2.1. given $Q(x) = \begin{cases} 0 & 0 \le x \le 1 \\ |\sin(6(x-1))| & 1 \le x \le 2 \\ \sqrt{x-2} - \sin(6) & 2 \le x \le 3 \\ 0 & 3 \le x \le 4 \end{cases}$

$$xy'' + 3y' + \lambda^2 xy = 0 \qquad y(0) = \text{finite} \quad y'(4) + 3y(4) = 0$$

Standard S-L form: $\frac{d}{dx}(py') + qy + \lambda^2 wy = 0$

$$py'' + qy + \lambda^2 wy = 0$$

multiply by x^2: $\underbrace{x^3 y'' + 3x^2 y'}$ $+ \lambda^2 x^3 y = 0$

$$\frac{d}{dx} x^3 y' = x^3 y'' + 3x^2 y' \longrightarrow \frac{d}{dx}(x^3 y') + \lambda^2 x^3 y = 0$$

$a = 3$
$b = \lambda^2$
$c = 3$

given bessel trick form $\longleftrightarrow$ $\frac{d}{dx}(x^a y') + bx^c y = 0$

$$v = \left(\frac{1-a}{c-a+2}\right) = \frac{1-3}{3-3+2} = -1$$

$$\alpha = \left(\frac{2}{c-a+2}\right) = 1$$

$$y = x^{v/\alpha} Z_v(\alpha\sqrt{b}\, x^{1/\alpha}) \longrightarrow \frac{1}{x}[J \pm (\lambda x)]$$

$$y(x) = x^{-1} J_1 \lambda x$$

given boundary condition

$$y_n(0) = \frac{d}{dx}(J_1(\lambda n x) / d/dx(x) = \frac{\lambda n}{2}$$

2.2.a

$$y_n(x) = \frac{J_1(\lambda_n x)}{x}$$

$$\begin{bmatrix} y(0) \text{ finite} \\ y'(4) = 0 \end{bmatrix}$$

part a

$\lambda_n = j^{th}$ root of $y_j'(x) = \frac{1}{2}\left[J_0(4k_j) - J_2(4k_j)\right]$

fzero to find roots

2.2.b

$$\frac{d}{dx}\left(x^3 y'\right) + \lambda^2 x^3 y = 0 \longrightarrow$$

part b

$$w(x) = x^3$$

2.2.c

part c

$$\hat{N}_n = \int_0^4 y_n^2\, w(x) = \int_0^4 \left(\frac{J_1(\lambda_n x)}{x}\right)^2 w(x)$$

$$Q(x) = \sum \frac{\hat{Q}_n y_n}{\hat{N}_n}$$