1.

$$\boxed{m\left(\frac{dv}{dt}\right) = mg - c(v)^2 \; ; \quad v(0) = 0}$$ ~ $\frac{dv}{dt} = 0 \; @ \; V_s$

$$\frac{dv}{dt} = g - \frac{c(v)^2}{m}$$

Variables: $v, t$
parameters: $g, c, m$

$$\tilde{v} = \frac{v - \cancel{v_r}}{v_s} \qquad \tilde{t} = \frac{t - \cancel{t_r}}{t_s}$$

$\_r$ = reference value
$\_s$ = scaling factor

$v(0) = 0 \rightarrow v_r, t_r = 0$

$$\frac{dv}{dt} = \frac{d(\tilde{v} \cdot v_s)}{d(\tilde{t} \cdot t_s)} \longrightarrow \frac{v_s}{t_s} \frac{d\tilde{v}}{d\tilde{t}}$$

non-dim

$$\left[ m \frac{v_s}{t_s} = mg \qquad t_s = \frac{v_s}{g} \right]$$

$$m \frac{v_s}{t_s} \frac{d\tilde{v}}{d\tilde{t}} = mg - c v_s^2 \tilde{v}^2$$

$$m \frac{v_s}{\frac{v_s}{g}} \frac{d\tilde{v}}{d\tilde{t}} = mg - c v_s^2 \tilde{v}^2$$

— divide by $mg$

$$\frac{d\tilde{v}}{d\tilde{t}} = 1 - \frac{c}{mg} v_s^2 \tilde{v}^2$$

— choose $v_c$ to be terminal velocity $\frac{dv}{dt} = 0 = g - \frac{cv^2}{m}$

$v_c^2 = \frac{mg}{c}$

$$\frac{d\tilde{v}}{d\tilde{t}} = 1 - \tilde{v}^2$$

- solve ODE and plot
- set $v = 0.95 \longrightarrow$ find $t$ value
- check $v = 1 \longrightarrow \frac{dv}{dt}$ should $= 0$

$$\frac{dv}{dt} = 1 - v^2 \qquad v(0) = 0$$

$$\frac{dv}{1-v^2} = dt \longrightarrow \int \frac{dv}{1-v^2} = \int dt$$

$$\int \frac{dv}{(1-v)(1+v)} = \int dt$$

partial fraction: $\frac{1}{(1-v)(1+v)} = \frac{A}{1+v} + \frac{B}{1-v}$

for $v = -1$
$A = \frac{1}{2}$

for $v = 1$
$B = \frac{1}{2}$

$$\int \frac{dv}{(1-v)(1+v)} = \int \frac{1}{2(1+v)}\, dv + \int \frac{1}{2(1-v)}\, dv$$

$$= \frac{1}{2}\ln|1+v| - \frac{1}{2}\ln|1-v|$$

$$\boxed{c' = e^{2c_1}}$$

$$\ln\left|\frac{1+v}{1-v}\right| = 2t + 2c_1 \longrightarrow \frac{1+v}{1-v} = c'e^{2t}$$

$$v = \frac{c'e^{2t} - 1}{c'e^{2t} + 1} \longrightarrow v(0) = 1$$

$$v(t) = \frac{e^{2t} - 1}{e^{2t} + 1}$$