

One-sample proportion tests

HYPOTHESIS TESTING IN PYTHON



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Chapter 1 recap

- Is a claim about an unknown population proportion feasible?
 1. Standard error of sample statistic from bootstrap distribution
 2. Compute a standardized test statistic
 3. Calculate a p-value
 4. Decide which hypothesis made most sense
- Now, calculate the test statistic without using the bootstrap distribution

Standardized test statistic for proportions

p : population proportion (unknown population parameter)

\hat{p} : sample proportion (sample statistic)

p_0 : hypothesized population proportion

$$z = \frac{\hat{p} - \text{mean}(\hat{p})}{\text{SE}(\hat{p})} = \frac{\hat{p} - p}{\text{SE}(\hat{p})}$$

Assuming H_0 is true, $p = p_0$, so

$$z = \frac{\hat{p} - p_0}{\text{SE}(\hat{p})}$$

Simplifying the standard error calculations

$$SE_{\hat{p}} = \sqrt{\frac{p_0 * (1 - p_0)}{n}} \rightarrow \text{Under } H_0, SE_{\hat{p}} \text{ depends on hypothesized } p_0 \text{ and sample size } n$$

Assuming H_0 is true,

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 * (1 - p_0)}{n}}}$$

- Only uses sample information (\hat{p} and n) and the hypothesized parameter (p_0)

Why z instead of t?

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{\sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}}$$

- s is calculated from \bar{x}
 - \bar{x} estimates the population mean
 - s estimates the population standard deviation
 - \uparrow uncertainty in our estimate of the parameter
- t-distribution - fatter tails than a normal distribution
- \hat{p} only appears in the numerator, so z-scores are fine

Stack Overflow age categories

H_0 : Proportion of Stack Overflow users under thirty = 0.5

H_A : Proportion of Stack Overflow users under thirty \neq 0.5

```
alpha = 0.01
```

```
stack_overflow['age_cat'].value_counts(normalize=True)
```

```
Under 30      0.535604  
At least 30   0.464396  
Name: age_cat, dtype: float64
```

Variables for z

```
p_hat = (stack_overflow['age_cat'] == 'Under 30').mean()
```

```
0.5356037151702786
```

```
p_0 = 0.50
```

```
n = len(stack_overflow)
```

```
2261
```

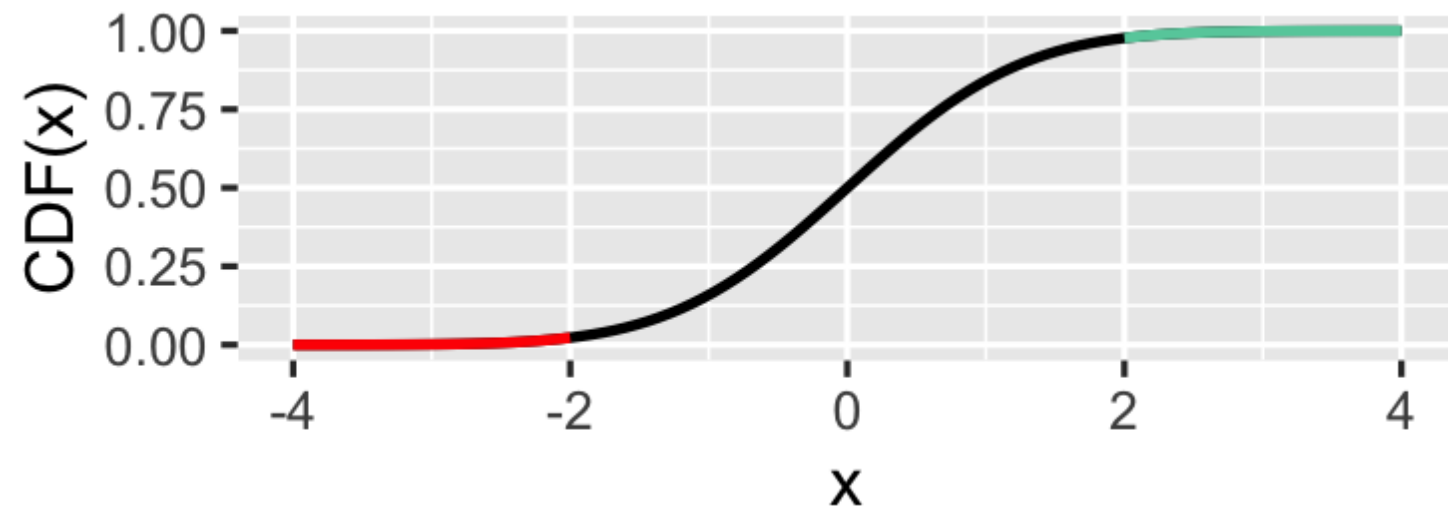
Calculating the z-score

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 * (1 - p_0)}{n}}}$$

```
import numpy as np
numerator = p_hat - p_0
denominator = np.sqrt(p_0 * (1 - p_0) / n)
z_score = numerator / denominator
```

```
3.385911440783663
```


Calculating the p-value



Left-tailed ("less than"):

```
from scipy.stats import norm
p_value = norm.cdf(z_score)
```

Right-tailed ("greater than"):

```
p_value = 1 - norm.cdf(z_score)
```

Two-tailed ("not equal"):

```
p_value = norm.cdf(-z_score) +
1 - norm.cdf(z_score)
```

```
p_value = 2 * (1 - norm.cdf(z_score))
```

```
0.0007094227368100725
```

```
p_value <= alpha
```

```
True
```

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Two-sample proportion tests

HYPOTHESIS TESTING IN PYTHON



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Comparing two proportions

H_0 : Proportion of hobbyist users is the same for those under thirty as those at least thirty

$$H_0: p_{\geq 30} - p_{< 30} = 0$$

H_A : Proportion of hobbyist users is different for those under thirty to those at least thirty

$$H_A: p_{\geq 30} - p_{< 30} \neq 0$$

```
alpha = 0.05
```

Calculating the z-score

- z-score equation for a *proportion test*:

$$z = \frac{(\hat{p}_{\geq 30} - \hat{p}_{< 30}) - 0}{\text{SE}(\hat{p}_{\geq 30} - \hat{p}_{< 30})}$$

- Standard error equation:

$$\text{SE}(\hat{p}_{\geq 30} - \hat{p}_{< 30}) = \sqrt{\frac{\hat{p} \times (1 - \hat{p})}{n_{\geq 30}} + \frac{\hat{p} \times (1 - \hat{p})}{n_{< 30}}}$$

- $\hat{p} \rightarrow$ weighted mean of $\hat{p}_{\geq 30}$ and $\hat{p}_{< 30}$

$$\hat{p} = \frac{n_{\geq 30} \times \hat{p}_{\geq 30} + n_{< 30} \times \hat{p}_{< 30}}{n_{\geq 30} + n_{< 30}}$$

- Only require $\hat{p}_{\geq 30}$, $\hat{p}_{< 30}$, $n_{\geq 30}$, $n_{< 30}$ from the sample to calculate the z-score

Getting the numbers for the z-score

```
p_hats = stack_overflow.groupby("age_cat")['hobbyist'].value_counts(normalize=True)
```

```
age_cat    hobbyist
At least 30  Yes      0.773333
             No      0.226667
Under 30     Yes      0.843105
             No      0.156895
Name: hobbyist, dtype: float64
```

```
n = stack_overflow.groupby("age_cat")['hobbyist'].count()
```

```
age_cat
At least 30    1050
Under 30       1211
Name: hobbyist, dtype: int64
```

Getting the numbers for the z-score

```
p_hats = stack_overflow.groupby("age_cat")['hobbyist'].value_counts(normalize=True)
```

```
age_cat    hobbyist
At least 30  Yes      0.773333
             No       0.226667
Under 30     Yes      0.843105
             No       0.156895
Name: hobbyist, dtype: float64
```

```
p_hat_at_least_30 = p_hats[("At least 30", "Yes")]
p_hat_under_30 = p_hats[("Under 30", "Yes")]
print(p_hat_at_least_30, p_hat_under_30)
```

```
0.773333 0.843105
```

Getting the numbers for the z-score

```
n = stack_overflow.groupby("age_cat")['hobbyist'].count()
```

```
age_cat
At least 30    1050
Under 30       1211
Name: hobbyist, dtype: int64
```

```
n_at_least_30 = n["At least 30"]
n_under_30 = n["Under 30"]
print(n_at_least_30, n_under_30)
```

```
1050 1211
```


Getting the numbers for the z-score

```
p_hat = (n_at_least_30 * p_hat_at_least_30 + n_under_30 * p_hat_under_30) /  
        (n_at_least_30 + n_under_30)  
  
std_error = np.sqrt(p_hat * (1-p_hat) / n_at_least_30 +  
                    p_hat * (1-p_hat) / n_under_30)  
  
z_score = (p_hat_at_least_30 - p_hat_under_30) / std_error  
print(z_score)
```

```
-4.223718652693034
```

Proportion tests using proportions_ztest()

```
stack_overflow.groupby("age_cat")["hobbyist"].value_counts()
```

```
age_cat    hobbyist
At least 30  Yes      812
             No      238
Under 30     Yes     1021
             No      190
Name: hobbyist, dtype: int64
```

```
n_hobbyists = np.array([812, 1021])
n_rows = np.array([812 + 238, 1021 + 190])
from statsmodels.stats.proportion import proportions_ztest
z_score, p_value = proportions_ztest(count=n_hobbyists, nobs=n_rows,
                                     alternative="two-sided")
```

```
(-4.223691463320559, 2.403330142685068e-05)
```

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Chi-square test of independence

HYPOTHESIS TESTING IN PYTHON



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Revisiting the proportion test

```
age_by_hobbyist = stack_overflow.groupby("age_cat")['hobbyist'].value_counts()
```

```
age_cat    hobbyist
At least 30  Yes      812
             No       238
Under 30     Yes     1021
             No       190
Name: hobbyist, dtype: int64
```

```
from statsmodels.stats.proportion import proportions_ztest
n_hobbyists = np.array([812, 1021])
n_rows = np.array([812 + 238, 1021 + 190])
stat, p_value = proportions_ztest(count=n_hobbyists, nobs=n_rows,
                                  alternative="two-sided")
```

```
(-4.223691463320559, 2.403330142685068e-05)
```

Independence of variables

Previous hypothesis test result: evidence that `hobbyist` and `age_cat` are associated

Statistical independence - proportion of successes in the response variable is the same across all categories of the explanatory variable

Test for independence of variables

```
import pingouin
expected, observed, stats = pingouin.chi2_independence(data=stack_overflow, x='hobbyist',
                                                    y='age_cat', correction=False)
print(stats)
```

	test	lambda	chi2	dof	pval	cramer	power
0	pearson	1.000000	17.839570	1.0	0.000024	0.088826	0.988205
1	cressie-read	0.666667	17.818114	1.0	0.000024	0.088773	0.988126
2	log-likelihood	0.000000	17.802653	1.0	0.000025	0.088734	0.988069
3	freeman-tukey	-0.500000	17.815060	1.0	0.000024	0.088765	0.988115
4	mod-log-likelihood	-1.000000	17.848099	1.0	0.000024	0.088848	0.988236
5	neyman	-2.000000	17.976656	1.0	0.000022	0.089167	0.988694

$$\chi^2 \text{ statistic} = 17.839570 = (-4.223691463320559)^2 = (z\text{-score})^2$$

Job satisfaction and age category

```
stack_overflow['age_cat'].value_counts()
```

```
Under 30      1211  
At least 30   1050  
Name: age_cat, dtype: int64
```

```
stack_overflow['job_sat'].value_counts()
```

```
Very satisfied      879  
Slightly satisfied  680  
Slightly dissatisfied 342  
Neither            201  
Very dissatisfied   159  
Name: job_sat, dtype: int64
```


Declaring the hypotheses

H_0 : Age categories are independent of job satisfaction levels

H_A : Age categories are not independent of job satisfaction levels

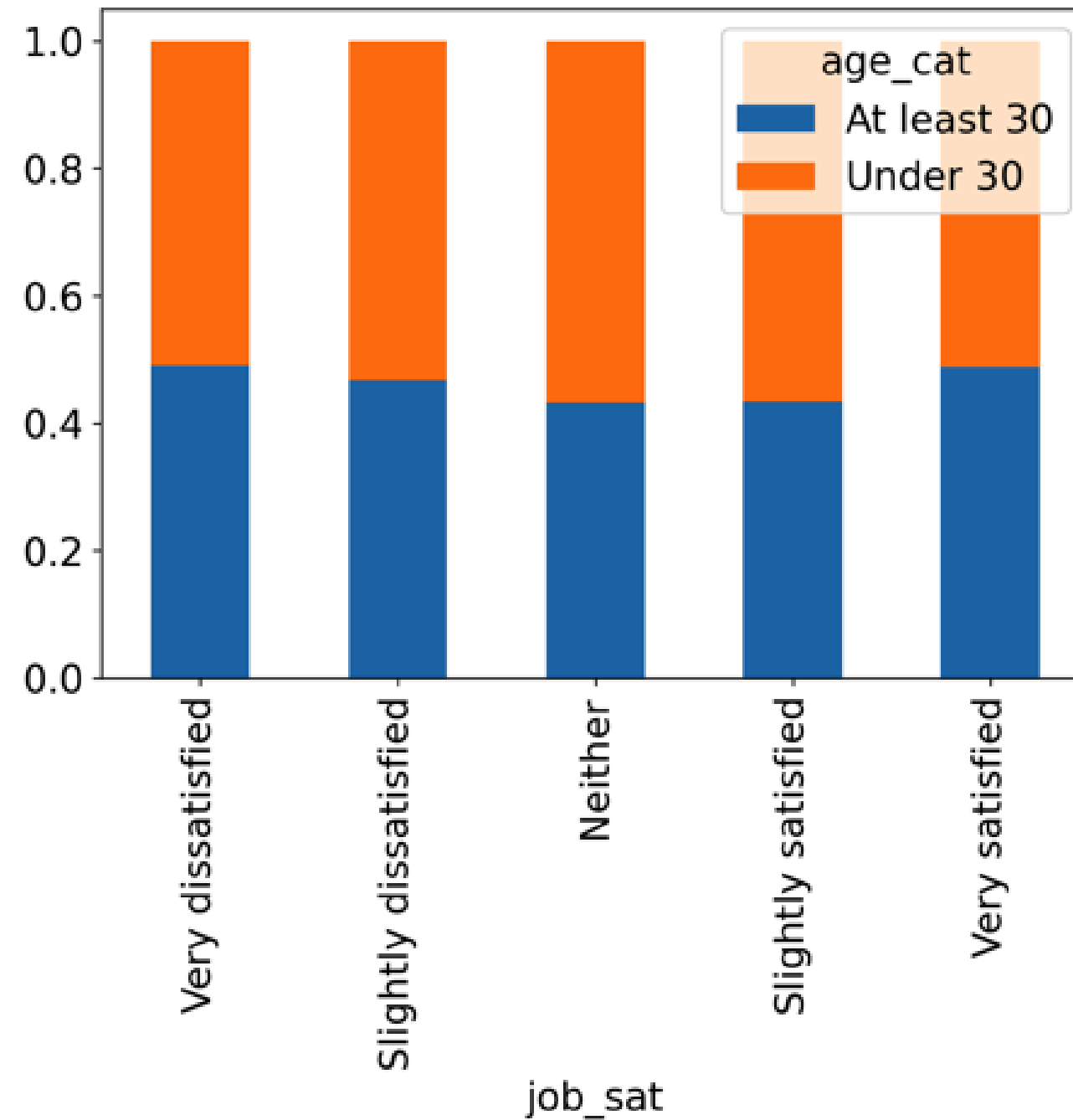
```
alpha = 0.1
```

- Test statistic denoted χ^2
- Assuming independence, how far away are the observed results from the expected values?

Exploratory visualization: proportional stacked bar plot

```
props = stack_overflow.groupby('job_sat')['age_cat'].value_counts(normalize=True)
wide_props = props.unstack()
wide_props.plot(kind="bar", stacked=True)
```

Exploratory visualization: proportional stacked bar plot



Chi-square independence test

```
import pingouin
expected, observed, stats = pingouin.chi2_independence(data=stack_overflow, x="job_sat", y="age_cat")
print(stats)
```

	test	lambda	chi2	dof	pval	cramer	power
0	pearson	1.000000	5.552373	4.0	0.235164	0.049555	0.437417
1	cressie-read	0.666667	5.554106	4.0	0.235014	0.049563	0.437545
2	log-likelihood	0.000000	5.558529	4.0	0.234632	0.049583	0.437871
3	freeman-tukey	-0.500000	5.562688	4.0	0.234274	0.049601	0.438178
4	mod-log-likelihood	-1.000000	5.567570	4.0	0.233854	0.049623	0.438538
5	neyman	-2.000000	5.579519	4.0	0.232828	0.049676	0.439419

Degrees of freedom:

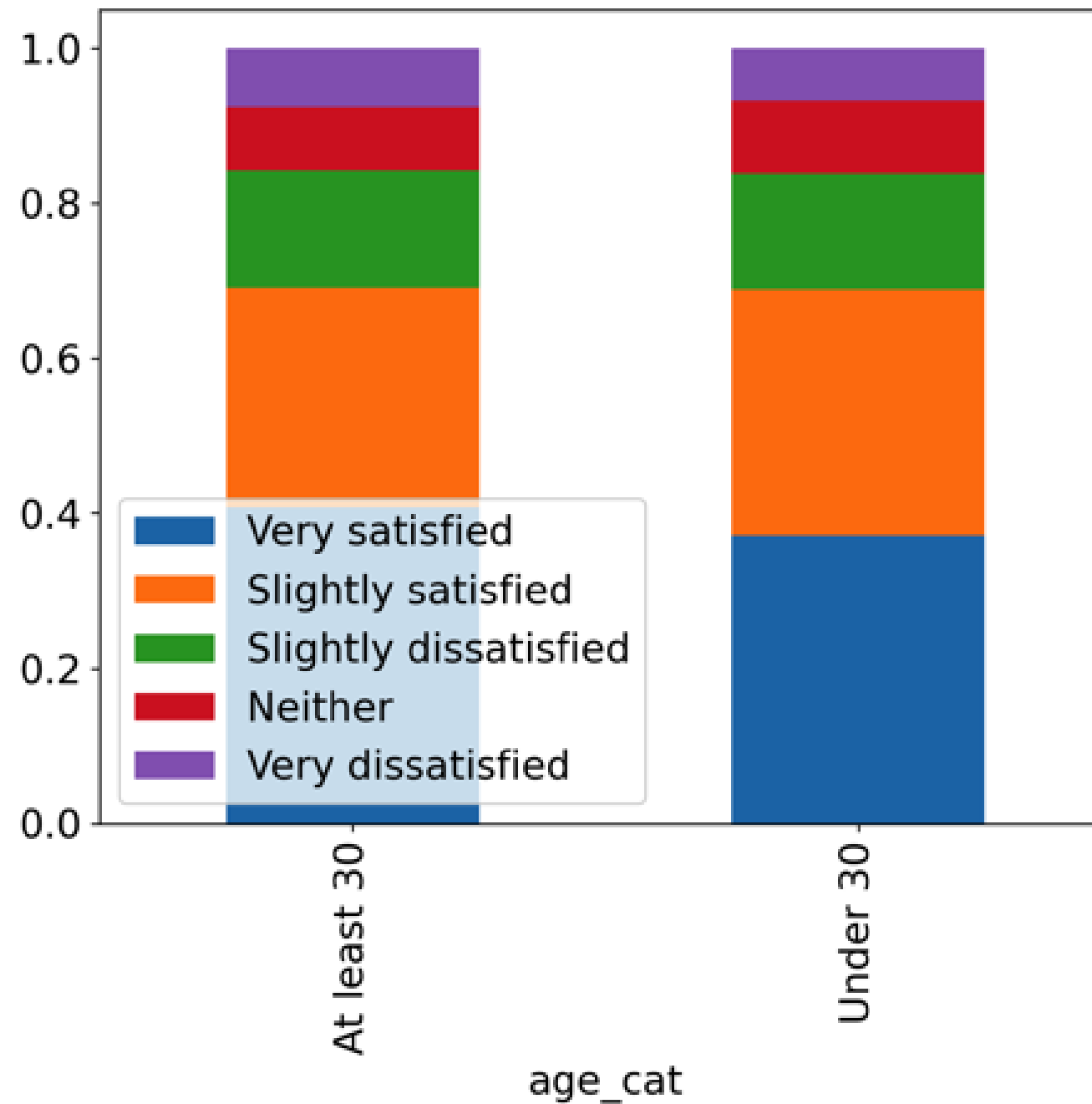
$(\text{No. of response categories} - 1) \times (\text{No. of explanatory categories} - 1)$

$$(2 - 1) * (5 - 1) = 4$$

Swapping the variables?

```
props = stack_overflow.groupby('age_cat')['job_sat'].value_counts(normalize=True)
wide_props = props.unstack()
wide_props.plot(kind="bar", stacked=True)
```

Swapping the variables?



chi-square both ways

```
expected, observed, stats = pingouin.chi2_independence(data=stack_overflow, x="age_cat", y="job_sat")  
print(stats[stats['test'] == 'pearson'])
```

	test	lambda	chi2	dof	pval	cramer	power
0	pearson	1.0	5.552373	4.0	0.235164	0.049555	0.437417

Ask: Are the variables X and Y independent?

Not: Is variable X independent from variable Y?

What about direction and tails?

- Observed and expected counts squared must be non-negative
- chi-square tests are almost always right-tailed ¹

¹ Left-tailed chi-square tests are used in statistical forensics to detect if a fit is suspiciously good because the data was fabricated. Chi-square tests of variance can be two-tailed. These are niche uses, though.

Let's practice!

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Chi-square goodness of fit tests

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Purple links

How do you feel when you discover that you've already visited the top resource?

```
purple_link_counts = stack_overflow['purple_link'].value_counts()
```

```
purple_link_counts = purple_link_counts.rename_axis('purple_link')\
                                .reset_index(name='n')\
                                .sort_values('purple_link')
```

	purple_link	n
2	Amused	368
3	Annoyed	263
0	Hello, old friend	1225
1	Indifferent	405

Declaring the hypotheses

```
hypothesized = pd.DataFrame({  
    'purple_link': ['Amused', 'Annoyed', 'Hello, old friend', 'Indifferent'],  
    'prop': [1/6, 1/6, 1/2, 1/6]})
```

	purple_link	prop
0	Amused	0.166667
1	Annoyed	0.166667
2	Hello, old friend	0.500000
3	Indifferent	0.166667

H_0 : The sample matches the hypothesized distribution

H_A : The sample does not match the hypothesized distribution

χ^2 measures how far observed results are from expectations in each group

```
alpha = 0.01
```

Hypothesized counts by category

```
n_total = len(stack_overflow)
hypothesized["n"] = hypothesized["prop"] * n_total
```

	purple_link	prop	n
0	Amused	0.166667	376.833333
1	Annoyed	0.166667	376.833333
2	Hello, old friend	0.500000	1130.500000
3	Indifferent	0.166667	376.833333

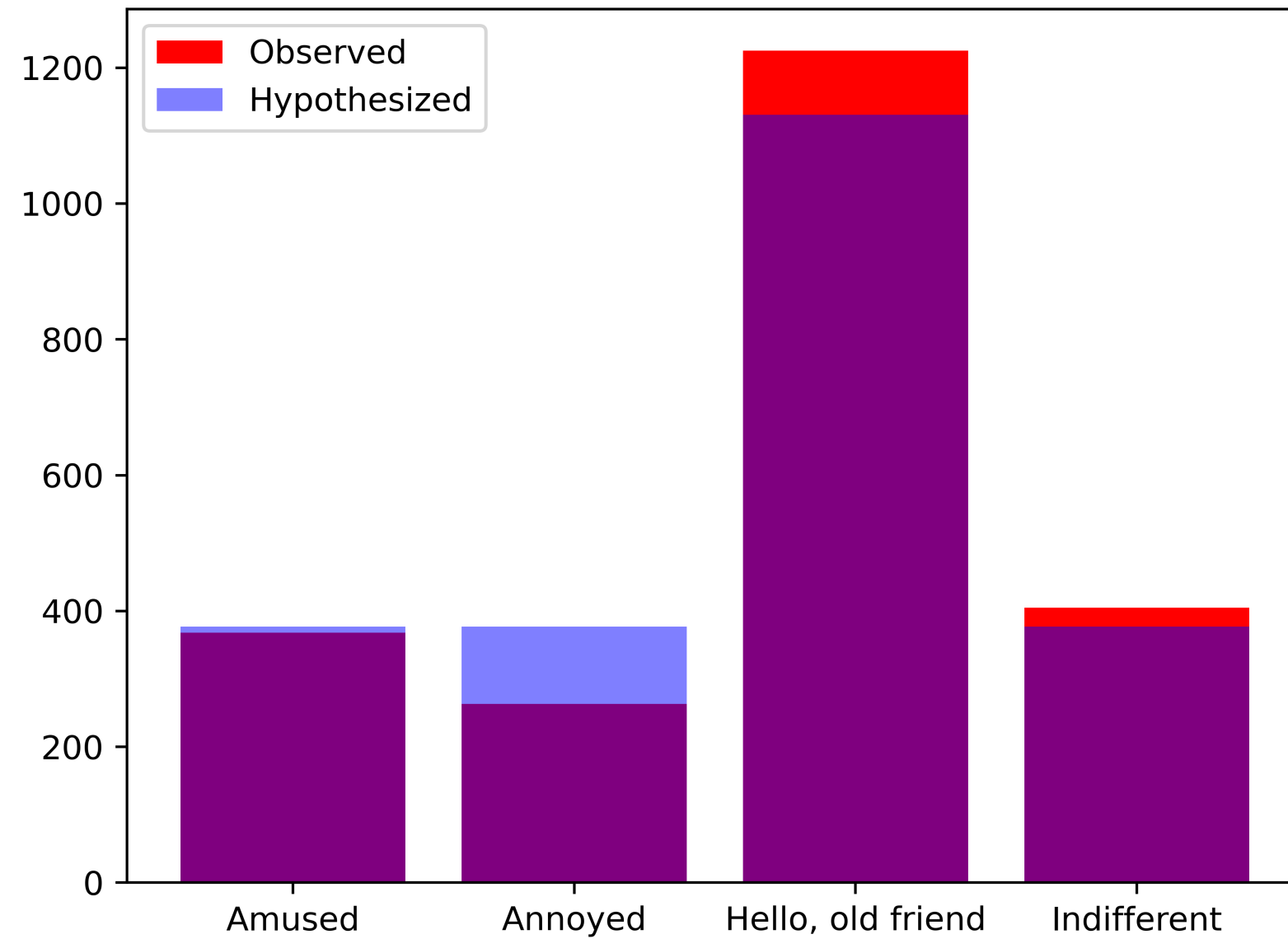
Visualizing counts

```
import matplotlib.pyplot as plt

plt.bar(purple_link_counts['purple_link'], purple_link_counts['n'],
        color='red', label='Observed')
plt.bar(hypothesized['purple_link'], hypothesized['n'], alpha=0.5,
        color='blue', label='Hypothesized')

plt.legend()
plt.show()
```

Visualizing counts



chi-square goodness of fit test

```
print(hypothesized)
```

	purple_link	prop	n
0	Amused	0.166667	376.833333
1	Annoyed	0.166667	376.833333
2	Hello, old friend	0.500000	1130.500000
3	Indifferent	0.166667	376.833333

```
from scipy.stats import chisquare  
chisquare(f_obs=purple_link_counts['n'], f_exp=hypothesized['n'])
```

```
Power_divergenceResult(statistic=44.59840778416629, pvalue=1.1261810719413759e-09)
```


Let's practice!

HYPOTHESIS TESTING IN PYTHON