Problem: Have employees who need to be assigned to 1 of the 4 groups based on their preferences, while meeting the following the following Constrains:

① Preference Max: Employees should ideally be assigned to groups they prefer → Ranked 1, 2, 3, 4

② Group size: Max 25 employees

③ Diversity:
- Gender & Race → Evenly Distributed
- Each group must include at least one employee from each section (Dep) ← volunteering grop

④ Fairness:
- Employee's pref should drive assignments as much as possible
- No section, gender, or race should dominate a single group disproportionately.

① sets & Idx

SO $i \in \{1, 2, \ldots, N\}$: index for employees

$j \in \{1, 2, \ldots, N\}$: index for groups

$k \in \{1, 2, \ldots, P\}$: Index for sections

$g \in \{1, 2, \ldots, Q\}$: Index for genders

$r \in \{1, 2, \ldots R\}$: Index for races

② Decision Var:

$$x_{ij} = \begin{cases} 1 & \text{if employee } i \text{ is assigned to group } j \\ 0 & \text{otherwise} \end{cases}$$

x_{ij}: which employee ($E_1, E_2, E_3, \dots, E_N$), which group ($G_1, G_2, G_3, G_4$)

③ Parameters:

S_{ij}: Preference score for employee i for group j.

C_j: Capacity of group j (25 max)

N_g: Total # of employees with g gender

N_r: Total # of employees with race r.

④ Objective Function:

We want to maximize the total preference i.e want to make sure an employee's preference to be in a specific group is prioritized.

i.e to calculate the total preference satisfaction across all employees i and group j

$$Z = \sum_{i=1}^{N} \sum_{j=1}^{M} S_{ij} \cdot x_{ij}$$

Decision Variable $x_{ij} = 1$ if emp i is assigned to group j, $x_{ij} = 0$ otherwise

preference Score i.e $S_{ij} = 4$ if group j is the employee's 1st choice

OR

$S_{ij} = 3$ if group j is the employee's 2nd choice

: so on

i: iterate over all employees $i = 1$ to N

j: iterate over all groups j, from $j = 1$ to M

- Here the product of $(S_{ij} \cdot x_{ij})$
  - → if $x_{ij} = 1$ it will contribute S_{ij} to Z
  - → if $x_{ij} = 0$ their prod contribute 0 to Z

Goal: Maximize Z

Example walk through:

Let's say that we have $N = 3$ employes
$M = 2$ groups

So the preference score S_{ij} for each i,j:

$$S = \begin{bmatrix} 4 & 3 \\ 2 & 4 \\ 1 & 3 \end{bmatrix}$$

← employees

↑ groups

Decision variable:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Apply the formula

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{2} S_{ij} \cdot x_{ij}$$

E1 $(i=1)$: $S_{11} \cdot x_{11} + S_{12} \cdot x_{12} = 4 \cdot 1 + 3 \cdot 0 = 4$

E2 $(i=2)$: $S_{21} \cdot x_{21} + S_{22} \cdot x_{22} = 2 \cdot 0 + 4 \cdot 1 = 4$

E3 $(i=3)$: $S_{31} \cdot x_{31} + S_{32} \cdot x_{32} = 1 \cdot 1 + 3 \cdot 0 = 1$

So we get

$Z = 4 + 4 + 1 = 9$ ← This will be the total satisfaction score for the given assignment.

Constrains:

① $\sum_{j=1}^{M} x_{ij} = 1 \quad \forall i$

Employee Assignment so each employee i must be assigned to exactly one group j

② $\sum_{i=1}^{N} x_{ij} \leq C_j \quad \forall j$

group capacity so each group j must not exceed it's max capacity C_j

③ $\sum_{i \in \text{section}_K} x_{ij} \geq 1 \quad \forall j, K$

section Representation so that each group j must have atleast 1 employee from each section K

④ $\sum_{i \in \text{Gender}_g} x_{ij} \geq \lfloor \frac{N_g}{M} \rfloor - 1 \quad \forall j, g$

gender Balance so the # of employees of each gender g in each group j

⑤ $\sum_{i \in \text{Race}_r} x_{ij} \geq \lfloor \frac{N_r}{M} \rfloor - 1 \quad \forall j, r$

Race Balance for each Race r ensure Balance across groups

How it works Example:

let's say we have $N = 6$ & $M = 2$ & these preferences

Employee	1st Choice	2nd Choice	3rd Choice	4th Choice	Section	Gender	Race
A	Group 1	Group 2	Group 1	Group 2	Sec 1	Male	Asian
B	Group 2	Group 1	Group 2	Group 1	Sec 1	Female	Asian
C	Group 1	Group 2	Group 1	Group 2	Sec 2	Male	White
D	Group 1	Group 2	Group 1	Group 2	Sec 2	Female	White
E	Group 2	Group 1	Group 2	Group 1	Sec 3	Male	Black
F	Group 2	Group 1	Group 2	Group 1	Sec 3	Female	Black

Group cap: $C_1 = C_2 = 3$

preference score for employee & groups:

Employee A has: $S_{A,1} = 4$, $S_{A,2} = 3$

Employee B has: $S_{B,1} = 3$, $S_{B,2} = 4$

Employee	group 1 score	group 2 score
A	4	3
B	3	4
C	4	3
D	4	4
E	3	3
F	3	4

- Objective Func to Maximize

$$Z = 4x_{A1} + 3x_{A2} + 3x_{B1} + 4x_{B2} + 4x_{C1} + 3x_{C2} + 4x_{D1} + 3x_{D2} + 3x_{E1} + 4x_{E2} + 3x_{F1} + 4x_{F2}$$

- Constraints:

Employee assignments:

$$x_{A1} + x_{A2} = 1, \quad x_{B1} + x_{B2} = 1, \ldots$$

Group capacity: $x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} + x_{F1} \leq 3$

Section: For Sec 1 (A, B):

$$x_{A1} + x_{B1} \geq 1, \quad x_{A2} + x_{B2} \geq 1$$

Can solve these using PuLP Optimizer

after get the Averge of the Preferen score

prob.solve()