ps1_1

February 6, 2023

[248]:
```
!pip install numpy
!pip install matplot
!pip install nbconvert
!pip install pandoc
```

```
Requirement already satisfied: numpy in c:\program files\python39\lib\site-
packages (1.24.1)


[notice] A new release of pip available: 22.3.1 -> 23.0
[notice] To update, run: python.exe -m pip install --upgrade pip
```

```
Requirement already satisfied: matplot in c:\program files\python39\lib\site-
packages (0.1.9)
Requirement already satisfied: matplotlib>=3.1.1 in c:\program
files\python39\lib\site-packages (from matplot) (3.6.3)
Requirement already satisfied: pyloco>=0.0.134 in c:\program
files\python39\lib\site-packages (from matplot) (0.0.139)
Requirement already satisfied: pillow>=6.2.0 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (9.4.0)
Requirement already satisfied: cycler>=0.10 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (0.11.0)
Requirement already satisfied: fonttools>=4.22.0 in c:\program
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Requirement already satisfied: numpy>=1.19 in c:\program
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Requirement already satisfied: pyparsing>=2.2.1 in c:\program
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Requirement already satisfied: contourpy>=1.0.1 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (1.0.7)
Requirement already satisfied: packaging>=20.0 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (23.0)
Requirement already satisfied: kiwisolver>=1.0.1 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (1.4.4)
Requirement already satisfied: python-dateutil>=2.7 in c:\program
files\python39\lib\site-packages (from matplotlib>=3.1.1->matplot) (2.8.2)
Requirement already satisfied: websocket-client in c:\program
files\python39\lib\site-packages (from pyloco>=0.0.134->matplot) (1.5.1)
Requirement already satisfied: twine in c:\program files\python39\lib\site-
```

```
packages (from pyloco>=0.0.134->matplot) (4.0.2)
Requirement already satisfied: SimpleWebSocketServer in c:\program
files\python39\lib\site-packages (from pyloco>=0.0.134->matplot) (0.1.2)
Requirement already satisfied: typing in c:\program files\python39\lib\site-
packages (from pyloco>=0.0.134->matplot) (3.7.4.3)
Requirement already satisfied: ushlex in c:\program files\python39\lib\site-
packages (from pyloco>=0.0.134->matplot) (0.99.1)
Requirement already satisfied: six>=1.5 in c:\program files\python39\lib\site-
packages (from python-dateutil>=2.7->matplotlib>=3.1.1->matplot) (1.16.0)
Requirement already satisfied: requests>=2.20 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (2.28.2)
Requirement already satisfied: readme-renderer>=35.0 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (37.3)
Requirement already satisfied: importlib-metadata>=3.6 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (6.0.0)
Requirement already satisfied: urllib3>=1.26.0 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot)
(1.26.14)
Requirement already satisfied: pkginfo>=1.8.1 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (1.9.6)
Requirement already satisfied: requests-toolbelt!=0.9.0,>=0.8.0 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (0.10.1)
Requirement already satisfied: keyring>=15.1 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot)
(23.13.1)
Requirement already satisfied: rich>=12.0.0 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (13.3.1)
Requirement already satisfied: rfc3986>=1.4.0 in c:\program
files\python39\lib\site-packages (from twine->pyloco>=0.0.134->matplot) (2.0.0)
Requirement already satisfied: zipp>=0.5 in c:\program files\python39\lib\site-
packages (from importlib-metadata>=3.6->twine->pyloco>=0.0.134->matplot)
(3.12.0)
Requirement already satisfied: pywin32-ctypes>=0.2.0 in c:\program
files\python39\lib\site-packages (from
keyring>=15.1->twine->pyloco>=0.0.134->matplot) (0.2.0)
Requirement already satisfied: jaraco.classes in c:\program
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keyring>=15.1->twine->pyloco>=0.0.134->matplot) (3.2.3)
Requirement already satisfied: docutils>=0.13.1 in c:\program
files\python39\lib\site-packages (from readme-
renderer>=35.0->twine->pyloco>=0.0.134->matplot) (0.19)
Requirement already satisfied: Pygments>=2.5.1 in c:\program
files\python39\lib\site-packages (from readme-
renderer>=35.0->twine->pyloco>=0.0.134->matplot) (2.14.0)
Requirement already satisfied: bleach>=2.1.0 in c:\program
files\python39\lib\site-packages (from readme-
renderer>=35.0->twine->pyloco>=0.0.134->matplot) (6.0.0)
Requirement already satisfied: charset-normalizer<4,>=2 in c:\program
```

```
files\python39\lib\site-packages (from
requests>=2.20->twine->pyloco>=0.0.134->matplot) (3.0.1)
Requirement already satisfied: certifi>=2017.4.17 in
c:\users\aaron\appdata\roaming\python\python39\site-packages (from
requests>=2.20->twine->pyloco>=0.0.134->matplot) (2022.9.24)
Requirement already satisfied: idna<4,>=2.5 in c:\program
files\python39\lib\site-packages (from
requests>=2.20->twine->pyloco>=0.0.134->matplot) (3.4)
Requirement already satisfied: markdown-it-py<3.0.0,>=2.1.0 in c:\program
files\python39\lib\site-packages (from
rich>=12.0.0->twine->pyloco>=0.0.134->matplot) (2.1.0)
Requirement already satisfied: webencodings in c:\program
files\python39\lib\site-packages (from bleach>=2.1.0->readme-
renderer>=35.0->twine->pyloco>=0.0.134->matplot) (0.5.1)
Requirement already satisfied: mdurl~=0.1 in c:\program files\python39\lib\site-
packages (from markdown-it-
py<3.0.0,>=2.1.0->rich>=12.0.0->twine->pyloco>=0.0.134->matplot) (0.1.2)
Requirement already satisfied: more-itertools in c:\program
files\python39\lib\site-packages (from
jaraco.classes->keyring>=15.1->twine->pyloco>=0.0.134->matplot) (9.0.0)


[notice] A new release of pip available: 22.3.1 -> 23.0
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Requirement already satisfied: nbconvert in c:\program files\python39\lib\site-
packages (7.2.9)
Requirement already satisfied: importlib-metadata>=3.6 in c:\program
files\python39\lib\site-packages (from nbconvert) (6.0.0)
Requirement already satisfied: markupsafe>=2.0 in c:\program
files\python39\lib\site-packages (from nbconvert) (2.1.2)
Requirement already satisfied: nbformat>=5.1 in c:\program
files\python39\lib\site-packages (from nbconvert) (5.7.3)
Requirement already satisfied: beautifulsoup4 in c:\program
files\python39\lib\site-packages (from nbconvert) (4.11.2)
Requirement already satisfied: bleach in c:\program files\python39\lib\site-
packages (from nbconvert) (6.0.0)
Requirement already satisfied: pandocfilters>=1.4.1 in c:\program
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Requirement already satisfied: mistune<3,>=2.0.3 in c:\program
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Requirement already satisfied: nbclient>=0.5.0 in c:\program
files\python39\lib\site-packages (from nbconvert) (0.7.2)
Requirement already satisfied: packaging in c:\program files\python39\lib\site-
packages (from nbconvert) (23.0)
Requirement already satisfied: tinycss2 in c:\program files\python39\lib\site-
packages (from nbconvert) (1.2.1)
Requirement already satisfied: defusedxml in c:\program files\python39\lib\site-
packages (from nbconvert) (0.7.1)
```

```
Requirement already satisfied: pygments>=2.4.1 in c:\program
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Requirement already satisfied: jupyter-core>=4.7 in c:\program
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Requirement already satisfied: traitlets>=5.0 in c:\program
files\python39\lib\site-packages (from nbconvert) (5.9.0)
Requirement already satisfied: jinja2>=3.0 in c:\program
files\python39\lib\site-packages (from nbconvert) (3.1.2)
Requirement already satisfied: jupyterlab-pygments in c:\program
files\python39\lib\site-packages (from nbconvert) (0.2.2)
Requirement already satisfied: zipp>=0.5 in c:\program files\python39\lib\site-
packages (from importlib-metadata>=3.6->nbconvert) (3.12.0)
Requirement already satisfied: platformdirs>=2.5 in
c:\users\aaron\appdata\roaming\python\python39\site-packages (from jupyter-
core>=4.7->nbconvert) (2.5.2)
Requirement already satisfied: pywin32>=1.0 in c:\program
files\python39\lib\site-packages (from jupyter-core>=4.7->nbconvert) (305)
Requirement already satisfied: jupyter-client>=6.1.12 in c:\program
files\python39\lib\site-packages (from nbclient>=0.5.0->nbconvert) (8.0.2)
Requirement already satisfied: jsonschema>=2.6 in c:\program
files\python39\lib\site-packages (from nbformat>=5.1->nbconvert) (4.17.3)
Requirement already satisfied: fastjsonschema in c:\program
files\python39\lib\site-packages (from nbformat>=5.1->nbconvert) (2.16.2)
Requirement already satisfied: soupsieve>1.2 in c:\program
files\python39\lib\site-packages (from beautifulsoup4->nbconvert) (2.3.2.post1)
Requirement already satisfied: six>=1.9.0 in c:\program files\python39\lib\site-
packages (from bleach->nbconvert) (1.16.0)
Requirement already satisfied: webencodings in c:\program
files\python39\lib\site-packages (from bleach->nbconvert) (0.5.1)
Requirement already satisfied: attrs>=17.4.0 in c:\program
files\python39\lib\site-packages (from
jsonschema>=2.6->nbformat>=5.1->nbconvert) (22.2.0)
Requirement already satisfied: pyrsistent!=0.17.0,!=0.17.1,!=0.17.2,>=0.14.0 in
c:\program files\python39\lib\site-packages (from
jsonschema>=2.6->nbformat>=5.1->nbconvert) (0.19.3)
Requirement already satisfied: python-dateutil>=2.8.2 in c:\program
files\python39\lib\site-packages (from jupyter-
client>=6.1.12->nbclient>=0.5.0->nbconvert) (2.8.2)
Requirement already satisfied: pyzmq>=23.0 in c:\program
files\python39\lib\site-packages (from jupyter-
client>=6.1.12->nbclient>=0.5.0->nbconvert) (25.0.0)
Requirement already satisfied: tornado>=6.2 in c:\program
files\python39\lib\site-packages (from jupyter-
client>=6.1.12->nbclient>=0.5.0->nbconvert) (6.2)


[notice] A new release of pip available: 22.3.1 -> 23.0
[notice] To update, run: python.exe -m pip install --upgrade pip
```

```
Collecting pandoc
  Downloading pandoc-2.3.tar.gz (33 kB)
  Preparing metadata (setup.py): started
  Preparing metadata (setup.py): finished with status 'done'
Collecting plumbum
  Downloading plumbum-1.8.1-py3-none-any.whl (126 kB)
     ------------------------------------- 126.7/126.7 kB 3.8 MB/s eta 0:00:00
Collecting ply
  Downloading ply-3.11-py2.py3-none-any.whl (49 kB)
     ---------------------------------------- 49.6/49.6 kB ? eta 0:00:00
Requirement already satisfied: pywin32 in c:\program files\python39\lib\site-
packages (from plumbum->pandoc) (305)
Installing collected packages: ply, plumbum, pandoc
  Running setup.py install for pandoc: started
  Running setup.py install for pandoc: finished with status 'done'
Successfully installed pandoc-2.3 plumbum-1.8.1 ply-3.11

  DEPRECATION: pandoc is being installed using the legacy 'setup.py install'
method, because it does not have a 'pyproject.toml' and the 'wheel' package is
not installed. pip 23.1 will enforce this behaviour change. A possible
replacement is to enable the '--use-pep517' option. Discussion can be found at
https://github.com/pypa/pip/issues/8559

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```

[2]:
```
import math
import numpy as np
import matplotlib.pyplot as plt
```

[3]:
```
import math
import numpy as np
```

[120]:
```
gravity = 32.2  # ft/s^2
initial_position = math.pi/4
initial_velocity = 0
arm_length = 2 # feet
end_time = 4  # seconds
time_step = 0.05  # seconds
time_range = np.arange(0,end_time,time_step)
```

$$\theta(t) = A\sin(\lambda t) + B\cos(\lambda t)$$

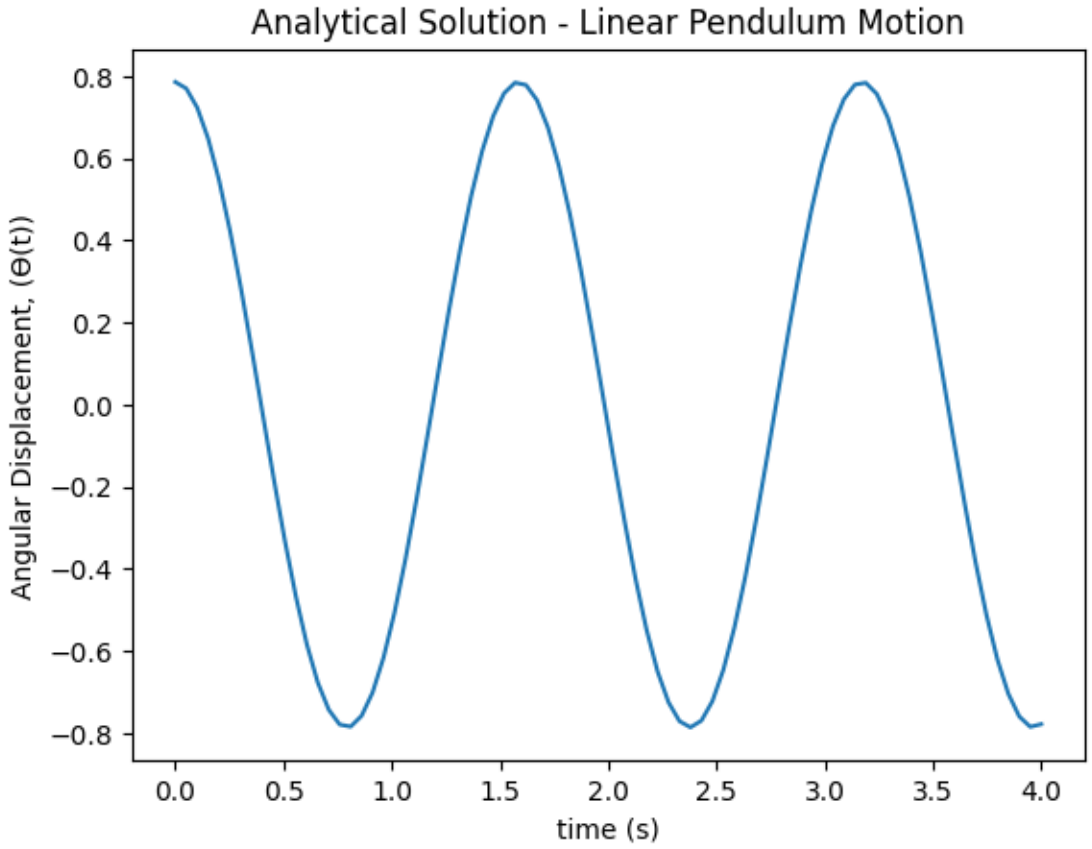
[121]:
```
def pendulum_analytical_angle_sol(velocity_naut, theta_naut, length, end_time):
    lambd = math.sqrt(gravity/length)
    A = velocity_naut / lambd
    B = theta_naut
```

```
    angle_in_time = []
    for time in time_range:
        angle_in_time.append((A * math.sin(lambd * time)) + (B * math.cos(lambd␣
↪* time)))
    return angle_in_time
```

[97]:
```
analytical_solution_output =␣
↪pendulum_analytical_angle_sol(initial_velocity,initial_position,arm_length,end_time)
```

[98]:
```
plt.xlabel('time (s)')
plt.ylabel('Angular Displacement, ( (t))')
plt.title('Analytical Solution - Linear Pendulum Motion')
plot_time = np.linspace(0, end_time, len(analytical_solution_output))
plt.plot(plot_time,analytical_solution_output)
```

[98]:
```
[<matplotlib.lines.Line2D at 0x22aae31eac0>]
```



$$v_{i+1} = v_i - \Delta t \lambda^2 \theta_i \theta_{i+1} = \theta_i + \Delta t v_i$$

[122]:
```
def pendulum_numerical_angle_sol_foward_difference(velocity_naut, theta_naut,␣
↪length, end_time, time_step):
    lambd = math.sqrt(gravity/length)
    velocity_series = [velocity_naut]
    angle_series = [theta_naut]

    for i in range(len(time_range)-1):
        velocity_series.append(velocity_series[i] - (time_step * lambd**2 *␣
↪angle_series[i]))
        angle_series.append(angle_series[i] + (time_step * velocity_series[i]))

    return angle_series
```

$$v_{i+1} = v_{i-1} - \Delta t \lambda^2 \theta_{i-1} \theta_{i+1} = \theta_{i-1} + \Delta t v_i$$

[123]:
```
def pendulum_numerical_angle_sol_backward_difference(velocity_naut, theta_naut,␣
↪length, end_time, time_step):
    lambd = math.sqrt(gravity/length)
    velocity_series = [velocity_naut]
    angle_series = [theta_naut]

    for i in range(1, len(time_range)):
        velocity_series.append(velocity_series[i-1] - (time_step * lambd**2 *␣
↪angle_series[i-1]))
        angle_series.append(angle_series[i-1] + (time_step *␣
↪velocity_series[i]))

    return angle_series
```

$$v_{i+1} = v_{i-1} - 2\Delta t \lambda^2 \theta_{i-1} \theta_{i+1} = \frac{\theta_{i-1} + \Delta t v_{i-1} + v_i}{2}$$

[138]:
```
def pendulum_numerical_angle_sol_central_difference(velocity_naut, theta_naut,␣
↪length, end_time, time_step):
    lambd = math.sqrt(gravity/length)
    velocity_series = [velocity_naut]
    angle_series = [theta_naut]

    for i in range(1, len(time_range) - 1):
        velocity_series.append(velocity_series[i-1] - (2 * time_step * lambd**2␣
↪* angle_series[i-1]))
        angle_series.append(angle_series[i-1] + (time_step *␣
↪(velocity_series[i-1] + velocity_series[i]) / 2))

    return angle_series
```

[125]:
```
numerical_solution_output_forward_difference =␣
↪pendulum_numerical_angle_sol_foward_difference(initial_velocity,initial_position,arm_length
```

[126]:
```
numerical_solution_output_backward_difference =␣
↪pendulum_numerical_angle_sol_backward_difference(initial_velocity,initial_position,arm_leng
```

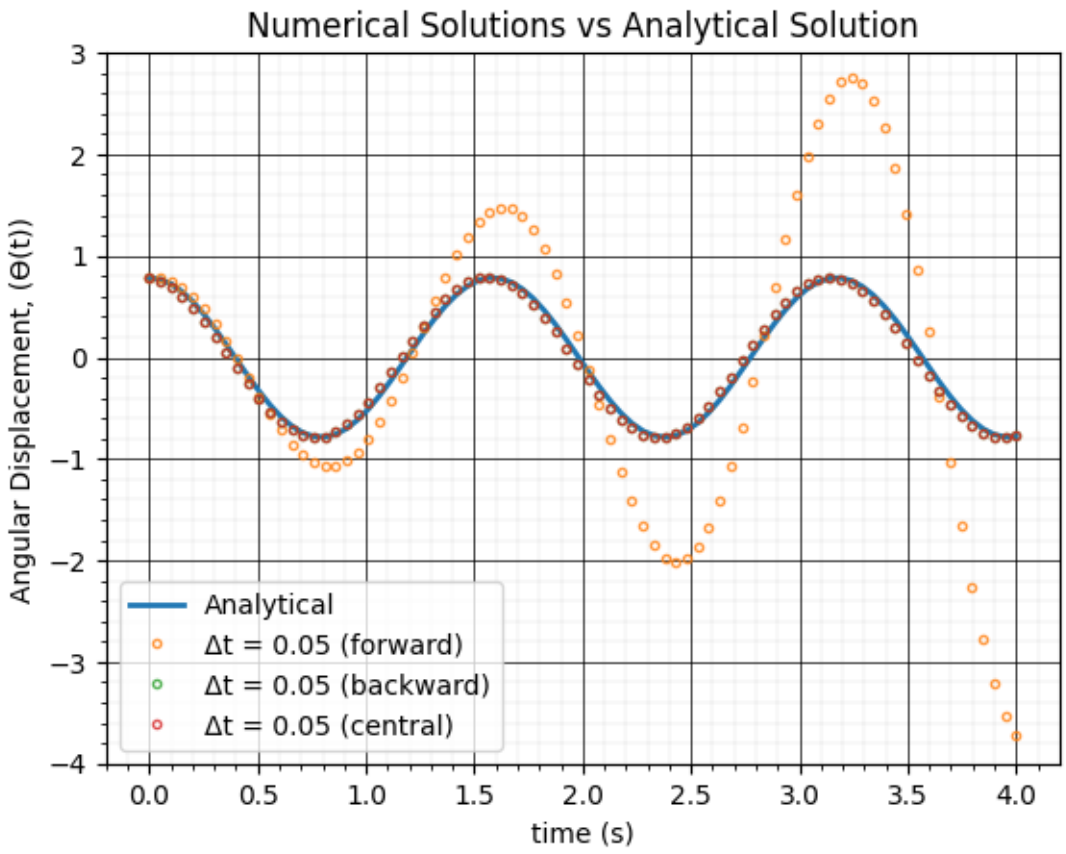
[139]:
```
numerical_solution_output_central_difference =␣
↪pendulum_numerical_angle_sol_backward_difference(initial_velocity,initial_position,arm_leng
```

[168]:
```
plt.xlabel('time (s)')
plt.ylabel('Angular Displacement, ( (t))')
plt.title('Numerical Solutions vs Analytical Solution')
plt.grid(color = 'black', which = 'major', linestyle = '-', linewidth = 0.5)
plt.grid(color = 'black', which = 'minor', linestyle = '--', linewidth = 0.05)
plt.minorticks_on()
plt.ylim([-4, 3])

plot_time = np.linspace(0, end_time, len(analytical_solution_output))
plt.plot(plot_time,analytical_solution_output, label='Analytical', linewidth=2)
plt.plot(plot_time,numerical_solution_output_forward_difference,"o" ,␣
↪label=f'Δt = {time_step} (forward)', markersize=3, alpha = 0.8, mfc='none')
plt.plot(plot_time,numerical_solution_output_backward_difference,"o" ,␣
↪label=f'Δt = {time_step} (backward)', markersize=3, alpha = 0.8, mfc='none')
plt.plot(plot_time,numerical_solution_output_central_difference,"o" ,␣
↪label=f'Δt = {time_step} (central)', markersize=3, alpha = 0.8, mfc='none')

plt.legend(loc="lower left")
```

[168]:
```
<matplotlib.legend.Legend at 0x22aafa32190>
```



[224]:
```
forward_error_plot = []
def calculate_error_forward(numerical_solution, analytical_solution):
    for i in range(len(time_range)):
        forward_error_plot.append(abs(numerical_solution[i] -␣
 ↪analytical_solution[i]))
        print( f'error at time {i*time_step:.2f} = {abs(numerical_solution[i] -␣
 ↪analytical_solution[i])}')
```

[225]:
```
backward_error_plot = []
def calculate_error_backward(numerical_solution, analytical_solution):
    for i in range(len(time_range)):
        backward_error_plot.append(abs(numerical_solution[i] -␣
 ↪analytical_solution[i]))
        print( f'error at time {i*time_step:.2f} = {abs(numerical_solution[i] -␣
 ↪analytical_solution[i])}')
```

[226]:
```
central_error_plot = []
def calculate_error_central(numerical_solution, analytical_solution):
    for i in range(len(time_range)):
```

```
        central_error_plot.append(abs(numerical_solution[i] -␣
↪analytical_solution[i]))
        print( f'error at time {i*time_step:.2f} = {abs(numerical_solution[i] -␣
↪analytical_solution[i])}')
```

[228]:
```
calculate_error_forward(numerical_solution_output_forward_difference,analytical_solution_outpu
```

```
error at time 0.00 = 0.0
error at time 0.05 = 0.015753192696299823
error at time 0.10 = 0.030768552614130584
error at time 0.15 = 0.04317560393873343
error at time 0.20 = 0.05121276571312727
error at time 0.25 = 0.053349060281358385
error at time 0.30 = 0.048396399490262976
error at time 0.35 = 0.03560588981590368
error at time 0.40 = 0.01474224556935479
error at time 0.45 = 0.013868632868641817
error at time 0.50 = 0.04932261033461188
error at time 0.55 = 0.09015301672624332
error at time 0.60 = 0.13437967802237116
error at time 0.65 = 0.17959013952137604
error at time 0.70 = 0.2230502176530479
error at time 0.75 = 0.2618393735831187
error at time 0.80 = 0.2930049235998783
error at time 0.85 = 0.3137278660292738
error at time 0.90 = 0.3214921811412097
error at time 0.95 = 0.31424890552711726
error at time 1.00 = 0.29056613641615847
error at time 1.05 = 0.2497564069424522
error at time 1.10 = 0.1919735933380795
error at time 1.15 = 0.11827265174979035
error at time 1.20 = 0.030626997820630372
error at time 1.25 = 0.06809982100945122
error at time 1.30 = 0.17422942701149818
error at time 1.35 = 0.2833899129084433
error at time 1.40 = 0.39067986854903103
error at time 1.45 = 0.4908696104415532
error at time 1.50 = 0.5786319987537754
error at time 1.55 = 0.6487933970182086
error at time 1.60 = 0.6965938609001185
error at time 1.65 = 0.7179446187727777
error at time 1.70 = 0.709670396162308
error at time 1.75 = 0.6697241851578973
error at time 1.80 = 0.5973626915930043
error at time 1.85 = 0.49327190434617835
error at time 1.90 = 0.3596339929501954
error at time 1.95 = 0.20012899588519176
error at time 2.00 = 0.019867431578619726
```

```
error at time 2.05 = 0.17474705592927242
error at time 2.10 = 0.3762227361759148
error at time 2.15 = 0.576247014253799
error at time 2.20 = 0.7660072026438961
error at time 2.25 = 0.936553315818389
error at time 2.30 = 1.0791888447907119
error at time 2.35 = 1.1858734624213776
error at time 2.40 = 1.249620227177052
error at time 2.45 = 1.2648691842506974
error at time 2.50 = 1.2278193717171004
error at time 2.55 = 1.1367021571595228
error at time 2.60 = 0.9919805538751035
error at time 2.65 = 0.7964616564244861
error at time 2.70 = 0.5553125183737855
error at time 2.75 = 0.27597356216181074
error at time 2.80 = 0.03203217684366602
error at time 2.85 = 0.35739117754930533
error at time 2.90 = 0.6873855485110446
error at time 2.95 = 1.008361260508396
error at time 3.00 = 1.306262439158779
error at time 3.05 = 1.567211972731814
error at time 3.10 = 1.77811568498602
error at time 3.15 = 1.9272651621821963
error at time 3.20 = 2.004913140286951
error at time 3.25 = 2.0037952491463322
error at time 3.30 = 1.9195729292596644
error at time 3.35 = 1.7511744915480607
error at time 3.40 = 1.501014540357213
error at time 3.45 = 1.175076235040019
error at time 3.50 = 0.7828459887929591
error at time 3.55 = 0.33709601457844934
error at time 3.60 = 0.1464835909098469
error at time 3.65 = 0.6497950307983171
error at time 3.70 = 1.1529706068507322
error at time 3.75 = 1.6351173470441598
error at time 3.80 = 2.075142140986203
error at time 3.85 = 2.4526263905586436
error at time 3.90 = 2.748715611863745
error at time 3.95 = 2.9469871549380766
```

[229]:
```
calculate_error_backward(numerical_solution_output_backward_difference,analytical_solution_out
```

```
error at time 0.00 = 0.0
error at time 0.05 = 0.015859083380447503
error at time 0.10 = 0.03118360542727494
error at time 0.15 = 0.045350467594074684
error at time 0.20 = 0.05777925670770967
error at time 0.25 = 0.06795601565181308
```

```
error at time 0.30 = 0.07545447965262289
error at time 0.35 = 0.07995390136287966
error at time 0.40 = 0.08125272049015995
error at time 0.45 = 0.07927749729772282
error at time 0.50 = 0.07408671748196821
error at time 0.55 = 0.06586928122529534
error at time 0.60 = 0.05493770341879067
error at time 0.65 = 0.041716266515042455
error at time 0.70 = 0.026724573511409666
error at time 0.75 = 0.010557137769489744
error at time 0.80 = 0.006140189031088039
error at time 0.85 = 0.022696016355472692
error at time 0.90 = 0.038440411129906704
error at time 0.95 = 0.05273200539904499
error at time 1.00 = 0.06498411851058
error at time 1.05 = 0.07468883008888416
error at time 1.10 = 0.0814380164445162
error at time 1.15 = 0.08494047993127851
error at time 1.20 = 0.08503445361815624
error at time 1.25 = 0.08169494651414766
error at time 1.30 = 0.0750356002403606
error at time 1.35 = 0.06530494828897082
error at time 1.40 = 0.052877195004501165
error at time 1.45 = 0.038237854045136466
error at time 1.50 = 0.02196479628792214
error at time 1.55 = 0.004705446338519548
error at time 1.60 = 0.012848972798989777
error at time 1.65 = 0.02999112241558599
error at time 1.70 = 0.046026049502966404
error at time 1.75 = 0.060299358445327966
error at time 1.80 = 0.07222391107032855
error at time 1.85 = 0.0813039659155558
error at time 1.90 = 0.08715576476842907
error at time 1.95 = 0.08952371248055929
error at time 2.00 = 0.08829146939271462
error at time 2.05 = 0.08348747761876812
error at time 2.10 = 0.07528466491406555
error at time 2.15 = 0.06399430400317518
error at time 2.20 = 0.05005424164793604
error at time 2.25 = 0.034011940836680576
error at time 2.30 = 0.016502991949021495
error at time 2.35 = 0.0017740641031128712
error at time 2.40 = 0.020085603662526874
error at time 2.45 = 0.037692367331442767
error at time 2.50 = 0.05387930421516196
error at time 2.55 = 0.06798460860152644
error at time 2.60 = 0.07942677008512028
error at time 2.65 = 0.08772853182342105
```

```
error at time 2.70 = 0.0925367704430084
error at time 2.75 = 0.09363747053961419
error at time 2.80 = 0.09096516050938133
error at time 2.85 = 0.08460639695600675
error at time 2.90 = 0.07479712336157984
error at time 2.95 = 0.06191397553461364
error at time 3.00 = 0.046459851611641456
error at time 3.05 = 0.02904429815210796
error at time 3.10 = 0.010359476604102635
error at time 3.15 = 0.008847342595556862
error at time 3.20 = 0.02780366535907275
error at time 3.25 = 0.04574284355245761
error at time 3.30 = 0.061935079249733405
error at time 3.35 = 0.07571710983175772
error at time 3.40 = 0.08651936663934717
error at time 3.45 = 0.0938894954983786
error at time 3.50 = 0.09751126853166188
error at time 3.55 = 0.09721809779379552
error at time 3.60 = 0.0930005752961929
error at time 3.65 = 0.08500770244570166
error at time 3.70 = 0.07354172528328762
error at time 3.75 = 0.05904675000665993
error at time 3.80 = 0.04209156567513095
error at time 3.85 = 0.023347337483247244
error at time 3.90 = 0.003561044899996979
error at time 3.95 = 0.016474284365165848
```

[230]:
```
calculate_error_central(numerical_solution_output_central_difference,analytical_solution_outpu
```

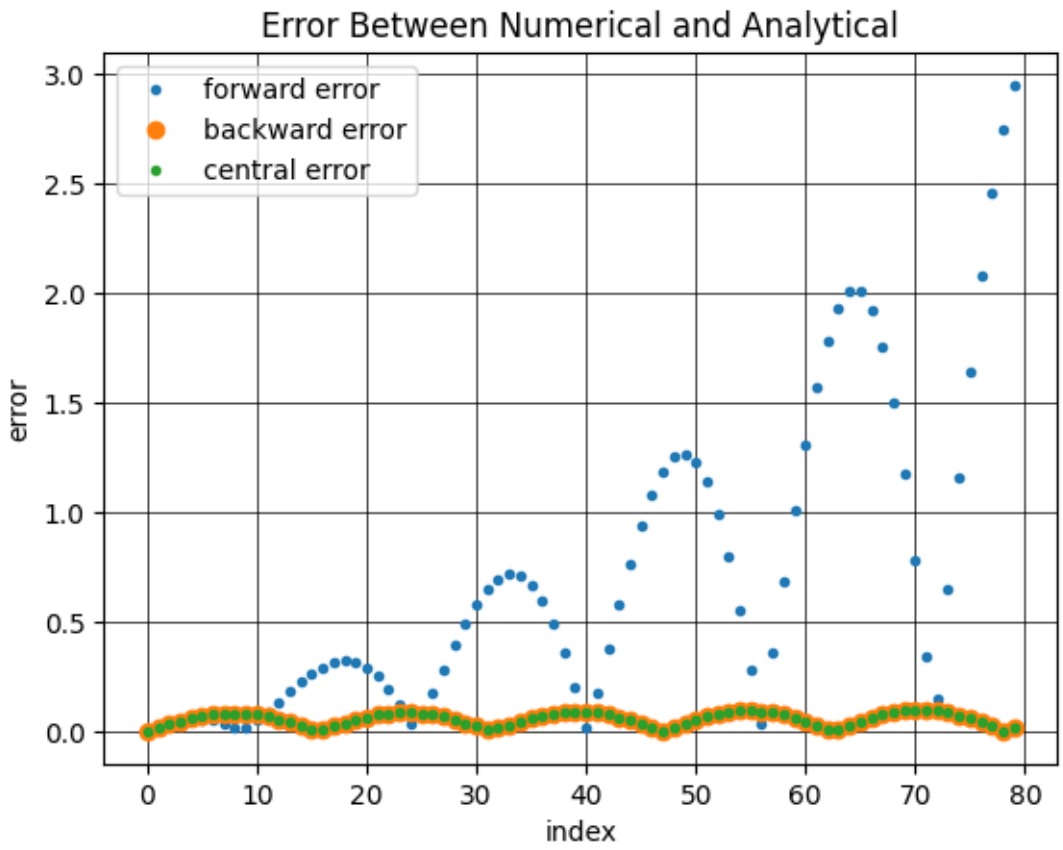
```
error at time 0.00 = 0.0
error at time 0.05 = 0.015859083380447503
error at time 0.10 = 0.03118360542727494
error at time 0.15 = 0.045350467594074684
error at time 0.20 = 0.05777925670770967
error at time 0.25 = 0.06795601565181308
error at time 0.30 = 0.07545447965262289
error at time 0.35 = 0.07995390136287966
error at time 0.40 = 0.08125272049015995
error at time 0.45 = 0.07927749729772282
error at time 0.50 = 0.07408671748196821
error at time 0.55 = 0.06586928122529534
error at time 0.60 = 0.05493770341879067
error at time 0.65 = 0.041716266515042455
error at time 0.70 = 0.026724573511409666
error at time 0.75 = 0.010557137769489744
error at time 0.80 = 0.006140189031088039
error at time 0.85 = 0.022696016355472692
error at time 0.90 = 0.038440411129906704
```

```
error at time 0.95 = 0.05273200539904499
error at time 1.00 = 0.06498411851058
error at time 1.05 = 0.07468883008888416
error at time 1.10 = 0.0814380164445162
error at time 1.15 = 0.08494047993127851
error at time 1.20 = 0.08503445361815624
error at time 1.25 = 0.08169494651414766
error at time 1.30 = 0.0750356002403606
error at time 1.35 = 0.06530494828897082
error at time 1.40 = 0.052877195004501165
error at time 1.45 = 0.038237854045136466
error at time 1.50 = 0.02196479628792214
error at time 1.55 = 0.004705446338519548
error at time 1.60 = 0.012848972798989777
error at time 1.65 = 0.02999112241558599
error at time 1.70 = 0.046026049502966404
error at time 1.75 = 0.060299358445327966
error at time 1.80 = 0.07222391107032855
error at time 1.85 = 0.0813039659155558
error at time 1.90 = 0.08715576476842907
error at time 1.95 = 0.08952371248055929
error at time 2.00 = 0.08829146939271462
error at time 2.05 = 0.08348747761876812
error at time 2.10 = 0.07528466491406555
error at time 2.15 = 0.06399430400317518
error at time 2.20 = 0.05005424164793604
error at time 2.25 = 0.034011940836680576
error at time 2.30 = 0.016502991949021495
error at time 2.35 = 0.0017740641031128712
error at time 2.40 = 0.020085603662526874
error at time 2.45 = 0.037692367331442767
error at time 2.50 = 0.05387930421516196
error at time 2.55 = 0.06798460860152644
error at time 2.60 = 0.07942677008512028
error at time 2.65 = 0.08772853182342105
error at time 2.70 = 0.0925367704430084
error at time 2.75 = 0.09363747053961419
error at time 2.80 = 0.09096516050938133
error at time 2.85 = 0.08460639695600675
error at time 2.90 = 0.07479712336157984
error at time 2.95 = 0.06191397553461364
error at time 3.00 = 0.046459851611641456
error at time 3.05 = 0.02904429815210796
error at time 3.10 = 0.010359476604102635
error at time 3.15 = 0.008847342595556862
error at time 3.20 = 0.02780366535907275
error at time 3.25 = 0.04574284355245761
error at time 3.30 = 0.061935079249733405
```

```
error at time 3.35 = 0.07571710983175772
error at time 3.40 = 0.08651936663934717
error at time 3.45 = 0.0938894954983786
error at time 3.50 = 0.09751126853166188
error at time 3.55 = 0.09721809779379552
error at time 3.60 = 0.0930005752961929
error at time 3.65 = 0.08500770244570166
error at time 3.70 = 0.07354172528328762
error at time 3.75 = 0.05904675000665993
error at time 3.80 = 0.04209156567513095
error at time 3.85 = 0.023347337483247244
error at time 3.90 = 0.003561044899996979
error at time 3.95 = 0.016474284365165848
```

[247]:
```
# plot errors
plt.plot(forward_error_plot,"o" , label='forward error', markersize=3)
plt.plot(backward_error_plot,"o" , label='backward error', markersize=6)
plt.plot(central_error_plot,"o" , label='central error', markersize=3)
plt.xlabel('index')
plt.ylabel('error')
plt.title('Error Between Numerical and Analytical')
plt.grid(color = 'black', which = 'major', linestyle = '-', linewidth = 0.5)
plt.grid(color = 'black', which = 'minor', linestyle = '--', linewidth = 0.05)
plt.legend(loc="upper left")
```

[247]:
```
<matplotlib.legend.Legend at 0x22ab001dcd0>
```


Error Between Numerical and Analytical
forward error
backward error
central error
error
index
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
10
20
30
40
50
60
70
80