

Performing t-tests

HYPOTHESIS TESTING IN PYTHON



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Two-sample problems

- Compare sample statistics across groups of a variable
- `converted_comp` is a numerical variable
- `age_first_code_cut` is a categorical variable with levels (`"child"` and `"adult"`)
- Are users who first programmed as a child compensated higher than those that started as adults?

Hypotheses

H_0 : The mean compensation (in USD) is **the same** for those that coded first as a child and those that coded first as an adult.

$$H_0: \mu_{child} = \mu_{adult}$$

$$H_0: \mu_{child} - \mu_{adult} = 0$$

H_A : The mean compensation (in USD) is **greater** for those that coded first as a child compared to those that coded first as an adult.

$$H_A: \mu_{child} > \mu_{adult}$$

$$H_A: \mu_{child} - \mu_{adult} > 0$$

Calculating groupwise summary statistics

```
stack_overflow.groupby('age_first_code_cut')['converted_comp'].mean()
```

```
age_first_code_cut
adult      111313.311047
child      132419.570621
Name: converted_comp, dtype: float64
```

Test statistics

- Sample mean estimates the population mean
- \bar{x} - a sample mean
- \bar{x}_{child} - sample mean compensation for coding first as a child
- \bar{x}_{adult} - sample mean compensation for coding first as an adult
- $\bar{x}_{child} - \bar{x}_{adult}$ - a *test statistic*
- z-score - a (standardized) test statistic

Standardizing the test statistic

$$z = \frac{\text{sample stat} - \text{population parameter}}{\text{standard error}}$$

$$t = \frac{\text{difference in sample stats} - \text{difference in population parameters}}{\text{standard error}}$$

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) - (\mu_{\text{child}} - \mu_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

Standard error

$$SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) \approx \sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}$$

s is the standard deviation of the variable

n is the sample size (number of observations/rows in sample)

Assuming the null hypothesis is true

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) - (\mu_{\text{child}} - \mu_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

$$H_0: \mu_{\text{child}} - \mu_{\text{adult}} = 0 \quad \rightarrow \quad t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{\sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}}$$

Calculations assuming the null hypothesis is true

```
xbar = stack_overflow.groupby('age_first_code_cut')['converted_comp'].mean()
```

```
adult    111313.311047  
child    132419.570621  
Name: converted_comp, dtype: float64 age_first_code_cut
```

```
s = stack_overflow.groupby('age_first_code_cut')['converted_comp'].std()
```

```
adult    271546.521729  
child    255585.240115  
Name: converted_comp, dtype: float64 age_first_code_cut
```

```
n = stack_overflow.groupby('age_first_code_cut')['converted_comp'].count()
```

```
adult    1376  
child     885  
Name: converted_comp, dtype: int64
```

Calculating the test statistic

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{\sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}}$$

```
import numpy as np
numerator = xbar_child - xbar_adult
denominator = np.sqrt(s_child ** 2 / n_child + s_adult ** 2 / n_adult)
t_stat = numerator / denominator
```

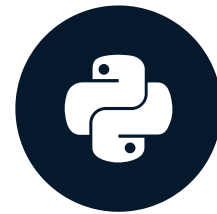
```
1.8699313316221844
```

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Calculating p-values from t-statistics

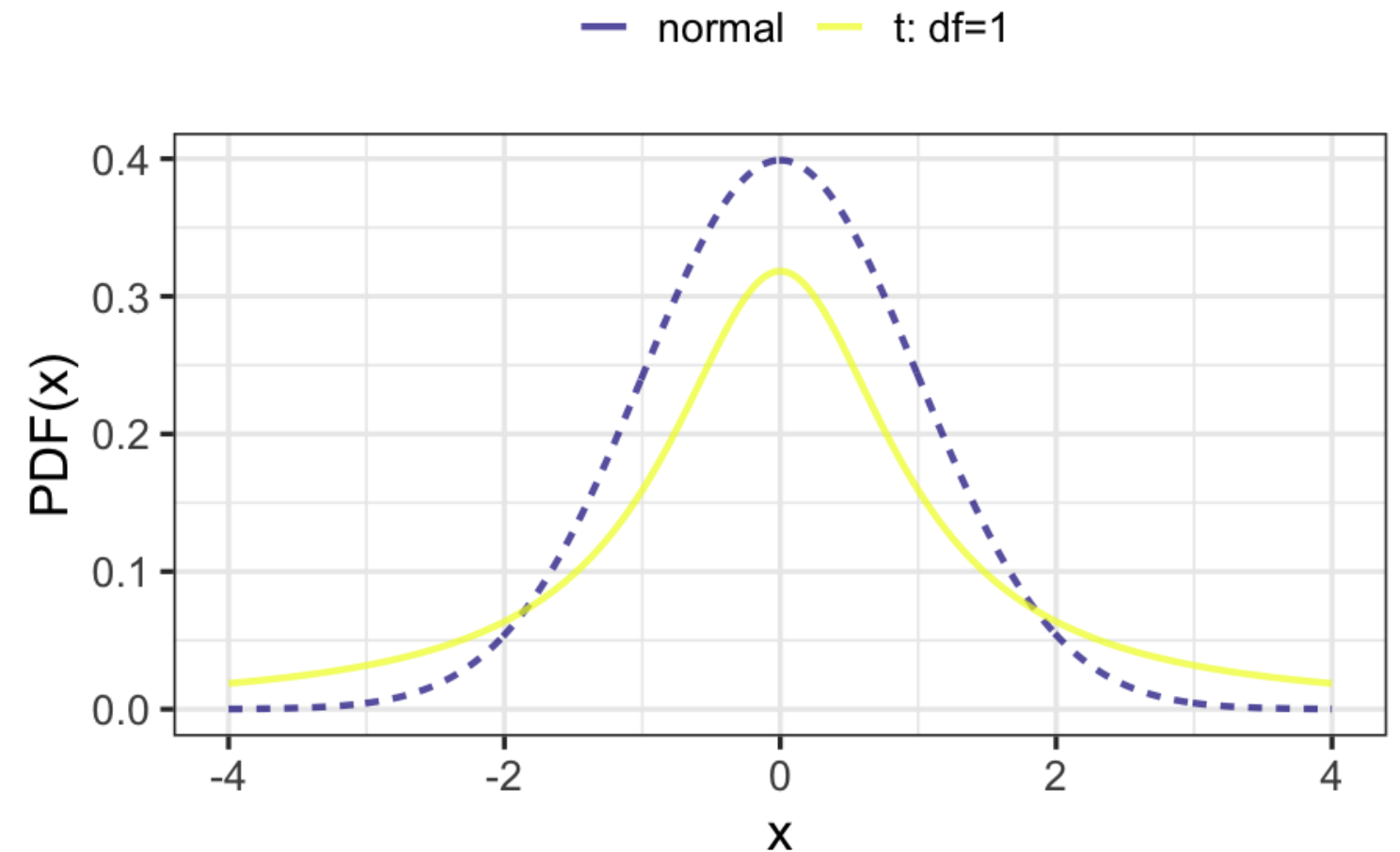
HYPOTHESIS TESTING IN PYTHON



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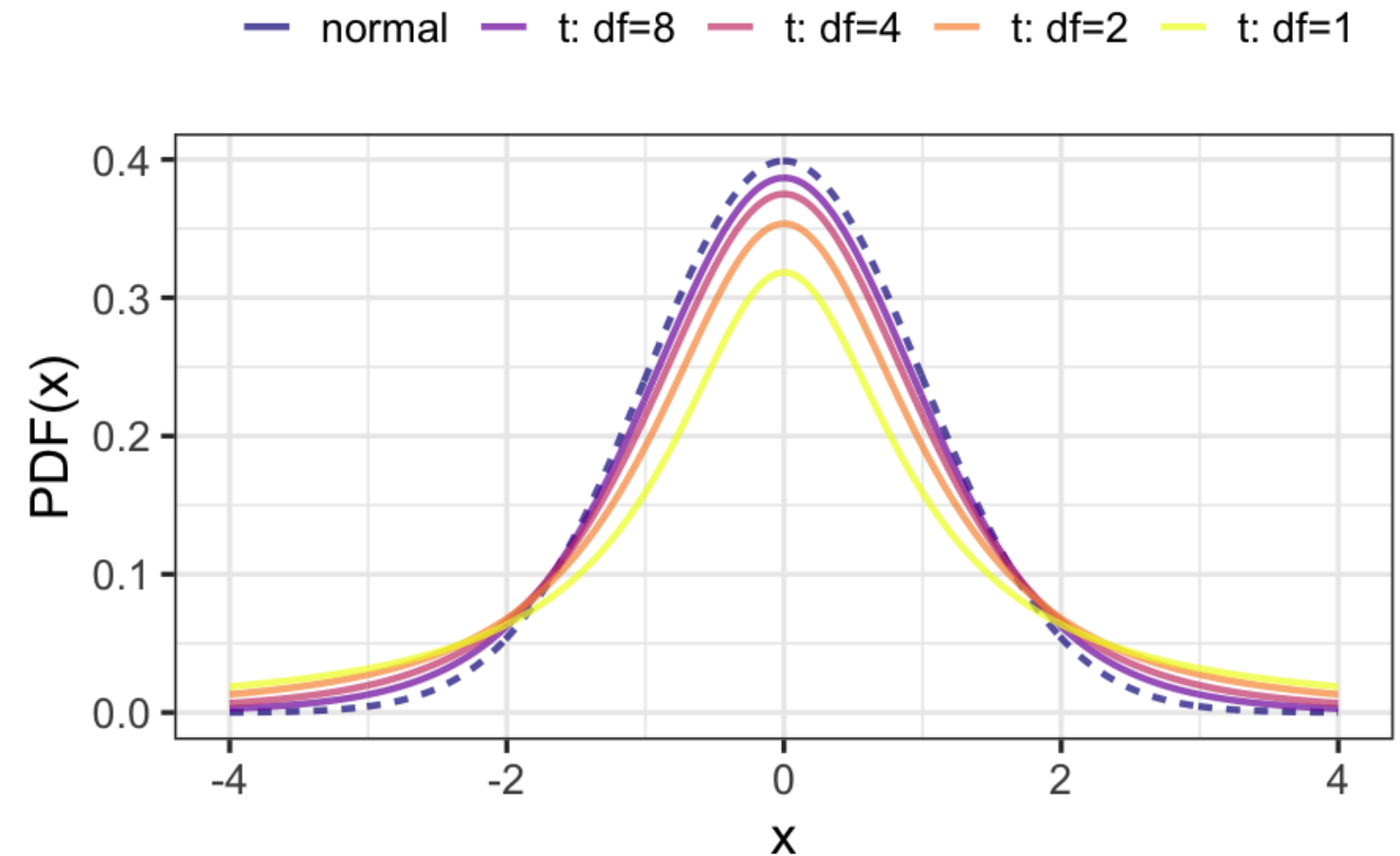
t-distributions

- t statistic follows a t-distribution
- Have a parameter named *degrees of freedom*, or *df*
- Look like normal distributions, with fatter tails



Degrees of freedom

- Larger degrees of freedom \rightarrow t-distribution gets closer to the normal distribution
- Normal distribution \rightarrow t-distribution with infinite df
- Degrees of freedom: maximum number of logically independent values in the data sample



Calculating degrees of freedom

- Dataset has 5 independent observations
- Four of the values are 2, 6, 8, and 5
- The sample mean is 5
- The last value must be 4
- Here, there are 4 degrees of freedom

- $df = n_{child} + n_{adult} - 2$

Hypotheses

H_0 : The mean compensation (in USD) is **the same** for those that coded first as a child and those that coded first as an adult

H_A : The mean compensation (in USD) is **greater** for those that coded first as a child compared to those that coded first as an adult

Use a right-tailed test

Significance level

$$\alpha = 0.1$$

If $p \leq \alpha$ then reject H_0 .

Calculating p-values: one proportion vs. a value

```
from scipy.stats import norm  
1 - norm.cdf(z_score)
```

$$SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) \approx \sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}$$

- z-statistic: needed when using one sample statistic to estimate a population parameter
- t-statistic: needed when using multiple sample statistics to estimate a population parameter

Calculating p-values: two means from different groups

```
numerator = xbar_child - xbar_adult  
denominator = np.sqrt(s_child ** 2 / n_child + s_adult ** 2 / n_adult)  
t_stat = numerator / denominator
```

```
1.8699313316221844
```

```
degrees_of_freedom = n_child + n_adult - 2
```

```
2259
```

Calculating p-values: two means from different groups

- Use t-distribution CDF not normal CDF

```
from scipy.stats import t  
1 - t.cdf(t_stat, df=degrees_of_freedom)
```

```
0.030811302165157595
```

- Evidence that Stack Overflow data scientists who started coding as a child earn more.

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Paired t-tests

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US Republican presidents dataset

```
   state      county  repub_percent_08  repub_percent_12
0  Alabama      Hale      38.957877      37.139882
1  Arkansas      Nevada      56.726272      58.983452
2  California      Lake      38.896719      39.331367
3  California      Ventura      42.923190      45.250693
..      ...      ...      ...
96  Wisconsin      La Crosse      37.490904      40.577038
97  Wisconsin      Lafayette      38.104967      41.675050
98  Wyoming      Weston      76.684241      83.983328
99  Alaska      District 34      77.063259      40.789626
```

```
[100 rows x 4 columns]
```

100 rows; each row represents county-level votes in a presidential election.

¹ <https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/VOQCHQ>

Hypotheses

Question: Was the percentage of Republican candidate votes lower in 2008 than 2012?

$$H_0: \mu_{2008} - \mu_{2012} = 0$$

$$H_A: \mu_{2008} - \mu_{2012} < 0$$

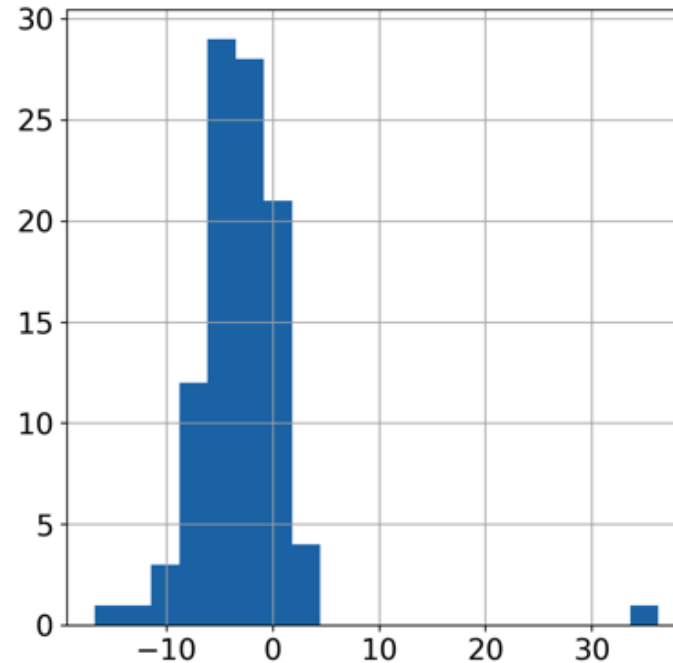
Set $\alpha = 0.05$ significance level.

- Data is **paired** → each voter percentage refers to the same county
 - Want to capture voting patterns in model

From two samples to one

```
sample_data = repub_votes_potus_08_12  
sample_data['diff'] = sample_data['repub_percent_08'] - sample_data['repub_percent_12']
```

```
import matplotlib.pyplot as plt  
sample_data['diff'].hist(bins=20)
```



Calculate sample statistics of the difference

```
xbar_diff = sample_data['diff'].mean()
```

```
-2.877109041242944
```

Revised hypotheses

Old hypotheses:

$$H_0: \mu_{2008} - \mu_{2012} = 0$$

$$H_A: \mu_{2008} - \mu_{2012} < 0$$

New hypotheses:

$$H_0: \mu_{\text{diff}} = 0$$

$$H_A: \mu_{\text{diff}} < 0$$

$$t = \frac{\bar{x}_{\text{diff}} - \mu_{\text{diff}}}{\sqrt{\frac{s_{\text{diff}}^2}{n_{\text{diff}}}}}$$

$$df = n_{\text{diff}} - 1$$

Calculating the p-value

```
n_diff = len(sample_data)
```

```
100
```

```
s_diff = sample_data['diff'].std()
```

```
t_stat = (xbar_diff-0) / np.sqrt(s_diff**2/n_diff)
```

```
-5.601043121928489
```

```
degrees_of_freedom = n_diff - 1
```

```
99
```

$$t = \frac{\bar{x}_{\text{diff}} - \mu_{\text{diff}}}{\sqrt{\frac{s_{\text{diff}}^2}{n_{\text{diff}}}}}$$

$$df = n_{\text{diff}} - 1$$

```
from scipy.stats import t  
p_value = t.cdf(t_stat, df=n_diff-1)
```

```
9.572537285272411e-08
```

Testing differences between two means using ttest()

```
import pingouin
pingouin.ttest(x=sample_data['diff'],
               y=0,
               alternative="less")
```

```
          T  dof alternative          p-val          CI95%  cohen-d  \
T-test -5.601043   99         less  9.572537e-08  [-inf, -2.02]  0.560104

          BF10  power
T-test  1.323e+05   1.0
```

¹ Details on Returns from pingouin.ttest() are available in the API docs for pingouin at <https://pingouin-stats.org/generated/pingouin.ttest.html#pingouin.ttest>.

ttest() with paired=True

```
pingouin.ttest(x=sample_data['repub_percent_08'],  
               y=sample_data['repub_percent_12'],  
               paired=True,  
               alternative="less")
```

	T	dof	alternative	p-val	CI95%	cohen-d	\
T-test	-5.601043	99	less	9.572537e-08	[-inf, -2.02]	0.217364	

	BF10	power
T-test	1.323e+05	0.696338

Unpaired ttest()

```
pingouin.ttest(x=sample_data['repub_percent_08'],  
               y=sample_data['repub_percent_12'],  
               paired=False, # The default  
               alternative="less")
```

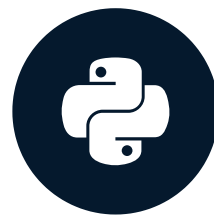
```
          T  dof alternative      p-val      CI95%  cohen-d  BF10  \  
T-test -1.536997 198         less  0.062945  [-inf, 0.22]  0.217364  0.927  
  
      power  
T-test  0.454972
```

- Unpaired t-tests on paired data increases the chances of false negative errors

Let's practice!
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ANOVA tests

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Job satisfaction: 5 categories

```
stack_overflow['job_sat'].value_counts()
```

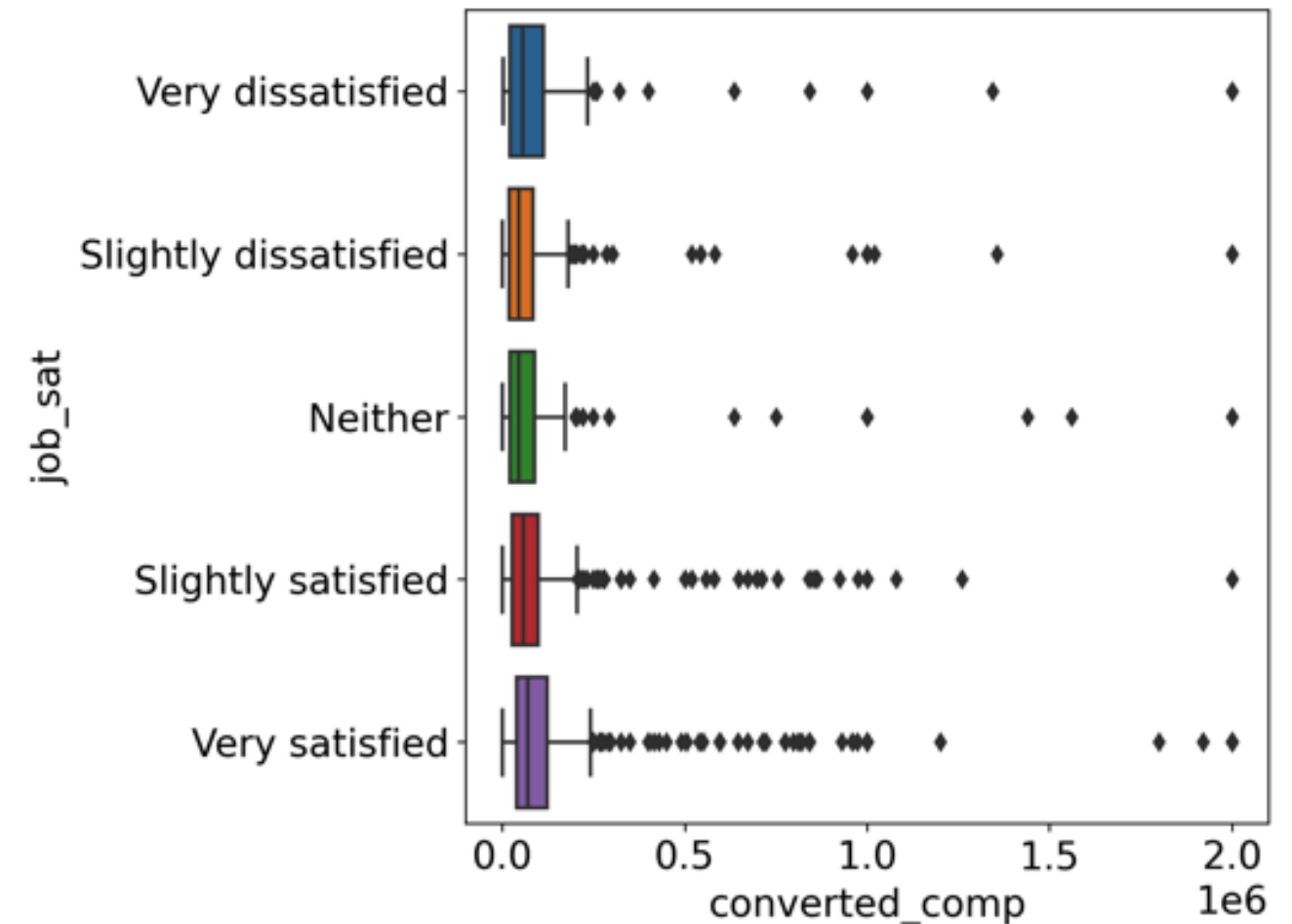
```
Very satisfied      879  
Slightly satisfied  680  
Slightly dissatisfied 342  
Neither            201  
Very dissatisfied   159  
Name: job_sat, dtype: int64
```

Visualizing multiple distributions

Is mean annual compensation different for different levels of job satisfaction?

```
import seaborn as sns
import matplotlib.pyplot as plt
sns.boxplot(x="converted_comp",
            y="job_sat",
            data=stack_overflow)

plt.show()
```



Analysis of variance (ANOVA)

- A test for differences *between* groups

```
alpha = 0.2
```

```
pingouin.anova(data=stack_overflow,  
               dv="converted_comp",  
               between="job_sat")
```

	Source	ddof1	ddof2	F	p-unc	np2
0	job_sat	4	2256	4.480485	0.001315	0.007882

- $0.001315 < \alpha$
- At least two categories have *significantly different* compensation

Pairwise tests

- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{slightly dissatisfied}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{neither}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{neither}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{neither}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{neither}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{slightly satisfied}} \neq \mu_{\text{very satisfied}}$

Set significance level to $\alpha = 0.2$.

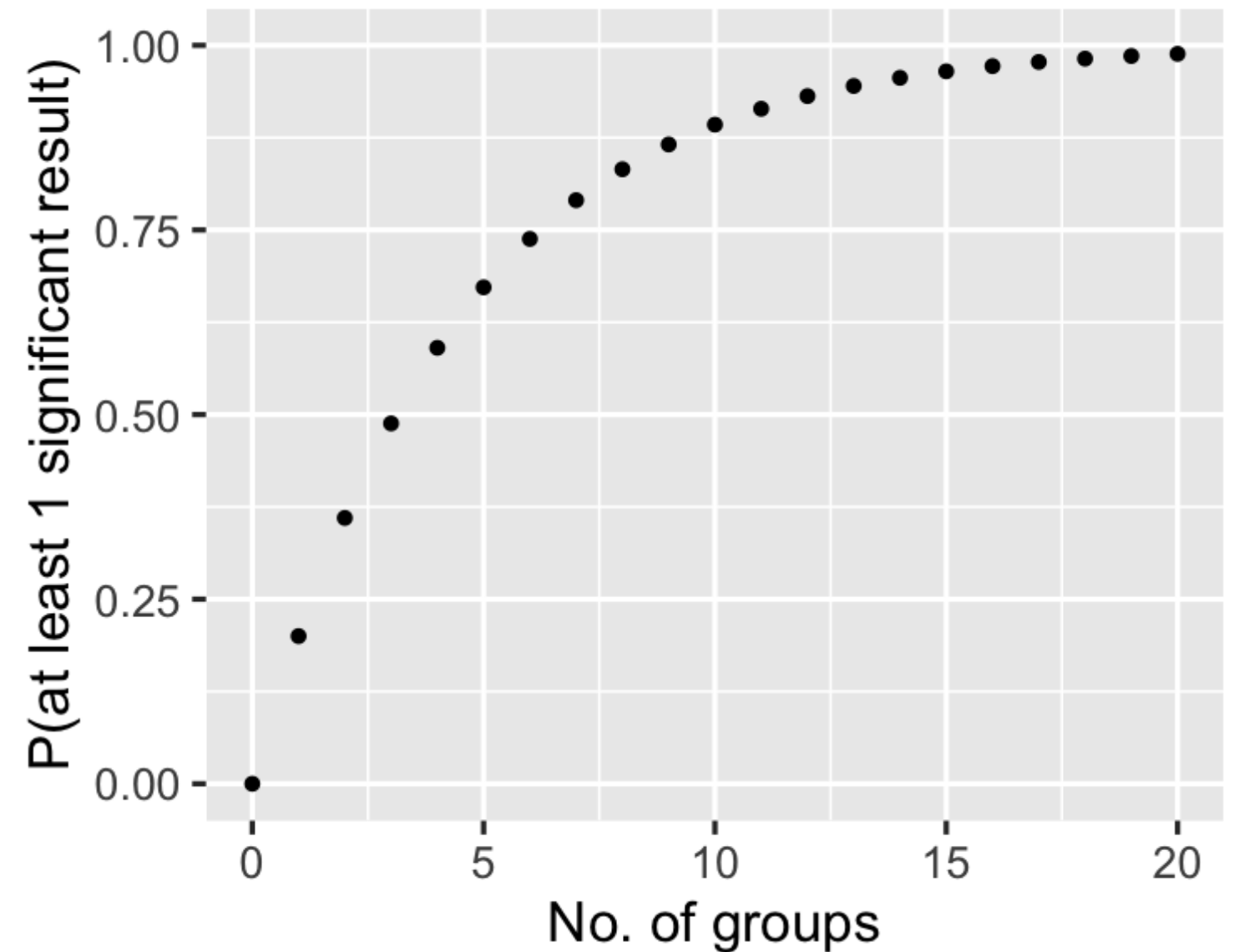
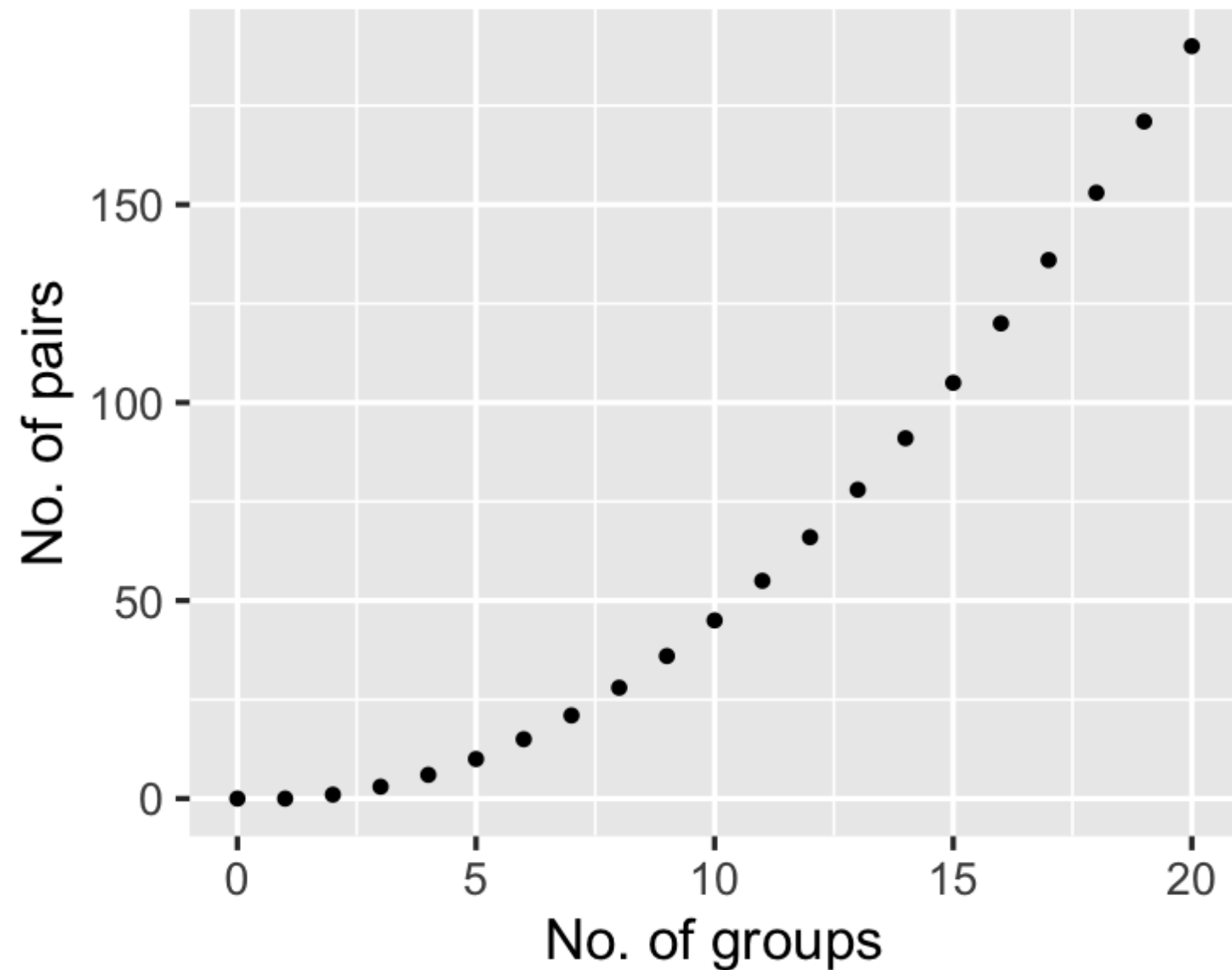
pairwise_tests()

```
pingouin.pairwise_tests(data=stack_overflow,  
                        dv="converted_comp",  
                        between="job_sat",  
                        padjust="none")
```

	Contrast	A	B	Paired	Parametric	...	dof	alternative	p-unc	BF10	hedges
0	job_sat	Slightly satisfied	Very satisfied	False	True	...	1478.622799	two-sided	0.000064	158.564	-0.192931
1	job_sat	Slightly satisfied	Neither	False	True	...	258.204546	two-sided	0.484088	0.114	-0.068513
2	job_sat	Slightly satisfied	Very dissatisfied	False	True	...	187.153329	two-sided	0.215179	0.208	-0.145624
3	job_sat	Slightly satisfied	Slightly dissatisfied	False	True	...	569.926329	two-sided	0.969491	0.074	-0.002719
4	job_sat	Very satisfied	Neither	False	True	...	328.326639	two-sided	0.097286	0.337	0.120115
5	job_sat	Very satisfied	Very dissatisfied	False	True	...	221.666205	two-sided	0.455627	0.126	0.063479
6	job_sat	Very satisfied	Slightly dissatisfied	False	True	...	821.303063	two-sided	0.002166	7.43	0.173247
7	job_sat	Neither	Very dissatisfied	False	True	...	321.165726	two-sided	0.585481	0.135	-0.058537
8	job_sat	Neither	Slightly dissatisfied	False	True	...	367.730081	two-sided	0.547406	0.118	0.055707
9	job_sat	Very dissatisfied	Slightly dissatisfied	False	True	...	247.570187	two-sided	0.259590	0.197	0.119131

[10 rows x 11 columns]

As the number of groups increases...



Bonferroni correction

```
pingouin.pairwise_tests(data=stack_overflow,  
                        dv="converted_comp",  
                        between="job_sat",  
                        padjust="bonf")
```

	Contrast	A	B	...	p-unc	p-corr	p-adjust	BF10	hedges
0	job_sat	Slightly satisfied	Very satisfied	...	0.000064	0.000638	bonf	158.564	-0.192931
1	job_sat	Slightly satisfied	Neither	...	0.484088	1.000000	bonf	0.114	-0.068513
2	job_sat	Slightly satisfied	Very dissatisfied	...	0.215179	1.000000	bonf	0.208	-0.145624
3	job_sat	Slightly satisfied	Slightly dissatisfied	...	0.969491	1.000000	bonf	0.074	-0.002719
4	job_sat	Very satisfied	Neither	...	0.097286	0.972864	bonf	0.337	0.120115
5	job_sat	Very satisfied	Very dissatisfied	...	0.455627	1.000000	bonf	0.126	0.063479
6	job_sat	Very satisfied	Slightly dissatisfied	...	0.002166	0.021659	bonf	7.43	0.173247
7	job_sat	Neither	Very dissatisfied	...	0.585481	1.000000	bonf	0.135	-0.058537
8	job_sat	Neither	Slightly dissatisfied	...	0.547406	1.000000	bonf	0.118	0.055707
9	job_sat	Very dissatisfied	Slightly dissatisfied	...	0.259590	1.000000	bonf	0.197	0.119131

[10 rows x 11 columns]

More methods

`padjust` : *string*

Method used for testing and adjustment of pvalues.

- `'none'` : no correction [default]
- `'bonf'` : one-step Bonferroni correction
- `'sidak'` : one-step Sidak correction
- `'holm'` : step-down method using Bonferroni adjustments
- `'fdr_bh'` : Benjamini/Hochberg FDR correction
- `'fdr_by'` : Benjamini/Yekutieli FDR correction

Let's practice!

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