1.1. given $Q_A = \begin{cases} 0 & 0 \le x \le 1 \\ |\sin(6(x-1))| & 1 \le x \le 2 \\ \sqrt{x-2} - \sin(6) & 2 \le x \le 3 \\ 0 & 3 \le x \le 4 \end{cases}$

1.2.a

$$y'' + \lambda^2 y = 0 \qquad y(0) = y'(4) = 0$$

(1) Standard S-L form: $\frac{d}{dx}(py') + qy + \lambda^2 wy = 0$

$$py'' + qy + \lambda^2 wy = 0$$

$$p=1 \quad q=0 \quad w=1$$

(2) solve $y = A\sin(\lambda x) + B\cos(\lambda x)$

B.C.1 $y(0) = A\sin(0) + B\cos(0) = B = 0 \quad \therefore B = c$

so the solution simplifies to $y = A\sin(\lambda x)$

$$y'(x) = A\lambda\cos(\lambda x)$$

B.C.2 $y'(4) = A\lambda\cos(\lambda 4) = 0$ where $A \neq 0$

odd multiples of $\frac{\pi}{2}$ makes cosine $= 0$

$$\lambda 4 = (2n-1)\frac{\pi}{2}$$

part a

eigenvalue $\lambda_n = \frac{(2n-1)\pi}{8}$

eigenfunction $y_n(x) = \sin\left(\frac{(2n-1)\pi x}{8}\right) \longleftrightarrow y_n = \sin(\lambda_n x)$

1.2.b

$$y'' + \lambda^2 y = 0 \qquad y(0) = y'(4) = 0$$

$$\frac{d}{dx}(py') + qy + \lambda^2 w y = 0$$

$$py'' + qy + \lambda^2 w y = 0$$

part b

$$p = 1 \quad q = 0 \quad w = 1 \longrightarrow \boxed{w(x) = 1}$$

1.2.c

part c

$$\boxed{Q(x) = \sum_0^\infty C_j Y_j \longrightarrow Q(x) Y_n = \sum_0^\infty C_j Y_j Y_n = \sum \left(\frac{\hat{Q}_n Y_n}{\hat{N}_n} \right)}$$

multiply both sides by w(x) and integrate between bounds

$$\hat{Q}_n \int_a^b Q(x) w(x) Y_n \, dx = \sum_0^\infty C_j \int_a^b Q_j Y_n w(x) \, dx$$

$\therefore C_n Y_n$

$\therefore 0$ if $j \neq n$

$\therefore \hat{N}_n$ if $j = n$

part c

$$\boxed{\hat{N}_n = \int_a^b = Y_n^2 \, w(x)}$$

$$\int_0^4 \sin(\lambda_n x)^2$$

given $y_n(x)$ from 1.2.a and Q(x) bounds from problem statement for both &