2. $y'' + 9y = 0 \qquad y(1) = 0 \qquad y(5) + 14y'(5) = 10$

power series:

derivatives

$$y = \sum_{j=0}^{\infty} c_j x^j = 0$$

$$y' = \sum_{j=0}^{\infty} j c_j x^{j-1} = 0$$

$$y'' = \sum_{j=0}^{\infty} j(j-1) c_j x^{j-2} = 0$$

substitution:

$$\sum_{j=0}^{\infty} j(j-1) c_j x^{j-2} + \sum_{j=0}^{\infty} 9 c_j x^j = 0$$

$k = j \qquad k = j-2$

match powers:

$$\sum_{k=0}^{\infty} k(k-1) c_k x^k + \sum_{k=2} 9 C_{k-2} x^{k-2} = 0$$

$\downarrow$

match K param:

$@k=0 \therefore 0$

$@k=1 \therefore 0$

$$y = \sum_{k=2}^{\infty} c_k \left[k(k-1) + 9 c_{k-2} \right] x^{k-2} = 0$$

recursion relation:

$$c_k = \frac{-9}{k(k-1)} c_{k-2}$$

for $f_N = \dfrac{-9}{k(k-1)}$ $\qquad c_k = f_N C_{k-2}$

Solutions: $\begin{bmatrix} y_1 = c_0 + c_2x^2 + c_4x^4 + c_{N+2}x^{N+2} \\ y_2 = c_1 + c_3x^3 + c_5x^5 + c_{N+1}x^{N+1} \end{bmatrix}$

let $c_0 = 1$
$c_1 = 1$

f_N

Nested series even: $E = y_1 = c_0\left(1 + \frac{c_2}{c_0}x^2\left(1 + \frac{c_4}{c_2}x^2\left(1 + \frac{c_6}{c_4}x^2 \ldots\right.\right.\right.$

Nested series odd: $O = y_2 = c_1\left(1 + \frac{c_3}{c_1}x^2\left(1 + \frac{c_5}{c_3}x^2\left(1 + \frac{c_7}{c_5}x^2 \ldots\right.\right.\right.$

define depth: $|f_N x^2| < 10^{-3}$

final solution: $y = C_A E + C_B O \longrightarrow y' = C_A E' + C_B O'$

boundary cond.: $y(1) = 0 \qquad y(5) + 14y'(5) = 0$

solve for C_A & C_B

$$y = \sum_{k=2}^{\infty} c_k\left[k(k-1) + 9c_{k-2}\right]x^{k-2} = 0$$

$$y' = \sum_{k=2}^{\infty} c_k(k-2)\left[k(k-1) + 9c_{k-2}\right]x^{k-3} = 0$$