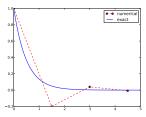
On Schemes for Exponential Decay

Hans Petter Langtangen^{1,2}

Center for Biomedical Computing, Simula Research Laboratory 1 Department of Informatics, University of Oslo 2

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Goal

The primary goal of this demo talk is to demonstrate how to write talks with doconce and get them rendered in numerous HTML formats.

Layout

This version utilizes beamer slides with the theme red2.

Mathematical problem

$$u'(t) = -au(t), \tag{1}$$

$$u(0) = I, (2)$$

- $t \in (0, T]$
- a, I, and T are prescribed parameters
- u(t) is the unknown function



Numerical solution method

- Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- ullet u^n : numerical approx to the exact solution at t_n

Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N - 1$$

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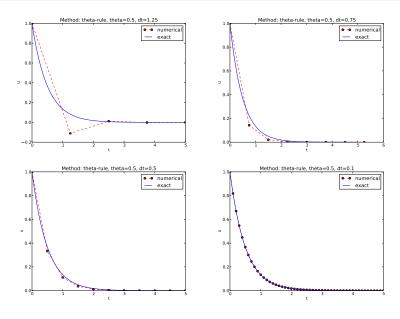
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Implementation

The numerical method is implemented in a Python function:

The Crank-Nicolson method



Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$

- Stability: |A| < 1
- No oscillations: A > 0
- ullet Always for Backward Euler (heta=1)
- $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- $\Delta t < 2/a$ for Crank-Nicolson $(\theta = 1/2)$

Concluding remarks

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