3. $x^2y'' + xy' - y = 9x^2 + 4x \qquad y(1) = 0 \quad y(5) = 14$

complimentary solution / homogenous portion : $x^2y'' + xy' - y = 0$

$m(m-1) + m - 1 = 0$

roots $\longrightarrow -1, 1$ $\qquad y_1 = x^{-1}$, $y_2 = x^1$

$y_c = c_1 x^{-1} + c_2 x^1$

particular solution / Variation of parameters : compute Wronskian

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} = y_1 \cdot y'_2 - y'_1 \cdot y_2$$

$$W(y_1, y_2) = x^{-1} \cdot 1 - (-x^{-2}) \cdot x = x^{-1} + x^{-1} = 2x^{-1}$$

$$W(y_1, y_2) = \frac{2}{x}$$

get u_1 and u_2 / non-homogenous : $9x^2 + 4x$

$$u_1 = -\int \frac{y_2 \cdot g(x)}{W} dx$$

$$u_2 = \int \frac{y_1 \cdot g(x)}{W} dx$$

$$u_1 = -\int \frac{x \cdot (9x^2 + 4x)}{\frac{2}{x}} dx$$

$$u_2 = \int \frac{x^{-1} \cdot (9x^2 + 4x)}{\frac{2}{x}} dx$$

$$u_1 = -\int \frac{x \cdot (9x^2 + 4x)}{\frac{2}{x}}\,dx = -\int \frac{9x^4 + 4x^3}{2}\,dx$$

$$u_1 = -\frac{9}{10}x^5 - \frac{1}{2}x^4$$

$$u_2 = \int \frac{x^{-1} \cdot (9x^2 + 4x)}{\frac{2}{x}}\,dx = \int \frac{9x^2 + 4x}{2}\,dx$$

$$u_2 = \frac{3}{2}x^3 + x^2$$

Complete particular solution: $y_p = u_1 y_1 + u_2 y_2$

$$y_p = -\frac{9}{10}x^4 - \frac{1}{2}x^3 + \frac{3}{2}x^4 + x^3$$

$$y_p = \frac{1}{2}x^3 + \frac{4}{5}x^4 = \frac{x^3(6x+5)}{10}$$

general solution: $y = y_c + y_p = c_1 x^{-1} + c_2 x^1 + \frac{x^3(6x+5)}{10}$

Solve for c_1/c_2:

$0 = c_1 + c_2 + \frac{11}{10}$ for $y(1) = 0$

$14 = \frac{c_1}{5} + 5c_2 + \frac{875}{10}$ for $y(5) = 14$

* solve system of equations