goertzel_test

November 14, 2023

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import bessel, lfilter, welch
```

Goertzel reference: https://www.embedded.com/the-goertzel-algorithm/ https://www.dsp-weimich.com/digital-signal-processing/goertzel-algorithm-and-c-implementation-using-the-octave-gnu-tool/

[ ]:
```
# Goertzel algorithm function
def goertzel(samples, sample_rate, target_freq, n):
    """
    The Goertzel algorithm is used to calculate the magnitude of a specific␣
 ↪frequency component within a signal.
    This is particularly efficient for computing the spectral content at␣
 ↪specific frequencies of interest.
    """
    k = int(0.5 + n * target_freq / sample_rate)
    omega = (2 * np.pi / n) * k
    coeff = 2 * np.cos(omega)
    q0, q1, q2 = 0, 0, 0
    for sample in samples:
        q0 = coeff * q1 - q2 + sample
        q2 = q1
        q1 = q0
    real = (q1 - q2 * np.cos(omega))
    imag = (q2 * np.sin(omega))
    return np.sqrt(real**2 + imag**2)

# Function to calculate magnitude response over a range of frequencies
def magnitude_response(signal, sample_rate, freq_range, n):
    magnitudes = []
    for freq in freq_range:
        magnitude = goertzel(signal, sample_rate, freq, n)
        magnitudes.append(magnitude)
    return magnitudes

# Function to calculate power response over a range of frequencies
def power_response(signal, sample_rate, freq_range, n):
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    powers = []
    for freq in freq_range:
        magnitude = goertzel(signal, sample_rate, freq, n)
        power = magnitude**2  # Squaring the magnitude to get power
        normalized_power = power / n**2  # Normalization by the square of the␣
↪number of samples
        powers.append(normalized_power)
    return powers

# Function to apply a 4th order Bessel filter
def apply_bessel_filter(signal, sample_rate, cutoff_freq):
    """
    This function applies a 4th order Bessel filter to the signal.
    The Bessel filter is known for its linear phase response but has a gentler␣
↪roll-off compared to other filters.
    The cutoff frequency determines the frequency point at which the filter␣
↪starts attenuating the signal.
    """
    b, a = bessel(N=4, Wn=cutoff_freq/(0.5*sample_rate), btype='low',␣
↪analog=False)
    return lfilter(b, a, signal)

def calculate_psd_rfu(signal, sample_rate, center_freq, bandwidth):
    """
    This function calculates the Power Spectral Density (PSD) which will be␣
↪used as RFU's.
    PSD provides a measure of the signal's power content versus frequency.
    RFU is calculated as the sum of PSD values within a specified frequency␣
↪range, indicating the signal's strength in that range.
    """

    """
    The function uses Welch's method (via the welch function from libraries␣
↪like SciPy)
    to estimate the PSD of the input signal. Welch's method divides the signal
    into overlapping segments, computes a modified periodogram for each␣
↪segment, and then averages
    these periodograms to estimate the PSD
    """
    f, Pxx = welch(signal, fs=sample_rate, nperseg=1024)   # nperseg is the␣
↪number of samples per segment (decrease for less noise)
    # Find the frequency range of interest
    freq_range = (f >= center_freq - bandwidth/2) & (f <= center_freq +␣
↪bandwidth/2)
    # Isolate the PSD values within the frequency range of interest
    Pxx_interest = Pxx[freq_range]
```

```
    f_interest = f[freq_range]
    # Calculate RFU as the sum of PSD in the frequency range of interest
    rfu = np.sum(Pxx_interest)
    return f_interest, Pxx_interest, rfu

# Function to add white Gaussian noise to a signal
def add_white_noise(signal, noise_level):
    """
    This function adds white Gaussian noise to the signal.
    The noise level is a fraction of the signal's standard deviation, allowing␣
 ↪control over the signal-to-noise ratio.
    Adding noise can be useful for testing the robustness of signal processing␣
 ↪algorithms.
    """
    mean_noise = 0
    std_noise = noise_level * np.std(signal)
    noise = np.random.normal(mean_noise, std_noise, len(signal))
    return signal + noise
```

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# Noise level as a fraction of the signal's standard deviation
noise_level = 0.3  # Adjust this to increase or decrease noise level

# Signal parameters
sampling_rate = 32000  # Hz
duration = 1           # seconds
frequencies = [2000, 2250, 2500, 2750, 3000]  # Hz
amplitude = 2.0  # Amplitude factor

# Time array
t = np.arange(0, duration, 1 / sampling_rate)

signals = [amplitude * np.sin(2 * np.pi * f * t) for f in frequencies]

# Plotting parameters
plot_duration = 0.03  # seconds to display in the plot
samples_to_plot = int(sampling_rate * plot_duration)

plt.figure(figsize=(12, 8))
# Plot each frequency component within a smaller time window
for i, signal in enumerate(signals):
    plt.subplot(len(frequencies), 1, i + 1)
    plt.plot(t[:samples_to_plot], signal[:samples_to_plot])
    plt.title(f"Frequency {frequencies[i]} Hz Component")
    plt.xlabel("Time (s)")
    plt.ylabel("Amplitude")
    plt.tight_layout()
```

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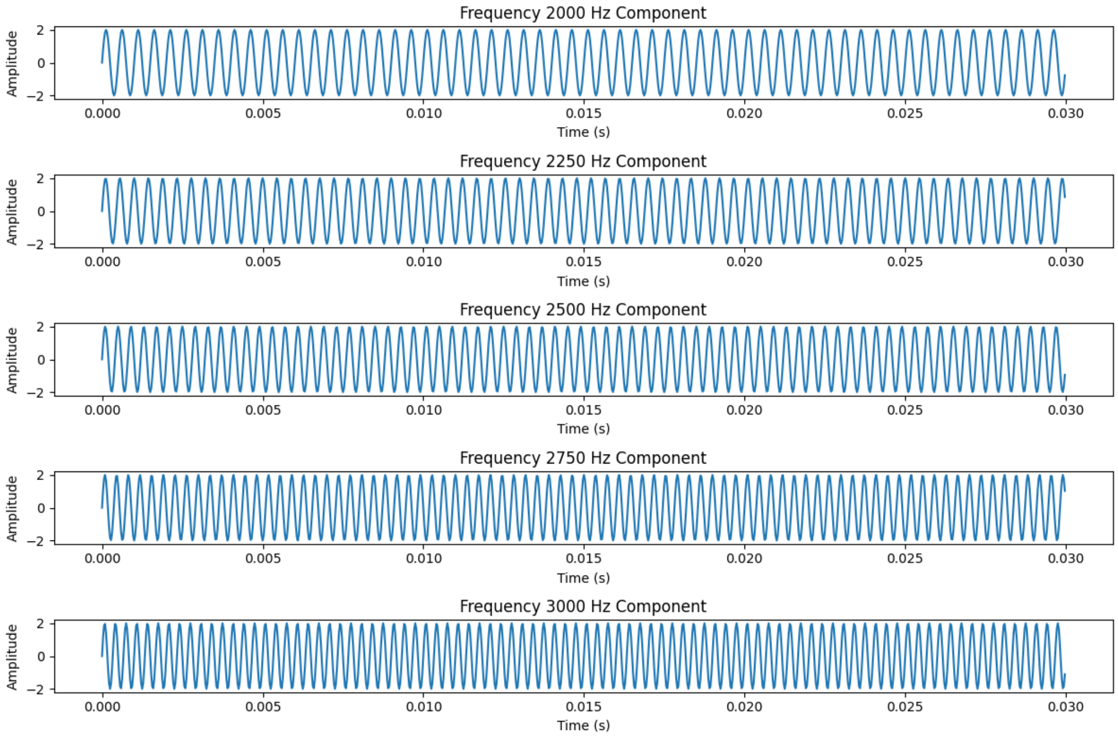
plt.show()
```



[ ]:
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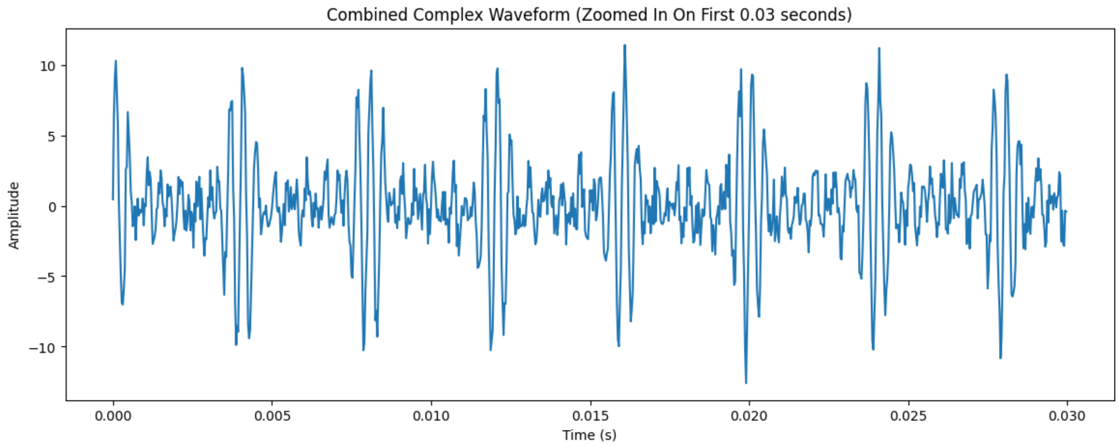
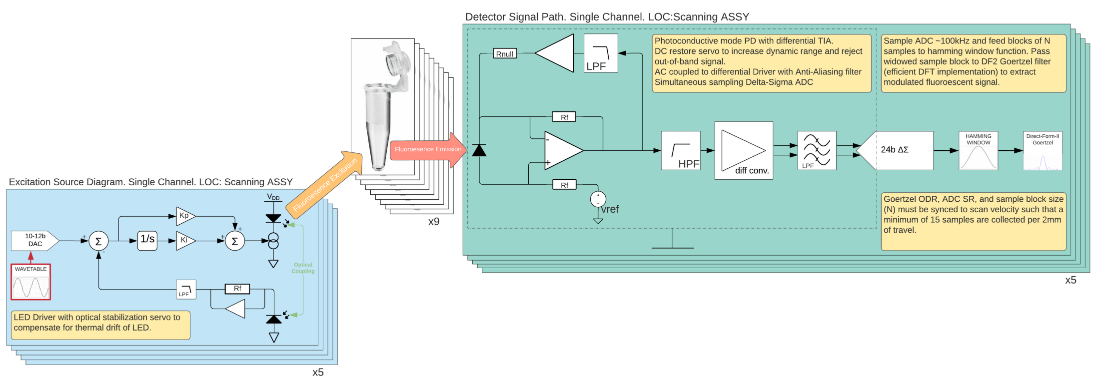
# Combining signals
combined_signal = np.sum(signals, axis=0)
combined_signal = add_white_noise(combined_signal, noise_level)

# Plotting the combined signal within the same smaller time window
plt.figure(figsize=(14, 5))
plt.plot(t[:samples_to_plot], combined_signal[:samples_to_plot])
plt.title(f"Combined Complex Waveform (Zoomed In On First {plot_duration}␣
 ↪seconds)")
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.show()
```





ADC sampled - Block of N samples (complex waveform) fed to MCU

1) 4th order Bessel filter with 0.05 cut-off frequency applied
2) Hamming window applied
3) Filtered complex waveform put through Goertzel to extract modulated fluorescent signal

[ ]:
```python
# Apply Bessel filter
''' low-pass filter that is used to remove high-frequency noise or components
 from signal'''
cutoff_frequency = 0.15 * sampling_rate  # Normalized cutoff frequency
# filtered_signal = combined_signal  # un-comment this line to skip filtering
filtered_signal = apply_bessel_filter(combined_signal, sampling_rate,
 cutoff_frequency)

# Apply Hamming window to the filtered signal
'''window function that is used to mitigate the spectral leakage in the Fourier
 transform of the signal'''
windowed_signal = filtered_signal * np.hamming(len(filtered_signal))

# Applying the Goertzel algorithm to the windowed signal
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```
n = len(windowed_signal)
results = {f: goertzel(windowed_signal, sampling_rate, f, n) for f in␣
↪frequencies}
```

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```
# Displaying the results
for f, magnitude in results.items():
    print(f"Frequency {f} Hz, Magnitude: {magnitude}")

# # Displaying the power results
# for f, power in power_results.items():
#     print(f"Frequency {f} Hz, Power: {power}")

# Define frequency range for the magnitude response
freq_start = 1750  # Start frequency in Hz
freq_end = 3250    # End frequency in Hz
freq_step = 10     # Frequency step in Hz
frequency_range = np.arange(freq_start, freq_end, freq_step)

# Calculate magnitude response of the windowed signal
n = len(windowed_signal)
magnitudes = magnitude_response(windowed_signal, sampling_rate,␣
↪frequency_range, n)

# Applying the Goertzel algorithm to the windowed signal
n = len(windowed_signal)
power_results = {f: goertzel(windowed_signal, sampling_rate, f, n)**2 / n**2␣
↪for f in frequencies}
# Calculate power response of the windowed signal
power_values = power_response(windowed_signal, sampling_rate, frequency_range,␣
↪n)

# Convert magnitudes to decibels
magnitudes_db = 20 * np.log10(magnitudes)

# Plotting setup
plt.figure(figsize=(14, 5))

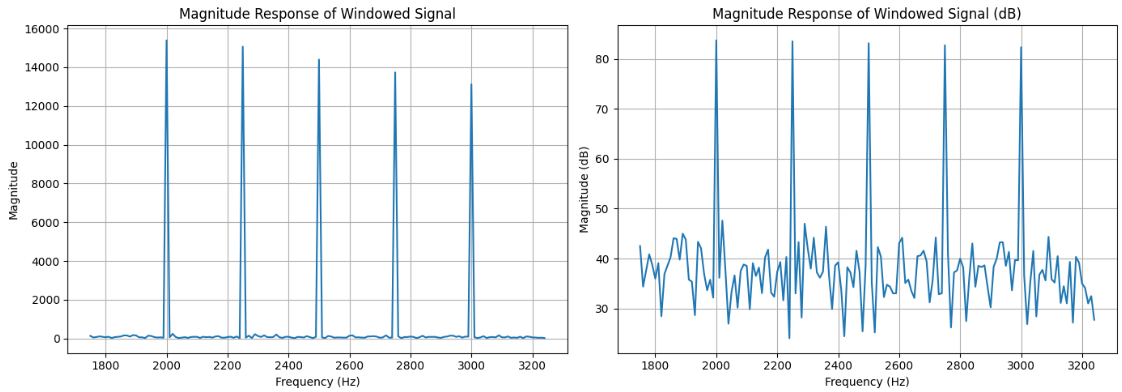

# Linear scale
plt.subplot(1, 2, 1)
plt.plot(frequency_range, magnitudes)
plt.title("Magnitude Response of Windowed Signal")
plt.xlabel("Frequency (Hz)")
plt.ylabel("Magnitude")
plt.grid(True)

# dB scale
```

```
plt.subplot(1, 2, 2)
plt.plot(frequency_range, magnitudes_db)
plt.title("Magnitude Response of Windowed Signal (dB)")
plt.xlabel("Frequency (Hz)")
plt.ylabel("Magnitude (dB)")
plt.grid(True)

plt.tight_layout()
plt.show()
```

```
Frequency 2000 Hz, Magnitude: 15391.877328545863
Frequency 2250 Hz, Magnitude: 15064.344640740394
Frequency 2500 Hz, Magnitude: 14400.167032960391
Frequency 2750 Hz, Magnitude: 13737.027195523766
Frequency 3000 Hz, Magnitude: 13123.12246181763
```



Interpreting PCR amplication (RFU's) as Power Spectral Density

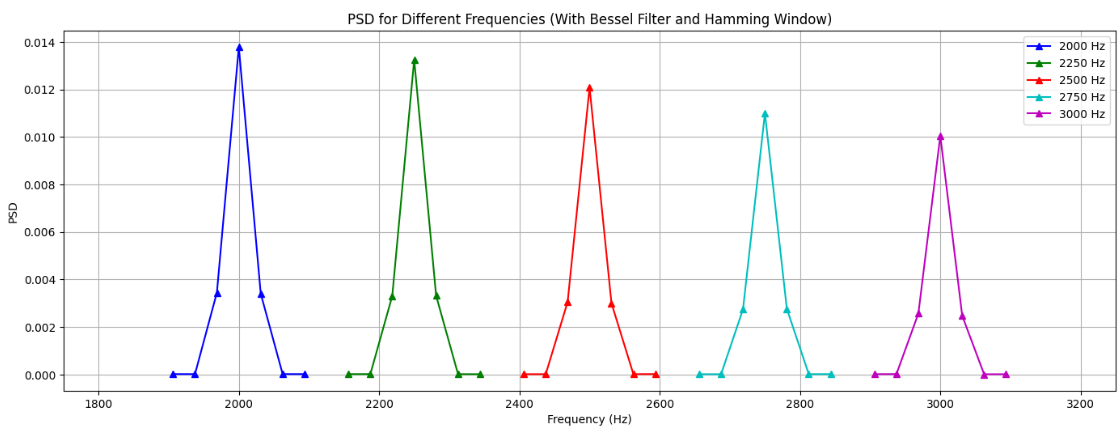
[ ]:
```
# Frequencies of interest and padding
frequencies_of_interest = frequencies
padding = 250  # Hz
bandwidth = 200  # Hz

# Plotting setup
plt.figure(figsize=(14,10))
colors = ['b', 'g', 'r', 'c', 'm']  # Different color for each frequency plot
# Plot 1: PSD with Bessel filter and Hamming window
plt.subplot(2, 1, 1)  # This creates a subplot in a 2-row, 1-column grid
for i, center_frequency in enumerate(frequencies_of_interest):
    freqs, psd, _ = calculate_psd_rfu(windowed_signal, sampling_rate,
 ↪center_frequency, bandwidth)
    plt.plot(freqs, psd, '^-', label=f'{center_frequency} Hz', color=colors[i %
 ↪len(colors)])
```

```
plt.title("PSD for Different Frequencies (With Bessel Filter and Hamming␣
↪Window)")
plt.xlabel("Frequency (Hz)")
plt.ylabel("PSD")
plt.xlim(min(frequencies_of_interest) - padding, max(frequencies_of_interest) +␣
↪padding)
plt.legend()
plt.grid(True)

# Plot 2: Power Response
# plt.subplot(2, 1, 2)  # This creates the second subplot
# plt.plot(frequency_range, power_values, label="Power Response")
# plt.title("Power Response of Windowed Signal")
# plt.xlabel("Frequency (Hz)")
# plt.ylabel("Normalized Power")
# plt.grid(True)
# plt.legend()

# Show the plots
plt.tight_layout()
plt.show()
```



Understanding the effects of the Bessel Filter the cut off freqeuncy steps attenuates the signal with increasing frequency

[ ]:
```
from scipy.signal import freqz

"""
The plot shows how the filter attenuates frequencies above the cutoff frequency.
↪
The Bessel filter is characterized by a smooth transition from the passband to␣
↪the stopband,
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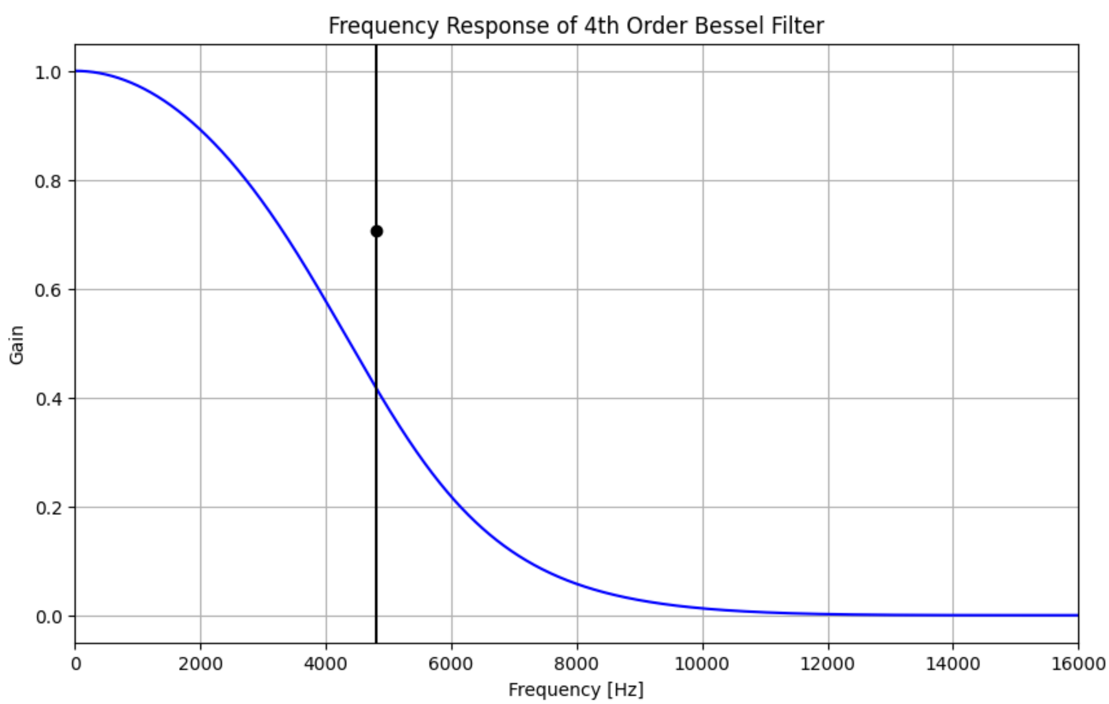
```
and the plot illustrates this with a gradual roll-off in gain beyond the cutoff␣
 ↪frequency.
The gain at the cutoff frequency is typically 0.707 (or -3dB), which is a␣
 ↪standard reference point for filter design.
"""

# Define the Bessel filter parameters
b, a = bessel(N=4, Wn=cutoff_frequency/(0.5*sampling_rate), btype='low',␣
 ↪analog=False)

# Compute the frequency response
w, h = freqz(b, a, worN=8000)

# Plot the frequency response
plt.figure(figsize=(10, 6))
plt.plot(0.5 * sampling_rate * w / np.pi, np.abs(h), 'b')
plt.plot(cutoff_frequency, 0.5*np.sqrt(2), 'ko')
print(cutoff_frequency)
plt.axvline(cutoff_frequency, color='k')
plt.xlim(0, 0.5 * sampling_rate)
plt.title("Frequency Response of 4th Order Bessel Filter")
plt.xlabel('Frequency [Hz]')
plt.ylabel('Gain')
plt.grid()
plt.show()
```

```
4800.0
```



1) the vertical line indicates the cut-off frequency value
2) the dot represents $0.5 * \sqrt{(}2)$ corresponding to -3dB, ~=0.707 which is 70.7% drop in signal

#———————————————————————— Next Steps 1) inegrate scanning velocity metrics 2) modify the function to have both white and pink noise 3) add simulated PCR curve data the data and run algorithm 4) create simulation to take block of data from scope and run through algorithm