3. $x^2y'' + xy' - y = 9x^2 + 41x$ $\quad y(1)=0 \quad y(5)=14$

find two linearly independent soln. $(y_1 \& y_2)$ for the homogenous part

$$x^2y'' + xy' - y = 0$$

$$\Downarrow$$

$$m(m-1) + m - 1 = 0$$

roots: $m_1 = 1 \quad m_2 = -1$ $\longrightarrow$ linearly independent solutions $\begin{bmatrix} y_1(x) = x \\ y_2(x) = 1/x \end{bmatrix}$

Cauchy-Euler equation!!

↳ soln, $y(x) = x^m$

Wronskian: $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$W(y_1, y_2) = \begin{vmatrix} x & 1/x \\ 1 & -1/x^2 \end{vmatrix} = -2$$

particular soln: $y_p(x) = y_1(x)\int \frac{y_2(x)(kx^2+4x)}{W(y_1,y_2)}\,dx - y_2(x)\int \frac{y_1(x)(kx^2+4x)}{W(y_1,y_2)}\,dx$

expanding: $u_1(x) = \int \frac{\frac{1}{x}(kx^2+4x)}{-2}\,dx$

$$u_2(x) = -\int \frac{x(kx^2+4x)}{-2}\,dx$$

$$y_p(x) = x \cdot u_1(x) + \frac{1}{x} \cdot u_2(x)$$

Complete solution with Simpson integration