

# Digital Closed-Loop Control Based on Adaptive Filter for Drive Mode of a MEMS Gyroscope

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**Abstract**—This paper describes the design of a digital closed drive loop for a MEMS vibratory, vacuum packaged gyroscope. The displacement of the gyroscope is demodulated by the adaptive filter-least mean square (LMS) though which the amplitude and phase of the displacement are separated for respective control. The amplitude is kept constant by the automatic gain control (AGC) method with a proportion-integral (PI) controller while the phase is controlled by phase locked loop (PLL). Results of experiments carried out on field-programmed-gate-array (FPGA) with a doubly decoupled bulk gyroscope are shown to be in close agreement with simulation results. The amplitude variance of the detect signal of drive mode is 28ppm in half an hour while the phase variance is 770ppm.

## I. INTRODUCTION

With the development of the micro-electronics technology, MEMS (Micro Electro Mechanical Systems) gyroscope, which is a sensor for measuring angular displacement, has found application in several field, including automotive industry, navigation system and also consumer electronics. Most MEMS gyroscopes are based on the Coriolis coupling effect, which transfers the energy from the drive mode to sense mode by inducing the Coriolis force in the sense direction. In the simplest implementation, shown in Fig.1, the device is realized by a single mass, suspended over a substrate by silicon spring. For operation, the mass is driven into vibration in the x-direction with the driving frequency. When the sensor rotates around the z-axis with an angular rate of  $\Omega$ , an alternating force in the y-direction is induced by the Coriolis force, according to the following equation:

$$F_c = -2m_x\Omega \times v_x \quad (1)$$

Where  $m_x$  is the effective mass of the drive mode,  $\Omega$  is the angular velocity around the z-axis and  $v_x$  is the vibration velocity of the drive mode. The vibration amplitude in the y-direction can be used as a measure of angular rate  $\Omega$ . The two-dimensional vibration system with two orthogonal vibration modes can be described as follows:

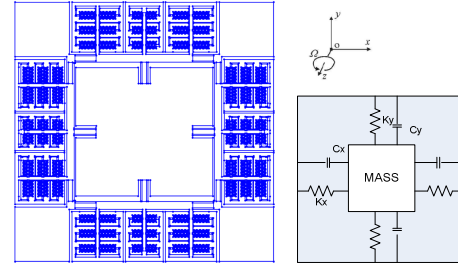


Figure 1. A simplified model for a vibratory gyroscope

$$m_x \frac{du_x^2}{dt^2} + c_x \frac{du_x}{dt} + k_x u_x = F_{el} = F_e \sin(w_d \cdot t) \quad (2)$$

$$m_y \frac{du_y^2}{dt^2} + c_y \frac{du_y}{dt} + k_y u_y = F_c \quad (3)$$

Where  $m_x, m_y$  are the effective mass in each mode,  $k_x, k_y$  are the stiffness coefficient,  $u_x, u_y$  are the displacement of the central mass,  $F_{el}$  is the external force in x-direction and  $F_e$  is the Coriolis force. Here a vacuum symmetrical doubly decoupled gyroscope is designed and used [1].

Since the Coriolis force is proportional to the external angular rate only on the premise of a non-varied velocity along the drive axis, it is fundamental for the control circuits to maintain a constant vibration. However, the gyroscope suffered from change of amplitude and phase of the vibration displacement as suspending springs' stiffness varies with environmental parameters. In recent years, many efforts have been concentrated on the closed-loop control of the gyroscope to develop more robust and better performance devices. A commonly used approach is utilizing AGC to adjust the displacement amplitude of the vibration to a target reference value. The design in [2] employs the analog AGC circuit to realize the self-oscillation of the gyroscope. These functions

can also be implemented by digital electronics, which are widely used nowadays because of the flexibility in algorithms and insensitivity to environmental parameters. And the amplitude and phase of the signals can be separated in digital electronics. In [3], a digital AGC controller with switching capacitors technology is used to fulfill the closed-loop control task.

The Least mean square (LMS) algorithm was proposed by Widrow and Hof in 1959 and started playing a very big role in adaptive technology. A considerable amount of research has done to improve the performance since then. In [4], a derivation of the normalized LMS algorithm is generalized, which can benefit parameter choosing for good performance. As a matter of fact, the coefficients of the normalized LMS filter can work as a good demodulator [5]. The demodulation with mutually orthogonal carrier can get results with separate amplitude and phase information.

In this paper, a digital closed-loop control system based on the adaptive filter for the gyroscope is proposed, in which a two-dimension LMS filter is used to separate amplitude from phase of the gyro's displacement for respective control. A printed circuit board composed of FPGA and relevant peripheral circuit is manufactured mounted with a Z-axis doubly decoupled bulk gyroscope to evaluate the control performance.

## II. CONTROL SYSTEM DESIGN

### A. Closed-Loop Control

Closed-loop control is needed since the driving mass has to oscillate with controlled amplitude and frequency in order to guarantee a stable sensitivity of the sensor and a synchronous demodulation can be performed at the sensing stage.

In the device we consider, the driving mass is electrostatically actuated using comb fingers electrodes, which act as a linear force transducer. Separated electrodes are employed to measure the motion of driving mass along drive axis. These electrodes are sensed by a C/V converter and an instrument amplifier, whose output is proportional to the displacement of the

driving mass. So, a closed-loop control on can be easily implemented.

Without regard to the non-ideal cross-coupling effect and nonlinear vibration phenomenon, the drive mode of the gyroscope is modeled as a second-order mass-spring-damping system with dynamics as (2) or:

$$\frac{du_x^2}{dt^2} + \left( \frac{c_x}{m_x} - \frac{F_e \sin(w_d \cdot t)}{m_x} \right) \frac{du_x}{dt} + w_x^2 u_x = 0 \quad (4)$$

To maintain the oscillations at constant amplitude, the damping must be regulated to zero then the system become a linear system with constant amplitude  $u$  and the analytic solution can be expressed as:

$$u_x = u \cos(w_x t + \theta) \quad (5)$$

Previous works have proposed a control scheme to realize the self-oscillation of the gyroscope with a AGC [2][3]. But it is difficult to obtain the phase information of the vibration displacement. As the quadrature error, which is mainly induced by stiffness coupling, is in phase with the vibration displacement of the drive mode and has a phase difference of about  $90^\circ$  with the Coriolis force[5], it can be eliminated for smaller zero bias if we can keep the phase information a known constant value.

So, unlike previous works, we proposed a LMS demodulator which can separate the phase from amplitude of the gyro's vibration for respective control and the control block diagram is presented in Fig.2. First the resonant amplitude and phase information of the vibration displacement of the gyroscope's drive mode is obtained by a sweep module in FPGA. Then the information is sent to the controller as the reference value the vibration is supposed to keep. As Fig.2 shows, the demodulate results include two independent parts: in-phase and quadrature components which represent the amplitude and phase of gyro's vibration respectively. The in-phase part is compared with the amplitude reference the vibration is supposed to have to provide error for an AGC circuit which adjusts the

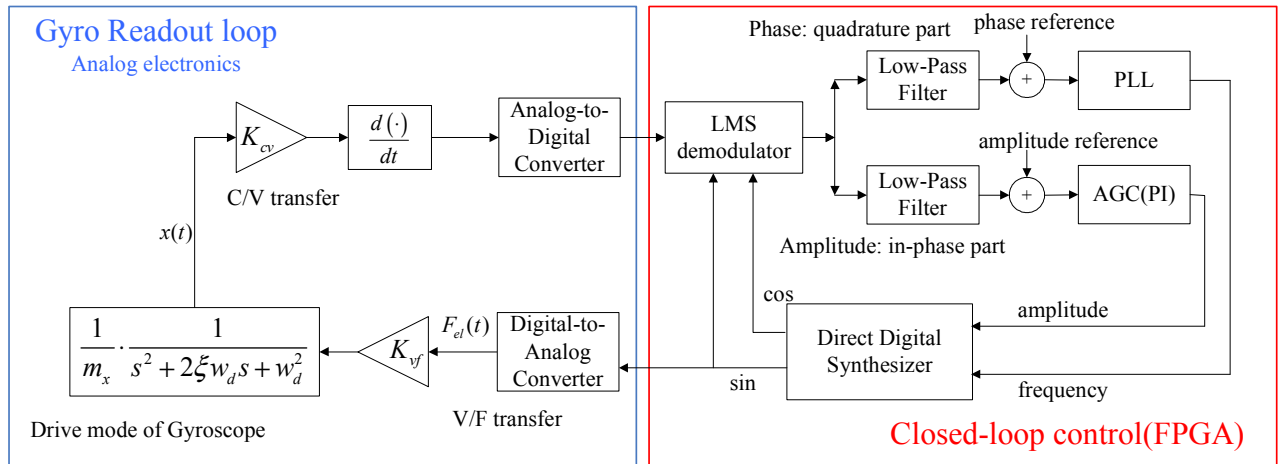


Figure 2. Closed-loop control for the drive mode of gyroscope

amplitude of drive signal. The quadrature part is compared with the phase reference to provide error for PLL to adjust the drive signal's frequency.

### B. LMS Demodulation

Fig.3 shows the principle of LMS demodulation. LMS algorithm works as an adaptive filter which makes the output of the filter approaching input signals through steepest descent method in the gradient direction of the difference between the input and reference. From another point of view, the coefficient of the filter reflects how the input signal is modulated by the reference signal. So demodulation can be realized if we set the modulation carrier as the reference and the system as a one-order filter. Then the filter coefficient is the demodulation results. The iteration process of the algorithm can be shown as follows:

#### 1) Initiation

$$w[0] = 0, r[k] = \sin[w_x k] \quad (6)$$

Where  $w$  is the one-order filter coefficient and  $r$  is the reference of the LMS algorithm playing the role of demodulation carrier.

#### 2) Step

$$e[k] = u_x[w_x k] - w[k-1] \cdot r[k] \quad (7)$$

$$w[k] = w[k-1] + 2\mu \cdot e[k] \cdot r[k] \quad (8)$$

Where  $e$  is the algorithm error and  $\mu$  is the step size or called convergence factor.

#### 3) Result

$$a[k] = w[k] \quad (9)$$

Where  $a$  is the demodulation result.

For one-dimension LMS demodulator, the demodulation result is the amplitude of the signal when the input is a sine wave of the same frequency as the reference. If the input signal  $u_x$  has a phase difference  $\theta$  from the reference sine

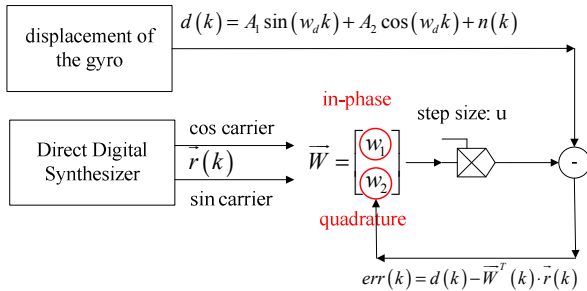


Figure 3. Principle of LMS algorithm

wave  $r$ , the demodulation result would be the in-phase component with the reference, which is the projection of the input on the reference as shown in Fig.4 (a). Furthermore, a two-dimension LMS demodulator with two orthogonal references - sine and cosine wave can get demodulation results including two independent parts: in-phase component  $w_1$  and quadrature component  $w_2$  which are the projection on the sine reference and cosine reference respectively as shown in Fig.4 (b). The in-phase part  $w_1$  which represents the amplitude of the gyroscope's vibration is compared with the amplitude reference to provide the error for an AGC circuit which adjusts the amplitude of drive signal, while quadrature part  $w_2$  is compared with the phase reference to provide error for PLL to adjust the drive signal's frequency. The process of a two-dimension LMS demodulator can be shown as:

#### 1) Initiation

$$\bar{w}[0] = [0 \ 0], \bar{r}[k] = [\sin[w_x k] \ \cos[w_x k]] \quad (10)$$

#### 2) Step

$$e[k] = u_x[w_x k] - \bar{w}^T[k-1] \cdot [\sin[w_x k] \ \cos[w_x k]] \quad (11)$$

$$\bar{w}[k] = \bar{w}[k-1] + 2\mu \cdot e[k] \cdot [\sin[w_x k] \ \cos[w_x k]] \quad (12)$$

#### 3) Result

$$\begin{bmatrix} a[k] \\ p[k] \end{bmatrix} = \bar{w}[k] = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (13)$$

Where  $a$  is the in-phase component and  $p$  is the quadrature component of the demodulation results.

The step size  $\mu$  affects the algorithm error and the convergence speed. When  $\mu$  is larger, the convergence time is reduced, whereas the control error will also be larger. So a balance should be found between them. This demodulator has the advantage of fast convenience time and can suppress noise from the mechanical structure of the gyroscope and the interface analog circuits as the structure of the demodulator is a good adaptive filter. It will also occupy fewer hardware

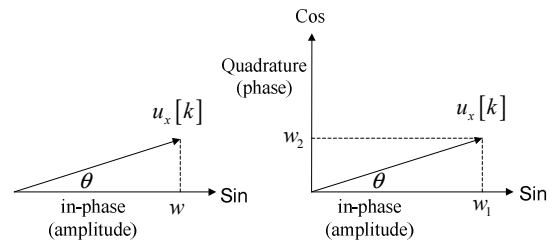


Figure 4. One-dimension LMS and two-dimension LMS

resources as one loop in the algorithm process only includes six multiplications and four additions.

Fig.5 shows the simulation results of a 2-dimension LMS demodulator. The first two figures represent the quadrature part  $p$  and the in-phase part  $a$  respectively, while the last represents the error  $e$ . The convergence time is less than 0.01 second.

### C. AGC and PLL

AGC is a kind of control strategy that employs the nonlinear function to regulate the difference between the actual signal and the desired one to the minimum. The AGC with only P controller is a simple and economical controller since the arrangement for the proportional term can be flexible but it will reserve a steady-state error. AGC with the PI controller can eliminate the error. Taking advantage of the proper and integral terms can ameliorate the transient response in evidence [6], AGC with a PI controller is chosen.

PLL mainly consists of a phase detector that is the LMS demodulator in this design and a controlled oscillator realized by

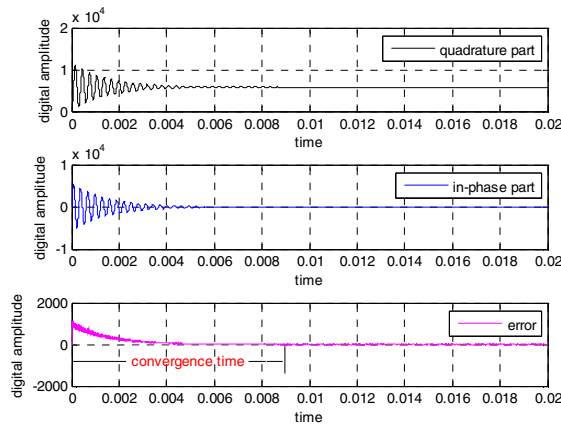


Figure 5. Simulation results of LMS demodulator

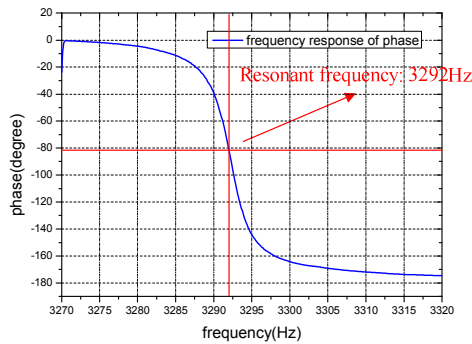


Figure 6. Frequency response of the displacement phase of the drive mode

a Direct digital frequency synthesizer (DDS) which has the advantage of high precision and SNR of 80dB according to the experiment results. The quadrature part of LMS demodulation results is compared with the phase reference to provide error for PLL to adjust the drive signal's frequency according to the frequency response of phase which is shown in Fig.6. For example, if the phase information in the demodulate results changes larger, that means the resonant frequency becomes higher and the vibration frequency is little than the resonant frequency. Then the controller would increase the driving frequency according to the phase error through PLL, and vice versa.

### III. TEST RESULTS FOR THE GYROSCOPE WITH THE CONTROLLER

To evaluate the control performance, a printed circuit board based on FPGA is manufactured mounted with a Z-axis symmetrical doubly decoupled bulk gyroscope, which is illustrated in Fig.7. The circuit board composes of analog circuits which include a C/V converter and an instrument amplifier to sense the motion of driving mass along drive axis through comb finger electrodes, an analog-to-digital converter (ADC), a FPGA in which the control loops, the demodulation circuitry, filtering and temperature compensation are implemented and a digital-to-analog converter (DAC) to generate the analog driving signal [7]. The LMS algorithm occupied 80 logic elements of FPGA, which are only 1.5% of the whole elements in the chip EP2C8T144 of Altera Company.

Fig.8 shows the transient response of the gyro's displacement of the drive mode with the controller. The rising time is measured to be less than 300ms. Fig.9 shows the test variance of the amplitude and phase of the displacement of the drive mode, which is 28ppm and 770 ppm respectively. Fig.10 shows the test nonlinearity of gyroscope which is 0.45% with the full scale of 400°/s and the scale factor being 5.2 mV/°/s.

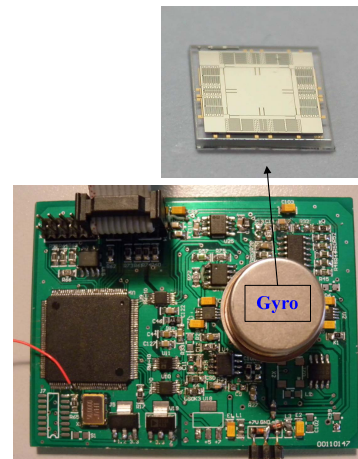


Figure 7. Test PCB

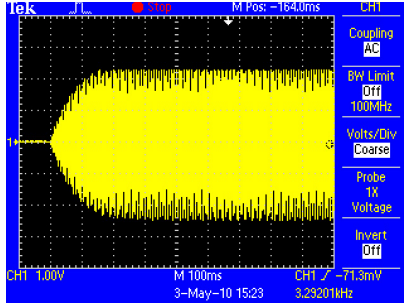
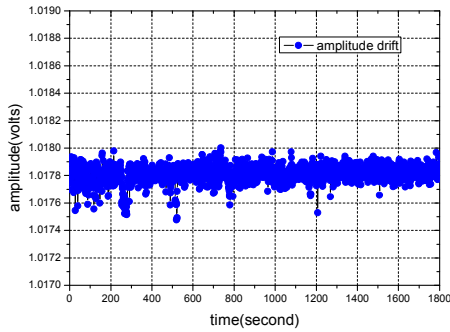
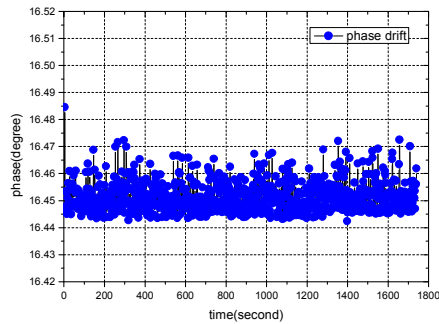


Figure 8. Close-loop drive waveform



(a)



(b)

Figure 9. Test variances of the displacement amplitude and phase of the drive mode

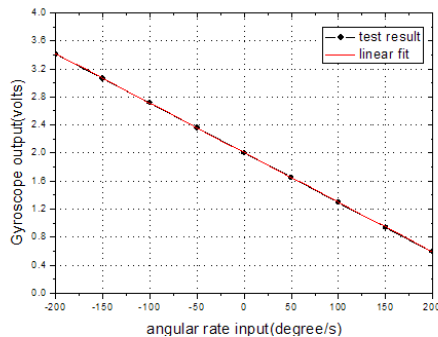


Figure 10. Nonlinearity of the gyroscope

#### IV. CONCLUSIONS

This paper presents the control system for a MEMS vibratory gyroscope in which the LMS algorithm is introduced as a demodulator to separate the amplitude from phase of the gyro's vibration for respective control. The LMS adaptive filter has fast convergent time and occupied only 80 logic elements in FPGA according to experiments. It is convenient that the demodulate results include two independent parts: in-phase and quadrature components which represent the amplitude and phase of gyro's vibration respectively. The in-phase part is compared with the amplitude reference the vibration is supposed to have to provide error for an AGC circuit which adjusts the amplitude of the drive signal. The quadrature part is compared with the phase reference to provide error for PLL to adjust the drive signal's frequency according to the frequency response of phase. The designed control loop has been tested with simulations and is implemented on a FPGA platform. The control system enjoys the advantages of fast start time as well as both constant amplitude and constant phase. And the phase information is important for the removal of quadrature error in the demodulation of the gyroscope's angular rate.

The AGC-PI control can suppress non-ideal interferences but are to be verified further through environmental variation experiments. Closed-loop for the sense mode will benefit the improvement of the zero drift and linearity. The relevant topics are under our current research work.

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