

Gaussian Legendre Quadrature - $f(x) = \frac{dx}{2\sqrt{x}}$

$$\int_0^5 f(x) dx = \int_0^{3.99} f(x) dx + \int_{3.99}^{4.01} f(x) dx + \int_{4.01}^5 f(x) dx$$

(1) (2) (3)

$$\int_0^5 f(x) dx = I_1 + I_2 + I_3$$

GLQ

$$\int_a^b f(x) dx \approx \frac{b-a}{2} \sum_{i=1}^n w_i f\left(\frac{b-a}{2}x_i + \frac{a+b}{2}\right)$$

for I_{1-3} over $[0, 3.99]$, $[3.99, 4.01]$, $[4.01, 5]$

$$I_1 \approx \frac{3.99-0}{2} \sum_{i=1}^n w_i f\left(\frac{3.99}{2}x_i + \frac{3.99}{2}\right)$$

$$I_2 \approx \frac{4.01-3.99}{2} \sum_{i=1}^n w_i f\left(\frac{0.02}{2}x_i + \frac{7.99}{2}\right)$$

$$I_3 \approx \frac{5-4.01}{2} \sum_{i=1}^n w_i f\left(\frac{0.99}{2}x_i + \frac{9.01}{2}\right)$$