Engr 5011: Homework #1

ODEs: series solutions and recipes

Please submit with this page first, and the problems attached in order.

Due 12 October in class (or by 11PM in canvas)

Name (print): ______________________ **TUID:**

I	**H**	**G**	**F**	**E**	**D**	**C**	**B**	**A**

USE THE ABOVE VALUES (A,B,C, ETC) WHEN A QUANTIFIED (NUMERICAL) ANSWER IS NEEDED.

"Technical points" possible for each question are shown; excellent documentation will recieve a bonus.

1. Problem 1 [**10 pt**] score/comments

2. Problem 2 [**15 pt**] score/comments

3. Problem 3 [**10 pt**] score/comments

4. Problem 4 [**15 pt**] score/comments

Engr-5011 Engineering Mathematics I

Homework #1: ODE recipes and series-solution method

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1. **[10 pt]** If a mass is dropped from a very large height the governing equation of motion for the (downward) velocity V is:

$$\boxed{m\,\frac{dV}{dt} = m\ g - c\,(V)^2\,; \quad V\,(0) = 0}$$

Here, c is the coefficient of drag in air.

(a) Put this governing equation in dimensionless form. (Hint, think about the terminal velocity V_{T}.)

(b) Solve the problem in its dimensionless form.

(c) Determine when the mass achieves 95% of its terminal velocity

2. **[15 pt]** Solve

$$\boxed{y'' + K\ y = 0; \quad y(1) = 0; \quad y(5) + \Gamma\ y'(5) = 10}$$

using the nested-series-solution method shown in class.
For the series-solution answer, please work out the derivative of $y(x)$ as a series, too, so you can satisfy the boundary condition.

Here $K = \max([A.B, C])$ and $\Gamma = E + F + G$, where A, B, C, D, E, F are taken from your TUID.

(a) Plot both your series solution and

(b) plot the exact solution on $1 \leq x \leq 5$ on the same graph for comparison.

3. **[10 pt]** If $\Gamma = E + F + G$ and $K = \max([A.B, C])$ (taken from your TUID), solve the following using the Wronskian recipe

$$\boxed{x^2 y'' + x\ y' - y \ = \ Kx^2 + 4x; \quad y(1) = 0; \quad y(5) = \Gamma}$$

(a) Write the solution in its integral form from the recipe.

(b) Using the simpson-rule integrator to evaluate the integrals, to plot the solution on $1 \leq x \leq 5$

4. **[15 pt]** If $K = \max([A.B, C])$, solve the following using the method of Frobenius.|
Here A, B, C, D are taken from your TUID.

$$\boxed{x\ y'' + 2y' + y = 0; \quad y(1) = -10; \quad y(5) = 2K\ \times (-1)^D}$$

(a) Compute your series solution using the nested recursion method, and plot this series-solution answer on $1 \leq x \leq 5$.

(b) Plot the "Bessel trick" solution on the same plot for comparison.