.390225	-0.388378
68	-0.639953	-0.429202	-0.427369
69	-0.671565	-0.467476	-0.465661
70	-0.702028	-0.504992	-0.5032
71	-0.731297	-0.541697	-0.53993
72	-0.759326	-0.57754	-0.575802
73	-0.786075	-0.612472	-0.610767
74	-0.811504	-0.646447	-0.644777
75	-0.835575	-0.67942	-0.677789
76	-0.858254	-0.711351	-0.709762
77	-0.879506	-0.742199	-0.740656
78	-0.899303	-0.771929	-0.770435
79	-0.917615	-0.800506	-0.799064
80	-0.934417	-0.827899	-0.826511
81	-0.949685	-0.854077	-0.852747
82	-0.9634	-0.879014	-0.877745
83	-0.975542	-0.902685	-0.901479
84	-0.986096	-0.925067	-0.923927
85	-0.995049	-0.946139	-0.945068
86	-1.00239	-0.965883	-0.964883
87	-1.00811	-0.984281	-0.983355
88	-1.01221	-1.00132	-1.00047
89	-1.01468	-1.01698	-1.01621
90	-1.01553	-1.03126	-1.03057
91	-1.01475	-1.04415	-1.04354
92	-1.01234	-1.05563	-1.05511
93	-1.00833	-1.0657	-1.06526
94	-1.00272	-1.07435	-1.07401
95	-0.995515	-1.08159	-1.08133
96	-0.986739	-1.0874	-1.08723
97	-0.976407	-1.09178	-1.09171
98	-0.96454	-1.09473	-1.09476
99	-0.95116	-1.09626	-1.09638

Таблица 2: Point comparison

n	AN	EE	RK
100	-0.936292	-1.09636	-1.09658
101	-0.919964	-1.09504	-1.09536
102	-0.902205	-1.0923	-1.09272
103	-0.883047	-1.08814	-1.08867
104	-0.862524	-1.08258	-1.0832
105	-0.840671	-1.07561	-1.07633
106	-0.817528	-1.06724	-1.06807
107	-0.793134	-1.05748	-1.05842
108	-0.767532	-1.04635	-1.0474
109	-0.740764	-1.03385	-1.03501
110	-0.712877	-1.01999	-1.02126
111	-0.683917	-1.0048	-1.00617
112	-0.653934	-0.988273	-0.989755
113	-0.622978	-0.970438	-0.97203
114	-0.591101	-0.951311	-0.953011
115	-0.558356	-0.930908	-0.932718
116	-0.524797	-0.909252	-0.91117
117	-0.49048	-0.886365	-0.888391
118	-0.455461	-0.86227	-0.864403
119	-0.419799	-0.836993	-0.839233
120	-0.383551	-0.810562	-0.812908
121	-0.346778	-0.783007	-0.785456
122	-0.309538	-0.754357	-0.756909
123	-0.271893	-0.724648	-0.727301
124	-0.233904	-0.693915	-0.696667
125	-0.195632	-0.662195	-0.665043
126	-0.15714	-0.629527	-0.63247
127	-0.118488	-0.595954	-0.598988
128	-0.0797403	-0.56152	-0.564642
129	-0.0409576	-0.526271	-0.529478
130	-0.0022025	-0.490253	-0.493543
131	0.0364633	-0.453519	-0.456886
132	0.0749781	-0.41612	-0.419561
133	0.113281	-0.378109	-0.38162
134	0.151311	-0.339543	-0.343119
135	0.189007	-0.300478	-0.304115
136	0.226312	-0.260975	-0.264666
137	0.263166	-0.221093	-0.224834
138	0.29951	-0.180893	-0.18468
139	0.335289	-0.14044	-0.144265
140	0.370447	-0.099795	-0.103654
141	0.404928	-0.059024	-0.0629098
142	0.438679	-0.0181915	-0.0220985
143	0.471649	0.0226374	0.0187156
144	0.503786	0.0633974	0.059467
145	0.535041	0.104023	0.100091
146	0.565366	0.14445	0.140522
147	0.594715	0.184614	0.180697
148	0.623044	0.224452	0.220552
149	0.650309	0.263899	0.260024

Таблица 3: Point comparison

n	AN	EE	RK
150	0.676469	0.302897	0.299052
151	0.701486	0.341383	0.337576
152	0.725322	0.379299	0.375536
153	0.747942	0.416589	0.412876
154	0.769313	0.453197	0.44954
155	0.789403	0.48907	0.485475
156	0.808184	0.524156	0.52063
157	0.825628	0.558406	0.554955
158	0.841711	0.591773	0.588402
159	0.856411	0.624212	0.620927
160	0.869707	0.655681	0.652487
161	0.881581	0.68614	0.683042
162	0.892019	0.715549	0.712554
163	0.901006	0.743875	0.740986
164	0.908531	0.771082	0.768305
165	0.914587	0.797141	0.794479
166	0.919168	0.822023	0.819481
167	0.922268	0.8457	0.843282
168	0.923887	0.868148	0.865859
169	0.924026	0.889345	0.887188
170	0.922689	0.90927	0.907248
171	0.91988	0.927905	0.926022
172	0.915608	0.945232	0.943491
173	0.909884	0.961237	0.959641
174	0.902719	0.975907	0.974459
175	0.894129	0.98923	0.987933
176	0.884131	1.0012	1.00005
177	0.872744	1.0118	1.01081
178	0.859989	1.02102	1.02019
179	0.845892	1.02887	1.0282
180	0.830476	1.03534	1.03483
181	0.813771	1.04042	1.04007
182	0.795805	1.04411	1.04393
183	0.77661	1.04641	1.0464
184	0.756221	1.04732	1.04748
185	0.734672	1.04685	1.04717
186	0.712001	1.04499	1.04549
187	0.688247	1.04175	1.04242
188	0.66345	1.03713	1.03798
189	0.637652	1.03115	1.03216
190	0.610897	1.0238	1.02499
191	0.58323	1.01509	1.01646
192	0.554698	1.00504	1.00658
193	0.525348	0.99366	0.995374
194	0.495228	0.980952	0.98284
195	0.46439	0.966934	0.968996
196	0.432883	0.951619	0.953855
197	0.40076	0.935024	0.937433
198	0.368074	0.917167	0.919747
199	0.334878	0.898064	0.900816

Таблица 4: Point comparison

n	AN	EE	RK
200	0.301226	0.877737	0.880658
201	0.267174	0.856208	0.859296
202	0.232777	0.833499	0.836753
203	0.19809	0.809637	0.813054
204	0.16317	0.784647	0.788225
205	0.128072	0.758559	0.762294
206	0.0928544	0.731402	0.735293
207	0.0575727	0.703211	0.707253
208	0.0222835	0.674018	0.678207
209	-0.0129567	0.643861	0.648194
210	-0.0480917	0.612777	0.617249
211	-0.0830657	0.580808	0.585414
212	-0.117823	0.547995	0.55273
213	-0.152309	0.514383	0.519242
214	-0.186469	0.480018	0.484994
215	-0.220249	0.444948	0.450036
216	-0.253596	0.409224	0.414417
217	-0.286458	0.372897	0.378188
218	-0.318783	0.33602	0.341402
219	-0.350521	0.298649	0.304114
220	-0.381624	0.26084	0.266381
221	-0.412042	0.222651	0.22826
222	-0.441729	0.184141	0.189809
223	-0.470638	0.14537	0.151089
224	-0.498727	0.106399	0.11216
225	-0.525951	0.0672899	0.0730839
226	-0.552269	0.0281048	0.0339226
227	-0.577642	-0.0110938	-0.0052615
228	-0.602031	-0.0502433	-0.0444059
229	-0.6254	-0.0892811	-0.083448
230	-0.647714	-0.128145	-0.122326
231	-0.668939	-0.166773	-0.160977
232	-0.689044	-0.205103	-0.19934
233	-0.708	-0.243076	-0.237356
234	-0.725779	-0.280633	-0.274964
235	-0.742356	-0.317713	-0.312106
236	-0.757707	-0.354262	-0.348725
237	-0.771811	-0.390223	-0.384765
238	-0.784647	-0.425543	-0.420173
239	-0.796199	-0.460169	-0.454896
240	-0.806451	-0.494052	-0.488885
241	-0.81539	-0.527143	-0.522089
242	-0.823004	-0.559396	-0.554464
243	-0.829285	-0.590766	-0.585964
244	-0.834227	-0.621212	-0.616547
245	-0.837823	-0.650694	-0.646173
246	-0.840072	-0.679174	-0.674804
247	-0.840973	-0.706616	-0.702404
248	-0.840528	-0.732988	-0.728939
249	-0.838741	-0.758257	-0.754379

Таблица 5: Point comparison

n	AN	EE	RK
250	-0.835619	-0.782395	-0.778693
251	-0.831168	-0.805375	-0.801854
252	-0.8254	-0.827172	-0.823837
253	-0.818328	-0.847763	-0.84462
254	-0.809964	-0.867126	-0.864179
255	-0.800326	-0.885244	-0.882497
256	-0.789433	-0.902098	-0.899556
257	-0.777304	-0.917673	-0.915339
258	-0.763963	-0.931955	-0.929834
259	-0.749432	-0.944933	-0.943026
260	-0.733739	-0.956594	-0.954906
261	-0.716912	-0.966931	-0.965465
262	-0.698979	-0.975936	-0.974693
263	-0.679972	-0.983602	-0.982585
264	-0.659924	-0.989925	-0.989136
265	-0.63887	-0.9949	-0.994342
266	-0.616845	-0.998526	-0.9982
267	-0.593888	-1.0008	-1.00071
268	-0.570036	-1.00173	-1.00187
269	-0.545331	-1.0013	-1.00168
270	-0.519814	-0.999529	-1.00015
271	-0.493527	-0.996414	-0.997268
272	-0.466515	-0.991961	-0.993054
273	-0.438822	-0.986175	-0.987506
274	-0.410495	-0.979064	-0.980634
275	-0.381579	-0.970636	-0.972444
276	-0.352123	-0.9609	-0.962946
277	-0.322175	-0.949866	-0.95215
278	-0.291783	-0.937548	-0.940068
279	-0.260999	-0.923958	-0.926714
280	-0.229871	-0.909111	-0.912101
281	-0.198451	-0.893023	-0.896244
282	-0.166789	-0.875711	-0.879163
283	-0.134936	-0.857194	-0.860874
284	-0.102944	-0.837493	-0.841398
285	-0.0708638	-0.816629	-0.820756
286	-0.0387476	-0.794627	-0.798973
287	-0.00664641	-0.771511	-0.776073
288	0.0253884	-0.747309	-0.752082
289	0.0573059	-0.72205	-0.727029
290	0.0890552	-0.695764	-0.700945
291	0.120586	-0.668483	-0.673861
292	0.151849	-0.640243	-0.645811
293	0.182794	-0.611078	-0.616832
294	0.213373	-0.581028	-0.58696
295	0.243538	-0.550131	-0.556235
296	0.273241	-0.518431	-0.524699
297	0.302436	-0.48597	-0.492394
298	0.331078	-0.452795	-0.459367
299	0.359123	-0.418952	-0.425663

Таблица 6: Point comparison

n	AN	EE	RK
300	0.386526	-0.384489	-0.391331
301	0.413246	-0.349459	-0.356422
302	0.439242	-0.313913	-0.320987
303	0.464473	-0.277904	-0.285079
304	0.488902	-0.241488	-0.248753
305	0.512492	-0.20472	-0.212065
306	0.535205	-0.167657	-0.175071
307	0.557009	-0.130359	-0.13783
308	0.577871	-0.0928824	-0.100399
309	0.59776	-0.0552882	-0.0628392
310	0.616645	-0.0176358	-0.0252087
311	0.6345	0.0200147	0.0124319
312	0.651299	0.057603	0.0500225
313	0.667016	0.095069	0.0875032
314	0.68163	0.132353	0.124814
315	0.69512	0.169396	0.161896
316	0.707466	0.206138	0.19869
317	0.718653	0.242523	0.235138
318	0.728664	0.278493	0.271184
319	0.737488	0.313992	0.30677
320	0.745111	0.348967	0.341843
321	0.751526	0.383363	0.37635
322	0.756725	0.417129	0.410238
323	0.760702	0.450216	0.443458
324	0.763454	0.482576	0.47596
325	0.764979	0.514161	0.507699
326	0.765278	0.544928	0.53863
327	0.764354	0.574835	0.56871
328	0.762211	0.60384	0.597898
329	0.758855	0.631906	0.626155
330	0.754295	0.658996	0.653445
331	0.74854	0.685075	0.679733
332	0.741603	0.710113	0.704986
333	0.733497	0.734078	0.729174
334	0.724239	0.756942	0.752269
335	0.713846	0.77868	0.774244
336	0.702337	0.799267	0.795074
337	0.689734	0.818681	0.814736
338	0.676059	0.836901	0.833211
339	0.661336	0.853909	0.850479
340	0.645591	0.869688	0.866522
341	0.628853	0.884223	0.881327
342	0.611151	0.8975	0.894877
343	0.592514	0.909509	0.907162
344	0.572975	0.920237	0.918171
345	0.552567	0.929677	0.927895
346	0.531325	0.937822	0.936326
347	0.509285	0.944664	0.943457
348	0.486485	0.9502	0.949285
349	0.462961	0.954426	0.953804

Таблица 7: Point comparison

n	AN	EE	RK
350	0.438755	0.957341	0.957014
351	0.413905	0.958942	0.958912
352	0.388453	0.959232	0.959499
353	0.362442	0.958211	0.958777
354	0.335914	0.955883	0.956748
355	0.308913	0.952251	0.953416
356	0.281483	0.947321	0.948786
357	0.253668	0.9411	0.942863
358	0.225516	0.933594	0.935656
359	0.19707	0.924813	0.927173
360	0.168378	0.914767	0.917424
361	0.139485	0.903468	0.90642
362	0.110439	0.890928	0.894174
363	0.0812859	0.877162	0.880699
364	0.0520728	0.862184	0.866009
365	0.0228466	0.846011	0.850123
366	-0.0063461	0.828663	0.833058
367	-0.0354587	0.810159	0.814832
368	-0.0644448	0.790519	0.795468
369	-0.0932584	0.769768	0.774987
370	-0.121854	0.74793	0.753415
371	-0.150186	0.72503	0.730775
372	-0.178211	0.701097	0.707097
373	-0.205883	0.676161	0.682409
374	-0.23316	0.650252	0.656742
375	-0.26	0.623404	0.630128
376	-0.286359	0.595652	0.602603
377	-0.312198	0.567033	0.574202
378	-0.337476	0.537584	0.544963
379	-0.362154	0.507347	0.514926
380	-0.386194	0.476363	0.484133
381	-0.40956	0.444676	0.452626
382	-0.432215	0.412331	0.420452
383	-0.454125	0.379376	0.387655
384	-0.475257	0.345859	0.354285
385	-0.495579	0.311829	0.320391
386	-0.51506	0.277339	0.286024
387	-0.533672	0.242441	0.251236
388	-0.551386	0.207188	0.21608
389	-0.568177	0.171636	0.180611
390	-0.58402	0.13584	0.144885
391	-0.598891	0.0998567	0.108957
392	-0.61277	0.0637431	0.0728845
393	-0.625637	0.0275566	0.0367248
394	-0.637473	-0.00864492	0.000535422
395	-0.648262	-0.0448036	-0.0356259
396	-0.657989	-0.0808617	-0.0717014
397	-0.666642	-0.116762	-0.107633
398	-0.674209	-0.152446	-0.143365
399	-0.68068	-0.187859	-0.178839

Таблица 8: Point comparison

n	AN	EE	RK
400	-0.686048	-0.222943	-0.214
401	-0.690307	-0.257645	-0.248792
402	-0.693453	-0.291909	-0.283162
403	-0.695483	-0.325684	-0.317055
404	-0.696397	-0.358917	-0.350421
405	-0.696196	-0.391558	-0.383209
406	-0.694883	-0.423559	-0.415369
407	-0.692463	-0.454873	-0.446855
408	-0.688943	-0.485453	-0.47762
409	-0.68433	-0.515258	-0.507621
410	-0.678634	-0.544244	-0.536816
411	-0.671868	-0.572373	-0.565164
412	-0.664044	-0.599605	-0.592626
413	-0.655179	-0.625905	-0.619167
414	-0.645287	-0.65124	-0.644752
415	-0.634388	-0.675576	-0.669348
416	-0.622502	-0.698884	-0.692924
417	-0.609649	-0.721136	-0.715453
418	-0.595853	-0.742304	-0.736906
419	-0.581138	-0.762366	-0.75726
420	-0.56553	-0.781298	-0.776491
421	-0.549056	-0.799079	-0.794578
422	-0.531743	-0.815691	-0.811502
423	-0.513623	-0.831117	-0.827245
424	-0.494725	-0.84534	-0.841792
425	-0.475082	-0.858347	-0.855127
426	-0.454727	-0.870126	-0.867238
427	-0.433695	-0.880666	-0.878114
428	-0.412019	-0.889957	-0.887746
429	-0.389738	-0.897992	-0.896124
430	-0.366886	-0.904765	-0.903243
431	-0.343504	-0.91027	-0.909097
432	-0.319629	-0.914504	-0.913682
433	-0.2953	-0.917464	-0.916995
434	-0.270557	-0.91915	-0.919035
435	-0.245442	-0.919561	-0.919801
436	-0.219995	-0.918699	-0.919296
437	-0.194257	-0.916566	-0.91752
438	-0.16827	-0.913167	-0.914477
439	-0.142077	-0.908506	-0.910173
440	-0.11572	-0.90259	-0.904613
441	-0.0892406	-0.895427	-0.897805
442	-0.0626823	-0.887025	-0.889756
443	-0.0360876	-0.877394	-0.880477
444	-0.00949899	-0.866546	-0.869979
445	0.0170411	-0.854494	-0.858274
446	0.0434903	-0.841251	-0.845375
447	0.0698067	-0.826834	-0.831298
448	0.0959484	-0.811258	-0.816059
449	0.121874	-0.794542	-0.799676

Таблица 9: Point comparison