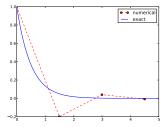
On Schemes for Exponential Decay

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1 Goal

The primary goal of this demo talk is to demonstrate how to write talks with doconce and get them rendered in numerous HTML formats.



Layout

This version utilizes latex document slides with the theme no theme.

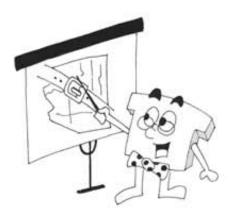
The talk investigates the accuracy of three finite difference schemes for the ordinary differential equation u' = -au with the aid of numerical experiments. Numerical artifacts are in particular demonstrated.

2 Mathematical problem

$$u'(t) = -au(t), \tag{1}$$

$$u(0) = I, (2)$$

- $t \in (0,T]$
- *a*, *I*, and *T* are prescribed parameters
- u(t) is the unknown function



3 Numerical solution method

- Mesh in time: $0 = t_0 < t_1 \cdots < t_N = T$
- Assume constant $\Delta t = t_n t_{n-1}$
- u^n : numerical approx to the exact solution at t_n

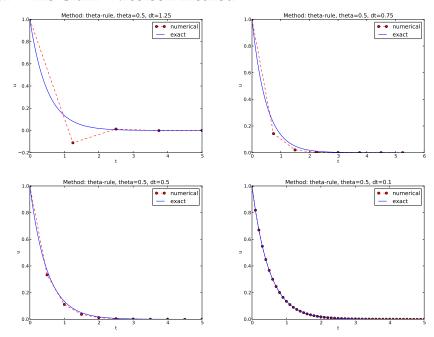
Numerical scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad n = 0, 1, \dots, N - 1$$

4 Implementation

The numerical method is implemented in a Python function:

4.1 The Crank-Nicolson method



4.2 The artifacts can be explained by some theory

Exact solution of the scheme:

$$u^n = A^n$$
, $A = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}$.

- Stability: |A| < 1
- No oscillations: A > 0
- Always for Backward Euler ($\theta = 1$)
- $\Delta t < 1/a$ for Forward Euler ($\theta = 0$)
- $\Delta t < 2/a$ for Crank-Nicolson ($\theta = 1/2$)



Concluding remarks:

Only the Backward Euler scheme is guaranteed to always give qualitatively correct results.