

$$\frac{dv}{dt} = 1 - v^2 \quad v(0) = 0$$

$$\frac{dv}{1-v^2} = dt \longrightarrow \int \frac{dv}{1-v^2} = \int dt$$

$$\int \frac{dv}{(1-v)(1+v)} = \int dt$$

partial fraction: $\frac{1}{(1-v)(1+v)} = \frac{A}{1+v} + \frac{B}{1-v}$

for $v = -1$
 $A = \frac{1}{2}$

for $v = 1$
 $B = \frac{1}{2}$

$$\int \frac{dv}{(1-v)(1+v)} = \int \frac{1}{2(1+v)} dv + \int \frac{1}{2(1-v)} dv$$

$$= \frac{1}{2} \ln|1+v| - \frac{1}{2} \ln|1-v|$$

$$\boxed{C' = e^{2C_1}}$$

$$\ln \left| \frac{1+v}{1-v} \right| = 2t + 2C_1 \longrightarrow \frac{1+v}{1-v} = C' e^{2t}$$

$$v = \frac{C' e^{2t} - 1}{C' e^{2t} + 1}$$

$$\longrightarrow v(0) = 1$$

$$v(t) = \frac{e^{2t} - 1}{e^{2t} + 1}$$